Songklanakarin J. Sci. Technol.
44 (3), 636-645, May - Jun. 2022

http://www.sjst.psu.ac.th

Original Article

Some novel correlation coefficients of spherical fuzzy sets with their application in pattern recognition

Abdul Haseeb Ganie*

School of Mathematics, Faculty of Sciences,
Shri Mata Vaishno Devi University, Katra, Jammu and Kashmir, 182320 India

Received: 24 March 2021; Revised: 11 October 2021; Accepted: 20 November 2021

Abstract

A spherical fuzzy set is a generalization of picture fuzzy sets, intuitionistic fuzzy sets, and fuzzy sets in which the square sum of the membership, non-membership, and neutrality values is at most one. The correlation coefficient is a crucial tool in fuzzy/non-standard fuzzy theory and has been applied in various fields such as clustering, pattern recognition, medical diagnosis, decision-making, etc. The existing correlation coefficients for spherical fuzzy sets give only the correlation degree and do not express the nature or direction of correlation between the spherical fuzzy sets. So, in this study, we propose two correlation coefficients for spherical fuzzy sets, which not only give the strength of correlation between two spherical fuzzy sets but also tell us whether the two spherical fuzzy sets are positively correlated or negatively correlated. We also discuss several properties of these correlation coefficients. We apply these correlation coefficients to solve a pattern recognition problem in the spherical fuzzy environment and compare the results with some existing measures.

Keywords: correlation coefficient, picture fuzzy set, spherical fuzzy set, linguistic hedge, pattern recognition

1. Introduction

The concept of the fuzzy set (FS) theory was put forward by Zadeh (1965) for handling imprecise and vague information. In an FS, each element is assigned a membership value lying between 0 and 1, indicating its degree of belongingness to the set. FSs have been applied in many fields such as pattern recognition, medical diagnosis, clustering, etc. Since in an FS, the non-membership value of an element cannot be chosen independently, so Atanassov (1986) introduced the concept of intuitionistic fuzzy sets (IFSs). In an intuitionistic fuzzy set (IFS), each element has a membership value and a non-membership value lying in the interval [0, 1] with their sum less or equal to one. This restriction on the sum of membership values limits the scope of IFSs and so Yager (2013) proposed the concept of Pythagorean fuzzy sets (PFSs). In a PFS, each element has a membership value and a non-membership value lying in the interval [0, 1] with their square sum less or equal to one. Though PFSs are more robust than FSs and IFSs they cannot handle the situations in which the square sum of membership grades exceeds one. So Yager (2017) introduced generalized orthopair fuzzy sets and termed them as q-rung orthopair fuzzy sets (q-ROPFSs). In a q-rung orthopair fuzzy set (q-ROPFS) each element has a membership value and a non-membership value lying in the interval [0, 1] with their q^{th} power sum less or equal to one. But all of these extensions of FSs lack an important concept i.e., degree of neutrality which plays an important role in many decision-making problems such as medical diagnosis, personnel selection, human voting, etc. So, realizing this a new generalization of FSs known as picture fuzzy sets (PIFSs) were suggested by Cuong and Kreinovich (2013). In a picture fuzzy set (PIFS) each element has a membership value, a non-membership value, and a neutrality value lying in the interval [0, 1] with their sum less or equal to one.

Like IFSs, the scope of PIFSs is also limited due to restriction on the sum of membership values. Realizing this, Mahmood, Ullah, Khan, and Jan (2019) proposed the concept of spherical fuzzy sets (SFSs). In a spherical fuzzy set (SFS), each element is characterized by a membership, non-membership, and neutrality degree with the square sum of the membership, non-membership, and neutrality less or equal to

*Corresponding author
Email address: ahg110605@gmail.com

one. This means that the space of SFSs is relatively broader than the space of FSs, IFSs, PFSs, *q*-ROPFSs, and PIFSs. The spherical fuzzy (SF) TOPSIS along with its utility in Hospital location selection was developed by Kahraman, Kutlu, Cevik, and Oztaysi (2019). Liu, Zhu, and Wang (2019) proposed a new multi-attribute decision-making (MADM) method in SF environment for a research and development project problem. Kutlu and Kahraman (2020a) introduced the Quality Function Development (QFD) in SF setting. The classical Analytical Hierarchical Process (AHP) was extended to SF environment by Gundogdu and Kahraman (2020b). A SF entropy measure and its application in MADM was proposed by Aydogdu and Gul (2020). By combining the AHP and TOPSIS, Mathew, Chakrabortty, and Ryan (2020) introduced a novel method in SF environment for an advanced manufacturing system selection problem. The classical VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method was extended to SF environment by Kutlu and Kahraman (2019). Kutlu (2020) extended the traditional Multi-Objective Optimization by a Ratio Analysis plus the Full Multiplicative Form (MULTIMOORA) to SF settings.

Application of some SF distance and similarity measures are available in the literature (Khan, Kumam, Deebani, Kumam, & Shah, 2020; Shishavan, Kutlu, Farrokhizadeh, Donyatalab, & Kahraman, 2020; Wei, Wang, Lu, Wu, & Wei, 2019). Ashraf, Abdullah, and Mahmood (2020) introduced some Dombi aggregation operators for SFSs and applied them in group decision-making. Some entropy-based Logarithmic aggregation operators in SF theory were proposed by Jin, Wu, Sun, Zeng, Luo, and Peng (2019). Some SF t-norms and t-conorms were introduced by Ashraf, Abdullah, Aslam, Qiyas, and Kutbi (2019). Some similarity measures and information measures for SFSs along with their applications were introduced by Mahmood, Ilyas, Ali, and Gumaei (2021). The application of some SF similarity measures in decision-making was shown by Rafiq, Ashraf, Abdullah, Mahmood, and Muhammad (2019). Some SF aggregation operators with their applicability in decision-making were suggested by Khan, Mahmood, and Ullah (2021). Mahmood *et al*., (2019) also proposed the concept of T-spherical fuzzy sets (T-SFSs) in which the sum of t^{th} power of membership, non-membership, and neutrality values is at most one. Some recent studies related to T-SFSs and their diverse applications are available in the literature (Mahmood, & Jan, 2018; Ullah, Hassan, Mahmood, Jan, & Hassan, 2019; Ullah, Ullah, Mahmood, Jan, & Ahmad, 2020; Zedam, Jan, Rak, Mahmood, & Ullah, 2020). The present study is related to the correlation coefficient for SFSs. The main contributions of this study are**:**

1. We propose two correlation coefficients for SFSs that receive their values in [-1, 1] and therefore give both the strength of correlation as well as the nature of the correlation between the SFSs.
2. We discuss their various properties.
3. We compare the suggested correlation coefficients with the existing correlation coefficients in the SF environment through the linguistic hedge aspect.
4. We demonstrate the application of the suggested correlation coefficients in pattern recognition and contrast the performance with the existing SF correlation coefficients.

2. Correlation Coefficients and Their Applications

In this section, we study the importance of correlation coefficients and their applications.

The correlation coefficient is used to describe the relationship between two objects. In fuzzy theory, the correlation coefficient is vital due to its application in MADM, pattern recognition, clustering, etc. Chiang and Lin (1999) introduced a correlation coefficient for FSs based on a statistical viewpoint. This correlation coefficient provides both strength as well as the nature of the correlation between two FSs. Chaudhuri and Bhattacharya (2001) introduced Spearman's rank-type correlation coefficient for FSs. By using Pearson's correlation coefficient, Wu and Hung (2016) introduced a method for calculating the correlation of interval fuzzy data. A generalized fuzzy correlation coefficient with its application in the ranking of Primary Health Centers was given by Sharma and Singh (2019).

The informational energy and correlation coefficient for IFSs was introduced by Gerstenkorn and Manko (1991). But this correlation coefficient for IFSs gives only the degree of correlation and not the nature of correlation. A correlation coefficient for IFSs indicating both nature and degree was proposed by Hung (2001). Some other studies concerning the IF correlation coefficients with their applicability in medical diagnosis, pattern recognition, clustering, etc. are available in the literature (Ejegwa & Onyeke, 2020; Hong & Hwang, 1995; Huang & Guo, 2019; Hung & Wu, 2002; Kumar, 2019; Liu, Shen, Mu, Chen, & Chen, 2016; Mitchell, 2004; Thao, Ali, & Smarandache, 2019).

Du (2019) proposed *q*-ROPFSs correlation coefficient on both bounded and unbounded continuous universes. Recently, Singh and Ganie (2020) introduced some robust correlation coefficients in the Pythagorean fuzzy environment and demonstrated their application in MADM, medical diagnosis, clustering, and pattern recognition. Based on a statistical viewpoint, Singh and Ganie (2021a) introduced some correlation coefficients for *q*-ROPFSs.

In the picture fuzzy (PIF) theory, Singh (2015) introduced two correlation coefficients with their applications in bidirectional approximate reasoning and clustering analysis. However, the PIF correlation measures due to Singh (2015) indicate only the strength of correlation between two PIFSs and do not tell us whether the two PIFSs are positively or negatively correlated. So, to find the nature of the correlation between two PIFSs, Ganie, Singh, and Bhatia (2020) proposed two correlation coefficients that express both nature and degree of correlation between two PIFSs. They also utilized these correlation coefficients for solving some problems related to pattern recognition, clustering, and medical diagnosis. A novel PIF correlation with its application in classification and decision-making was proposed by Singh and Ganie (2021b). For SFSs, Mahmood *et al.* (2021) developed correlation coefficients and applied them in pattern recognition and medical diagnosis.

Some novel methods of constructing the correlation coefficients from similarity and dissimilarity functions were suggested by Batyrshin (2019). The definitions of association measures that generalize Pearson's correlation coefficient as well as the general methods for constructing such measures were proposed by Batyrshin (2015b). Some similarity-based correlation coefficients for binary data were introduced by Batyrshin, Ramirez, Batyrshin, and Solovyev (2020). Batyrshin (2015a, 2019) studied the similarity measures and construction of association measures on [0, 1].

Most of the existing correlation coefficients in fuzzy/non-standard fuzzy theory receive their value in [0, 1] and thus indicate only the degree of correlation and remain silent about the nature of correlation. From an application point of view, a correlation coefficient indicating both degree and nature of correlation is more suitable than the correlation coefficient indicating only the degree of correlation. Also, the correlation coefficients for SFSs given by Mahmood *et al*. (2021) do not inform us about the nature of the correlation between two SFSs. So, keeping these points in mind, we, in this study, propose two novel correlation coefficients for SFSs. These correlation coefficients indicate both nature as well as the strength of correlation between two SFSs.

The content of the paper is structured as follows. Section 3 covers the preliminary concepts. Section 4 introduces two novel correlation coefficients in SF settings. A comparative analysis based on linguistic hedges is shown in Section 5. The application and advantage of the proposed correlation coefficients in pattern recognition are shown in Section 6. Finally, Section 7 concludes the paper.

3. Preliminaries

In this section, we introduce the basic definitions of fuzzy and non-standard FSs.

Definition 1. (Zadeh, 1965) A fuzzy set *G* in a universe of discourse $T=\{t_1,t_2,\ldots,t_l\}$ is defined as

$$G=\left\{\left(t_k,m_G(t_k)\right),t_k\in T,k=1,2,\ldots,l.\right\}$$

where $m_G:T\to[0,1]$ is a membership function, and $m_G(t_k)$ is the membership degree of $t_k\in T$ in the set *G*.

Definition 2. (Atanassov, 1986) An intuitionistic fuzzy set *G* in a universe of discourse $T=\{t_1,t_2,\ldots,t_l\}$ is given as

$$G=\left\{\left(t_k,m_G(t_k),n_G(t_k)\right),t_k\in T,k=1,2,\ldots,l.\right\}$$

where $m_G:T\to[0,1]\text{and}n_G:T\to[0,1]$ are the membership and non-membership functions respectively. Here, $m_G(t_k)$ represents the membership degree, $n_G(t_k)$ represents the non-membership degree of the element $t_k\in T$ in the set *G*, and $0\le m_G(t_k)+n_G(t_k)\le 1$.

Definition 3. (Cuong & Kreinovich, 2013) A picture fuzzy set *G* in a universe of discourse $T=\{t_1,t_2,\ldots,t_l\}$ is given as

$$G=\left\{\left(t_k,m_G(t_k),n_G(t_k),h_G(t_k)\right),t_k\in T,k=1,2,\ldots,l.\right\}$$

where $m_G:T\to[0,1], n_G:T\to[0,1]$ and $h_G:T\to[0,1]$ are the membership, non-membership, and neutrality functions respectively. Here, $m_G(t_k)$ represents the membership degree, $n_G(t_k)$ represents the non-membership degree, $h_G(t_k)$ represents the neutrality degree of the element $t_k\in T$ in the set *G*, and $0\le m_G(t_k)+n_G(t_k)+h_G(t_k)\le 1$.

Definition 4. (Kutlu Gündoğdu & Kahraman, 2020b) A spherical fuzzy set *G* in a universe of discourse $T=\{t_1,t_2,\ldots,t_l\}$ is given as

$$G=\left\{\left(t_k,m_G(t_k),n_G(t_k),h_G(t_k)\right),t_k\in T,k=1,2,\ldots,l.\right\}$$

where $m_G:T\to[0,1], n_G:T\to[0,1]\text{and}h_G:T\to[0,1]$ are the membership, non-membership, and neutrality functions respectively. Here, $m_G(t_k)$ represents the membership degree, $n_G(t_k)$ represents the non-membership degree, $h_G(t_k)$ represents the neutrality degree of the element $t_k\in T$ in the set *G* and $0\le\left(m_G(t_k)\right)^2+\left(n_G(t_k)\right)^2+\left(n_G(t_k)\right)^2\le 1$. Also, the degree of refusal of an element $t_k\in T$ in the spherical fuzzy set *G* is given by

$$e_G(t_k)=\sqrt{1-\left(m_G(t_k)\right)^2-\left(n_G(t_k)\right)^2-\left(h_G(t_k)\right)^2}.$$

Definition 5. (Mahmood *et al*., 2021) The two correlation coefficients for spherical fuzzy sets are given as:

$$C_{MIAG1}=\frac{\sum_{k=1}^{l}\left\{\begin{array}{c}\left(m_G(t_k)m_H(t_k)\right)^2+\left(n_G(t_k)n_H(t_k)\right)^2\\+\left(h_G(t_k)h_H(t_k)\right)^2+\left(e_G(t_k)e_H(t_k)\right)^2\end{array}\right\}}{\left\{\begin{array}{c}\left(\sum_{k=1}^{l}\left(\left(m_G(t_k)\right)^4+\left(n_G(t_k)\right)^4+\left(h_G(t_k)\right)^4+\left(e_G(t_k)\right)^4\right)\right)\\\times\left(\sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^4+\left(n_H(t_k)\right)^4+\left(h_H(t_k)\right)^4+\left(e_H(t_k)\right)^4\right)\right)\end{array}\right\}^{\frac{1}{2}}} \quad (1)$$

$$C_{MIAG2}=\frac{\sum_{k=1}^{l}\left\{\begin{matrix}\left(m_G(t_k)m_H(t_k)\right)^2+\left(n_G(t_k)n_H(t_k)\right)^2\\+\left(h_G(t_k)h_H(t_k)\right)^2+\left(e_G(t_k)e_H(t_k)\right)^2\end{matrix}\right\}}{max\left\{\begin{matrix}\sum_{k=1}^{l}\left(\left(m_G(t_k)\right)^4+\left(n_G(t_k)\right)^4+\left(h_G(t_k)\right)^4+\left(e_G(t_k)\right)^4\right),\\ \sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^4+\left(n_H(t_k)\right)^4+\left(h_H(t_k)\right)^4+\left(e_H(t_k)\right)^4\right)\end{matrix}\right\}}. \quad (2)$$

4. New Correlation Coefficients for SFSs

In this section, we introduce some novel SF correlation coefficients.

Let $V(T)$ denote the set of all SFSs in the universe of discourse $T=\{t_1,t_2,\dots,t_l\}$ throughout this paper. Here we propose two correlation coefficients for SFSs, which, together with strength, also express the nature of the correlation between two SFSs.

Definition 5. For any $G,H\in V(T)$, a correlation coefficient based on membership, non-membership, and neutrality degree is defined as

$$C_{G1}(G,H)=\frac{1}{3}(\theta_1+\theta_2+\theta_3), \quad (3)$$

where $\theta_1=\frac{\sum_{k=1}^{l}\left(\left(m_G(t_k)\right)^2-(\overline{m_G})^2\right)\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)}{\sqrt{\sum_{k=1}^{l}\left(\left(m_G(t_k)\right)^2-(\overline{m_G})^2\right)^2}\times\sqrt{\sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)^2}}$,

$$\theta_2=\frac{\sum_{k=1}^{l}\left(\left(n_G(t_k)\right)^2-(\overline{n_G})^2\right)\left(\left(n_H(t_k)\right)^2-(\overline{n_H})^2\right)}{\sqrt{\sum_{k=1}^{l}\left(\left(n_G(t_k)\right)^2-(\overline{n_G})^2\right)^2}\times\sqrt{\sum_{k=1}^{l}\left(\left(n_H(t_k)\right)^2-(\overline{n_H})^2\right)^2}},$$

$$\theta_3=\frac{\sum_{k=1}^{l}\left(\left(h_G(t_k)\right)^2-(\overline{h_G})^2\right)\left(\left(h_H(t_k)\right)^2-(\overline{h_H})^2\right)}{\sqrt{\sum_{k=1}^{l}\left(\left(h_G(t_k)\right)^2-(\overline{h_G})^2\right)^2}\times\sqrt{\sum_{k=1}^{l}\left(\left(h_H(t_k)\right)^2-(\overline{h_H})^2\right)^2}}, \overline{m_G}=\frac{1}{l}\sum_{k=1}^{l}m_G(t_k), \overline{n_G}=\frac{1}{l}\sum_{k=1}^{l}n_G(t_k),$$

$\overline{h_G}=\frac{1}{l}\sum_{k=1}^{l}h_G(t_k), \overline{m_H}=\frac{1}{l}\sum_{k=1}^{l}m_H(t_k), \overline{n_H}=\frac{1}{l}\sum_{k=1}^{l}n_H(t_k), \overline{h_H}=\frac{1}{l}\sum_{k=1}^{l}h_H(t_k).$

Definition 6. For any $G,H\in V(T)$, a correlation coefficient based on membership, non-membership, neutrality, and refusal degree is defined as

$$C_{G2}(G,H)=\frac{1}{4}(\theta_1+\theta_2+\theta_3+\theta_4), \quad (4)$$

where $\theta_4=\frac{\sum_{k=1}^{l}\left(\left(e_G(t_k)\right)^2-(\overline{e_G})^2\right)\left(\left(e_H(t_k)\right)^2-(\overline{e_H})^2\right)}{\sqrt{\sum_{k=1}^{l}\left(\left(e_G(t_k)\right)^2-(\overline{e_G})^2\right)^2}\times\sqrt{\sum_{k=1}^{l}\left(\left(e_H(t_k)\right)^2-(\overline{e_H})^2\right)^2}}$, $\overline{e_G}=\frac{1}{l}\sum_{k=1}^{l}e_G(t_k), \overline{e_H}=\frac{1}{l}\sum_{k=1}^{l}e_H(t_k)$, and the rest of the terms are the same as in Equation (3).

Next, we discuss the properties of the proposed SF correlation coefficients $C_{G1}(G,H)$, and $C_{G2}(G,H)$ in the following:

Theorem 1. *For any* $G,H\in V(T)$, *we have,*

(1) $C_{G1}(G,H)=C_{G1}(H,G)$
(2) $-1\le C_{G1}(G,H)\le 1$
(3) $C_{G1}(G,H)=1$, if $G=H$.

Proof. (1) It is obvious.

(2) By Cauchy Schwarz inequality, we have

$$(\theta_1)^2=\left(\frac{\sum_{k=1}^{l}\left(\left(m_G(t_k)\right)^2-(\overline{m_G})^2\right)\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)}{\sqrt{\sum_{k=1}^{l}\left(\left(m_G(t_k)\right)^2-(\overline{m_G})^2\right)^2}\times\sqrt{\sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)^2}}\right)^2$$

$$=\frac{\left(\sum_{k=1}^{l}\left(\left(m_G(t_k)\right)^2-(\overline{m_G})^2\right)\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)\right)^2}{\left(\sqrt{\sum_{k=1}^{l}\left(\left(m_G(t_k)\right)^2-(\overline{m_G})^2\right)^2}\times\sqrt{\sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)^2}\right)^2}$$

$$\leq \frac{\sum_{k=1}^{l}\left(\left(m_G(t_k)\right)^2-(\overline{m_G})^2\right)^2 \sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)^2}{\sum_{k=1}^{l}\left(\left(m_G(t_k)\right)^2-(\overline{m_G})^2\right)^2 \sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)^2} = 1.$$

$$\Rightarrow |\theta_1| \leq 1.$$

Similarly, $|\theta_2| \leq 1$ and $|\theta_3| \leq 1$.

So, $|C_{G1}(G,H)| = \left|\frac{1}{3}(\theta_1+\theta_2+\theta_3)\right| \leq \frac{1}{3}\{|\theta_1|+|\theta_2|+|\theta_3|\} \leq \frac{1}{3}(1+1+1) = 1$,

i.e., $-1 \leq C_{SG2}(G,H) \leq 1$.

(3) Let $G = H$, i.e., $m_G(t_k) = m_H(t_k), n_G(t_k) = n_H(t_k)$ and $h_G(t_k) = h_H(t_k)$, then we have

$$\theta_1 = \frac{\sum_{k=1}^{l}\left(\left(m_G(t_k)\right)^2-(\overline{m_G})^2\right)\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)}{\sqrt{\sum_{k=1}^{l}\left(\left(m_G(t_k)\right)^2-(\overline{m_G})^2\right)^2} \times \sqrt{\sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)^2}}$$

$$= \frac{\sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)}{\sqrt{\sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)^2} \times \sqrt{\sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)^2}}$$

$$= \frac{\sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)}{\sqrt{\sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)^2} \times \sqrt{\sum_{k=1}^{l}\left(\left(m_H(t_k)\right)^2-(\overline{m_H})^2\right)^2}} = 1.$$

Similarly, $\theta_2 = 1$ and $\theta_3 = 1$. Therefore, $C_{G1}(G,H) = \frac{1}{3}(\theta_1+\theta_2+\theta_3) = \frac{1}{3}(1+1+1) = 1$.

Theorem 2. *For any* $G, H \in V(T)$, *we have,*

(1) $C_{G2}(G,H) = C_{G2}(H,G)$

(2) $-1 \leq C_{G2}(G,H) \leq 1$

(3) $C_{G2}(G,H) = 1$, if $G = H$.

Proof. On the same lines as that of Theorem 1.

5. A Comparative Study based on Linguistic Hedges

In this section, we compare the suggested SF correlation coefficients with the existing SF correlation coefficients by using linguistic hedges.

Definition 10. (Mathew *et al*., 2020) For any $G \in V(T)$, the n^{th} power of *G*, where *n* is any positive real number is defined as

$$G^n = \left\{\left(\frac{t_k, \left(m_G(t_k)\right)^n, \sqrt{1-\left(1-\left(n_G(t_k)\right)^2\right)^n},}{\sqrt{\left(1-\left(n_G(t_k)\right)^2\right)^n - \left(1-\left(n_G(t_k)\right)^2-\left(h_G(t_k)\right)^2\right)^n}}\right) \mid t_k \in T\right\}, n > 0.$$

Example 1. Consider an SFS *G* in $T = \{t_1, t_2, t_3, t_4, t_5\}$ as

$$G = \left\{\begin{matrix}(t_1, 0.3, 0.1, 0.1), (t_2, 0.3, 0.4, 0.1), (t_3, 0.3, 0.2, 0.2), \\ (t_4, 0.1, 0.1, 0), (t_5, 0.4, 0.5, 0.3)\end{matrix}\right\}.$$

By considering the above definition of the modifier of an SFS, we define the SFSs

LARGE = *G*,

Very LARGE = G^2,

Very very LARGE = G^4,

Not very LARGE = $G^{2'}$ and More or less LARGE = $G^{\frac{1}{2}}$.

We use these SFSs to compare our proposed methods with some existing ones for calculating the correlation coefficients. The comparison results are given in Tables 1-3. The following notations are used in Tables1-3.

LARGE: L.

Very LARGE: V.L.

Very very LARGE: V.V.L.

More or less LARGE: M.L.L.

Not Very LARGE: N.V.L.

From the characterization of linguistic variables, a correlation coefficient *K* should satisfy the following requirements.

$$\left.\begin{array}{c} K(M.L.L.,L.) > K(M.L.L.,V.L.) > K(M.L.L.,V.V.L.) \\ K(V.V.L.,V.L.) > K(V.V.L.,L.) > K(V.V.L.,M.L.L.) \\ K(L.,M.L.L.) > K(L.,V.L.) > K(L.,V.V.L.) \\ K(V.L.,L.) > K(V.L.,V.V.L.) > K(V.L.,M.L.L.) \end{array}\right] \quad (5)$$

From Tables 2-3, we observe that

$$C_{MIAGi}(M.L.L.,L.) > C_{MIAGi}(M.L.L.,V.L.) > C_{MIAGi}(M.L.L.,V.V.L.)$$
$$C_{MIAGi}(V.V.L.,V.L.) > C_{MIAGi}(V.V.L.,L.) > C_{MIAGi}(V.V.L.,M.L.L.)$$
$$C_{MIAGi}(L.,M.L.L.) < C_{MIAGi}(L.,V.L.) > C_{MIAGi}(L.,V.V.L.)$$
$$C_{MIAGi}(V.L.,L.) < C_{MIAGi}(V.L.,V.V.L.) > C_{MIAGi}(V.L.,M.L.L.), i = 1, 2.$$
$$C_{Gi}(M.L.L.,L.) > C_{Gi}(M.L.L.,V.L.) > C_{Gi}(M.L.L.,V.V.L.)$$
$$C_{Gi}(V.V.L.,V.L.) > C_{Gi}(V.V.L.,L.) > C_{Gi}(V.V.L.,M.L.L.)$$
$$C_{Gi}(L.,M.L.L.) > C_{Gi}(L.,V.L.) > C_{Gi}(L.,V.V.L.)$$
$$C_{Gi}(V.L.,L.) > C_{Gi}(V.L.,V.V.L.) > C_{Gi}(V.L.,M.L.L.), i = 1, 2.$$

Thus, we observe that only the suggested SF correlation coefficients C_{G1} and C_{G2} satisfy all the conditions given in Equation (5). On the other hand, the existing SF correlation coefficients C_{MIAG1} and C_{MIAG2} fail to satisfy all the requirements given in Equation (5). This shows that from a linguistic hedge aspect, the suggested SF correlation coefficients are more effective than the available SF correlation coefficients.

Table 1. Values of the correlation coefficients C_{MIAG1} and C_{MIAG2}.

	M.L.L.	L.	V.L.	V.V.L.	N.V.L.
M.L.L.	1.0000	0.9636	0.8798	0.8020	0.9566
L.	0.9636	1.0000	0.9728	0.9187	0.9596
V.L.	0.8798	0.9728	1.0000	0.9811	0.8976
V.V.L.	0.8020	0.9187	0.9811	1.0000	0.8157
N.V.L.	0.9566	0.9596	0.8976	0.8157	1.0000

Table 2. Values of the correlation coefficient C_{G1}

	M.L.L.	L.	V.L.	V.V.L.	N.V.L.
M.L.L.	1.0000	0.9917	0.9448	0.8409	0.8455
L.	0.9917	1.0000	0.9785	0.8972	0.8768
V.L.	0.9448	0.9785	1.0000	0.9641	0.8982
V.V.L.	0.8409	0.8972	0.9641	1.0000	0.8623
N.V.L.	0.8455	0.8768	0.8982	0.8623	1.0000

Table 3. Values of the correlation coefficient C_{G2}

	M.L.L.	L.	V.L.	V.V.L.	N.V.L.
M.L.L.	1.0000	0.9840	0.9375	0.8580	0.8630
L.	0.9840	1.0000	0.9810	0.9162	0.9047
V.L.	0.9375	0.9810	1.0000	0.9709	0.9236
V.V.L.	0.8580	0.9162	0.9709	1.0000	0.8946
N.V.L.	0.8630	0.9047	0.9236	0.8946	1.0000

6. Application in Pattern Recognition

In this section, we discuss the application of our proposed SF correlation coefficients in pattern recognition. Classifying an unknown pattern into some known patterns is referred to as pattern recognition. Here we utilize our proposed SF correlation coefficients for solving some problems of pattern recognition and contrast the results with the existing SF correlation coefficients. First, we formulate a pattern recognition problem in the SF environment.

Problem: Given some known patterns $G_i\,(i = 1, 2, \dots, m)$ and an unknown pattern H in the form of SFSs as

$$G_i = \{\langle t_k, m_{G_i}(t_k), n_{G_i}(t_k), h_{G_i}(t_k)\rangle | t_k \in T, k = 1, 2, \dots, l\}, i = 1, 2, \dots, m.$$

$$H = \{\langle t_k, m_H(t_k), n_H(t_k), h_H(t_k)\rangle | t_k \in T, k = 1, 2, \dots, l\}.$$

Aim: To classify the unknown pattern H into one of the known patterns $G_i (i = 1, 2, \dots, m)$.

Recognition principle: The unknown pattern H can be assigned to the known pattern with which it has maximum correlation.
Now, we solve some pattern recognition problems in view of SF information with the help of our proposed SF correlation measures given in Equations (3) and (4). Furthermore, we compare the results with the existing SF correlation measures given in the Equations (1)-(2).

Example 1. (Ganie *et al*., 2020) Consider three known patterns G_1, G_2, G_3, and an unknown pattern H in the form of SFSs as

$$G_1 = \left\{ \begin{matrix} (t_1, 0.4, 0.3, 0.1), (t_2, 0.5, 0.3, 0.2), (t_3, 0.4, 0.3, 0.0), \\ (t_4, 0.7, 0.0, 0.2), (t_5, 0.6, 0.1, 0.1) \end{matrix} \right\},$$

$$G_2 = \left\{ \begin{matrix} (t_1, 0.7, 0.1, 0.1), (t_2, 0.2, 0.3, 0.4), (t_3, 0.2, 0.1, 0.5), \\ (t_4, 0.1, 0.5, 0.2), (t_5, 0.3, 0.3, 0.3) \end{matrix} \right\},$$

$$G_3 = \left\{ \begin{matrix} (t_1, 0.1, 0.3, 0.4), (t_2, 0.4, 0.3, 0.1), (t_3, 0.3, 0.4, 0.2), \\ (t_4, 0.2, 0.5, 0.3), (t_5, 0.5, 0.3, 0.1) \end{matrix} \right\} \text{ and}$$

$$H = \left\{ \begin{matrix} (t_1, 0.6, 0.2, 0.1), (t_2, 0.3, 0.4, 0.2), (t_3, 0.4, 0.3, 0.2), \\ (t_4, 0.7, 0.1, 0.0), (t_5, 0.4, 0.2, 0.2) \end{matrix} \right\}.$$

The calculated values of the SF correlation measures between the known patterns $G_i, i = 1, 2, 3$, and the unknown pattern H are summarized in Table 4 and also shown in Figure 1.

Table 4. Calculated values of various SF correlation measures regarding Example 1.

	(G_1, H)	(G_2, H)	(G_3, H)	Result
C_{MIAG1} (Mahmood *et al*., 2021)	0.9815	0.9432	0.9373	G_1
C_{MIAG2} (Mahmood *et al*., 2021)	0.9776	0.9410	0.9329	G_1
C_{G1} (Proposed)	0.3281	0.2549	-0.5880	G_1
C_{G2} (Proposed)	0.3634	0.2932	-0.3626	G_1

From Table 4, it is clear that all the SF correlation measures given in the Equations (1)-(4) assign the unknown pattern H to the known pattern G_1. This shows that our proposed SF correlation measures are consistent with the existing SF correlation measures. We also note that the pattern H has a negative correlation with the pattern G_3 as indicated by the proposed correlation coefficients C_{G1}, and C_{G2}.

Example 2. Let

$$G_1 = \left\{ \begin{matrix} (t_1, 0.40, 0.67, 0.22), (t_2, 0.41, 0.74, 0.41), \\ (t_3, 0.67, 0.58, 0.44), (t_4, 0.55, 0.56, 0.37) \end{matrix} \right\},$$

$$G_2 = \left\{ \begin{matrix} (t_1, 0.44, 0.65, 0.50), (t_2, 0.50, 0.59, 0.59), \\ (t_3, 0.59, 0.80, 0.00), (t_4, 0.59, 0.48, 0.39) \end{matrix} \right\},$$

$$G_3 = \left\{ \begin{matrix} (t_1, 0.67, 0.51, 0.39), (t_2, 0.74, 0.67, 0.00), \\ (t_3, 0.66, 0.67, 0.31), (t_4, 0.16, 0.82, 0.47) \end{matrix} \right\},$$

be some known patterns and

$$H = \left\{ \begin{matrix} (t_1, 0.38, 0.80, 0.44), (t_2, 0.59, 0.50, 0.38), \\ (t_3, 0.38, 0.59, 0.31), (t_4, 0.38, 0.74, 0.50), \end{matrix} \right\},$$

be an unknown pattern.

The calculated values of the SF correlation measures between the known patterns $G_i, i = 1, 2, 3$, and the unknown pattern H are summarized in Table 5 and also shown in Figure 2.

Table 5. Calculated values of various SF correlation measures regarding Example 2.

	(G_1, H)	(G_2, H)	(G_3, H)	Result
C_{MIAG1} (Mahmood *et al*., 2021)	0.7639	0.7639	0.7510	Not Classified
C_{MIAG2} (Mahmood *et al*., 2021)	0.7477	0.7289	0.7477	Not Classified
C_{G1} (Proposed)	-0.4240	-0.2230	0.5179	G_3
C_{G2} (Proposed)	-0.5492	-0.3512	0.2793	G_3

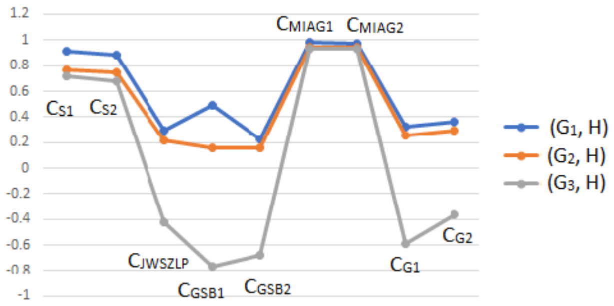

Figure 1. Correlation coefficient between the patterns regarding Example 1

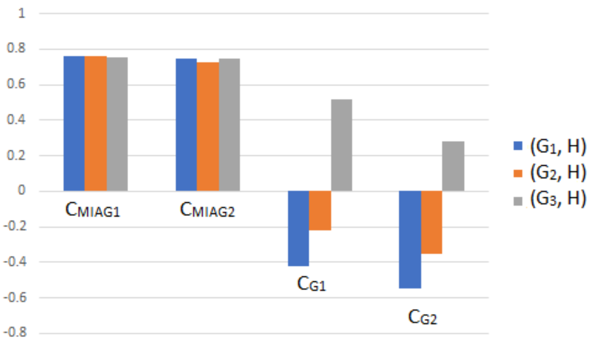

Figure 2. Correlation coefficient between the patterns regarding Example 2

From Table 5, it is clear that the existing SF correlation measures given in Equations (1)-(2) fail to classify the pattern H to one of the patterns G_i, $i = 1, 2, 3$. Our proposed SF correlation coefficients given in Equations (3) and (4) classify the pattern H into the pattern G_3.

Thus, from Examples 1 and 2, we conclude that the suggested SF correlation coefficients are better than the SF correlation coefficients due to Mahmood *et al*. (2021) in computing the correlation degree and determining the nature of correlation.

7. Conclusions

In this study, we have proposed two correlation coefficients for SFSs. It has been observed that the proposed correlation coefficients indicate the degree as well as the nature of correlation (positive or negative correlation) in two SF data sets. Also, we found that from a linguist hedge viewpoint the suggested correlation coefficients are robust than the existing correlation coefficients for SFSs. We also applied our proposed SF correlation coefficients for solving some pattern recognition problems. We found that our proposed correlation methods perform better than the existing SF correlation methods.

In the future, we will study the application of the suggested SF correlation coefficients in decision-making problems, clustering analysis, medical diagnosis, etc., and extend them to an interval-valued SF environment.

References

Ashraf, S., Abdullah, S., Aslam, M., Qiyas, M., & Kutbi, M. A. (2019). Spherical fuzzy sets and its representation of spherical fuzzy t-norms and t-conorms. *Journal of Intelligent and Fuzzy Systems*, *36*(6), 6089–6102. doi:10.3233/JIFS-181941

Ashraf, S., Abdullah, S., & Mahmood, T. (2020). Spherical fuzzy Dombi aggregation operators and their application in group decision making problems. *Journal of Ambient Intelligence and Humanized Computing*, *11*(7), 2731–2749. doi:10.1007/s12652-019-01333-y

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96. doi:10.1016/S0165-0114(86)80034-3

Aydoğdu, A., & Gül, S. (2020). A novel entropy proposition for spherical fuzzy sets and its application in multiple attribute decision-making. *International Journal of Intelligent Systems*, *35*(9), 1354–1374. doi:10.1002/int.22256

Batyrshin, I. Z. (2015a). Association measures on [0,1]. *Journal of Intelligent and Fuzzy Systems*, *29*(3), 1011–1020. doi:10.3233/IFS-151714

Batyrshin, I. Z. (2015b). On definition and construction of association measures. *Journal of Intelligent and Fuzzy Systems*, *29*(6), 2319–2326. doi:10.3233/IFS-151930

Batyrshin, I. Z. (2019). Constructing correlation coefficients from similarity and dissimilarity functions. *Acta Polytechnica Hungarica*, *16*(10), 191–204.

Batyrshin, I. Z., Ramirez-Mejia, I., Batyrshin, I. I., & Solovyev, V. (2020). *Similarity-based correlation functions for binary data* (pp. 224–233). doi:10.1007/978-3-030-60887-3_20

Chaudhuri, B. B., & Bhattacharya, A. (2001). On correlation between two fuzzy sets. *Fuzzy Sets and Systems*, *118*(3), 447–456. doi:10.1016/S0165-0114(98)00347-9

Chiang, D. A., & Lin, N. P. (1999). Correlation of fuzzy sets. *Fuzzy Sets and Systems*, *102*(2), 221–226. doi:10.1016/S0165-0114(97)00127-9

Cuong, B. C., & Kreinovich, V. (2013). Picture fuzzy sets - A new concept for computational intelligence problems. *2013 Third World Congress on Information and Communication Technologies (WICT 2013)*, 1–6. doi:10.1109/WICT.2013.71130 99

Du, W. S. (2019). Correlation and correlation coefficient of generalized orthopair fuzzy sets. *International Journal of Intelligent Systems*, *34*(4), 564–583. doi:10.1002/int.22065

Ejegwa, P. A., & Onyeke, I. C. (2020). Medical diagnostic analysis on some selected patients based on modified Thao *et al.*'s correlation coefficient of intuitionistic fuzzy sets via an algorithmic approach. *Journal of Fuzzy Extension and Applications*, *1*(2), 130–141. doi:10.22105/JFEA.2020.250108.1014

Ganie, A. H., Singh, S., & Bhatia, P. K. (2020). Some new correlation coefficients of picture fuzzy sets with applications. *Neural Computing and Applications*, *32*(16), 12609–12625. doi:10.1007/s00521-020-04715-y

Gerstenkorn, T., & Mańko, J. (1991). Correlation of intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *44*(1), 39–43. doi:10.1016/0165-0114(91)90031-K

Hong, D. H., & Hwang, S. Y. (1995). Correlation of intuitionistic fuzzy sets in probability spaces. *Fuzzy Sets and Systems*, *75*(1), 77–81. doi:10.1016/0165-0114(94)00330-A

Huang, H.-L., & Guo, Y. (2019). An improved correlation coefficient of intuitionistic fuzzy sets. *Journal of Intelligent Systems*, *28*(2), 231–243. doi:10.1515/jisys-2017-0094

Hung, W.-L. (2001). Using statistical viewpoint in developing correlation of intuitionistic fuzzy sets. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, *09*(04), 509–516. doi:10.1142/S0218488501000910

Hung, W. L., & Wu, J. W. (2002). Correlation of intuitionistic fuzzy sets by centroid method. *Information Sciences*, *144*(1–4), 219–225. doi:10.1016/S0020-0255(02)00181-0

Jin, Y., Ashraf, S., & Abdullah, S. (2019). Spherical fuzzy logarithmic aggregation operators based on entropy and their application in decision support systems. *Entropy*, *21*(7), 628. doi:10.3390/e21070628

Kahraman, C., Kutlu Gundogdu, F., Cevik Onar, S., & Oztaysi, B. (2019). Hospital location selection using spherical fuzzy TOPSIS. *Proceedings of the 2019 Conference of the International Fuzzy Systems Association and the European Society for Fuzzy Logic and Technology (EUSFLAT 2019)*. doi:10.2991/eusflat-19.2019.12

Khan, M. J., Kumam, P., Deebani, W., Kumam, W., & Shah, Z. (2020). Distance and similarity measures for spherical fuzzy sets and their applications in selecting mega projects. *Mathematics*, *8*(4), 519. doi:10.3390/math8040519

Khan, Q., Mahmood, T., & Ullah, K. (2021). Applications of improved spherical fuzzy Dombi aggregation operators in decision support system. *Soft Computing*, *25*(14), 9097–9119. doi:10.1007/s00500-021-05829-8

Kumar, M. (2019). Intuitionistic fuzzy measures of correlation coefficient of intuitionistic fuzzy numbers under weakest triangular norm. *International Journal of Fuzzy System Applications*, *8*(1), 48–64. doi:10.4018/IJFSA.2019010103

Kutlu Gündoğdu, F. (2020). A spherical fuzzy extension of MULTIMOORA method. *Journal of Intelligent and Fuzzy Systems*, *38*(1), 963–978. doi:10.3233/JIFS-179462

Kutlu Gündoğdu, F., & Kahraman, C. (2019). A novel VIKOR method using spherical fuzzy sets and its application to warehouse site selection. *Journal of Intelligent and Fuzzy Systems*, *37*(1), 1197–1211. doi:10.3233/JIFS-182651

Kutlu Gündoğdu, F., & Kahraman, C. (2020a). A novel spherical fuzzy QFD method and its application to the linear delta robot technology development. *Engineering Applications of Artificial Intelligence*, *87*, 103348. doi:10.1016/j.engappai.2019.103348

Kutlu Gündoğdu, F., & Kahraman, C. (2020b). A novel spherical fuzzy analytic hierarchy process and its renewable energy application. *Soft Computing*, *24* (6), 4607–4621. doi:10.1007/s00500-019-04222-w

Liu, B., Shen, Y., Mu, L., Chen, X., & Chen, L. (2016). A new correlation measure of the intuitionistic fuzzy sets. *Journal of Intelligent and Fuzzy Systems*, *30*(2), 1019–1028. doi:10.3233/IFS-151824

Liu, P., Zhu, B., & Wang, P. (2019). A multi-attribute decision-making approach based on spherical fuzzy sets for Yunnan Baiyao's R&D project selection problem. *International Journal of Fuzzy Systems*, *21*(7), 2168–2191. doi:10.1007/s40815-019-00687-x

Mahmood, T., Ilyas, M., Ali, Z., & Gumaei, A. (2021). Spherical fuzzy sets-based cosine similarity and information measures for pattern recognition and medical diagnosis. *IEEE Access*, *9*, 25835–25842. doi:10.1109/ACCESS.2021.3056427

Mahmood, T., Ullah, K., Khan, Q., & Jan, N. (2019). An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. *Neural Computing and Applications*, *31*(11), 7041–7053. doi:10.1007/s00521-018-3521-2

Mathew, M., Chakrabortty, R. K., & Ryan, M. J. (2020). A novel approach integrating AHP and TOPSIS under spherical fuzzy sets for advanced manufacturing system selection. *Engineering Applications of Artificial Intelligence*, *96*, 103988. doi:10.1016/j.engappai.2020.103988

Mitchell, H. B. (2004). A correlation coefficient for intuitionistic fuzzy sets. *International Journal of Intelligent Systems*, *19*(5), 483–490. doi:10.1002/int.20004

Rafiq, M., Ashraf, S., Abdullah, S., Mahmood, T., & Muhammad, S. (2019). The cosine similarity measures of spherical fuzzy sets and their applications in decision making. *Journal of Intelligent and Fuzzy Systems*, *36*(6), 6059–6073. doi:10.3233/JIFS-181922

Sharma, S., & Singh, S. (2019). On some generalized correlation coefficients of the fuzzy sets and fuzzy soft sets with application in cleanliness ranking of public health centres. *Journal of Intelligent and Fuzzy Systems*, *36*(4), 3671–3683. doi:10.3233/JIFS-181838

Shishavan, S. A. S., Kutlu Gündoğdu, F., Farrokhizadeh, E., Donyatalab, Y., & Kahraman, C. (2020). Novel similarity measures in spherical fuzzy environment and their applications. *Engineering Applications of Artificial Intelligence*, *94*, 103837. doi:10.1016/j.engappai.2020.103837

Singh, P. (2015). Correlation coefficients for picture fuzzy sets. *Journal of Intelligent and Fuzzy Systems*, *28*(2), 591–604. doi:10.3233/IFS-141338

Singh, S., & Ganie, A. H. (2020). On some correlation coefficients in Pythagorean fuzzy environment with applications. *International Journal of Intelligent Systems*, *35*(4), 682–717. doi:10.1002/int.22222

Singh, S., & Ganie, A. H. (2021a). Some novel q-rung orthopair fuzzy correlation coefficients based on the statistical viewpoint with their applications. *Journal of Ambient Intelligence and Humanized Computing*. doi:10.1007/s12652-021-02983-7

Singh, S., & Ganie, A. H. (2021b). Applications of a picture fuzzy correlation coefficient in pattern analysis and decision-making. *Granular Computing*. doi:10.1007/s41066-021-00269-z

Thao, N. X., Ali, M., & Smarandache, F. (2019). An intuitionistic fuzzy clustering algorithm based on a new correlation coefficient with application in medical diagnosis. *Journal of Intelligent and Fuzzy Systems*, *36*(1), 189–198. doi:10.3233/JIFS-181084

Ullah, K., Hassan, N., Mahmood, T., Jan, N., & Hassan, M. (2019). Evaluation of investment policy based on multi-attribute decision-making using interval valued t-spherical fuzzy aggregation operators. *Symmetry*, *11*(3), 357. doi:10.3390/sym11030357

Ullah, K., Mahmood, T., & Jan, N. (2018). Similarity measures for t-spherical fuzzy sets with applications in pattern recognition. *Symmetry*, *10*(6), 193. doi:10.3390/sym10060193

Ullah, K., Mahmood, T., Jan, N., & Ahmad, Z. (2020). Policy decision making based on some averaging aggregation operators of t-spherical fuzzy sets; a multi-attribute decision making approach. *Annals of Optimization Theory and Practice*, *3*(3), 69–92. doi:10.22121/AOTP.2020.241244.1035

Wei, G., Wang, J., Lu, M., Wu, J., & Wei, C. (2019). Similarity measures of spherical fuzzy sets based on cosine function and their applications. *IEEE Access*, *7*, 159069–159080. doi:10.1109/ACCESS.2019.2949296

Wu, B., & Hung, C. F. (2016). Innovative correlation coefficient measurement with fuzzy data. *Mathematical Problems in Engineering*, *2016*, 1–11. doi:10.1155/2016/9094832

Yager, R. R. (2017). Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*, *25*(5), 1222–1230. doi:10.1109/TFUZZ.2016.2604005

Yager, R. R. (2013). Pythagorean fuzzy subsets. *2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS)*, 57–61. doi:10.1109/IFSA-NAFIPS.2013.6608375

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8* (3), 338–353. doi:10.1016/S0019-9958(65)90241-X

Zedam, L., Jan, N., Rak, E., Mahmood, T., & Ullah, K. (2020). An approach towards decision-making and shortest path problems based on t-spherical fuzzy information. *International Journal of Fuzzy Systems*, *22*(5), 1521–1534. doi:10.1007/s40815-020-00820-1