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Original Article

Radiation effect on MHD flow through porous medium past a vertical plate with time dependent surface temperature and concentration

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Abstract

An MHD unsteady start-up (i.e. motion from rest) boundary layer flow past a moving vertical plate in a saturated porous medium with variable surface temperature and concentration is studied. The convective contributions to flow and heat transfer process are neglected due to slow motion. These render the governing equations linear, paving the way for solution by Laplace Transform. The novelty of this study is laid down as (i) Embedding the vertical plate in a saturated uniformly porous medium. (ii) Considering the thermal radiation in case of large temperature gradient. Some important findings of the study are**:** The presence of a porous matrix opposes the decelerating effect of resistive electromotive force as well as viscous drag of the boundary surface. The hiking of the velocity profiles near the plate may be attributed to overriding the effect of buoyancy forces on viscous drag.

Keywords: MHD flow, thermal radiation, chemical reaction, porous matrix, variable surface temperature

1. Introduction

Problems of fluid flow and mass transfer past a heated vertical surface through porous media are not only the interest of Mathematicians but also to Chemical engineers. The porous media are very widely used for a heated body to maintain its temperature augmenting free convection effectively. The civil engineers are confronted with the problem of saltwater encroachment of costal aquifers and chemical engineers are concentrated with miscible displacement processes. The study of magnetohydrodynamics has been a subject of curiosity due to its phenomenal response to petroleum industries, magnetohydrodynamics power generators, plasma studies, crystal growth, nuclear reactors cooling and the boundary layer control in aerodynamics. (Muthukumarswamy & Janakiraman, 2006; Muthu kumarswamy, Sathappan, & Natarajan, 2008; Nayak, Dash, & Singh, 2016; Kataria, Patel, & Singh, 2017) have studied the magnetohydrodynamics flow on a vertical plate in which one boundary is fixed and other one is force imposing certain conditions on the plate itself i.e. its permeability, variable temperature and considering the thermal radiation. Heat and mass transfer analysis is an open problem of interest to many researchers such as (Elbashbeshy & Emam, 2011; Kataria & Mittal, 2017; Makinde, 2005; Pal, 2011, 2012; Olajuown, Oahimire, & Ferdow, 2014; Prasad, Reddy, & Muthu kumarswamy, 2007; Rajput & Kumar, 2011). They have considered the flow to be two dimensional. They have also accounted for the radiation and mass diffusion leading to solutal variation.

The chemical reaction between a foreign mass and the fluid in which the plate is moving occurs in many chemical engineering processes. Numerous industrial applications are there where these processes taking place such as food processing, polymer production and manufacturing of ceramics or glassware. (Kandasamy, Periasamy, & Sivagnana, 2005) studied the problem in a stretching surface with thermal stratification. Considering chemical reaction and magnetic field, the flow of a viscous fluid over a non-linearly stretching sheet is studied by (Raptis & Perdikis, 2006). The unsteady viscous incompressible electrically conducting fluid flow

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along a semi-infinite vertical plate with thermal and concentration buoyancy effects has been solved using the implicit finite difference method and discussed by Reddy and Srihari (2009). A buoyancy effect due to temperature difference has been accounted for by (Mahapatra, Dash, Panda, & Acharya, 2010). Further, (Kar, Sahoo, & Dash, 2014; Sahoo & Dash, 2012) have studied flow past a stretching surface considering the Hall current. Kataria and Patel (2016) have analyzed the influence of chemical reaction and thermal radiation on magnetohydrodynamics Casson fluid past an oscillating vertical plate embedded in a saturated porous medium. (Pakdee, Yuvakanit, & Hussein, 2017) investigated the unsteady magnetrohydrodynamic flow of compressible fluid with variable thermal properties. Their result shows that the magnetic field and variable properties considerably influence the flows that are compressible thereby affecting the heat transfer as well as the wall shear stress. (Khan, Rasheed, & Salahuddin, 2020) analyzed the heat and mass transfer effect in a doubly stratified medium of mixed convective flow with viscous dissipation. They have considered hyperbolic tangent fluid with chemical reaction and thermal radiation. They noticed that the skin friction coefficient is decreasing with the increase in power law index. Again, (Khan, Rasheed, Ali, & Azim, 2021) have offered an intensive study to know about the impact of MHD and solar radiation on three dimensional boundary layer Jeffery fluid flow over a non-uniform stretching sheet. The sheet is of variable thickness, the medium is of porous, and the chemical reaction is of 1st order. Their study reveals a bi-folded behavior of velocity due to variable thickness of the sheet. It is also noticed that with the increase of Deborah number the transverse and axial velocities are increasing but the reverse effect has been marked for the temperature and concentration of the fluid.

In order to reduce the natural free convection on vertical heated surface, the insulation is useful. The combined effect of flow through porous medium and free/forced convection flow plays a vital role in the petroleum industry in extracting pure petrol from the crude and in agricultural engineering. Partha and Raja Sekhar (2005) have studied in a non-Darcy porous medium with thermal radiation the mixed convection heat and mass transfer. Further, Dey and Raja Sekhar (2016) have presented the hydrodynamics of a time dependent flow and transfer of mass through a channel asymmetrically lined with a deformable porous layer analytically. (Rout, Sahoo, & Dash, 2016) have studied the heat source as well as chemical reaction effect in magnetohydrodynamics flow past a vertical plate with time dependent surface temperature. They remarked that the heat source and the dominating effect of kinematic viscosity over molecular diffusivity have a decelerating effect on fluid velocity. Kataria and Patel (2018) have discussed the characteristics of the flow, heat and mass transfer of MHD Casson fluid past over an oscillating vertical plate which is embedded in a saturated porous medium with ramped wall temperature. They have used the Laplace transform method to solve the basic constitutive flow, heat and mass transfer equations.

Problems on fluid flow and mass transfer through porous media are useful to overcome the problems related to reacting and absorbing species in miscible displacement processes, in salt water encroachment of coastal aquifers and effective heat insulation of surfaces with large temperature gradient. The embedding of the vertical bounding surface in a saturated porous medium with thermal radiation justifies and corroborates to physical applications. The suggestive as well as remedial types of conclusion such as higher Prandtl-number flows with moderately increasing magnetic field intensity in the presence of heavier species and heat source favor the reduction of skin-friction and higher radiation parameter retards the rate of heat transfer at the surface which is desirable for flow stability and effective insulation. Also, porous media are very widely used for a heated body to maintain its temperature. Moreover, for effective insulation of the bounding surface, it is necessary to study the free convection effects on the flow through porous medium. The free convection effects are usually considered flow past a vertical surface. Further, the vertical surfaces act as heat exchanger between fluid layers adjacent to the plate surface and the ambient state/potential flow.

The present study supplements to Rout *et al*. (2016) considering the thermal radiation due to high temperature gradient and permeability of the saturated porous medium embedding the vertical plate. Inclusion of these aspects modifies the momentum equation as well as energy equation which characterize the present flow problem appreciably. The above extension serves as the novelty of the present discussion and supplements the earlier studies. We have used the linear Darcy model to account for the flow through porous media and Rosseland approximations for thermal radiation.

2. Mathematical Analysis

The unsteady start-up flow of a viscous incompressible fluid with free convection and mass transfer through a porous medium bounded by an infinite vertical plate is considered. When forced flow through a parallel plate exchanger ceases, a free-convection flow structure prevails since the bounding surface or fluid remain at an elevated temperature. The considered flow is influenced by an imposed magnetic field as the fluid is a conductor of electricity but the plate is electrically non-conducting. Cartesian co-ordinate system is chosen such that x, y-axes respectively, are in the vertical upward and horizontal direction as shown in Fig.1. We also assume that the fluid has constant properties as the influence of the density variation with temperature is considered only in the body force term. It is also assumed that the magnetic Reynolds number is so small that the induced magnetic field can be neglected in comparison with the applied one (Pai (1962)) so that $B = (0, B_0, 0)$, where B_0 is a constant. It is also assumed that no applied and polarization voltage exists (i.e. $E = 0$). This then corresponds to the case when no energy is added to or extracted from due to electric field. Since the plate is infinite in extent, all the physical variables are functions of y and t only. Initially ($t \leq 0$), both the fluid and the plate are assumed to be at rest. When $t > 0$, the plate starts moving with a velocity, u_0, temporal temperature and concentration as prescribed.

Taking into account the Boussinesq approximation and neglecting the Soret-Dufour (thermal-diffusion and diffusion-thermal) effects because of low concentration level, the equations of momentum, energy and mass diffusion are given by

$$\frac{\partial u}{\partial t}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho}-\frac{\upsilon}{K}u+g\beta(T-T_\infty)+g\hat{\beta}(C-C_\infty) \tag{1}$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{Q_0}{\rho C_p}\left(T-T_\infty\right)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2}-K_l(C-C_\infty) \tag{3}$$

The appropriate initial and boundary conditions are

$$\begin{aligned}&t\le 0: u=0, T=T_\infty, C=C_\infty \text{ for all } y\\ &t>0:\begin{cases}y=0:\ u=u_0, T=T_\infty+(T_w-T_\infty)\dfrac{u_0^2 t}{\upsilon}, C=C_\infty+(C_w-C_\infty)\dfrac{u_0^2 t}{\upsilon}\\ y\to\infty: u\to 0, T\to T_\infty, C\to C_\infty\end{cases}\end{aligned} \tag{4}$$

Using the Rosseland approximation for radiation (Brewster (1972)), we write

$$q_r=-\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \text{ and } T^4\equiv 4T_\infty^{\ 3}T-3T_\infty^{\ 4}.$$

Therefore, the Equation (2) is simplified to

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{Q_0}{\rho C_p}\left(T-T_\infty\right)+\frac{16\sigma T_\infty^{\ 3}}{3k^*\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{5}$$

3. Solution

Following non-dimensional quantities are introduced.

$$y^*=\frac{yu_0}{\upsilon}, u^*=\frac{u}{u_0}, t^*=\frac{tu_0^2}{\upsilon}, M=\frac{\sigma B_0^2\upsilon}{\rho u_0^2}, Kp=\frac{Ku_0^{\ 2}}{\upsilon^2}, Pr=\frac{\upsilon\rho C_p}{k}, Sc=\frac{\upsilon}{D}, T^*=\frac{T-T_\infty}{T_w-T_\infty},$$
$$C^*=\frac{C-C_\infty}{C_w-C_\infty}, Gr=\frac{g\beta\upsilon(T_w-T_\infty)}{u_0^3}, Gm=\frac{g\hat{\beta}\upsilon(C_w-C_\infty)}{u_0^3}, \phi=\frac{\upsilon Q_0}{\rho C_p u_0^2}, Kc=\frac{K_l\upsilon}{u_0^{\ 2}},\ Rd=\frac{16\sigma T_\infty^{\ 3}}{3k^* k}$$

With the help of the above non-dimensional quantities and omitting the asterisks, equations (1), (5) and (3) respectively can be written as

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}-(M+\frac{1}{Kp})u+GrT+GmC \tag{6}$$

$$\frac{\partial T}{\partial t}=\frac{1+Rd}{Pr}\frac{\partial^2 T}{\partial y^2}-\phi T \tag{7}$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-KcC \tag{8}$$

and the boundary conditions (4) become

$$\begin{aligned}&t\le 0: u=0, T=0, C=0 \text{ for all } y=0\\ &t>0:\begin{cases}y=0: u=1, T=t, C=t\\ y\to\infty: u\to 0, T\to 0, C\to 0\end{cases}\end{aligned} \tag{9}$$

$$\frac{\partial^2 \bar{u}}{\partial y^2} - (s + M + \frac{1}{Kp})\bar{u} = -\frac{Gr}{s^2} e^{-\sqrt{\frac{Pr}{1+Rd}(s+\varphi)}y} - \frac{Gm}{s^2} e^{-\sqrt{Sc(s+Kc)}y} \tag{10}$$

$$\frac{\partial^2 \bar{T}}{\partial y^2} - \frac{Pr}{1+Rd}(s+\phi)\bar{T} = 0 \tag{11}$$

$$\frac{\partial^2 \bar{C}}{\partial y^2} - Sc(s+Kc)\bar{C} = 0 \tag{12}$$

The corresponding boundary conditions for $t > 0$ are

$$\left.\begin{array}{l} y = 0:\ \bar{u} = 1/s,\ \bar{T} = 1/s^2,\ \bar{C} = 1/s^2 \\ y \to \infty:\ \bar{u} \to 0,\ \bar{T} \to 0,\ \bar{C} \to 0 \end{array}\right\} \tag{13}$$

The solution of the resulting semi-infinite domain, equations (10), (11) and (12) satisfying boundary conditions (13) are given by

$$T(y,t) = \frac{1}{2}\left[\left(t + (y/2)\sqrt{\frac{Pr}{\phi(1+Rd)}}\right) exp\left(y\sqrt{Pr\,\phi/(1+Rd)}\right) erfc\left((y/2)\sqrt{\frac{Pr}{t(1+Rd)}} + \sqrt{\phi t}\right) + \left(t - (y/2)\sqrt{\frac{Pr}{\phi(1+Rd)}}\right) exp\left(-y\sqrt{Pr\phi/(1+Rd)}\right) erfc\left((y/2)\sqrt{\frac{Pr}{t(1+Rd)}} - \sqrt{\phi t}\right)\right],$$

(14)

$$C(y,t) = \frac{1}{2}\left[\left(t + (y/2)\sqrt{\frac{Sc}{Kc}}\right) exp\left(y\sqrt{ScKc}\right) erfc\left((y/2)\sqrt{\frac{Sc}{t}} + \sqrt{Kct}\right) + \left(t - (y/2)\sqrt{\frac{Sc}{Kc}}\right) exp\left(-y\sqrt{ScKc}\right) erfc\left((y/2)\sqrt{\frac{Sc}{t}} - \sqrt{Kct}\right)\right], \tag{15}$$

$$u(y,t) = \frac{1}{2}\left[exp\left(y\sqrt{L}\right) erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{Lt}\right) + exp\left(-y\sqrt{L}\right) erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{Lt}\right)\right] - \alpha_1 F(y,t) - \alpha_2 R(y,t), \tag{16}$$

where

$$\begin{aligned} F(y,t) = \frac{1}{2}\Bigg[& \frac{1}{\beta_1}\left(t + \frac{1}{\beta_1} + \frac{y}{2\sqrt{L}}\right) e^{y\sqrt{L}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{Lt}\right) + \frac{1}{\beta_1}\left(t + \frac{1}{\beta_1} - \frac{y}{2\sqrt{L}}\right) e^{-y\sqrt{L}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{Lt}\right) \\ & - \frac{1}{\beta_1}\left(t + \frac{1}{\beta_1} + (y/2)\sqrt{\frac{Pr}{\phi(1+Rd)}}\right) exp\left(y\sqrt{Pr\,\phi/(1+Rd)}\right) erfc\left((y/2)\sqrt{\frac{Pr}{(1+Rd)t}} + \sqrt{\phi t}\right) \\ & - \frac{1}{\beta_1}\left(t + \frac{1}{\beta_1} + (y/2)\sqrt{\frac{Pr}{\phi(1+Rd)}}\right) exp\left(-y\sqrt{Pr\,\phi(1+Rd)}\right) erfc\left((y/2)\sqrt{\frac{Pr}{(1+Rd)t}} - \sqrt{\phi t}\right) \\ & - \frac{e^{\beta_1 t}}{\beta_1^{\,2}}\Bigg\{ exp\left(y\sqrt{L+\beta_1}\right) erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(L+\beta_1)t}\right) + exp\left(-y\sqrt{L+\beta_1}\right) erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(L+\beta_1)t}\right) \\ & - exp\left(y\sqrt{\frac{\Pr(\phi+\beta_1)}{1+Rd}}\right) erfc\left((y/2)\sqrt{\frac{Pr}{(1+Rd)t}} + \sqrt{(\phi+\beta_1)t}\right) \\ & - exp\left(-y\sqrt{\frac{\Pr(\phi+\beta_1)}{1+Rd}}\right) erfc\left((y/2)\sqrt{\frac{Pr}{(1+Rd)t}} - \sqrt{(\phi+\beta_1)t}\right)\Bigg\}\Bigg], \end{aligned}$$

$$\begin{aligned} R(y,t) = \frac{1}{2}\Bigg[& \frac{1}{\beta_2}\left(t + \frac{1}{\beta_2} + \frac{y}{2\sqrt{L}}\right) e^{y\sqrt{L}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{Lt}\right) + \frac{1}{\beta_2}\left(t + \frac{1}{\beta_2} - \frac{y}{2\sqrt{L}}\right) exp\left(-y\sqrt{L}\right) erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{Lt}\right) \\ & - \frac{1}{\beta_2}\left(t + \frac{1}{\beta_2} + (y/2)\sqrt{\frac{Sc}{Kc}}\right) exp\left(y\sqrt{ScKc}\right) erfc\left((y/2)\sqrt{\frac{Sc}{t}} + \sqrt{Kct}\right) \end{aligned}$$

$$-\frac{1}{\beta_2}\left(t+\frac{1}{\beta_2}+(y/2)\sqrt{\frac{Sc}{Kc}}\right)exp\left(-y\sqrt{ScKc}\right)erfc\left((y/2)\sqrt{\frac{Sc}{t}}-\sqrt{Kct}\right)$$
$$-\frac{e^{\beta_2 t}}{\beta_2^{\,2}}\left\{e^{y\sqrt{L+\beta_2}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(L+\beta_2)t}\right)+e^{-y\sqrt{L+\beta_2}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(L+\beta_2)t}\right)\right.$$
$$\left.\left.-e^{y\sqrt{Sc(Kc+\beta_2)}}erfc\left(\frac{y}{2}\sqrt{\frac{Sc}{t}}+\sqrt{(Kc+\beta_2)t}\right)-e^{-y\sqrt{ScKc+\beta_2)}}erfc\left(\frac{y}{2}\sqrt{\frac{Sc}{t}}-\sqrt{(Kc+\beta_2)t}\right)\right\}\right].$$

The rate of heat transfer at the plate i.e. Nusselt number (Nu) is

$$Nu=-\frac{\partial T}{\partial y}\bigg]_{y=0}=\frac{1}{2}\sqrt{\frac{Pr}{\phi(1+Rd)}}\left\{1-erfc\left(\sqrt{\phi t}\right)\right\}+t\sqrt{\frac{Pr\,\phi}{1+Rd}}\ +t\left\{\sqrt{\frac{Pr}{(1+Rd)\pi t}}e^{-\phi t}-\sqrt{\frac{Pr}{1+Rd}}\,erfc\left(\sqrt{\phi t}\right)\right\}.$$

The rate of mass transfer at the wall i.e. Sherwood number (Sh) is

$$Sh=-\frac{\partial C}{\partial y}\bigg]_{y=0}=\frac{1}{2}\sqrt{\frac{Sc}{K_c}}\left\{1-erfc\left(\sqrt{Kct}\right)\right\}+t\sqrt{KcSc}+t\left\{\sqrt{\frac{Sc}{\pi t}}e^{-K_c t}-\sqrt{KcSc}\,erfc\left(\sqrt{Kct}\right)\right\}.$$

The shear stress at the plate i.e. skin-friction (τ) is

$$\tau=\frac{\partial u}{\partial y}\bigg]_{y=0}=\sqrt{L}\left(erfc(\sqrt{Lt}\,)-1\right)-\frac{1}{\sqrt{\pi t}}e^{-Lt}+\alpha_1 F_2(y,t)-\alpha_2 R_2(y,t).$$

where

$$F_2(y,t)=\frac{1}{2\beta_1\sqrt{L}}(1\text{-}erfc(\sqrt{Lt}))+\frac{1}{\beta_1}(t+\frac{1}{\beta_1})\left\{\sqrt{L}(1\text{-}erfc(\sqrt{Lt})+\frac{1}{\sqrt{\pi t}}e^{-Lt}\right\}$$
$$-\frac{1}{\beta_1}\left[\left(t+\frac{1}{\beta_1}\right)\sqrt{\frac{Pr\,\phi}{1+Rd}}\left(1-erfc\sqrt{\phi t}\right)+\frac{1}{2}\left(t+\frac{1}{\beta_1}\right)\sqrt{\frac{Pr}{(1+Rd)\pi t}}\left(2-erfc\sqrt{\phi t}\right)\right.$$
$$\left.-\frac{1}{2}\sqrt{\frac{Pr}{\phi(1+Rd)}}+\left(t+\frac{1}{\beta_1}\right)\sqrt{\frac{Pr}{(1+Rd)\pi t}}e^{-\phi t}\right]$$
$$+\frac{1}{\beta_1^{\,2}}e^{\beta_1 t}\left[\sqrt{L+\beta_1}\left(erfc\left(\sqrt{(L+\beta_1)t}-1\right)\right)-\frac{1}{\sqrt{\pi t}}e^{-(L+\beta_1)t}\right.$$
$$\left.+\sqrt{\frac{Pr(\phi+\beta_1)}{1+Rd}}\left(1-erfc\left(\sqrt{(\phi+\beta_1)t}\right)\right)+\frac{Pr}{\sqrt{\pi t}(1+Rd)}e^{-(\phi+\beta_1)t}\right],$$

$$R_2(y,t)=\frac{1}{2\beta_2\sqrt{L}}\left(1\text{-}erfc(\sqrt{Lt})\right)+\frac{1}{\beta_2}\left(t+\frac{1}{\beta_2}\right)\left\{\sqrt{L}\left(1\text{-}erfc(\sqrt{Lt})\right)+\frac{1}{\sqrt{\pi t}}e^{-Lt}\right\}$$
$$-\frac{1}{\beta_2}\left[\left(t+\frac{1}{\beta_2}\right)\sqrt{ScKc}\left(1-erfc\sqrt{Kct}\right)+\frac{1}{2}\left(t+\frac{1}{\beta_2}\right)\sqrt{\frac{Sc}{\pi t}}\left(2-erfc\sqrt{Kct}\right)\right.$$
$$\left.-\frac{1}{2}\sqrt{\frac{Sc}{Kc}}+\left(t+\frac{1}{\beta_2}\right)\sqrt{\frac{Sc}{\pi t}}e^{-Kct}\right]$$
$$+\frac{1}{\beta_2^{\,2}}e^{\beta_2 t}\left[\sqrt{L+\beta_2}\left(erfc\left(\sqrt{(L+\beta_2)t}-1\right)\right)-\frac{1}{\sqrt{\pi t}}e^{-(L+\beta_2)t}\right.$$
$$\left.+\sqrt{Sc(Kc+\beta_2)}\left(1-erfc\left(\sqrt{(Kc+\beta_2)t}\right)\right)+\frac{Sc}{\sqrt{\pi t}}e^{-(Kc+\beta_2)t}\right].$$

where

$L=M+1/Kp$, $\alpha_1=Gr/(1-Pr/(1+Rd))$, $\beta_1=((Pr/(1+Rd))\phi\text{-}1)/(1\text{-}Pr/(1+Rd))$,
$\alpha_2=Gm/(1\text{-}Sc)$, $\beta_2=(ScKc-1)/(1-Sc)$

4. Results and Discussion

Laplace Transformation has been applied to solve the coupled linear system of partial differential equations. The contribution of convective terms in momentum, heat and mass transfer equations have not been taken in to account due to slow motion of the fluid model considered. The computation is carried out assigning the values of the parameters from the earlier studies and the text.

Figure 2 shows the variation of velocity for different values of *Pr*, *M* and *Kp*. It is seen that an increase in *Pr* and *M* reduces the velocity distribution where as *Kp* enhances it. A common characteristic of the distribution is that the hike in velocity distribution is marked near the plate exceeding the prescribed value. Physically, it is explained as: the unsteady start up-velocity of the bounding surface associated with escalated prescribed surface temperature and concentration both proportional to product of square of the initial velocity and time which produces greater shearing stress at the plate surface. The velocity reduces asymptotically span wise to attain the ambient state. Further, it is explained that the resistive force due to applied magnetic field and low thermal conductivity of higher value of *Pr* reduces the velocity across the flow domain where as an increase in *Kp*, reduces the inter layer resistance contributing to increase in velocity ($Kp \rightarrow \infty$)indicates free flow without porous matrix). The visible difference on comparing Figure 2 of the present study with that of Figures 2 and 3 of Rout *et al*. (2016) due to inclusion of additional body force (arising out of permeability of the porous medium) and radiation effect, greater momentum transport is marked near the boundary surface so that the velocity is hiked in the neighborhood layers which was not marked in case of Rout *et al*., (2016). This is a very significant contribution. Further, it is seen that attainment of ambient state of the flow requires a greater span wise variation. Figure 3 elucidates the velocity distribution indicating escalation for higher thermal and solutal buoyancies (increase in *Gr* and *Gm*) and reduction in case of increasing *Sc* (higher value of *Sc* indicates the heavier diffusing species). Form Figure 4 it is observed that higher thermal radiation enhances the velocity contributing to thicker boundary layer where as an increase in heat source and chemical reaction decelerate the velocity distribution producing a progressive thinning of boundary layer span wise. The physical reasoning can be attributed as: the non-dimensional parameters ϕ and *Kc* are inversely proportional to square of the initial velocity of the plate where as *Rd* is directly proportional to the cube of the characteristic temperature T_∞. Figure 5 shows the temperature distribution across the flow domain. The sudden fall of temperature near the plate is marked for higher Prandtl number fluid producing low thermal conductivity as discussed earlier. On the other hand, smooth variation is marked for ϕ and *Rd* decreasing and increasing the temperature respectively. Figure 6 shows the smooth variation of solutal concentration across the flow domain indicating fall in concentration level due to higher chemical reaction parameter, *Kc* and heavier diffusing species (higher value of *Sc*). Comparing Figures 5 and 6 of present study with Figures 10 and 11 of Rout *et al*. (2016), it is revealed that sharp fall of temperature is not visible in earlier study. Similarly, variation of solutal concentration across the flow domain is slowed down affecting the processes in chemical industry significantly.

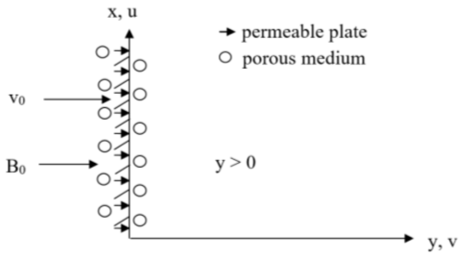

Figure 1. Flow geometry

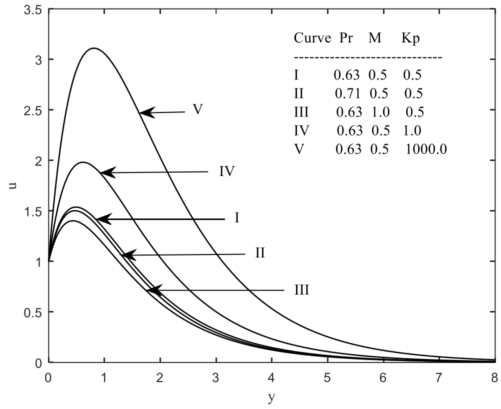

Figure 2. Variation of velocity with *Pr*, *M* and *Kp*

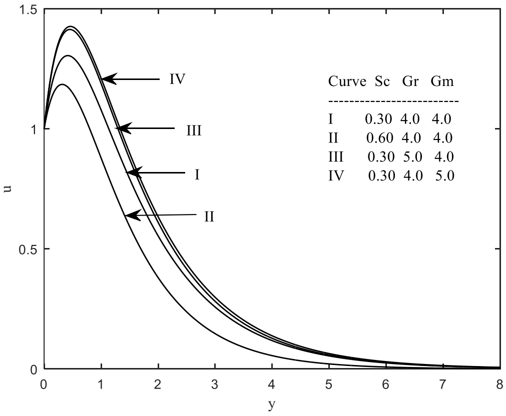

Figure 3. Variation of velocity with *Sc*, *Gr* and *Gm*

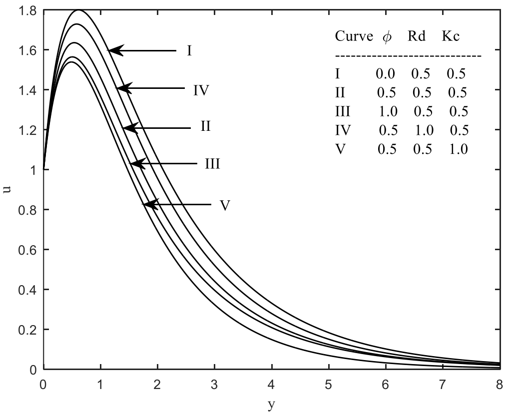

Figure 4. Variation of velocity with ϕ, *Rd* and *Kc*

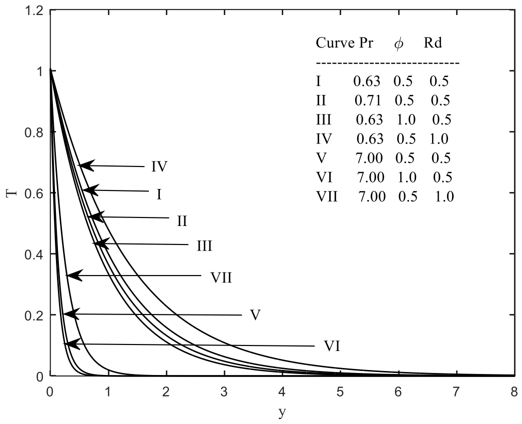

Figure 5. Variation of *T* with *Pr*, ϕ and *Rd*

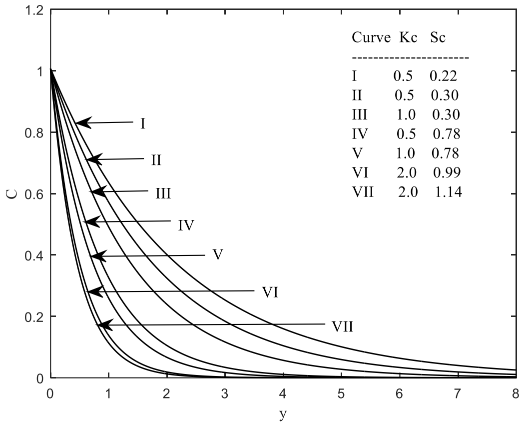

Figure 6. Variation of *C* with *Kc* and *Sc*

Table 1 shows the variation of skin-friction (Force Coefficient) for different values of flow parameters. The skin-friction remains negative for all the parameters involved in the analysis. From the table it is observed that the magnitude of skin-friction increases with the increase in the parameters *Pr*, *Gr*, *Gm* and ϕ and the reverse effect is observed for rest of the parameters such as *M*, *Sc*, *Kc* and *Rd*. The increase in skin-friction indicates a progressive thinning of velocity boundary layer and decreasing indicates the thickening of boundary layer. Thus, it is suggested that, as the growth of boundary layer is not conducive for maintaining laminarity of flow, hence necessary measures may be taken to control the growth of boundary layer. Tables 2 and 3 show the heat transfer and mass transfer coefficients at the plate surface. All the entries are positive unlike the skin-friction. The positive value indicates that the heat flows from the plate to the fluid. Similarly, for solutal concentration, the level of concentration decreases at the boundary and the diffusing species moves from plate surface to the fluid mass enhancing the concentration level of fluid. Further, it is seen that an increase in *Pr* and ϕ (heat source) increases the Nusselt number but radiation parameter decreases it. Thus, physically it is interpreted as fluid with higher Prandtl number and embedded heat source with thermal power enhance the heat transfer serving as a coolant of the boundary surface. Similar explanation can be attributed to variation of solutal concentration of diffusing species. Tables 4 and 5 present a comparison of skin friction (τ) and the Nusselt number (*Nu*) of the present work with that of Rout *et al*. (2016) when *Kp*=1000.0 ($Kp \rightarrow \infty$) i.e. without porous medium) and *Rd*=0.0 (without thermal radiation). It shows a good agreement and serves as a benchmark of validation.

Table 2. Effect of flow parameters on Nusselt number (*Nu*)

Pr	ϕ	*Rd*	*Nu*
0.63	0.50	0.50	0.9297
0.71	0.50	0.50	1.0205
0.63	1.00	0.50	1.1081
0.63	0.50	1.00	0.7465

Table 3. Effect of flow parameters on Sherwood number (*Sh*)

Sc	*Kc*	*Sh*
0.30	0.50	0.5704
0.78	0.50	1.1373
0.30	1.00	0.7238

Table 1. Effect of flow parameters on skin-friction (τ)

Pr	*M*	*Gr*	*Gm*	*Sc*	*Kp*	*Kc*	ϕ	*Rd*	τ
0.63	0.50	4.00	4.00	0.30	0.50	0.50	0.50	0.50	-6.6475
0.71	0.50	4.00	4.00	0.30	0.50	0.50	0.50	0.50	-6.7501
0.63	1.00	4.00	4.00	0.30	0.50	0.50	0.50	0.50	-5.5983
0.63	0.50	5.00	4.00	0.30	0.50	0.50	0.50	0.50	-8.4729
0.63	0.50	4.00	5.00	0.30	0.50	0.50	0.50	0.50	-7.8970
0.63	0.50	4.00	4.00	0.60	0.50	0.50	0.50	0.50	-5.3253
0.63	0.50	4.00	4.00	0.30	1.00	0.50	0.50	0.50	-9.2323
0.63	0.50	4.00	4.00	0.30	0.50	1.00	0.50	0.50	-6.0451
0.63	0.50	4.00	4.00	0.30	0.50	0.50	1.00	0.50	-6.9042
0.63	0.50	4.00	4.00	0.30	0.50	0.50	0.50	1.00	-6.5910

Table 4. Comparison of skin friction (τ)

Pr	*M*	*Gr*	*Gm*	*Sc*	ϕ	*Kc*	Skin friction (τ)	
							Rout *et at*., (2016) *Kp*=1000.0 (*Kp*→∞) and *Rd*=0.0	Present result
0.50	0.50	0.50	0.50	0.60	0.50	0.50	-1.7319	-1.7288
0.50	1.00	0.50	0.50	0.60	0.50	0.50	-1.2672	-1.2698
0.50	0.50	1.00	0.50	0.60	0.50	0.50	-4.5659	-4.5701
0.50	0.50	0.50	1.00	0.60	0.50	0.50	-4.6922	-4.7001
0.50	0.50	0.50	0.50	0.60	1.50	0.50	-3.9529	-3.9783

Table 5. Comparison of Nusselt number (*Nu*)

Pr	ϕ	Nusselt number (*Nu*)	
		Rout *et al*., (2016) *Kp*=1000.0 (*Kp*→∞) and *Rd*=0.0	Present result
0.50	0.50	0.5491	0.5512
0.71	0.50	0.6543	0.6556
0.71	1.00	0.7109	0.7151
7.00	0.50	3.1566	3.1584

5. Conclusions

a. The hiking of the velocity profiles near the plate may be attributed to overriding the effect of buoyant forces on viscous drag.
b. The higher thermal radiation enhances the velocity contributing to thicker boundary layer.
c. The increase in heat source, decelerate the velocity distribution producing a progressive thinning of boundary layer span wise.
d. The increase in skin-friction indicates a progressive thinning of velocity boundary layer.
e. Fluid with higher Prandtl number and embedded heat source with thermal power enhance the heat transfer serving as a coolant of the boundary surface.

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Appendix

Nomenclature

k^*	absorption coefficient
g	acceleration due to gravity
C_∞	ambient concentration
T_∞	ambient temperature
E	applied and polarization voltage
C_w	concentration at the plate
B_0	constant magnetic flux density
x, y	coordinate axes
K_l	dimensional chemical reaction parameter
C^*	dimensional concentration
K	dimensional porosity parameter
T^*	dimensional temperature
t^*	dimensional time variable
u^*	dimensional velocity component along x-axis
y^*	dimensional y-axis
Q_0	dimensional heat source parameter
u	dimensionless velocity component along x- axis
B	external magnetic field
k	fluid thermal conductivity
$\bar{C}$	Laplace transform of C
$\bar{T}$	Laplace transform of T
s	Laplace transform of t
$\bar{u}$	Laplace transform of u
M	magnetic field parameter
D	mass diffusion coefficient
Gm	mass Grashof number
Kc	non-dimensional chemical reaction parameter
C	non-dimensional concentration
Kp	non-dimensional porosity parameter
T	non-dimensional temperature
t	non-dimensional time variable
Pr	Prandtl number
q_r	radiative heat flux
u_0	start-up velocity
T_w	temperature at the plate
Gr	thermal Grashof number
Rd	thermal radiation parameter
Sc	Schmidt number
C_p	specific heat at constant pressure

Greek symbols

μ	dynamic viscosity
ρ	fluid density
υ	kinematics viscosity
ϕ	non-dimensional heat source parameter
$\hat{\beta}$	volumetric coefficient of concentration expansion
β	volumetric coefficient of thermal expansion
Σ	Stefan-Boltzmann constant