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Original Article

On interior bases of a semigroup

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Abstract

The main purpose of this paper is to introduce the concept of interior bases of a semigroup. In addition, we give a characterization when a non-empty subset of a semigroup is an interior base of a semigroup and give necessary and sufficient conditions of an interior base of a semigroup to be a subsemigroup.

Keywords: semigroup, interior ideal, interior base, quasi-order

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1. Introduction and Preliminaries

The notion of interior ideals of a semigroup has been introduced by Lajos (1976). Muhiuddin (2019) applied the cubic set theory to interior ideals of a semigroup. Muhiuddin and Mahboob (2020) introduced and studied int-soft interior ideals over the soft sets in ordered semigroups. Muhiuddin studied the concept of different types of ideals in semigroups, see (Muhiuddin, 2018; Muhiuddin, Mahboob, & Mohammad Khan, 2019). Based on the notion of interior ideals of a semigroup generated by a non-empty subset of a semigroup. The notion of one-sided bases of a semigroup was first introduced by Tamura (1955). Later, Fabrici (1972) studied the structure of a semigroup containing one-sided bases. After that, the concept of two-sided bases of a semigroup was studied by Fabrici (1975). Changphas and Summaprab (2014) introduced the concept of two-sided bases of an ordered semigroup. Recently, Kummoon and Changphas (2017) introduced the concept of bi-bases of a semigroup. The main purpose of this paper is to introduce the concept which is called interior bases of a semigroup. Also, we give a characterization when a non-empty subset of a semigroup is an interior base of the semigroup. Finally, we give necessary and sufficient conditions of an interior base of a semigroup to be a subsemigroup.

A semigroup is a pair $(S,\cdot)$ in which S is a non-empty set and $\cdot$ is a binary associative operation on S, i.e., the equation $(x\cdot y)\cdot z = x\cdot(y\cdot z)$ holds for all $x,y,z\in S$.

Throughout this paper, unless stated otherwise, we write the semigroup operation as multiplication and we mostly omit it typographically, i.e., we write S instead of $(S,\cdot)$, xy instead of $x\cdot y$, $x(yz)$ instead of $x\cdot(y\cdot z)$ and so on.

For A and B are non-empty subsets of a semigroup S, we define the set product AB of A and B, by

$$AB = \{ab \mid a\in A,\, b\in B\}.$$

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For $a \in S,$ we write Ba for $B\{a\},$ and similarly for $aB.$

Definition 1.1. (Lajos, 1976) A non-empty subset A of a semigroup S is called a *subsemigroup* of S if $AA \subseteq A.$

Definition 1.2. (Lajos, 1976) A subsemigroup A of a semigroup S is called an *interior ideal* of S if $SAS \subseteq A.$

Lemma 1.3. *Let S be a semigroup and A_i be a subsemigroup of S for all $i \in I.$ If $\bigcap_{i \in I} A_i \neq \varnothing,$ then $\bigcap_{i \in I} A_i$ is a subsemigroup of $S.$*

Proof. Assume that $\bigcap_{i \in I} A_i \neq \varnothing.$ Let $a, b \in \bigcap_{i \in I} A_i.$ Then $a, b \in A_i$ for all $i \in I.$ Since A_i is a subsemigroup of S for all $i \in I,$ so $ab \in A_i$ for all $i \in I.$ Thus $ab \in \bigcap_{i \in I} A_i.$ Therefore, $\bigcap_{i \in I} A_i$ is a subsemigroup of $S.$

Lemma 1.4. *Let S be a semigroup and A_i be an interior ideal of S for all $i \in I.$ If $\bigcap_{i \in I} A_i \neq \varnothing,$ then $\bigcap_{i \in I} A_i$ is an interior ideal of $S.$*

Proof. Assume that $\bigcap_{i \in I} A_i \neq \varnothing.$ By Lemma 1.3, $\bigcap_{i \in I} A_i$ is a subsemigroup of $S.$ Next, we will show that $S(\bigcap_{i \in I} A_i)S \subseteq \bigcap_{i \in I} A_i.$ Let $x \in S(\bigcap_{i \in I} A_i)S.$ Then $x = s_1 a s_2$ for some $s_1, s_2 \in S$ and $a \in \bigcap_{i \in I} A_i.$ Since $a \in \bigcap_{i \in I} A_i,$ we have $a \in A_i$ for all $i \in I,$ where A_i is an interior ideal of S for all $i \in I.$ So we have $x = s_1 a s_2 \in S(A_i)S \subseteq A_i$ for all $i \in I.$ Thus $x \in \bigcap_{i \in I} A_i.$ Therefore, $\bigcap_{i \in I} A_i$ is an interior ideal of $S.$

Definition 1.5. Let S be a semigroup and let A be a non-empty subset of $S.$ Then, the intersection of all interior ideals of S containing A is the smallest interior ideal of S generated by $A,$ denoted by $(A)_I.$

Lemma 1.6. *Let S be a semigroup and let A be a non-empty subset of $S.$ Then,*

$$(A)_I = A \cup AA \cup SAS.$$

Proof. Let $B = A \cup AA \cup SAS.$ Consider,

$$\begin{aligned} BB &= (A \cup AA \cup SAS)(A \cup AA \cup SAS) \\ &= AA \cup AAA \cup ASAS \cup AAA \cup AAAA \cup AASAS \cup (SAS)A \cup SASAA \cup SASSAS \\ &\subseteq AA \cup SAS \cup SAS \cup SAS \cup SAS \cup SAS \cup SAS \cup SAS \cup SAS \\ &= AA \cup SAS \subseteq B. \end{aligned}$$

Thus B is a subsemigroup of $S.$ Next, consider

$$\begin{aligned} SBS &= S(A \cup AA \cup SAS)S \\ &= (SA \cup SAA \cup SSAS)S \\ &= SAS \cup SAAS \cup SSASS \\ &\subseteq SAS \cup SAS \cup SAS = SAS \subseteq B. \end{aligned}$$

Thus $SBS \subseteq B.$ Hence, B is an interior ideal of S containing $A.$ Finally, let C be an interior ideal of S containing $A.$ Clearly, $A \subseteq C.$ Since C is a subsemigroup of $S,$ we have $AA \subseteq CC \subseteq C.$ Since C is an interior ideal of $S,$ we have $SAS \subseteq SCS \subseteq C.$ Thus $B = A \cup AA \cup SAS \subseteq C.$ Hence, B is the smallest interior ideal of S containing $A.$

2. Main Results

In this part, the definition of interior bases of a semigroup and the algebraic structure of a semigroup containing interior bases will be presented.

Definition 2.1. Let S be a semigroup. A non-empty subset A of S is called an *interior base* of S if it satisfies the following two conditions:

(1) $S = A \cup AA \cup SAS,$ i.e., $S = (A)_I$;

(2) if B is a subset of A such that $S = (B)_I$, then $B = A.$

Example 2.2. (Bussaban & Changhas, 2016) Let $S = \{a,b,c,d,f\}$ be a semigroup with the binary operation defined by:

$\times$	a	b	c	d	f
a	a	a	a	a	a
b	a	b	a	d	a
c	a	f	c	c	f
d	a	b	d	d	b
f	a	f	a	c	a

The interior bases of S are $\{b\}$, $\{c\}$, $\{d\}$, and $\{f\}$.

Example 2.3. (Yaqoob, Aslam, & Chinram, 2012) Let $S = \{0,1,2,3\}$ be a semigroup with the binary operation defined by:

$\times$	0	1	2	3
0	0	2	2	3
1	2	1	2	3
2	2	2	2	3
3	3	3	3	3

The interior base of S is $\{0,1\}$. But $\{0\}$ and $\{1\}$ are not interior bases of S.
First, we have the following useful lemma.

Lemma 2.4. *Let A be an interior base of a semigroup S, and let $a,b \in A$. If $a \in bb \cup SbS$, then $a = b$.*

Proof. Assume that $a \in bb \cup SbS,$ and suppose that $a \neq b.$ Let $B = A \setminus \{a\}.$ Then $B \subset A.$ Since $a \neq b,$ we have $b \in B.$ We will show that $(B)_I = S.$ Clearly, $(B)_I \subseteq S.$ Next, let $x \in S.$ Then, by $(A)_I = S,$ we have $x \in A \cup AA \cup SAS.$ There are three cases to consider:

Case 1: $x \in A.$

Subcase 1.1: $x \neq a.$ Then $x \in B \subseteq (B)_I.$

Subcase 1.2: $x = a.$ By assumption, we have $x = a \in bb \cup SbS \subseteq BB \cup SBS \subseteq (B)_I.$

Case 2: $x \in AA.$ Then $x = a_1 a_2$ for some $a_1, a_2 \in A.$

Subcase 2.1: $a_1 \neq a$ and $a_2 \neq a.$ Then $a_1, a_2 \in B.$ We have $x = a_1 a_2 \in BB \subseteq (B)_I.$

Subcase 2.2: $a_1 = a$ and $a_2 \neq a.$ By assumption and $a_2 \in B$, we have

$$x = a_1 a_2 = a_1 a \in (bb \cup SbS)B \subseteq (BB \cup SBS)B = BBB \cup SBSB \subseteq SBS \cup SBS = SBS \subseteq (B)_I.$$

Subcase 2.3: $a_1 \neq a$ and $a_2 = a.$ Then $a_1 \in B$ and by assumption, we have

$$x = a_1 a_2 = a a_2 \in B(bb \cup SbS) \subseteq B(BB \cup SBS) = BBB \cup BSBS \subseteq SBS \cup SBS = SBS \subseteq (B)_I.$$

Subcase 2.4: $a_1 = a$ and $a_2 = a.$ By assumption, we have

$$\begin{aligned} x = a_1 a_2 = aa &\in (bb \cup SbS)(bb \cup SbS) = bbbb \cup bbSbS \cup SbSbb \cup SbSSbS \\ &\subseteq BBBB \cup BBSBS \cup SBSBB \cup SBSSBS \\ &\subseteq SBS \cup SBS \cup SBS \cup SBS = SBS \subseteq (B)_I. \end{aligned}$$

Case 3: $x \in SAS.$ Then $x = s_1 a_3 s_2$ for some $s_1, s_2 \in S$ and $a_3 \in A.$

Subcase 3.1: $a_3 \neq a.$ Then $a_3 \in B.$ We have $x = s_1a_3s_2 \in SBS \subseteq (B)_I.$

Subcase 3.2: $a_3 = a.$ By assumption, we have

$$x = s_1a_3s_2 \in S(bb \cup SbS)S \subseteq S(BB \cup SBS)S = (SBB \cup SSBS)S = SBBS \cup SSBSS \subseteq SBS \subseteq (B)_I.$$

So, we obtain $S \subseteq (B)_I.$ This implies $(B)_I = S,$ which is a contradiction since A is an interior base of $S.$ Thus $a = b.$

Lemma 2.5. *Let A be an interior base of a semigroup $S,$ and let $a,b,c \in A.$ If $a \in cb \cup ScbS,$ then $a = b$ or $a = c.$*

Proof. Assume that $a \in cb \cup ScbS.$ Suppose that $a \neq b$ and $a \neq c.$ Setting $B = A \setminus \{a\}.$ Then $B \subset A.$ Since $a \neq b$ and $a \neq c,$ we have $b,c \in B.$ To show that $(A)_I \subseteq (B)_I.$ Let $x \in (A)_I.$ Then $x \in A \cup AA \cup SAS.$ There are three cases to consider:

Case 1: $x \in A.$

Subcase 1.1: $x \neq a.$ Then $x \in B \subseteq (B)_I.$

Subcase 1.2: $x = a.$ By assumption, we have

$$x = a \in cb \cup ScbS \subseteq BB \cup SBBS \subseteq BB \cup SBS \subseteq (B)_I.$$

Case 2: $x \in AA.$ Then $x = a_1a_2$ for some $a_1, a_2 \in A.$

Subcase 2.1: $a_1 \neq a$ and $a_2 \neq a.$ Then $a_1, a_2 \in B.$ We have $x = a_1a_2 \in BB \subseteq (B)_I.$

Subcase 2.2: $a_1 = a$ and $a_2 \neq a.$ By assumption and $a_2 \in B,$ we have

$$x = a_1a_2 \in (cb \cup ScbS)B \subseteq (BB \cup SBBS)B = BBB \cup SBBSB \subseteq SBS \subseteq (B)_I.$$

Subcase 2.3: $a_1 \neq a$ and $a_2 = a.$ Then $a_1 \in B$ and by assumption, we have

$$x = a_1a_2 \in B(cb \cup ScbS) \subseteq B(BB \cup SBBS) = BBB \cup BSBBS \subseteq SBS \subseteq (B)_I.$$

Subcase 2.4: $a_1 = a$ and $a_2 = a.$ By assumption, we have

$$x = a_1a_2 \in (cb \cup ScbS)(cb \cup ScbS) = cbcb \cup cbScbS \cup ScbScb \cup ScbSScbS$$
$$\subseteq BBBB \cup BBSBBS \cup SBBSBB \cup SBBSSBBS \subseteq SBS \subseteq (B)_I.$$

Case 3: $x \in SAS.$ Then $x = s_1a_3s_2$ for some $s_1, s_2 \in S$ and $a_3 \in A.$

Subcase 3.1: $a_3 \neq a.$ Then $a_3 \in B.$ We have $x = s_1a_3s_2 \in SBS \subseteq (B)_I.$

Subcase 3.2: $a_3 = a.$ By assumption, we have

$$x = s_1a_3s_2 \in S(cb \cup ScbS)S \subseteq S(BB \cup SBBS)S = SBBS \cup SSBBSS \subseteq SBS \subseteq (B)_I.$$

From both cases, we obtain $(A)_I \subseteq (B)_I.$ Since A is an interior base of $S,$ we have $S = (A)_I \subseteq (B)_I \subseteq S.$ Thus $S = (B)_I,$ which is a contradiction. Therefore, $a = b$ or $a = c.$

To give a characterization when a non-empty subset of a semigroup is an interior base of a semigroup, we need the concept of a quasi-order defined as follows:

Definition 2.6. Let S be a semigroup. Define a *quasi-order* $\leq_I$ on S by, for any $a,b \in S,$

$$a \leq_I b \Leftrightarrow (a)_I \subseteq (b)_I.$$

The following example shows that the order $\leq_I$ defined above is not, in general, a partial order.

Example 2.7. From Example 2.2, we have that $(b)_I \subseteq (c)_I$ (i.e., $b \leq_I c$) and $(c)_I \subseteq (b)_I$ (i.e., $c \leq_I b$), but $b \neq c.$ Thus $\leq_I$ is not a partial order on $S.$

Lemma 2.8. *Let A be an interior base of a semigroup $S.$ If $a,b \in A$ such that $a \neq b,$ then neither $a \leq_I b$ nor $b \leq_I a.$*

Proof. Assume that $a,b \in A$ such that $a \neq b.$ Suppose that $a \leq_I b.$ Then $a \in (a)_I \subseteq (b)_I.$ Since $a \in (b)_I = b \cup bb \cup SbS$ and $a \neq b,$ so we have $a \in bb \cup SbS.$ By Lemma 2.4, $a = b.$ This is a contradiction. The case $b \leq_I a$ can be proved similarly. Thus $a \leq_I b$ and $b \leq_I a$ are false.

Lemma 2.9. *Let A be an interior base of a semigroup $S.$ Let $a,b,c \in A$ and $s \in S.$*

(1) If $a \in bc \cup bcbc \cup SbcS,$ *then* $a = b$ *or* $a = c.$

(2) If $a \in sbcs \cup sbcssbcs \cup SsbcsS,$ *then* $a = b$ *or* $a = c.$

Proof. (1) Assume that $a \in bc \cup bcbc \cup SbcS,$ and suppose that $a \neq b$ and $a \neq c.$ Let $B = A \setminus \{a\}.$ Then $B \subset A.$ Since $a \neq b$ and $a \neq c,$ we have $b, c \in B.$ We will show that $(A)_I \subseteq (B)_I$. It suffices to show that $A \subseteq (B)_I.$ Let $x \in A.$ If $x \neq a,$ then $x \in B \subseteq (B)_I.$ So $x \in (B)_I.$ If $x = a,$ then by assumption, we have

$$x = a \in bc \cup bcbc \cup SbcS \subseteq BB \cup BBBB \cup SBBS \subseteq BB \cup SBS \subseteq (B)_I.$$

Thus $A \subseteq (B)_I.$ This implies $(A)_I \subseteq (B)_I.$ So $S = (A)_I \subseteq (B)_I \subseteq S.$ Hence, $S = (B)_I.$ This is a contradiction. Therefore, $a = b$ or $a = c.$

(2) Assume that $a \in sbcs \cup sbcssbcs \cup SsbcsS,$ and suppose that $a \neq b$ and $a \neq c.$ Let $B = A \setminus \{a\}.$ Then $B \subset A.$ Since $a \neq b$ and $a \neq c,$ we have $b, c \in B.$ We will show that $(A)_I \subseteq (B)_I$. It suffices to show that $A \subseteq (B)_I.$ Let $x \in A.$ If $x \neq a,$ then $x \in B \subseteq (B)_I.$ So $x \in (B)_I.$ If $x = a,$ then by assumption, we have

$$x = a \in sbcs \cup sbcssbcs \cup SsbcsS \subseteq SBBS \cup SBBSSBBS \cup SSBBSS \subseteq SBS \subseteq (B)_I.$$

Thus $x \in (B)_I,$ and so $A \subseteq (B)_I.$ This implies $(A)_I \subseteq (B)_I.$ So $S = (A)_I \subseteq (B)_I \subseteq S,$ and hence, $S = (B)_I.$ This is a contradiction. Therefore, $a = b$ or $a = c.$

Lemma 2.10. *Let* A *be an interior base of a semigroup* $S.$

(1) For any $a, b, c \in A,$ *if* $a \neq b$ *and* $a \neq c,$ *then* $a \not\leq_I bc.$

(2) For any $a, b, c \in A$ *and* $s \in S,$ *if* $a \neq b$ *and* $a \neq c,$ *then* $a \not\leq_I sbcs.$

Proof. (1) Let $a, b, c \in A,$ where $a \neq b$ and $a \neq c.$ Suppose that $a \leq_I bc.$ Then $a \in (a)_I \subseteq (bc)_I.$ Since $a \in (bc)_I = bc \cup bcbc \cup SbcS,$ by Lemma 2.9(1), we have $a = b$ or $a = c.$ This contradicts to assumption. Thus $a \not\leq_I bc.$

(2) Let $a, b, c \in A$ and $s \in S,$ where $a \neq b$ and $a \neq c.$ Suppose that $a \leq_I sbcs.$ Then $a \in (a)_I \subseteq (sbcs)_I = sbcs \cup sbcssbcs \cup SsbcsS.$ By Lemma 2.9(2), it follows that $a = b$ or $a = c.$ This contradicts to assumption. Thus $a \not\leq_I sbcs.$

Lemma 2.11. *Let* A *be an interior base of a semigroup* $S.$ *For any* $a, b \in A$ *and* $s_1, s_2 \in S,$ *if* $a \neq b,$ *then* $a \not\leq_I s_1bs_2.$

Proof. Let $a, b \in A$ and $s_1, s_2 \in S.$ Assume that $a \neq b$ and suppose that $a \leq_I s_1bs_2.$ Then $a \in (a)_I \subseteq (s_1bs_2)_I,$ and so $a \in (s_1bs_2)_I = s_1bs_2 \cup s_1bs_2s_1bs_2 \cup Ss_1bs_2S.$ Setting $B = A \setminus \{a\}.$ Then $B \subset A.$ Since $a \neq b,$ we have $b \in B.$ We will show that $(A)_I \subseteq (B)_I$. It suffices to show that $A \subseteq (B)_I.$ Let $x \in A.$ If $x \neq a,$ then $x \in B \subseteq (B)_I.$ So $x \in (B)_I.$ If $x = a,$ then by assumption, we have

$$x = a \in s_1bs_2 \cup s_1bs_2s_1bs_2 \cup Ss_1bs_2S \subseteq SBS \cup SBSSBS \cup SSBSS \subseteq SBS \subseteq (B)_I.$$

Thus $x \in (B)_I,$ and so $A \subseteq (B)_I.$ This implies $(A)_I \subseteq (B)_I.$ So $S = (A)_I \subseteq (B)_I \subseteq S,$ and hence, $S = (B)_I.$ This is a contradiction. Therefore, $a \not\leq_I s_1bs_2.$

The following theorem characterizes when a non-empty subset of a semigroup S is an interior base of $S.$

Theorem 2.12. *A non-empty subset* A *of a semigroup* S *is an interior base of* S *if and only if* A *satisfies the following conditions:*

(1) For any $x \in S,$

(1.1) there exists $a \in A$ *such that* $x \leq_I a$*; or*

(1.2) there exist $a_1, a_2 \in A$ *such that* $x \leq_I a_1a_2$*; or*

(1.3) there exist $a_3 \in A,$ $s_1, s_2 \in S$ *such that* $x \leq_I s_1a_3s_2.$

(2) For any $a, b, c \in A,$ *if* $a \neq b$ *and* $a \neq c,$ *then* $a \not\leq_I bc.$

(3) For any $a, b \in A$ *and* $s_1, s_2 \in S,$ *if* $a \neq b,$ *then* $a \not\leq_I s_1bs_2.$

Proof. Assume that A is an interior base of S. Then $S=(A)_I$. To show that (1) holds, let $x\in S$. We have $x\in(A)_I=A\cup AA\cup SAS$. There are three cases to consider:

Case 1: $x\in A$. Then $x\leq_I x$.

Case 2: $x\in AA$. Then $x=a_1a_2$ for some $a_1,a_2\in A$. This implies $(x)_I=(a_1a_2)_I$. Hence, $x\leq_I a_1a_2$.

Case 3: $x\in SAS$. Then $x=s_1a_3s_2$ for some $a_3\in A$, $s_1,s_2\in S$. We obtain $(x)_I=(s_1a_3s_2)_I$. Hence, $x\leq_I s_1a_3s_2$.

From both cases, we conclude that the condition (1) holds. The validity of (2) and (3) follow, respectively, from Lemma 2.10(1) and Lemma 2.11.

Conversely, assume that the conditions (1), (2) and (3) hold. We will show that A is an interior base of S. First, we claim that $S=(A)_I$. Clearly, $(A)_I\subseteq S$. By (1.1), we have $S\subseteq A$. From (1.2), we obtain $S\subseteq AA$. By (1.3), we have $S\subseteq SAS$. So $S\subseteq A\cup AA\cup SAS=(A)_I$. Thus $S=(A)_I$. Next, to show that A is a minimal subset of S with the property $S=(A)_I$, we suppose that $S=(B)_I$ for some $B\subset A$. Since $B\subset A$, there exists $x\in A$ such that $x\notin B$. Since $x\in A\subseteq S=(B)_I$ and $x\notin B$, we have $x\in BB\cup SBS$. There are two cases to consider:

Case 1: $x\in BB$. Then $x=a_1a_2$ for some $a_1,a_2\in B$. We have $a_1,a_2\in A$. Since $x\notin B$, so $x\neq a_1$ and $x\neq a_2$. Since $x=a_1a_2$, we obtain $(x)_I\subseteq(a_1a_2)_I$. Thus $x\leq_I a_1a_2$. This contradicts to (2).

Case 2: $x\in SBS$. Then $x=s_1a_3s_2$ for some $s_1,s_2\in S$ and $a_3\in B$. We have $a_3\in A$. Since $x\notin B$, so $x\neq a_3$. Since $x=s_1a_3s_2$, we obtain $(x)_I\subseteq(s_1a_3s_2)_I$. Thus $x\leq_I s_1a_3s_2$. This contradicts to (3).

Therefore, A is an interior base of S.

In Example 2.3, we have that $\{0,1\}$ is an interior base of S where as it is not a subsemigroup of S. The following theorem we find a condition for an interior base is a subsemigroup.

Theorem 2.13. *Let A be an interior base of a semigroup S. Then A is a subsemigroup of S if and only if for any $a,b\in A$, $ab=a$ or $ab=b$.*

Proof. Assume that A is a subsemigroup of S. Suppose that $ab\neq a$ and $ab\neq b$. Let $c=ab$. Then $c\neq a$ and $c\neq b$. Since $c=ab\in ab\cup SabS$, by Lemma 2.5, we have $c=a$ or $c=b$. This is a contradiction.

The converse statement is obvious.

3. Conclusions

In this paper, we introduce the concept of interior bases of a semigroup by using the concepts of bases and interior ideals of a semigroup. The main theorems of this paper are Theorem 2.12 and Theorem 2.13. In Theorem 2.12, we give the necessary and sufficient condition for a non-empty subset of a semigroup to be an interior base. In Theorem 2.13, we give the necessary and sufficient condition for an interior base of a semigroup to be a subsemigroup.

In the future work, we can introduce other bases of a semigroup by using concepts of bases and other ideals of a semigroup.

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