Danita Chunarom 2006: Expansions of Real Numbers in Non-Integer Bases. Master of Science (Mathematics), Major Field: Mathematics, Department of Mathematics. Thesis Advisor: Associate Professor Vichian Laohakosol, Ph.D. 36 pages.

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By a q-expansion of 1, $q \in (1,2]$, we mean a sequence $(e_i)_{i\geq 1}$ of integers in $\{0,1\}$ satisfying the equality $1 = \sum_{i=1}^{\infty} \frac{e_i}{q^i}$. There exist two particular expansions, known as the greedy and the lazy expansions. In the greedy case, we always choose the biggest possible value for e_i, while in the lazy case, we always choose the smallest possible value for e_i. The study on q-expansion of 1 started in 1990 by Erdös, Joo and Komornik who investigated the problem of uniqueness. In 1998, Komornik and Loreti determined the smallest base number q for which the q-expansion of 1 is unique. In 1999, Komornik gave conditions for sequences representing numbers having unique and having exactly two different q-expansions.

In this thesis, our overall objective is to investigate how far these results continue to hold for positive number x replacing the number 1. General results about greedy and lazy q-expansions are first derived. It is found that most results about q-expansions for real numbers greater than or equal to 1 are in somewhat opposite direction to those for real numbers less than or equal to 1. Then through the concept of U-sequences, the situation when a real number has a unique q-expansion, and when it has exactly two q-expansions are investigated. Finally, the smallest base number q yielding unique q-expansion of certain positive number is determined and a particular sequence is constructed which becomes the smallest sequence representing certain positive number with corresponding base number q having exactly two q-expansions.

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