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Numerical Simulation for Activation Energy Impact on Darcy-Forchheimer Flow of Casson Fluid Suspended with Nano Particles over a Stretching Cylinder

Deepak Sarve[1], Pradip Kumar Gaur[2,*], Vishnu Kumar Sharma[2]

[1]*Department of Mathematics, University of Mumbai, Mumbai, Maharashtra 400098, India*
[2]*Department of Mathematics, JECRC University, Jaipur, Rajasthan 303905, India*

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ABSTRACT

This present work is concentrating on the influence of energy of activation on Darcy-Forchheimer flow of Casson fluid with nano particles. The formulation of the mathematical model is done by considering Casson fluid with $TiO_2$and Cu as nanoparticles dispersed in the base Ethylene glycol. The governing equations are reduced by using standard similarity variables and boundary conditions to obtain ODE's. The numerical solution is approximated by using Rung-Kutta Fehlberg's5th order technique. The influence of numerous pertinent physical parameters on the velocity profile and the thermal profile are plotted by using BVP4C condition in MATLAB software.

Keywords: Activation energy; Casson hybrid nanofluid; Chemical reaction; Darcy-Forchheimer flow

1. Introduction

In recent years, due to growing engineering and scientific applications, then on-Newtonian fluid flows attract more attention from researchers. The major uses of non-Newtonian fluids are coolant or heat exchangers. Therefore, the vital study of the non-Newtonian fluids flow in stretching cylinders is very important and plays major role in transfer of heat and dynamics. Materials like melts, sludge, adhesives, photographic ink, paints, shampoos and tomato Sauce, exhibits non-Newtonian fluid properties. The features of non-Newtonian fluids are different from other fluids. The equations which govern the flow of Newtonian fluids are less non-linear when compared to non- Newtonian fluids. Casson [1] to discuss the flow behavior of suspensions of pigments in oil

***Corresponding author:** pradeep.gaur@jecrcu.edu.in

and this is the initial article of Casson fluid. The discussion of gyrating Casson liquid flow with radiation effect was given by Archana et al. [2]. Kumar et al. [3] scrutinized the mixed convective flow of Casson liquid via vertical plate with the impact of non-linear radiation. Sadiq and Hayat [4] examined the Casson with fluid flow with the impact of ohmic heating. The Casson fluid flow of various fluids with several influencing effects through different surfaces was deliberated by some researchers [5-7].

The estimation of fluid flow and heat transmission through a cylinder has been the focus of significant research interests because of its various industrial applications. Chamkha and Selimefendigil, [8] explained the convective flow of a nanofluid via cylinder with rotation in three dimensions. Ramesha et .al [9] explained the hybrid nanofluid flow via cylinder with the influence of homogeneous and heterogeneous chemical reactions. Aminian et .al [10] studied the effect of magnetic field for the forced convective flow of nanofluids through a cylinder. The impact of viscous dissipation and ohmic heating through a stretching cylinder for the flow of nanofluid is presented by Mishra et al. [11].

Wide-ranging surveys are performed on Darcy-Forchheimer flow and transfer of heat over porous media. Kumar and Chamka [12] discussed the heat transference in Darcy-Forchheimer flow of nano fluid with particle shape impact. Reddy et al. [13] scrutinized the heat transfer phenomenon in Darcy-Forchheimer flow of fluid. Ramzan et .al [14] scrutinized the Darcy flow on a motive thin needle with inhomogeneous heat source/sink with chemical responses.

The minimal quantity of energy which is essential to initiate a chemical response is known as energy of activation. Scientist S. Arrhenius from Swedish country gave the term energy of activation. Khan et al. [15] used nanofluid to explore the influence of energy of activation on fluid flow through different surfaces with other influencing effects. Mabood et al. [16] studied the influence of activation energy on micropolar liquid via thin needle on taking account of dyadic chemical response. Khan et al. [17] exposed the influence of energy of activation in convective flow of nanofluid with gyrotactic microorganisms definition. The heat transfer intensification with activation energy of Maxwell fluid in two dimensions was examined by Ijaz and Ayub [18]. Kumar [19] suggests a new analytical modelling for fractional telegraph equation via Laplace transform. Goufo et al. [20] studied Similarities in a fifth-order evolution equation with and with no singular kernel. Kumar and Rashidi [21] have developed a new analytical method for gas dynamics equation arising in shock fronts. Kumar et al. [22] studied heat equations arises in diffusion process using new Yang-Abdel-Aty-Cattani fractional operator

This paper fundamentally pivots on the emphasis of energy of activation in Darcy-Forchheimer flow of Casson nano fluid through cylinder placed in porous medium.

2. Mathematical Formulation

Consider Darcy-Forchheimer flow of casson fluid with suspended Titanium dioxide nano with ethylene glycol as base fluid over stretched cylinder on taking account of energy of activation. Energy activation, to stimulate chemical reaction, under this circumstance energy, momentum, thermal and concentration equations which govern the flow are stated as follows:

$$\frac{\partial(ru)}{\partial(x)}+\frac{\partial(rv)}{\partial(r)}=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{\mu_{hnf}}{\rho_{hnf}}\left(1+\frac{1}{\beta *}\right)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-\frac{\mu_{hnf}}{\rho_{hnf}K *}u-\frac{C_b}{\sqrt{K *}}u^2, \tag{2.2}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=\frac{k_{hnf}}{\left(\rho C_p\right)_{hnf}}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right), \tag{2.3}$$

$$\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial r}\right)=D_{hnf}\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right),- k_r^2\left(\frac{T}{T_\infty}\right)e^{\frac{-E_a}{KT}}\left(C-C_\infty\right). \tag{2.4}$$

The corresponding boundary conditions are at $r=a$; $u=cx, v=0, T=T_w, C=C_w$,

at $r\to\infty$; $u\to 0, T\to T_\infty, C=C_\infty$. (2.5)

To change the governing equations of flow problem into dimensionless form, the following similarity variables are used:

$$\eta=\frac{r^2-a^2}{2a}\sqrt{\frac{c}{\upsilon_f}},\ u=cx\ \mathrm{f}'(\eta),$$

$$v=-\frac{a}{r}\sqrt{c\upsilon_{\mathrm{f}}},\mathrm{f}(\eta),\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$$

$$\chi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}.$$

By using above similarity transformations Eqs. (2.1)-(2.5) reduced to fallowing ODE's

$$\varepsilon_1\left(\left(1+\frac{1}{\beta *}\right)\left(\left(2\eta\omega+1\right)\mathrm{f}'''+2\omega\mathrm{f}''\right)-\lambda\mathrm{f}'\right)- \left(\left(\mathrm{f}'\right)^2-\mathrm{f}\,\mathrm{f}'+\mathrm{F}_r\left(\mathrm{f}'\right)^2\right)=0, \tag{2.6}$$

$$\frac{k_{hnf}}{k_f}\frac{1}{\mathrm{Pr}}\varepsilon_2\left(\left(2\eta\omega+1\right)\theta''+2\omega\theta'\right)+\mathrm{f}\,\theta'=0, \tag{2.7}$$

$$\left(1-\Phi_2\right)^{2.5}\left(\left(2\eta\omega+1\right)\chi''+2\omega\chi'\right)+ \mathrm{f}_{\chi'}-Sc\sigma\left(1+\delta\theta\right)^n\exp\left(-\frac{E}{\left(1+\delta\theta\right)}\right)\chi=0, \tag{2.8}$$

where

$$\varepsilon_1=\frac{1}{\left(1-\Phi_1\right)^{2.5}\left(1-\Phi_2\right)^{2.5}\left(\left(1-\Phi_2\right)\left\{\left(1-\Phi_1\right)+\Phi_1\left(\frac{\rho_{s_1}}{\rho_{\mathrm{f}}}\right)\right\}+\Phi_2\left(\frac{\rho_{s_2}}{\rho_{\mathrm{f}}}\right)\right)},$$

$$\varepsilon_2=\frac{1}{\left(1-\Phi_2\right)\left\{\left(1-\Phi_1\right)+\Phi_1\left(\frac{\left(\rho C_p\right)_{s_1}}{\left(\rho C_p\right)_{\mathrm{f}}}\right)\right\}\left(\Phi_2\left(\frac{\left(\rho C_p\right)_{s_2}}{\left(\rho C_p\right)_{\mathrm{f}}}\right)\right)}.$$

Similarly, the reduced boundary conditions are

$$\mathrm{f}'(0)=1,\mathrm{f}(0)=0,\theta(0)=1,\chi(0)=1,$$
$$\mathrm{f}'(\infty)\to 0,\theta(\infty)\to 0,\chi(\infty)\to 0, \tag{2.9}$$

where $\lambda=\frac{\upsilon_{\mathrm{f}}}{K*c},F_r=\frac{C_b x}{\sqrt{K*}},\mathrm{Pr}=\frac{\left(\rho C_p\right)_{\mathrm{f}}\upsilon_{\mathrm{f}}}{k_{\mathrm{f}}},$

$$Sc=\frac{\upsilon_{\mathrm{f}}}{D_A},\sigma=\frac{k_r^2}{\Omega},E=\frac{E_a}{KT_\infty},\delta=\frac{T_w-T_\infty}{k_{\mathrm{f}}}.$$

2.1 Thermophysical properties of hybrid nanofluids are given by

$$\mu_{hnf}=\frac{\mu_{\mathrm{f}}}{\left(1-\Phi_1\right)^{2.5}\left(1-\Phi_2\right)^{2.5}},$$

$$D_{hnf}=D_{\mathrm{f}}\left(1-\Phi_1\right)^{2.5}\left(1-\Phi_2\right)^{2.5},$$

$$\rho_{hnf}=\left(1-\Phi_2\right)\left\{\left(1-\Phi_1\right)\rho_{\mathrm{f}}+\Phi_1\rho_{s_1}\right\}+\Phi_2\rho_{s_2},$$

$$\left(\rho C_p\right)_{hnf}=\left(1-\Phi_2\right)\left\{\left(1-\Phi_1\right)\left(\rho C_p\right)_{\mathrm{f}}+\Phi_1\left(\rho C_p\right)_{s_1}\right\}+ \Phi_2\left(\rho C_p\right)_{s_2},$$

$$\frac{k_{hnf}}{k_{bf}}=\frac{k_{s_2}+2k_{bf}-2\Phi_2\left(k_{bf}-k_{s_2}\right)}{k_{s_2}+2k_{bf}+\Phi_2\left(k_{bf}-k_{s_2}\right)},$$

$$\frac{k_{bf}}{k_{\mathrm{f}}}=\frac{k_{s_1}+2k_{bf}-2\Phi_1\left(k_{\mathrm{f}}-k_{s_1}\right)}{k_{s_1}+2k_{bf}+\Phi_2\left(k_{\mathrm{f}}-k_{s_1}\right)}.$$

Non-dimensional variables are obtained are:

$$Re^{-\frac{1}{2}}Sh=-\chi'(0),Re^{-\frac{1}{2}}Nu=Re^{\frac{1}{2}}C_{\mathrm{f}}=-\frac{k_{hnf}}{k_{bf}}\theta'(0),$$

$$\left(1+\frac{1}{\beta}\right)\frac{f''(0)}{\left(1-\Phi_1\right)^{2.5}\left(1-\Phi_2\right)^{2.5}}.$$

where $Re=\frac{\Omega r^2}{\upsilon_{\mathrm{f}}}$.

3. Result and Discussion

In this part, we elucidate the features of diverse values of physical parameters on velocity, thermal, concentration, rate of heat and mass transfer in graphical form. To clearly check the insight of proposed model, the results, behavior and the numerical calculations are restricted thoroughly for numerous values of physical parameters such as, Darcy-Forchheimer, porous, Casson parameters, Schmidt number, Reaction rate, Non-dimensional activation energy. We have considered Casson hybrid nanofluid flow brought by cylinder containing $Cu - TiO2/EG$.

Table 1. Thermo physical characteristics in base liquid and nanoparticles**.**

Thermo-Physical Properties	EG	Cu	TiO_2
ρ	1114	8933	4250
C_p	2415	385	686.2
k	0.252	400	8.9538
Pr	6.2		

Fig. 1 and Fig. 2 illustrate the impact of Forchheimer parameter over v and thermal gradients. Fig.1. is depicted for velocity profile over η for numerous values of F_r. It is noted that the increase in parameter declines the velocity of the fluid flow. Fig. 2. Reveals that for enhance in values of the parameter enhances the thermal gradients and its boundary thickness associated with it. Resistive forces are created within the shoot up values of Forchheimer parameter which results in decline of fluid motion and hence velocity reduces and this drag force rise the temperature of the fluid which obviously inclines the thermal gradients. Figs. 3, 4 and 5 portray the influence of porous parameter over velocity, thermal and concentration gradients. Increase in value of λ decreases the velocity of the fluid which automatically increases the temperature and concentration gradients. Due to the existence of porosity in the medium resistance in the fluid motion is generated which obviously declines the velocity and enhances the thermal and concentration profiles and boundary layer thickness associated with it. Fig. 6 scrutinize the importance of Prandtl number subjected to thermal profile. It is noticed that with increase in Prandtl Number thermal gradient and its boundary layer thickness decays gradually. Figs.7, 8 and 9 exemplify the behavior of concentration gradient over various parameters such as chemical Schmidt number, reaction rate, and activation energy, respectively. With the rise in values of Sc concentration gradient decays the concentration of the fluid and its boundary thickness associated with it. Sc is the ratio of viscosity and mass diffusivity of the fluid. With the shoot up values of Sc which decreases the mass diffusivity and increases the viscosity of the fluid, due to the reason the concentration profile decays (see Fig. 7). Fig. 8 illustrates the influence of chemical reaction rate parameter over concentration gradient. Close observation is made

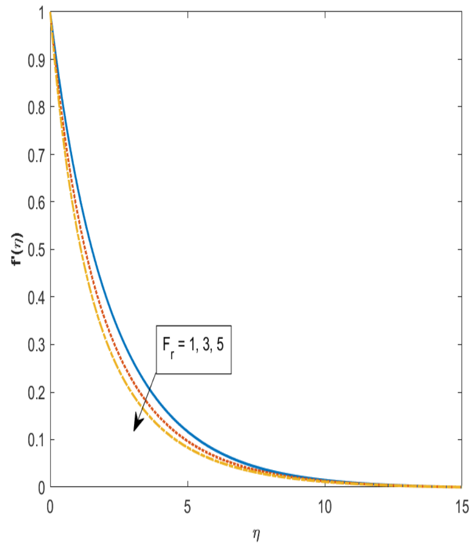

Fig. 1. Impact of F_r on f' versus η.

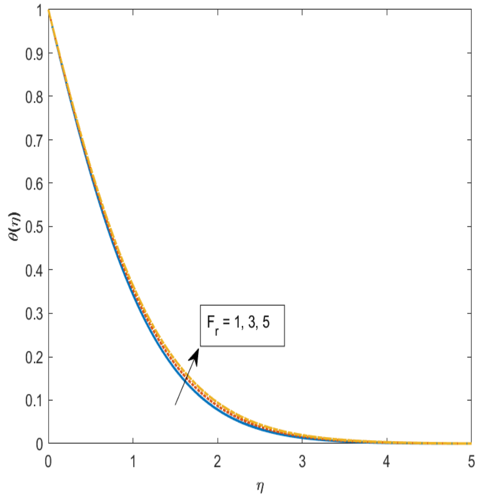

Fig. 2. Impact of F_r on θ versus η.

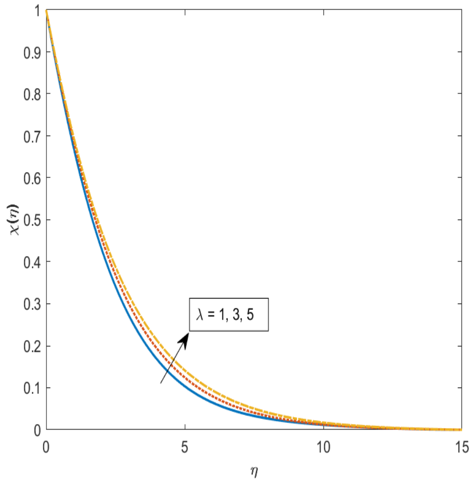

Fig. 5. Influence of λ on χ versus.

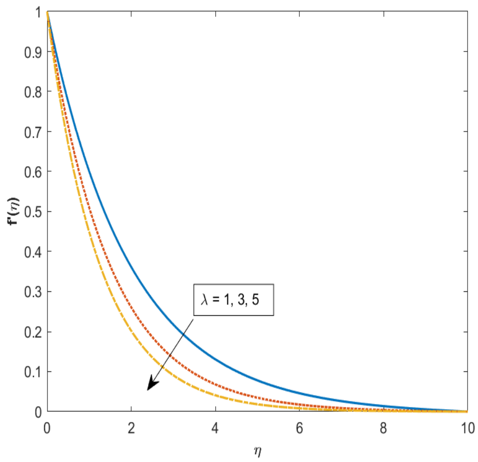

Fig. 3. Influence of λ on f versus.

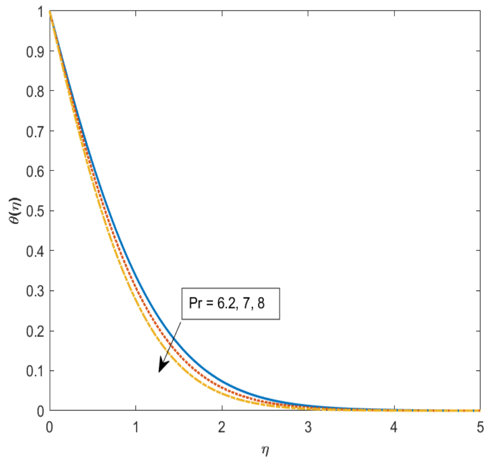

Fig. 6. Variation in thermal profile for different values of $\Pr$.

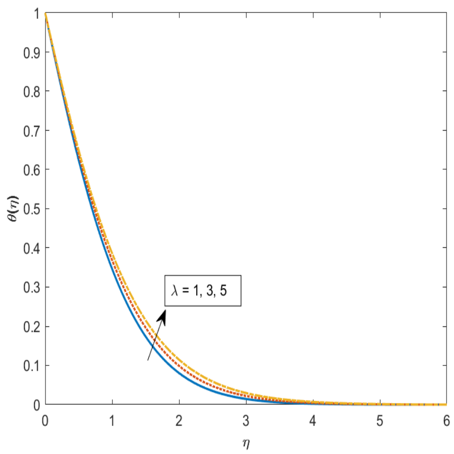

Fig. 4. Impact of F_r on θ versus η.

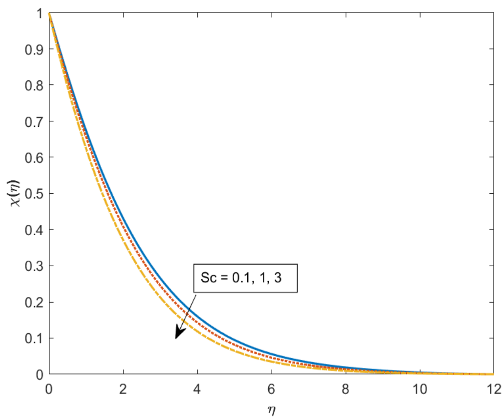

Fig. 7. Variation in Concentration profile for different values of Sc.

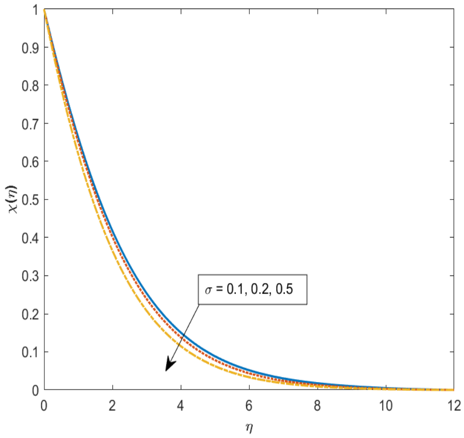

Fig. 8. χ versus η for several values of σ.

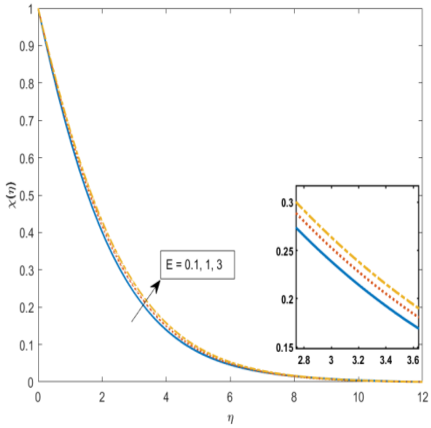

Fig. 9. χ versus η for several values of E.

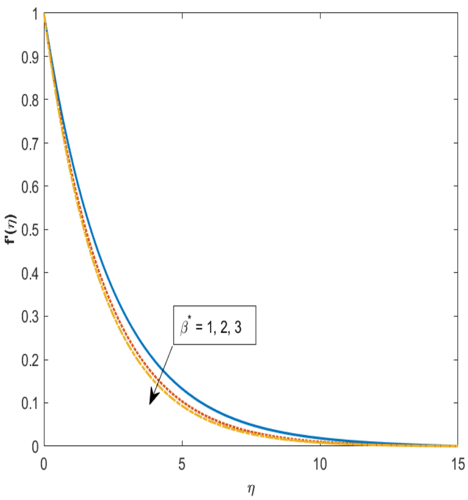

Fig. 10. Influence of casson parameter on f'

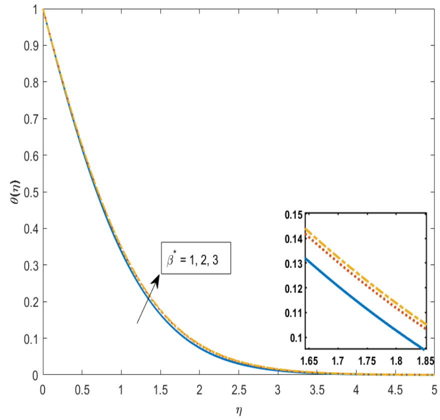

Fig. 11. Influence of casson parameter on θ.

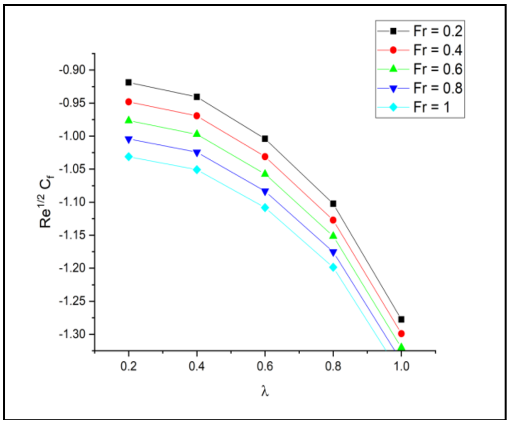

Fig. 12. Influence of porosity parameter on $Re^{\frac{1}{2}}C_f$.

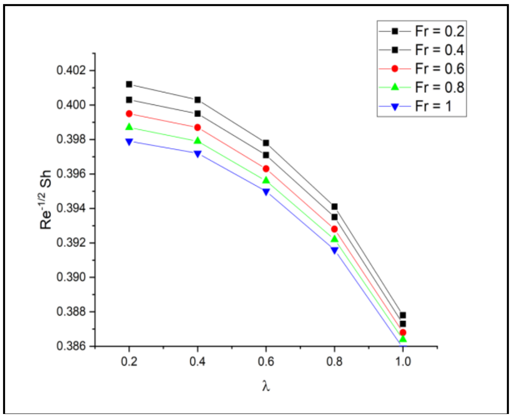

Fig. 13. Influence of porosity parameter on $Re^{-\frac{1}{2}}Sh$.

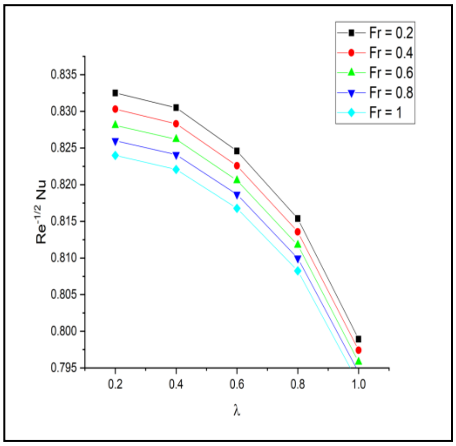

Fig. 14. Influence of porosity parameter on $Re^{-\frac{1}{2}}Nu$.

4. Conclusions

In this study, the flow regime features of Casson fluid are reported. The Casson hybrid nanofluid containing Cu and T_iO_2 is equipped above. By using the suitable similarity variables, the PDE's which governing the flow are reduced into non-linear ODE's and obtained a numerical solution by shooting techniques with RKF-45 order scheme. The results are analyzed for dimensionless velocity, thermal and concentration gradients for various parameters and are presented in the form of graphs. The C_f, Nu and Sh are also presented in the graphical form. From these we concluded that

- Enhance in Forchheimer parameter declines the velocity profile and inclines the thermal gradients.
- Shoot up values of activation energy increases the rate of growth of concentration profile gradually.
- Increase in value of Schmidt number decreases the concentration profile.
- Rise in porosity parameter slowdowns the velocity of the fluid and increases the thermal concentration gradients and boundary layer thickness associated with it.

References

[1] N. Casson, A flow equation for pigment oil suspensions of the printing ink type. In: Rheology of Sisperse Systems (C.C. Mill Ed.) Pergamon Press, Oxford, 1959, 84-102.

[2] M. Archana, B. J. Gireesha, B. C. Prasannakumaraand R.S.R. Gorla, Influence of nonlinear thermal radiation on rotating flow of Casson nanofluid, 7(2), (2017), 91-101.

[3] K.G. Kumar, M. Archana, B.J. Gireesha, M.R. Krishanamurthy and N.G. Rudraswamy,Cross diffusion effect on MHD mixed convection flow of nonlinear radiative heat and mass transfer of Casson fluid over a vertical plate. Results in Physics, 8, (2018), 694-701.

[4] M.A. Sadiq and T. Hayat, Characterization of Marangoni Forced Convection in CassonNanoliquid Flow with Joule Heating and Irreversibility. Entropy, 22(4), (2020), 433.

[5] K.G. Kumar, B.J. Gireesha, B. C. Prasannakumara, G.K. Ramesh and O.D. Makinde, Phenomenon of Radiation and Viscous Dissipation on CassonNanoliquid Flow Past a Moving Melting Surface. Diffusion Foundations, 11, (2017), 33-42.

[6] M. Archana, B. J. Gireesha, B. C. Prasannakumara,Triple diffusive flow of Casson nanofluid with buoyancy forces and nonlinear thermal radiation over a horizontal plate, Archives of Thermodynamics, 40(1), (2019), 49-69.

[7] B.J. Gireesha B.M. Shankaralingappa, B.C. Prasannakumarand B. Nagaraja, MHD flow and melting heat transfer of dusty Casson fluid over a stretching sheet with CattaneoChristov heat flux model, International Journal of Ambient Energy, (2020), 1-22.

[8] A.J. Chamkhaand F. Selimefendigil, MHD mixed convection of nanofluid due to an inner rotating cylinder in a 3D enclosure with a phase change material, International Journal of Numerical Methods for Heat & Fluid Flow, 29(10), (2019), 559-3583.

[9] Ramesh, G.K., Manjunatha, S., Roopa, G.S. and Ali J. Chamkha Hybrid (ND-Co3O4/EG) nanoliquid through a permeable cylinder under homogeneous-heterogeneous reactions and slip effects. J Therm Anal Calorim, (2020).

[10] E. Aminian, H. Moghadasi, and H. Saffari, Magnetic field effects on forced convection flow of a hybrid nanofluid in a cylinder filled with porous media: a numerical study, J. Therm. Anal. Calorim., (2020), 0123456789.

[11] A. Mishra and M. Kumar,Velocity and thermal slip effects on MHD nanofluid flow past a stretching cylinder with viscous dissipation and Joule heating,SN Appl. Sci., 2(8), 2020,

[12] K.G. Kumar and A.J. Chamkha, Darcy-Forchheimer flow and heat transfer of water-based Cu nanoparticles in convergent/divergent channel subjected to particle shape effect. The European Physical Journal Plus, 134(3), (2019), 614-22.

[13] M.G. Reddy, M.V.V.N.L. Sudharani, K.G. Kumar,A.J. Chamkha and G. Lorenzini, Physical aspects of Darcy–Forchheimer flow and dissipative heat transfer of Reiner–Philippoff fluid. J Therm Anal Calorim 141, 829-38 (2020).

[14] M. Ramzan, N. Shaheen, S. Kadry, Y. Ratha, Y. Nam, Thermally Stratified Darcy Forchheimer Flow on a Moving Thin Needle with Homogeneous Heterogeneous Reactions and Non-Uniform Heat Source/Sink, Applied Sciences, 10(2), (2020), 432.

[15] M.I. Khan, T. Hayat, M.I. Khan and A. Alsaedi, Activation energy impact in nonlinear radiative stagnation point flow of Cross nanofluid. International Communications in Heat and Mass Transfer, 91, (2018), 216-24.

[16] F. Mabood, M.K. Nayak and A.J. Chamkha, Heat transfer on the cross flow of micropolar fluids over a thin needle moving in a parallel stream influenced by binary chemical reaction and Arrhenius activation energy. The European Physical Journal Plus, 134(9), (2019), 427

[17] M.I. Khan, F. Haq, S.A. Khan, T. Hayatand M.I. Khan, Development of thixotropic nanomaterial in fluid flow with gyrotactic microorganisms, activation energy, mixed convection. Computer Methods and Programs in Biomedicine, 187, (2019), 105186.

[18] M. Ijaz and M. Ayub, Nonlinear convective stratified flow of Maxwell nanofluid with activation energy, (2019). Heliyon 5, e01121.

[19] Sunil Kumar, A new analytical modelling for fractional telegraph equation via Laplace transform,Applied Mathematical Modelling, 38, 13, (2014), 3154-63.

[20] Emile, F. Doungmo Goufo Sunil Kumar, and S.B.Mugisha, Similarities in a fifth-order evolution equation with and with no singular kernel, Chaos, Solitons & Fractals, 130, (2020), 109467.

[21] Sunil Kumar, and Mohammad Mehdi Rashidi, New analytical method for gas dynamics equation arising in shock fronts, Computer Physics Communications, 185, 7, (2014), 1947-54.

[22] Sunil Kumar, Surat Ghosh, Bessem and Emile Franc Doungmo Goufo, An analysis for heat equations arises in diffusion process using new Yang-Abdel-Aty-Cattani fractional operator, Mathematical methods in the applied Science, (2020).

Appendix

(u,v)	Constituents of Velocity
(x,r)	Axial and radial Directions
$f'(\eta)$	Dimensionless Fluid velocity
θ	Non-dimensional fluid temperature
χ	Dimensionless fluid concentration
μ	Dynamic viscosity
ρ	Density
k	Thermal conductivity
C_p	Drag co-efficient
(ρC_p)	Heat capacitance
T	Temperature
$K*$	Porous medium permeability
F_r	Forchheimer parameter
δ	Temperature difference
$\beta*$	Casson parameter
k_r	Chemical reaction rate constant
n	Fitted rate constant
Φ_1, Φ_2	Solid volume fractions of nano particles
E	Non-dimensional activation energy
D	Diffusion co-efficient
K	Boltzmann's constant
C	Specific heat
Re	Local Reynolds number
λ	Porous parameter
Nu	Nusselt number
Sc	Schmid number
Sh	Sherwood number
K	Stokes drag coefficient
C_f	Skin friction coefficient
σ	Reaction rate
N	Kinematic viscosity

Subscripts

f	Fluid
bf	Base fluid
hnf	Hybrid nanofluid
s_1, s_2	Solid particle
∞	Ambient
w	Wall/surface
nf	Nanofluid