New Expansion Scheme to Solitary Wave Solutions for a Model of Wave-Wave Interactions in Plasma

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ABSTRACT

In this article, a new expansion technique is executed for travelling wave solutions of Generalized Zakharov System (GZS) as a model of wave-wave interaction in plasma. New Solitary and periodic wave solutions with arbitrary parameters are obtained and expressed in terms of the hyperbolic, trigonometric, and rational functions. Some of the acquisition solutions are graphically sketched.

Keywords: Generalized Zakharov System (GZS); Periodic wave; Solitary wave solution; solution; Travelling wave solution

1. Introduction

Nonlinearity is a mesmerizing component of nature, Almost, in all the scientific and engineering fields nonlinear wave phenomena appears in one or other ways. There are of more practical use of it if one can find the travelling wave solutions for nonlinear evolution equations (NLEEs). The wave-wave interactions can also be explained by these solutions. The model of wave-wave interaction in plasma may be written as [1-3]

$$iE_t + E_{xx} - 2\beta |E|^2 E + 2\upsilon E = 0,$$

$$\upsilon_{tt} - \upsilon_{xx} + |E|^2_{xx} = 0, \qquad (1.1)$$

where, $E(x,t)$ and $\upsilon(x,t)$ are the envelope of the high-frequency electric field and plasma density measured from its equilibrium value respectively. Eq. (1.1) reduced to the classical Zakharov equations [4, 5] of plasma physics whenever $\beta = 0$. In the past three decades, several methods explored which includes-the homogeneous balance method [6], Tanh-coth scheme [5, 7], sine-cosine method [8] and so on [9-21]. Here in this article the study involving to the New Expansion Scheme for solving the generalized Zakharov system (GZS) is presented. Solitary wave propagation is also discussed to explain the wave interactions phenomenon arising in NLEEs.

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2. Explanation of Scheme

Let us consider nonlinear partial differential equation (PDE) as

$$\Phi(v, v_t, v_x, v_{xx}, v_{tt}, v_{tx}, ...) = 0. \quad (2.1)$$

This equation involving Φ is a polynomial of unknown function $v(x,t)$ with its derivatives and nonlinear terms.

Step 1: We introduce a variable $\eta = x \pm V t$,

$$v(x,t) = v(\eta), \quad (2.2)$$

where travelling wave speed is V. So, Eq. (2.1) will be

$$\Psi(v, v', v'', v''', ...) = 0, \quad (2.3)$$

where Ψ is a polynomial of unknown function, its derivatives with respect to η and nonlinear terms.

Step 2: let the solution of Eq. (2.3) may be written as :

$$v(\eta) = \sum_{i=0}^{N} \alpha_i (d + M)^i + \sum_{i=1}^{N} \beta_i (d + M)^{-i}, \quad (2.4)$$

where $\alpha_i (i = 0, 1, 2, ..., N),\ d$ and $\beta_i (i = 1, 2, ..., N)$ are constant may be determined and $M(\eta)$ is as

$$M(\eta) = (G'/G), \quad (2.5)$$

where $G = G(\eta)$ solution of auxiliary nonlinear equation:

$$AGG'' - BGG' - EG^2 - C(G')^2 = 0, \quad (2.6)$$

where A, B, C and E are parameters will be determined.

The General solution of Eq. (2.6) put in Eq. (2.4) with the value of N find by homogeneous balance between nonlinear term and highest order derivative term in Eq. (2.3), so we get solutions of Eq. (2.3) and we may derive the value of $M(\eta)$ for Eq. (2.5):

Family 1: When $B \neq 0$, $\omega = A - C$ and $\Omega = B^2 + 4E(A - C) > 0$,

$$M(\eta) = \frac{B}{2\omega} + \frac{\sqrt{\Omega}}{2\omega} \frac{C_1 \sinh\left(\frac{\sqrt{\Omega}}{2A}\eta\right) + C_2 \cosh\left(\frac{\sqrt{\Omega}}{2A}\eta\right)}{C_1 \cosh\left(\frac{\sqrt{\Omega}}{2A}\eta\right) + C_2 \sinh\left(\frac{\sqrt{\Omega}}{2A}\eta\right)}. \quad (2.7)$$

Family 2: When $B \neq 0$, $A - C = \omega$ and $B^2 + 4E(A - C) = \Omega < 0$,

$$M(\eta) = \frac{B}{2\omega} + \frac{\sqrt{-\Omega}}{2\omega} \frac{-C_1 \sin\left(\frac{\sqrt{-\Omega}}{2A}\eta\right) + C_2 \cosh\left(\frac{\sqrt{-\Omega}}{2A}\eta\right)}{C_1 \cos\left(\frac{\sqrt{-\Omega}}{2A}\eta\right) + C_2 \sinh\left(\frac{\sqrt{-\Omega}}{2A}\eta\right)}. \quad (2.8)$$

Family 3: $B \neq 0$, $A - C = \omega$ and $B^2 + 4E(A - C) = \Omega = 0$,

$$M(\eta) = \frac{B}{2\omega} + \frac{C_2}{C_1 + C_2\eta}. \quad (2.9)$$

Family 4: $B = 0$, $A - C = \omega$ and $\Delta = \omega E > 0$,

$$M(\eta) = \frac{\sqrt{\Delta}}{\omega} \frac{C_1 \sinh(\frac{\sqrt{\Delta}}{A}\eta) + C_2 \cosh(\frac{\sqrt{\Delta}}{A}\eta)}{C_1 \cosh(\frac{\sqrt{\Delta}}{A}\eta) + C_2 \sinh(\frac{\sqrt{\Delta}}{A}\eta)}. \quad (2.10)$$

Family 5: When $B = 0$, $A - C = \omega$ and $\Delta = \omega E < 0$,

$$M(\eta) = \frac{\sqrt{-\Delta}}{\omega} \frac{C_1 \sin(\frac{\sqrt{-\Delta}}{A}\eta) + C_2 \cos(\frac{\sqrt{\Delta}}{A}\eta)}{C_1 \cos(\frac{\sqrt{-\Delta}}{A}\eta) + C_2 \sin(\frac{\sqrt{-\Delta}}{A}\eta)}. \quad (2.11)$$

3. Application of Scheme to Model of Wave-Wave Interactions in Plasma

Let for Eq. (1.1)

$$E(t,x) = u(\eta)e^{i\theta}, \quad \upsilon(t,x) = \psi(\eta),$$

$$\theta = kx + qt, \ \eta = (x - 2kt)\, p.$$

We get

$$u(k^2 + q) - p^2 u'' + 2\beta u^3 - 2u\psi = 0, \ (3.1)$$
$$(4k^2 - 1)\psi'' + u'' = 0. \qquad (3.2)$$

Twice time integral of Eq. (3.2) with first integration constant zero, we get

$$\psi(\eta) = \frac{c - u^2}{4k^2 - 1}. \qquad (3.3)$$

Its use in eq. (3.1), we get

$$au - p^2 u'' + 2bu^3 = 0, \qquad (3.4)$$

where

$$a = \left(k^2 + q - \frac{2c}{4k^2 - 1} \right), \quad b = \left(\beta + \frac{1}{4k^2 - 1} \right),$$

the value of N by homogeneous balance between nonlinear term u^3 and highest order derivative term u'' in Eq. (3.4) such that $3N = N + 2$ i.e., $N = 1$. So, we get from Eq. (2.4):

$$u(\eta) = \alpha_0 + \alpha_1 (d + M) + \beta_1 (d + M)^{-1} \qquad (3.5)$$

It put with Eqs. (2.5) and (2.6) into Eq. (3.4), we yields a set of simultaneous algebraic equations, if we equate each

coefficient of resultant polynomial to zero. Solve these algebraic equations and got three set of solutions:

Set 1:

$$\alpha_0 = \frac{Aa - 4bhd}{2Ab}, \quad \alpha_1 = 0, \ d = d,$$

$$\beta_1 = -\frac{p^2}{bA}(d^2\omega + Bd - E), \qquad (3.6)$$

where $A - C = \omega, \ h = -\dfrac{p^2}{2b}$ other parameters are unrestricted.

Set 2:

$$\alpha_0 = \frac{1}{2b} - \frac{hB + 2hd\omega}{A},$$

$$\beta_1 = 0, \alpha_1 = \frac{2h\omega}{A}, d = d. \qquad (3.7)$$

where $A - C = \omega, \ h = -\dfrac{p^2}{2b}$ other parameters are unrestricted.

Set 3:

$$\alpha_0 = \frac{a}{2b}, \ \alpha_1 = \frac{p^2\omega}{bA}, \ d = -\frac{B}{2\omega},$$

$$\beta_1 = \frac{p^2}{2bA\omega}(4E\omega + B^2), \qquad (3.8)$$

where $A - C = \omega,$ other parameters are unrestricted.

Solution for set 1 with Family-1

If $C_1 = 0$ but $C_2 \neq 0,$

$$u_{1_1}(\eta) = \frac{Aa - 4bhd}{2Ab} - \frac{p^2}{bA}(d^2\omega + Bd - E)\left(d + \frac{B}{2\omega} + \frac{\sqrt{\Omega}}{2\omega}\coth(\frac{\sqrt{\Omega}}{2A}\eta) \right)^{-1}.$$

If $C_2 = 0$ but $C_1 \neq 0,$

$$u_{1_2}(\eta) = \frac{Aa - 4bhd}{2Ab} - \frac{p^2}{bA}(d^2\omega + Bd - E)\left(d + \frac{B}{2\omega} + \frac{\sqrt{\Omega}}{2\omega}\tanh(\frac{\sqrt{\Omega}}{2A}\eta) \right)^{-1}.$$

Solution for set 1 with Family-2
If $C_1 = 0$ but $C_2 \neq 0,$

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$$u_{1_3}(\eta) = \frac{Aa - 4bhd}{2Ab} - \frac{p^2}{bA}(d^2\omega + Bd - E) \times \left(d + \frac{B}{2\omega} + \frac{\sqrt{-\Omega}}{2\omega}\coth(\frac{\sqrt{-\Omega}}{2A}\eta) \right)^{-1}.$$

If $C_2 = 0$ but $C_1 \neq 0$,

$$u_{1_4}(\eta) = \frac{Aa - 4bhd}{2Ab} - \frac{p^2}{bA}(d^2\omega + Bd - E) \times \left(d + \frac{B}{2\omega} + \frac{\sqrt{-\Omega}}{2\omega}\tan(\frac{\sqrt{\Omega}}{2A}\eta) \right)^{-1}.$$

Solution for set 1 with Family-3

$$u_{1_5}(\eta) = \frac{Aa - 4bhd}{2Ab} - \frac{p^2}{bA}(d^2\omega + Bd - E) \left(d + \frac{B}{2\omega} + \frac{C_2}{C_1 + C_2\eta} \right)^{-1}.$$

Solution For set 1 with Family-4
If $C_1 = 0$ but $C_2 \neq 0$;

$$u_{1_6}(\eta) = \frac{Aa - 4bhd}{2Ab} - \frac{p^2}{bA}(d^2\omega + Bd - E) \times (d + \frac{\sqrt{\Delta}}{\omega}\coth(\frac{\sqrt{\Delta}}{A}\eta))^{-1}.$$

If $C_2 = 0$ but $C_1 \neq 0$

$$u_{1_7}(\eta) = \frac{Aa - 4bhd}{2Ab} - \frac{p^2}{bA}(d^2\omega + Bd - E) \times (d + \frac{\sqrt{\Delta}}{\omega}\tanh(\frac{\sqrt{\Delta}}{A}\eta))^{-1}.$$

Solution For set 1 with Family-5
If $C_1 \neq 0, C_2 = 0$,

$$u_{1_8}(\eta) = \frac{Aa - 4bhd}{2Ab} - \frac{p^2}{bA}(d^2\omega + Bd - E) \times (d + \frac{\sqrt{-\Delta}}{\omega}\cot(\frac{\sqrt{-\Delta}}{A}\eta))^{-1},$$

$$u_{1_9}(\eta) = \frac{Aa - 4bhd}{2Ab} - \frac{p^2}{bA}(d^2\omega + Bd - E) \times (d - \frac{\sqrt{-\Delta}}{\omega}\tan(\frac{\sqrt{-\Delta}}{A}\eta))^{-1}.$$

Similarly, Solution For set 2 with Family-1

If $C_1 = 0$ but $C_2 \neq 0$, $u_{2_1}(\eta) = \frac{1}{2b} + \frac{1}{A}h\sqrt{\Omega}\coth(\frac{\sqrt{\Omega}}{2A}\eta).$

If $C_2 = 0$ but $C_1 \neq 0$, $u_{2_2}(\eta) = \frac{1}{2b} + \frac{1}{A}h\sqrt{\Omega}\tanh(\frac{\sqrt{\Omega}}{2A}\eta)).$

Solution For set 2 with Family-2

If $C_1 = 0$ but $C_2 \neq 0$, $u_{2_3}(\eta) = \frac{1}{2b} + \frac{1}{A}h\sqrt{\Omega}\coth(\frac{\sqrt{-\Omega}}{2A}\eta)).$

If $C_2 = 0$ but $C_1 \neq 0$, $u_{2_4}(\eta) = \frac{1}{2b} + \frac{1}{A}h\sqrt{\Omega}\tan(\frac{\sqrt{-\Omega}}{2A}\eta).$

Solution For set 2 with Family-3

$$u_{2_5}(\eta) = \frac{1}{2b} - \frac{2h\omega}{A}\left(\frac{C_2}{C_1 + C_2\eta}\right).$$

Solution For set 2 with Family-4

If $C_1 = 0$ but $C_2 \neq 0$, $u_{2_6}(\eta) = \frac{1}{2b} - \frac{h}{A}\left(B - 2\sqrt{\Delta}\coth(\frac{\sqrt{\Delta}}{A}n)\right)$.

If $C_2 = 0$ but $C_1 \neq 0$, $w_{2_7}(\eta) = \frac{1}{2b} - \frac{h}{A}\left(B - 2\sqrt{\Delta}\tanh(\frac{\sqrt{\Delta}}{A}n)\right)$.

Solution For set 2 with Family-5

If $C_1 = 0$ but $C_2 \neq 0$, $u_{2_8}(\eta) = \frac{1}{2b} - \frac{h}{A}\left(B - 2i\sqrt{\Delta}\cot(\frac{\sqrt{-\Delta}}{A}n)\right)$.

If $C_2 = 0$ but $C_1 \neq 0$, $u_{2_9}(\eta) = \frac{1}{2b} - \frac{h}{A}\left(B - 2i\sqrt{\Delta}\tan(\frac{\sqrt{-\Delta}}{A}n)\right)$.

Similarly, we can find the **Solution For set 3 with Family-1**
If $C_1 = 0$ but $C_2 \neq 0$,

$$u_{3_1}(\eta) = \frac{a}{2b} + \frac{p^2\sqrt{\Omega}}{2bA} \times \coth(\frac{\sqrt{\Omega}}{2A}\eta) + \frac{p^2}{bA\sqrt{\Omega}}(4E\omega + B^2) \times \coth(\frac{\sqrt{\Omega}}{2A}\eta).$$

If $C_2 = 0$ but $C_1 \neq 0$,

$$u_{3_2}(\eta) = \frac{a}{2b} + \frac{p^2\sqrt{\Omega}}{2bA} \times \tanh(\frac{\sqrt{\Omega}}{2A}\eta) + \frac{p^2}{bA\sqrt{\Omega}}(4E\omega + B^2) \times \tanh(\frac{\sqrt{\Omega}}{2A}\eta)).$$

Solution For set 3 with Family-2
If $C_1 = 0$ but $C_2 \neq 0$,

$$u_{3_3}(\eta) = \frac{a}{2b} + i\frac{p^2\sqrt{\Omega}}{2bA}\coth(\frac{\sqrt{-\Omega}}{2A}\eta) - i\frac{p^2}{bA\sqrt{\Omega}}(4E\omega + B^2)\cot(\frac{\sqrt{-\Omega}}{2A}\eta).$$

If $C_2 = 0$ but $C_1 \neq 0$,

$$u_{3_4}(\eta) = \frac{a}{2b} - i\frac{p^2\sqrt{\Omega}}{2bA}\tan(\frac{\sqrt{-\Omega}}{2A}\eta) + i\frac{p^2}{bA\sqrt{\Omega}}(4E\omega + B^2)\tanh(\frac{\sqrt{-\Omega}}{2A}\eta).$$

Solution For set 3 with Family-3

$$u_{3_5}(\eta) = \frac{a}{2b} + \frac{p^2\omega}{bA}\left(\frac{C_2}{C_1 + C_2\eta}\right) + \frac{p^2}{2bA\omega}(4E\omega + B^2)\left(\frac{C_2}{C_1 + C_2\eta}\right)^{-1}.$$

Solution For set 3 with Family-4

If $C_1 = 0$ but $C_2 \neq 0$,

$$u_{3_6}(\eta) = \frac{a}{2b} + \frac{p^2}{bA}\left(\frac{-B}{2} + \sqrt{\Delta}\coth(\frac{\sqrt{\Delta}}{A}n)\right) + \frac{p^2}{4bA}(4E\omega + B^2)\left(-B + 2\sqrt{\Delta}\coth(\frac{\sqrt{\Delta}}{A}n)\right)^{-1}.$$

If $C_2 = 0$ but $C_1 \neq 0$,

$$u_{3_7}(\eta) = \frac{a}{2b} + \frac{p^2}{bA}\left(\frac{-B}{2} + \sqrt{\Delta}\tanh(\frac{\sqrt{\Delta}}{A}n)\right) + \frac{p^2}{4bA}(4E\omega + B^2)\left(-B + 2\sqrt{\Delta}\tanh(\frac{\sqrt{\Delta}}{A}n)\right)^{-1}.$$

Solution For set 3 with Family-5

If $C_1 = 0$ but $C_2 \neq 0$,

$$u_{3_8}(\eta) = \frac{a}{2b} + \frac{p^2}{2bA}\left(-B + 2i\sqrt{\Delta}\cot(\frac{\sqrt{-\Delta}}{A}n)\right) + \frac{p^2}{4bA}(4E\omega + B^2)\left(-B + 2i\sqrt{\Delta}\cot(\frac{\sqrt{\Delta}}{A}\eta)\right)^{-1}.$$

If $C_2 = 0$ but $C_1 \neq 0$,

$$u_{3_9}(\eta) = \frac{a}{2b} + \frac{p^2}{2bA}\left(-B + 2i\sqrt{\Delta}\tan(\frac{\sqrt{-\Delta}}{A}n)\right) + \frac{p^2}{4bA}(4E\omega + B^2)\left(-B + 2i\sqrt{\Delta}\tan(\frac{\sqrt{\Delta}}{A}\eta)\right)^{-1}.$$

4. Results and Discussion

It is worth announcement that the some of our possessed solutions are in good approximation with already done results which are relevant as following:

(1) With $\left(B^2 + 4E\right) > 0$, $C = 0$, $\omega = A = 1$ $a = \left(k^2 + q - \frac{2c}{4k^2 - 1}\right)$, $b = \left(\beta + \frac{1}{4k^2 - 1}\right)$,

$$u_{3_1}(\eta) = \frac{a}{2b} + \frac{3p^2}{b}\sqrt{\left(B^2 + 4E\right)}\coth\left(\frac{\sqrt{\left(B^2 + 4E\right)}}{2}p(x - 2kt)\right),$$

then

$$E(x,t) = \left\{\frac{a}{2b} + \frac{3p^2}{b}\sqrt{\left(B^2 + 4E\right)}\coth\left(\frac{\sqrt{\left(B^2 + 4E\right)}}{2}p(x - 2kt)\right)\right\}e^{i(kx + qt)},$$

$$v(x,t) = \frac{c}{4k^2-1} - \frac{1}{4k^2-1}\left\{ \frac{a}{2b} + \frac{3p^2}{b}\sqrt{(B^2+4E)}\coth\left(\frac{\sqrt{(B^2+4E)}}{2}p(x-2kt)\right)\right\}^2.$$

(2) With $B=0$, $d=0$ $AE>0$, $C=0$, $\omega=A$,

$$a = \left(k^2+q-\frac{2c}{4k^2-1}\right), b = \left(\beta+\frac{1}{4k^2-1}\right), u_{1_6}(\eta) = \frac{a}{2b} + \frac{p^2}{b}\sqrt{\frac{E}{A}}\tanh\left(\sqrt{\frac{E}{A}}p(x-2kt)\right).$$

then

$$E(x,t) = \left\{\frac{a}{2b} + \frac{p^2}{b}\sqrt{\frac{E}{A}}\tanh\left(\sqrt{\frac{E}{A}}p(x-2kt)\right)\right\}e^{i(kx+qt)},$$

$$v(x,t) = \frac{c}{4k^2-1} - \frac{1}{4k^2-1}\left\{\frac{a}{2b} + \frac{p^2}{b}\sqrt{\frac{E}{A}}\tanh\left(\sqrt{\frac{E}{A}}p(x-2kt)\right)\right\}^2.$$

(3) With $B=0$, $d=0$ $AE<0$, $C=0$, $\omega=A$,

$$a = \left(k^2+q-\frac{2c}{4k^2-1}\right), b = \left(\beta+\frac{1}{4k^2-1}\right), u_{2_9}(\eta) = \frac{1}{2b} - \frac{p^2}{b}\sqrt{\frac{-E}{A}}\tan\left(\sqrt{\frac{-E}{A}}p(x-2kt)\right),$$

then

$$E(x,t) = \left\{\frac{1}{2b} - \frac{p^2}{b}\sqrt{\frac{-E}{A}}\tan\left(\sqrt{\frac{-E}{A}}p(x-2kt)\right)\right\}e^{i(kx+qt)},$$

$$v(x,t) = \frac{c}{4k^2-1} - \frac{1}{4k^2-1}\left\{\frac{1}{2b} - \frac{p^2}{b}\sqrt{\frac{-E}{A}}\tan\left(\sqrt{\frac{-E}{A}}p(x-2kt)\right)\right\}^2.$$

(4) With $B=0$, $\Delta=\omega E=AE>0$, $C=0$, $\omega=A$,

$$a = \left(k^2+q-\frac{2c}{4k^2-1}\right), b = \left(\beta+\frac{1}{4k^2-1}\right),$$

$$u_{3_7}(\eta) = \frac{a}{2b} + \frac{p^2\sqrt{\Delta}}{bA}\left(2\tanh(\frac{\sqrt{\Delta}}{A}n) + \coth(\frac{\sqrt{\Delta}}{A}n)\right).$$

then

$$E(x,t) = \left\{\frac{a}{2b} + \frac{p^2\sqrt{\Delta}}{bA}\left(2\tanh(\frac{\sqrt{\Delta}}{A}n) + \coth(\frac{\sqrt{\Delta}}{A}n)\right)\right\}e^{i(kx+qt)},$$

$$v(x,t) = \frac{c}{4k^2-1} - \frac{1}{4k^2-1}\left\{\frac{a}{2b} + \frac{p^2\sqrt{\Delta}}{bA}\left(2\tanh(\frac{\sqrt{\Delta}}{A}n) + \coth(\frac{\sqrt{\Delta}}{A}n)\right)\right\}^2.$$

(5) With $B = 0$, $d = 0$, $\Delta = \omega E = AE < 0$, $C = 0$, $\omega = A$,

$$a = \left(k^2 + q - \frac{2c}{4k^2 - 1} \right), b = \left(\beta + \frac{1}{4k^2 - 1} \right),$$

$$u_{3_9}(\eta) = \frac{1}{2b} - \frac{ip^2\sqrt{\Delta}}{2Ab}\left\{ 2\tan\left(\frac{\sqrt{-\Delta}}{A} p(x - 2kt) \right) - \cot\left(\frac{\sqrt{-\Delta}}{A} p(x - 2kt) \right) \right\},$$

then

$$E(x,t) = \left\{ \frac{1}{2b} - \frac{ip^2\sqrt{\Delta}}{2Ab}\left\{ 2\tan\left(\frac{\sqrt{-\Delta}}{A} p(x - 2kt) \right) - \cot\left(\frac{\sqrt{-\Delta}}{A} p(x - 2kt) \right) \right\} \right\} e^{i(kx+qt)},$$

$$v(x,t) = \frac{c}{4k^2 - 1} - \frac{1}{4k^2 - 1}\left\{ \frac{1}{2b} - \frac{ip^2\sqrt{\Delta}}{2Ab}\left\{ 2\tan\left(\frac{\sqrt{-\Delta}}{A} p(x - 2kt) \right) - \cot\left(\frac{\sqrt{-\Delta}}{A} p(x - 2kt) \right) \right\} \right\}^2.$$

Besides this enumeration, we may gain more new solutions (travelling and solitary wave solutions) like $u_{2_2}(\eta)$, $u_{2_4}(\eta)$, $u_{2_5}(\eta) - u_{2_9}(\eta)$, $u_{1_1}(\eta) - u_{1_9}(\eta)$, $u_{3_2}(\eta) - u_{3_6}(\eta)$, which not established in the prevalence literature. But
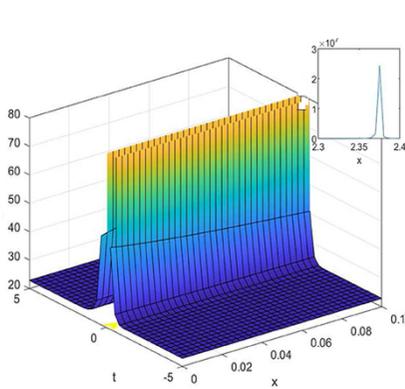
$u_{3_1}(\eta)$, $u_{1_6}(\eta)$ and $u_{2_6}(\eta)$ are as same the solution of [5].

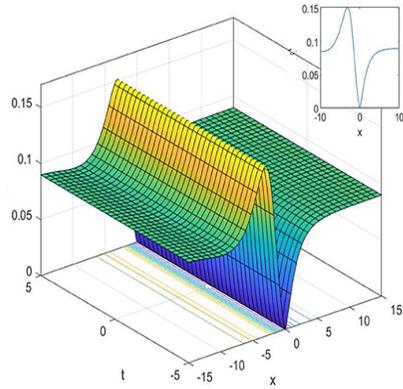### 4.1. Graphically sketched of the solution

The u_{2_2} plotting to explore the soliton solutions for the model Eq. (1.1).
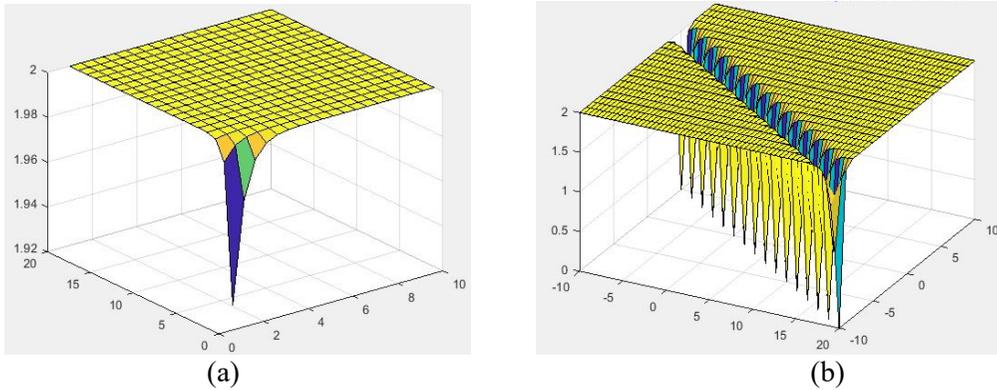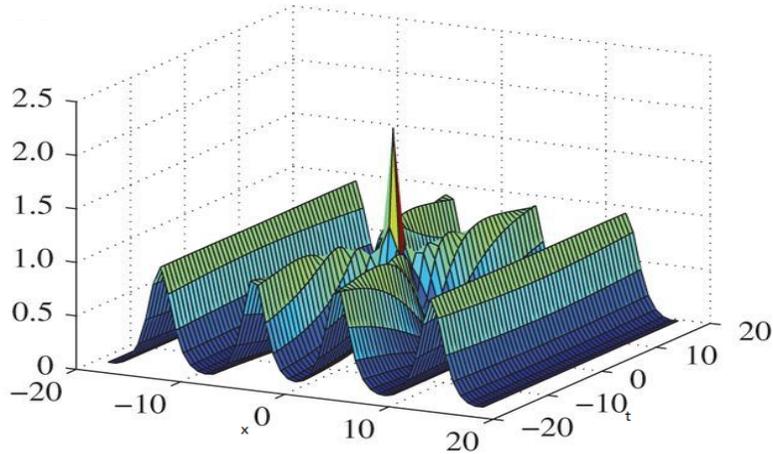

Fig. 1. (a) The one-solitary wave propagation (b) More parallel solitons plot for Eq. (1.1) with $-15 \le x \le 15$, $-5 \le t \le 5$, $\beta \ne 0$, $-q = d = -1/2, A = B = 1$, $C = 0$, $c = 0, k = p = 1/\sqrt{2}$.

Fig. 2. Modular in 3D, 2D plot of Kink wave Shape of $u_{1_2}(\eta)$ when $A = B = 1,\ C = 0,\ c = 0,$
$k = p = 1/\sqrt{2},\ -q = d = -1/2,$
$-5 \le t \le 5,\ 0 \le x \le 0.1,\ \beta \ne 0.$

Fig. 3. Modular plot of Kink wave Shape of $u_{3_7}(\eta)$ when $A = E = 1,\ B = 0,\ C = 0,\ d = 0,\ c = 0,$
$k = p = 1/\sqrt{2},\ -q = d = -1/2,$
$-15 \le x \le 15,\ -5 \le t \le 5,\ \beta \ne 0.$

Fig. 4. Mesh 3D graph for solitons interaction $\left\{ |E|^2 + v \right\}$ for $\beta = -1$.

5. Conclusion

A new expansion scheme is exhibited to locate solitary wave solution of the model of wave-wave interactions in plasma for generalized Zakharov equations (GZE). The solitary wave solution Figs. 1, 2, 3 and its interaction in Fig. 4 of this system are exist. Therefore, we learn that this method well emulates development of many exact travelling wave solutions are gotten which contain the new soliton-like hyperbolic function solutions $u_{3_7}(\eta)$, $u_{2_2}(\eta)$ trigonometric function solutions $u_{2_9}(\eta)$,

$u_{3_9}(\eta)$ and rational solutions $u_{2_5}(\eta)$ $u_{3_5}(\eta)$. Solitons are caused by a cancellation of nonlinear and dispersive effects in wave interactions. The term $\left\{ |E|^2 + v \right\}$ is plotted for $\beta = -1$ for the results mentioned in results and discussion section. The given plot is of the last results where E and v both have iota and tan & cot terms. In calculating the $\left\{ |E|^2 + v \right\}$, the *iota* terms are neutralized but the tan and cot

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terms together form the solitons in travelling wave. The terms under *cot* function in $u_{2_8}(\eta)$ and under *tan* function in $u_{2_9}(\eta)$ create the instability. So, it is estimated that there may be the possibility of modulational instability due to this nature, but it is the further topic of research. This scheme has shown the way for a long-playing computing truthfulness.

References

[1]   Kavita Sharma Arun Kumar and Ram Dayal Pankaj, Development of Modulational instabilities in traveling wave solutions for wave-wave interaction in ionic media. ISSN: 2320-5407 int. J. Adv. Res. 2017,5(4). 1323-30.

[2]   Kumar, A. and Pankaj R. D. Finite Difference Scheme for the Zakharov Equation as a Model for Nonlinear Wave-Wave Interaction in Ionic Media, International Journal of Scientific & Engineering Research, Volume 2014. 5(2) 759 ISSN 2229-5518.

[3]   Bekir, Application of the (G′/G) -expansion method for nonlinear evolution equations, Phys. Lett. A, 2008. 372, pg. 3400-6.

[4]   Malomed B, Anderson D, Lisak M, Quiroga-Teixeiro ML. Dynamics of solitary waves in the Zakharov model equations. Phys Rev E 1977; 55:962-8.

[5]   Arun Kumar, Ram Dayal Pankaj Tanh-coth scheme for traveling wave solutions for Nonlinear Wave Interaction model, Journal of the Egyptian Mathematical Society. 23(2) 2014, 282-5.

[6]   E. G. Fan, Two new applications of the homogeneous balance method. Phys. Lett. A. 2000. 265 pg. 353-7.

[7]   S. A. El-Wakil, M. A. Abdou, New exact travelling wave solutions using modified extended tanh-function method. Chaos Solitons Fractals 2007, 31(4) pg. 840-52.

[8]   M. Wazwaz, A sine-cosine method for handle nonlinear wave equations, Applied Mathematics and Computer Modeling, 2004,40, pg.499-508.

[9]   M. Usman, A. Nazir, T. Zubair, I. Rashid, Z. Naheed, S.T. Mohyud-Din, Solitary wave solutions of (3+1)-dimensional Jimbo Miwa and Pochhammer-Chree equations by modified Exp-function method, Int. J. Modern Math. Sci. 2013, 5(1) pg.27-36.

[10] D. Liu, Jacobi elliptic function solutions for two variant Boussinesq equations, Chaos solitons Fractals, 2005. 24, pg. 1373-85.

[11] Ram Dayal Pankaj Bhawani Singh, Arun Kumar New extended Jacobi elliptic function expansion scheme for wave-wave interaction in ionic media. Nanosystems: physics, chemistry, mathematics, 2019. 9(5), Pg.581-5.

[12] Kumar, A. and Pankaj R. D. Some Solitary and Periodic Solutions of the non-linear wave Equation by Variational Approach, Journals of Rajasthan Academy of Physical Sciences,2012, 11(2) pp 133-139, ISSN:0972-6306.

[13] T. L. Bock and M. D. Kruskal, A two-parameter Miura transformation of the Benjamin-One equation, Phys. Lett. A 1979, 74, pg.173-6.

[14] M. L. Wang, X. Z. Li and J. Zhang, The (G′/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics, Phys. Lett. A, 2008 372, pg. 417-23.

[15] M. A. Akbar, N. H. M. Ali and E. M. E. Zayed, A generalized and improved (G′/G) -expansion method for nonlinear evolution equations, Math. Prob. Engr., 2012, Vol 2012 22 pages. doi: 10.1155/2012/459879.

[16] Pankaj R. D. and Sindhi, Chandrawati Traveling Wave Solutions for Wave-Wave Interaction Model in Ionic Media. International Educational Scientific Research Journal E-ISSN No: 2455-295X 2016. 2(6) 70-2.

[17] Pankaj, Ram Dayal, Laplace - Modified Decomposition Method to Study Solitary Wave Solutions of Coupled Nonlinear Klein-Gordon Schrödinger Equation, International Journal of Statistika and Mathematika, 2013, 5(1) pp 01-05, ISSN: 2277- 90.

[18] Kumar, A. and Pankaj R. D. Solitary Wave Solutions of Schrödinger Equation by Laplace−Adomian Decomposition Method, Physical Review & Research International 2013, 3(4): 702-12.

[19] Kumar, A. and Pankaj R. D. Finite Difference Scheme for the Zakharov Equation as a Model for Nonlinear Wave-Wave Interaction in Ionic Media, International Journal of Scientific & Engineering Research, 2014. 5(2) 759 ISSN 2229-5518.

[20] Ram Dayal Pankaj and Chandrawati Sindhi. A Study for Zakharov system by Laplace-Modified Decomposition Method JETIR) 2019, 5(11) ISSN-2349-5162 http://www.jetir.org/papers/JETIR181155 0.pdf\

[21] Ram Dayal Pankaj, A novel narration for propagation of wave in relaxing medium by expansion scheme. Advances And Applications in Mathematical Sciences. 2021. 20(8), 1517-23.

[22] P. Veeresha, D. G. Prakasha, Devendra Kumar, Dumitru Baleanu, Jagdev Singh An efficient computational technique for fractional model of generalized Hirota-Satsuma coupled KdV and coupled mKdV equations, Journal of Computational and Nonlinear Dynamics, 15 (2020), 071003.

[23] R. D. Pankaj et al Evolution of modulational instability in non-linear hirota types equation Science & Technology Asia 2020.25(1) 123-7.

[24] Zakharov, V. E Collapse of Langmuir waves. Zh Eksp Teor Fiz 1972 62: 1745-51.