Songklanakarin J. Sci. Technol.
43 (4), 1103-1114, Jul. - Aug. 2021

http://www.sjst.psu.ac.th

Original Article

Application of interval-valued neutrosophic soft sets in decision making based on game theory

Somen Debnath*

Department of Mathematics, Tripura University, Suryamaninagar, Agartala, Tripura, 799022 India

Received: 30 June 2020; Revised: 16 August 2020; Accepted: 22 August 2020

Abstract

The main objective of this paper is to present the notion of interval-valued neutrosophic soft sets in game theory strategy. Interval-valued neutrosophic soft sets (in short ivn-soft sets) is a generalization of interval-valued intuitionistic fuzzy soft sets. In this article, after giving the definition of ivn-soft sets and their basic operations we define two-person ivn-soft game, which can be applied to problems containing uncertain, incomplete, inconsistent and imprecise data. We then give solution method of the games which is based on ivn-soft saddle points, ivn-soft lower values, ivn-soft upper values, ivn-soft dominated strategy and ivn-soft Nash equilibrium. We also extend the two-person ivn-soft game to n-person ivn-soft game. Finally we develop a decision-making method.

Keywords: soft set, neutrosophic soft set, ivn-soft payoff function, ivn-soft dominated strategy

1. Introduction

The concept of soft set introduced by Molodtsov (Molodsov, 1999) in 1999 emerged as a general mathematical tool of vagueness. Mathematical researchers use the concept of soft set in various fields such as economics, engineering, topology, game theory, operation research, soft analysis etc to solve complex problems in a simple manner. The novelty of the use of soft set theory lies in the fact that there is no requirement for assigning membership function as sometimes we find difficulties in setting membership function in case of fuzzy set. Also it is difficult to construct a mathematical model and to get the exact solution. To get rid of this problem, soft set was introduced. There is no restriction on the approximate description, which means it is very effective and easily applicable in practice. We also handle parametric data using soft set theory.

Soft set theory has been progressing rapidly in recent years and it has been expanded by embedding the idea of fuzzy sets (Zadeh, 1965), vague sets (Gau & Buehrer, 1993), intuitionistic fuzzy sets (Atanassov, 1986; Shyamal & Pal, 2004), interval valued fuzzy set (Gorzalczany, 1987), interval-valued intuitionistic fuzzy sets (Atanassov & Gargov, 1989; Zhenhua, Jingyu, Youpei & Qian Sheng, 2011), neutrosophic sets (Smarandache, 2005), rough sets (Pawlak, 1982) etc. Therefore the generalization of soft sets are fuzzy soft sets (Maji, Biswas & Roy, 2001; Roy & Maji, 2007; Yao, Liu & Yan, 2008), intuitionistic fuzzy soft sets (Maji, Biswas & Roy, 2004), interval-valued fuzzy soft sets (Chetia & Das, 2010; Yang, Lin, Yang, Li & Yu, 2009), interval-valued intuitionistic fuzzy soft sets (Jiang, Tang, Chen, Liu & Tung, 2010), rough soft sets (Moinuddin, 2017), neutrosophic soft sets (Maji, 2012, 2013), interval valued neutroso-phic soft sets (Deli, 2014) etc. By transforming the function F into a multi-argument function, Smarandache (2018) generalized the soft set to the Hypersoft set.

The term ‘neutrosophy’ is used for the philosophical point of view. The notion of neutrosophic set (NS) was introduced by Smarandache (2005). It is a generalization of intuitionistic fuzzy set (Atanassov, 1986; Shyamal & Pal, 2004). NS is described by membership function, indeterminate function and non-membership function and they are independently related to each other. It is a general framework for handling problems involving imprecise, indeterministic and inconsistent data. Neutrosophic set is growing rapidly and it is used in different directions. Some of

*Corresponding author
Email address: somen008@rediffmail.com

the contributions of neutrosophic set are mentioned in Broumi, Bakali, Talea and Smarandache (2017); Broumi, Nagarajan, Bakali, Talea, Smarandache and Lathamaheswari (2019); Broumi, Son, Bakali, Talea, Smarandache and Selvachandran (2017); Broumi, Talea, Bakali, Singh and Smarandache (2019); Edalatpanah (2020); Mullai, sangeetha, Surya, Madhan Kumar, Jeyabalan and Broumi (2020); Rattana and Chinram (2020); Salama,Bondok Henawy and Alhabib (2020). By embedding the idea of neutrosophic set and the existing sets the contributions of several researchers are given in Deli (2014); Sahin, Alkhazaleh and Ulucay (2015); Smarandache (1995, 1998); Smarandache and Broumi (2015); and Wang, Smarandache, Zhang and Sunderraman (2005). Moreover in (Maji, 2013), P.K.Maji introduced the concept of neutrosophic soft set (NSS) and in (Maji, 2012), he used NSS in decision making problem.

We all know about different types of games (indoor/outdoor) and all these games have different rules and the players who are participating and bound to follow the rules otherwise they will exit from the game. Before starting a game, players work out a game plan so that they will be able to defeat their opposition. To prepare a proper game plan we need to know about some technique or method. Techniques used in the game not only gives us a better chance to win a game but they are also useful in our real life situation. That is why, first of all, we need to know about Theory of Games. The original edition of Theory of Games and Economic Behavior was published in 1944 by Neumann and Morgenstern (1944) and its revised edition in 1947. Game theory has been successfully used in logic, decision making problems, economics etc. In 1950, game theory was extensively developed by many scholars. In 1953, Borel introduced the concept of pure and mixed strategies, but Neumann points out that without minimax theorem no theory of games can be exist. Over the years many scholars have worked on the concept of classical game theory and extended this concept, which leads to the introduction of Cagman and Deli (2013, 2015, 2016); Deli (2013), and Mukherjee and Debnath (2016, 2018). The formal definition of games pays out the players, their preferences, their information, the strategy available to them and how these influence the outcome.

After the introduction of game theory by Neumann and Morgenstern, they have started to work on the modern game theory. It has many applications in logic, economics, sociology, comp uter science, political science etc. By embedding the ideas of fuzzy sets many applications of game theory have been proposed in Cagman and Deli (2013, 2015, 2016); and Deli (2013).

In Cagman and Deli (2013) and Cagman and Deli (2015), the authors presented the notion of soft games and fuzzy soft games respectively. Anjan and Somen in Mukherjee and Debnath (2018) introduced intuitionistic fuzzy soft game theory. Also, Anjan et al. in Mukherjee and Debnath (2016) discussed generalized fuzzy soft sets in decision making based on game theory and their applications.

In this article we are mainly concerned with ivn-soft sets (Deli, 2014). Wang et al. was introduced the concept of ivn-set in Wang, Smarandache, Zhang and Sunderraman (2005). In this work, we propose a game model for dealing with imprecise, indeterminacy and inconsistent data. The proposed new game is ivn-soft game as it is based on interval-valued neutrosophic set and soft set. We then give one solution method of the game which is ivn-soft saddle point. We apply this concept in decision making which shows that the method can be successfully applied to a real-life problem and extend the two-person ivn-soft game to n-person ivn-soft game. We also introduce ivn-soft upper and lower values of a two-person ivn-soft game.

2. Preliminaries

In this section we recapitulate some basic definitions and examples which we use in the later section of this paper.

Definition 2.1 (Zadeh, 1965) Let X be a non empty set. Then a fuzzy set A is a set having the form $A=\{(x,\mu_A(x)):x\in X\}$, where the function $\mu_A:X\to[0,1]$ is called the membership function and $\mu_A(x)$ is called the degree of membership of each element $x\in X$.

Definition 2.2 (Cagman & Enginoglu,2010; Feng, Liu, Leoreanu & Young, 2011; Maji, Biswas& Roy, 2003; Molodtsov, 1999) Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes power set of U and $A\subseteq E$. Then the pair (F,A) is called a soft set over U , where F is a mapping given by $F:A\to P(U)$.

Definition 2.3 (Atanassov, 1986; Shyamal & Pal, 2004) Let X be a non empty set. An intuitionistic fuzzy set A in X is an object $A=\{(x,\mu_A(x),\gamma_A(x)):x\in X\}$, where the functions $\mu_A:X\to[0,1]$, and denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x\in X$ to the set A respectively and $0\le\mu_A(x)+\gamma_A(x)\le 1$ for each $x\in X$.

Definition 2.4 (Maji, Biswas & Roy, 2001; Roy & Maji, 2007; Yao, Liu & Yan, 2008) Let U be an initial universe and E be a set of parameters. Let I^U be the set of all fuzzy subsets of U and $A\subseteq E$. Then the pair (F,A) is called a fuzzy soft set over U, where F is a mapping given by $F:A\to I^U$.

Definition 2.5 (Maji, Biswas & Roy, 2004) Let U be an initial universe and E be a set of parameters. Let IF^U be the set of all intuitionistic fuzzy subsets of U and $A \subseteq E$. Then the pair (F,A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by $F:A \to IF^U$.

Example 2.6 (Maji, Biswas & Roy, 2004) Consider an intuitionistic fuzzy soft set (F,A) where U is a set of five houses under the consideration of a decision maker to purchase, which is denoted by $U=\{h_1,h_2,h_3,h_4,h_5\}$ and A is a parameter set where $A=\{e_1,e_2,e_3,e_4,e_5\}$ ={expensive, beautiful, wooden, in good repair, in the green surroundings}. The intuitionistic fuzzy soft set (F,A) describes the "attractiveness of the houses" to the decision maker.

$$F(e_1)=\{(h_1,0.6,0.1),(h_2,0.8,0.05),(h_3,0.6,0.2),(h_4,0.65,0.15),(h_5,0.56,0.2)\}$$
$$F(e_2)=\{(h_1,0.7,0.15),(h_2,0.6,0.15),(h_3,0.5,0.2),(h_4,0.7,0.15),(h_5,0.6,0.25)\}$$
$$F(e_3)=\{(h_1,0.75,0.1),(h_2,0.5,0.2),(h_3,0.6,0.1),(h_4,0.68,0.1),(h_5,0.7,0.1)\}$$
$$F(e_4)=\{(h_1,0.8,0.01),(h_2,0.65,0.2),(h_3,0.66,0.2),(h_4,0.69,0.1)(h_5,0.72,0.1)\}$$
$$F(e_5)=\{(h_1,0.77,0.05),(h_2,0.6,0.2),(h_3,0.6,0.2),(h_4,0.63,0.15),(h_5,0.7,0.1)\}$$

The intuitionistic fuzzy soft set (F,A) is a parameterized family $\{F(e_i,i=1,2,3,4,5)\}$ of interval valued intuitionistic fuzzy sets on U and

$$(F, A)=\{\text{expensive houses}=\{(h_1,0.6,0.1),(h_2,0.8,0.05),(h_3,0.6,0.2),(h_4,0.65,0.15),(h_5,0.56,0.2)\},$$
$$\text{beautiful houses}=\{(h_1,0.7,0.15),(h_2,0.6,0.15),(h_3,0.5,0.2),(h_4,0.7,0.15),(h_5,0.6,0.25)\}$$
$$\text{wooden houses}=\{(h_1,0.75,0.1),(h_2,0.5,0.2),(h_3,0.6,0.1),(h_4,0.68,0.1),(h_5,0.7,0.1)\}$$
$$\text{in good repair houses}=\{(h_1,0.8,0.01),(h_2,0.65,0.2),(h_3,0.66,0.2),(h_4,0.69,0.1)(h_5,0.72,0.1)\}$$
$$\text{in the green surroundings}=\{(h_1,0.77,0.05),(h_2,0.6,0.2),(h_3,0.6,0.2),(h_4,0.63,0.15),(h_5,0.7,0.1)\}$$

Definition 2.7 (Jiang, Tang, Chen, Liu & Tung, 2010) Let U be an initial universe and E be a set of parameters. Let $IVIFS^U$ be the set of all interval valued intuitionistic fuzzy sets on U and $A \subseteq E$. Then the pair (F,A) is called an interval valued intuitionistic fuzzy soft set (IVIFSS for short) over U, where F is a mapping given by $F:A \to IVIFS^U$.

Definition 2.8 (Smarandache, 2005) Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic set (N-set) A in U is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(u)$,$I_A(u)$ and $F_A(u)$ are real standard or nonstandard subsets of [0,1].
There is no restriction on the sum of $T_A(u)$,$I_A(u)$ and $F_A(u)$, so, $0 \le \sup T_A(u)+\sup I_A(u)+ \sup F_A(u) \le 3$.

Definition 2.9 (Smarandache, 2005) Let U be a space of points (objects), with a generic element in U denoted by u. An interval-valued neutrosophic set (IVN-sets) A in U is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity- membership function F_A. For each point $u \in U$; T_A,I_A and $F_A \subseteq [0,1]$.

Definition 2.10 (Maji, 2012, 2013) Let U be an initial universe set and E be a set of parameters. Consider $A \subset E$.Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F,A) is termed the soft neutrosophic set over U, where F is a mapping given by $F:A \to P(U)$.

Example 2.11 Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider E ={beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive}. In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that there are five houses in the universe

U given by, $U=\{h_1,h_2,h_3,h_4,h_5\}$ and the set of parameters $A=\{e_1,e_2,e_3,e_4\}$, where e_1 stands for the parameter 'beautiful', e_2 stands for the parameter 'wooden', e_3 stands for the parameter 'costly' and the parameter e_4 stands for 'moderate'. Suppose that

$$F(\text{beautiful})=\{\langle h_1,0.5,0.6,0.3\rangle,\langle h_2,0.4,0.7,0.6\rangle,\langle h_3,0.6,0.2,0.3\rangle,\langle h_4,0.7,0.3,0.2\rangle,\langle h_5,0.8,0.2,0.3\rangle\},$$

$$F(\text{wooden})=\{\langle h_1,0.6,0.3,0.5\rangle,\langle h_2,0.7,0.4,0.3\rangle,\langle h_3,0.8,0.1,0.2\rangle,\langle h_4,0.7,0.1,0.3\rangle,\langle h_5,0.8,0.3,0.6\rangle\},$$

$$F(\text{costly})=\{\langle h_1,0.7,0.4,0.3\rangle,\langle h_2,0.6,0.7,0.2\rangle,\langle h_3,0.7,0.2,0.5\rangle,\langle h_4,0.5,0.2,0.6\rangle,\langle h_5,0.7,0.3,0.4\rangle\},$$

$$F(\text{moderate})=\{\langle h_1,0.8,0.6,0.4\rangle,\langle h_2,0.7,0.9,0.6\rangle,\langle h_3,0.7,0.6,0.4\rangle,\langle h_4,0.7,0.8,0.6\rangle,\langle h_5,0.9,0.5,0.7\rangle\}.$$

The neutrosophic soft set (NSS) (F,E) is a parameterized family $\{F(e_i),i=1,2,.....10\}$ of all neutrosophic sets of U and describes a collection of approximations of an object. The mapping F here is 'houses(.)',where dot(.) is to be filled up by a parameter $e\in E$ .Therefore, $F(e_1)$ means 'houses(beautiful)' whose functional-value is the neutrosophic set

$$\{\langle h_1,0.5,0.6,0.3\rangle,\langle h_2,0.4,0.7,0.6\rangle,\langle h_3,0.6,0.2,0.3\rangle,\langle h_4,0.7,0.3,0.2\rangle,\langle h_5,0.8,0.2,0.3\rangle\}.$$

Thus we can view the neutrosophic soft set (NSS) (F,A) as a collection of approximations as below:

$$(F,A)=\left\{\begin{array}{l}\text{beautiful houses}=\{\langle h_1,0.5,0.6,0.3\rangle,\langle h_2,0.4,0.7,0.6\rangle,\langle h_3,0.6,0.2,0.3\rangle,\langle h_4,0.7,0.3,0.2\rangle,\langle h_5,0.8,0.2,0.3\rangle\},\\ \text{wooden houses}=\{\langle h_1,0.6,0.3,0.5\rangle,\langle h_2,0.7,0.4,0.3\rangle,\langle h_3,0.8,0.1,0.2\rangle,\langle h_4,0.7,0.1,0.3\rangle,\langle h_5,0.8,0.3,0.6\rangle\},\\ \text{costly houses}=\{\langle h_1,0.7,0.4,0.3\rangle,\langle h_2,0.6,0.7,0.2\rangle,\langle h_3,0.7,0.2,0.5\rangle,\langle h_4,0.5,0.2,0.6\rangle,\langle h_5,0.7,0.3,0.4\rangle\},\\ \text{moderate houses}=\{\langle h_1,0.8,0.6,0.4\rangle,\langle h_2,0.7,0.9,0.6\rangle,\langle h_3,0.7,0.6,0.4\rangle,\langle h_4,0.7,0.8,0.6\rangle,\langle h_5,0.9,0.5,0.7\rangle\}\end{array}\right\},$$

where each approximation has two parts: (i) a predicate p, and (ii) an approximate value-set v (or simply to be called value-set v).

The tabular representation of the neutrosophic soft set (F,A) is shown as follows in Table 1.

Table 1. Neutrosophic soft set (F,A)

U	beautiful	wooden	costly	moderate
h_1	(0.5,0.6,0.3)	(0.6,0.3,0.5)	(0.7,0.4,0.3)	(0.8,0.6,0.4)
h_2	(0.4,0.7,0.6)	(0.7,0.4,0.3)	(0.6,0.7,0.2)	(0.7,0.9,0.6)
h_3	(0.6,0.2,0.3)	(0.8,0.1,0.2)	(0.7,0.2,0.5)	(0.7,0.6,0.4)
h_4	(0.7,0.3,0.2)	(0.7,0.1,0.3)	(0.5,0.2,0.6)	(0.7,0.8,0.6)
h_5	(0.8,0.2,0.3)	(0.8,0.3,0.6)	(0.7,0.3,0.4)	(0.9,0.5,0.7)

Definition 2.12 (Deli, 2014) Let U be an initial universe set, IVN(U) denotes the set of all interval-valued neutrosophic sets of U and E be a set of parameters that are describe the elements of U. An interval-valued neutrosophic soft sets (ivn-soft sets) over U is a set defined by a set valued function ν_k representing a mapping $\nu_k : E \rightarrow IVN(U)$. It can be written as a set of ordered pairs $\Upsilon_k=\{(x,\nu_k(x)\text{:}x\in E\}$

Here, ν_k, which is interval-valued neutrosophic sets, is called approximate function of the ivn-soft sets Υ_k and $\nu_k(x)$ is called x-approximate value of $x\in E$. The sets of all ivn-soft sets over U will be denoted by IVNS (U).

Let us give the following example for ivn-soft sets.

Example 2.13 (Deli, 2014) Let U= {u_1, u_2} be the set of houses under consideration and E be a set of parameters, where each parameter is a neutrosophic word.

Considering, E={x_1=cheap, x_2=beautiful, x_3=green surroundings, x_4=costly, x_5=large}. In this case, we give an (ivn-soft sets) Υ_k over U as;

$$\Upsilon_k = \{(x_1, \{\langle[0.6,0.8],[0.8,0.9],[0.1,0.5]\rangle / u_1, \langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle / u_2\}),$$
$$(x_2, \{\langle[0.1,0.4],[0.5,0.8],[0.3,0.7]\rangle / u_1, \langle[0.1,0.9],[0.6,0.9],[0.2,0.3]\rangle / u_2\}),$$
$$(x_3, \{\langle[0.2,0.9],[0.1,0.5],[0.7,0.8]\rangle / u_1, \langle[0.4,0.9],[0.1,0.6],[0.5,0.7]\rangle / u_2\}),$$
$$(x_4, \{\langle[0.6,0.9],[0.6,0.9],[0.6,0.9]\rangle / u_1, \langle[0.5,0.9],[0.6,0.8],[0.1,0.8]\rangle / u_2\}),$$
$$((x_1, \{\langle[0.0,0.9],[0.1,1.0],[0.1,1.0]\rangle / u_1, \langle[0.0,0.9],[0.8,1.0],[0.2,0.5]\rangle / u_2\})\}$$

The tabular representation of the (ivn-soft sets) Υ_k is shown in Table 2.

Table 2. Interval neutrosophic soft set

U	u_1	u_2
x_1	$\langle[0.6,0.8],[0.8,0.9],[0.1,0.5]\rangle$	$\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle$
x_2	$\langle[0.1,0.4],[0.5,0.8],[0.3,0.7]\rangle$	$\langle[0.1,0.9],[0.6,0.9],[0.2,0.3]\rangle$
x_3	$\langle[0.2,0.9],[0.1,0.5],[0.7,0.8]\rangle$	$\langle[0.4,0.9],[0.1,0.6],[0.5,0.7]\rangle$
x_4	$\langle[0.6,0.9],[0.6,0.9],[0.6,0.9]\rangle$	$\langle[0.5,0.9],[0.6,0.8],[0.1,0.8]\rangle$
x_5	$\langle[0.0,0.9],[0.1,1.0],[0.1,1.0]\rangle$	$\langle[0.0,0.9],[0.8,1.0],[0.2,0.5]\rangle$

3. Two Person Interval-Valued Neutrosophic Soft Games (tp-ivn-soft games)

In the soft game (Cagman & Deli, 2013) the strategy sets and the soft payoffs are crisp but in the fuzzy soft game (Cagman & Deli, 2015) the strategy sets are crisp but the fuzzy soft payoffs are fuzzy subsets of the universe of discourse or alternatives U. Here we extend the definitions and results on game theory defined in Cagman and Deli (2013, 2015, 2016); Deli (2013); Mukherjee and Debnath (2016, 2018). Now we construct tp-ivn-soft game by using interval-valued neutrosophic soft sets.

Definition 3.1 Let E be a set of strategy and $X,Y \subseteq E$. Then the set of all available ivn-actions in ivn-soft game is denoted by $X \times Y$.

Definition 3.2 Let U be a set of alternatives, E be a set of strategies and $X,Y \subseteq E$.If IVN(U) denotes all interval-valued neutrosophic sets over U, then a set valued function $\nu_{X\times Y}: X\times Y \to IVN(U)$ is called an interval-valued neutrosophic soft payoff(ivn-payoff) function and the value $\nu_{X\times Y}(x,y)$ is called an ivn-payoff for each $(x,y) \in X\times Y$.

Definition 3.3 Let $X \times Y$ be all possible ivn-action pairs. Then an ivn-action $(x^*,y^*) \in X\times Y$ is called an optimal ivn-action if $\nu_{X\times Y}(x^*,y^*) \supseteq \nu_{X\times Y}(x,y)$ for all $(x,y) \in X\times Y$.

Definition 3.4 Suppose X and Y be two sets of strategies for player 1 and player 2 respectively and U be a set of alternatives. Let $\nu^k_{X\times Y}: X\times Y \to IVN(U)$ be an interval-valued neutrosophic soft payoff (ivns-payoff) function for player k=1, 2. Then for each k, a tp-ivns-game is defined by an ivn-soft set over U is given by $\Gamma^k_{X\times Y} = \left\{\left((x,y), \nu^k_{X\times Y}\right) : (x,y) \in X\times Y\right\}$, $\nu^k_{X\times Y} = \left(\left[T^k_{-X\times Y}, T^k_{+X\times Y}\right], \left[I^k_{-X\times Y}, I^k_{+X\times Y}\right], \left[F^k_{-X\times Y}, F^k_{+X\times Y}\right]\right)$ where the 1st interval is for truth- membership, 2nd is for indeterminacy-membership and the 3rd is for falsity- membership and each interval is a subset of [0,1] and sum of the supremum of these three intervals is greater or equal to zero and less or equal to three as we have considered the standard unit interval.

If $X=\{x_1,x_2,x_3,\ldots\ldots\ldots,x_m\}$ and $Y=\{y_1,y_2,y_3,\ldots\ldots,y_n\}$ be the sets of strategies of player 1 and player 2, then the interval-valued neutrosophic soft payoffs $\nu^k_{X\times Y}$ of the tp-ivns game $\Gamma^k_{X\times Y}$ can be represented in the form of $m \times n$ matrix as shown in Table 3.

Example 3.5 Let U= $\{u_1,u_2,u_3,u_4\}$ be a set of alternatives, and IVN(U) denotes the set of all interval-valued neutrosophic sets over U. Again, let $E=\{e_1,e_2,e_3,e_4,e_5\}$be a set of strategies and $X=\{x_1,x_3,x_4\}$and $Y=\{x_1,x_2\}$ be a set of strategies of player 1 and player 2 respectively. If we construct a tp-ivns-games as follows;

$$
\begin{aligned}
\Gamma^{1}_{X\times Y} = \{ & ((x_1,x_1),\{\langle[0.6,0.8],[0.8,0.9],[0.1,0.5]\rangle/u_1,\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle/u_3,\langle[0.5,0.7],[0.2,0.8],[0.1,0.6]\rangle/u_4\}),\\
& ((x_1,x_2)\{\langle[0.1,0.4],[0.5,0.8],[0.3,0.7]\rangle/u_1,\langle[0.1,0.9],[0.6,0.9],[0.2,0.3]\rangle/u_2,\langle[0.6,0.8],[0.4,0.9],[0.1,0.7]\rangle/u_3\}),\\
& ((x_3,x_1)\{\langle[0.2,0.9],[0.1,0.5],[0.7,0.8]\rangle/u_1,\langle[0.4,0.9],[0.1,0.6],[0.5,0.7]\rangle/u_2,\langle[0.2,0.5],[0.6,0.9],[0.4,0.7]\rangle/u_3\}),\\
& ((x_3,x_2)\{\langle[0.6,0.9],[0.6,0.9],[0.6,0.9]\rangle/u_1,\langle[0.5,0.9],[0.6,0.8],[0.1,0.8]\rangle/u_3\}),((x_4,x_1)\{\langle[0.2,0.9],[0.1,0.8],[0.3,0.6]\rangle/u_1,\\
& \langle[0.2,0.9],[0.3,0.8],[0.2,0.5]\rangle/u_2,\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle/u_4\})\},((x_4,x_2)\{\langle[0.1,0.9],[0.1,0.8],[0.2,0.6]\rangle/u_1,\\
& \langle[0.3,0.9],[0.5,0.7],[0.2,0.5]\rangle/u_2,\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle/u_3,\langle[0.6,0.8],[0.3,0.9],[0.4,0.7]\rangle/u_4\})\}
\end{aligned}
$$

Then the matrix representation of tp-ivns-game of player 1 can be represented in matrix form, which is shown in Table 4.

Table 3. Two person interval nutrosophic soft game

$\Gamma^{k}_{X\times Y}$	y_1	y_2	…………	y_n
x_1	$\nu^{k}_{X\times Y}(x_1,y_1)$	$\nu^{k}_{X\times Y}(x_1,y_2)$	…………	$\nu^{k}_{X\times Y}(x_1,y_n)$
x_2	$\nu^{k}_{X\times Y}(x_2,y_1)$	$\nu^{k}_{X\times Y}(x_2,y_2)$	…………	$\nu^{k}_{X\times Y}(x_2,y_n)$
…….	……….	…………..	…………..	…………
x_m	$\nu^{k}_{X\times Y}(x_m,y_1)$	$\nu^{k}_{X\times Y}(x_m,y_2)$	…………	$\nu^{k}_{X\times Y}(x_m,y_n)$

Table 4. Two person interval neutrosophic soft game of player 1

$\Gamma^{1}_{X\times Y}$	x_1	x_2
x_1	$\langle[0.6,0.8],[0.8,0.9],[0.1,0.5]\rangle/u_1$, $\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle/u_3$, $\langle[0.5,0.7],[0.2,0.8],[0.1,0.6]\rangle/u_4$	$\langle[0.1,0.4],[0.5,0.8],[0.3,0.7]\rangle/u_1$, $\langle[0.1,0.9],[0.6,0.9],[0.2,0.3]\rangle/u_2$, $\langle[0.6,0.8],[0.4,0.9],[0.1,0.7]\rangle/u_3$
x_3	$\langle[0.2,0.9],[0.1,0.5],[0.7,0.8]\rangle/u_1$, $\langle[0.4,0.9],[0.1,0.6],[0.5,0.7]\rangle/u_2$, $\langle[0.2,0.5],[0.6,0.9],[0.4,0.7]\rangle/u_3$	$\langle[0.6,0.9],[0.6,0.9],[0.6,0.9]\rangle/u_1$, $\langle[0.5,0.9],[0.6,0.8],[0.1,0.8]\rangle/u_3$
x_4	$\langle[0.2,0.9],[0.1,0.8],[0.3,0.6]\rangle/u_1$, $\langle[0.2,0.9],[0.3,0.8],[0.2,0.5]\rangle/u_2$, $\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle/u_4$	$\langle[0.1,0.9],[0.1,0.8],[0.2,0.6]\rangle/u_1$, $\langle[0.3,0.9],[0.5,0.7],[0.2,0.5]\rangle/u_2$, $\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle/u_3$, $\langle[0.6,0.8],[0.3,0.9],[0.4,0.7]\rangle/u_4$

Now, if player 2 constructs a tp-ivns-game then interval-valued neutrosophic soft payoffs of the game one given in the following Table 5.

Table 5. Two person interval neutrosophic soft game of player 2

$\Gamma^{1}_{X\times Y}$	x_1	x_2
x_1	$\langle[0.5,0.8],[0.6,0.8],[0.3,0.5]\rangle/u_3$, $\langle[0.2,0.8],[0.2,0.8],[0.1,0.6]\rangle/u_4$	$\langle[0.1,0.7],[0.5,0.6],[0.3,0.8]\rangle/u_1$, $\langle[0.2,0.9],[0.4,0.9],[0.1,0.3]\rangle/u_2$, $\langle[0.3,0.8],[0.6,0.9],[0.3,0.7]\rangle/u_3$
x_2	$\langle[0.3,0.9],[0.1,0.4],[0.6,0.8]\rangle/u_1$, $\langle[0.3,0.9],[0.4,0.6],[0.6,0.7]\rangle/u_2$, $\langle[0.2,0.8],[0.3,0.8],[0.1,0.7]\rangle/u_4$	$\langle[0.4,0.6],[0.3,0.4],[0.3,0.9]\rangle/u_1$, $\langle[0.6,0.8],[0.3,0.6],[0.2,0.8]\rangle/u_2$
x_3	$\langle[0.5,0.9],[0.6,0.8],[0.4,0.6]\rangle/u_1$, $\langle[0.6,0.9],[0.4,0.8],[0.4,0.5]\rangle/u_2$, $\langle[0.3,0.8],[0.2,0.9],[0.4,0.7]\rangle/u_3$	$\langle[0.6,0.9],[0.4,0.8],[0.2,0.6]\rangle/u_2$, $\langle[0.4,0.9],[0.5,0.8],[0.3,0.8]\rangle/u_3$, $\langle[0.2,0.8],[0.5,0.9],[0.3,0.7]\rangle/u_4$

Definition 3.6 A tp-ivns-game is called a two-person empty intersection interval-valued neutrosophic soft game if intersection of the ivns-payoffs of players is an empty set for each ivn-action pairs.

Definition 3.7 Let $\nu^{k}_{X\times Y}$ be an ivns-payoffs function of a tp-ivns-game $\Gamma^{k}_{X\times Y}$. Then if the following properties hold

a) $\bigcup_{i=1}^{m} \nu^{k}_{X\times Y}(x_i,y_j) = \nu^{k}_{X\times Y}(x,y) =$

$$\left\{\left(\left[\max T^{k}_{-X\times Y}(x_i,y_j), \max T^{k}_{+X\times Y}(x_i,y_j)\right], \left[\max(I^{k}_{+X\times Y}(x_i,y_j) - I^{k}_{-X\times Y}(x_i,y_j)/2), \min(I^{k}_{+X\times Y}(x_i,y_j) + I^{k}_{-X\times Y}(x_i,y_j)/2)\right], \left[\min F^{k}_{-X\times Y}(x_i,y_j), \min F^{k}_{+X\times Y}(x_i,y_j)\right]\right)\right\}/u_t$$

b) $\bigcap_{j=1}^{n} \nu^{k}_{X\times Y}(x_i,y_j) = \nu^{k}_{X\times Y}(x,y) =$

$$\left\{\left(\left[\min T^{k}_{-X\times Y}(x_i,y_j), \min T^{k}_{+X\times Y}(x_i,y_j)\right], \left[\min(I^{k}_{+X\times Y}(x_i,y_j) - I^{k}_{-X\times Y}(x_i,y_j)/2), \max(I^{k}_{+X\times Y}(x_i,y_j) + I^{k}_{-X\times Y}(x_i,y_j)/2)\right], \left[\max F^{k}_{-X\times Y}(x_i,y_j), \min F^{k}_{+X\times Y}(x_i,y_j)\right]\right)\right\}/u_t$$

then $\nu^{k}_{X\times Y}(x,y)$ is called an interval-valued neutrosophic soft saddle point value and (x,y) is called neutrosophic soft saddle point of player k's in the tp-ivns-game.

Example 3.8 Let $U=\{u_1,u_2,u_3,u_4\}$ be a set of alternatives and $X=\{x_1,x_2,x_3\}$ and $Y=\{y_1,y_2\}$ be the set of strategies of player 1 and player 2 respectively. We consider the tp-ivns-payoffs matrix of player 1 given by

Now,

$$\bigcup_{i=1}^{3} \nu^{1}_{X\times Y}(x_i,y_1) = \left\{\langle[0.6,0.9],[0.35,0.45],[0.1,0.5]\rangle/u_1, \langle[0.4,0.9],[0.25,0.35],[0.2,0.5]\rangle/u_2, \langle[0.5,0.8],[0.35,0.55],[0.1,0.7]\rangle/u_3\right\}$$

$$\bigcup_{i=1}^{3} \nu^{1}_{X\times Y}(x_i,y_2) = \left\{\langle[0.6,0.9],[0.35,0.45],[0.2,0.6]\rangle/u_1, \langle[0.3,0.9],[0.15,0.6],[0.2,0.3]\rangle/u_2, \langle[0.6,0.9],[0.35,0.55],[0.1,0.7]\rangle/u_3, \langle[0.6,0.8],[0.35,0.55],[0.4,0.7]\rangle/u_4\right\}$$

$$\bigcap_{j=1}^{2} \nu^{1}_{X\times Y}(x_1,y_j) = \left\{\langle[0.1,0.4],[0.05,0.85],[0.3,0.7]\rangle/u_1, \langle[0.5,0.8],[0.35,0.65],[0.1,0.7]\rangle/u_3\right\}$$

$$\bigcap_{j=1}^{2} \nu^{1}_{X\times Y}(x_2,y_j) = \left\{\langle[0.2,0.9],[0.15,0.75],[0.7,0.9]\rangle/u_1, \langle[0.2,0.5],[0.15,0.7],[0.4,0.8]\rangle/u_3\right\}$$

$$\bigcap_{j=1}^{2} \nu^{1}_{X\times Y}(x_1,y_j) = \left\{\langle[0.1,0.9],[0.35,0.45],[0.3,0.6]\rangle/u_1, \langle[0.2,0.9],[0.1,0.6],[0.2,0.5]\rangle/u_2, \langle[0.5,0.8],[0.3,0.6],[0.4,0.7]\rangle/u_4\right\}$$

In the above example, tp-ivns-game does not have an interval neutrosophic soft saddle point. But we may find some cases where we get interval neutrosophic soft saddle point. So, it is left for the reader for the motivation to find such example where interval neutrosophic soft saddle point exists.

Definition 3.9 Let $\Gamma_{X\times Y}$ be a tp-ivns-game with its ivns-payoff function $\nu_{X\times Y}$ where

$$\nu_{X\times Y}(x,y) = \left(\left[T_{-X\times Y}(x,y), T_{+X\times Y}(x,y)\right], \left[I_{-X\times Y}(x,y), I_{+X\times Y}(x,y)\right], \left[F_{-X\times Y}(x,y), F_{+X\times Y}(x,y)\right]\right)$$

Where the 1st ,2nd and 3rd interval represent the truth membership value, indeterminacy membership value and the falsity membership value respectively. Then,

a) Interval neutrosophic soft upper value of the tp-ivns-game is denoted by V^* and it is defined as

$$V^* = \left\{\cap\left(\cup \nu_{X\times Y}(x,y)\right)\right\}/u_t$$

$$= \left\{\left[\cap\left(\cup T_{-X\times Y}(x,y)\right), \cap\left(\cup T_{+X\times Y}(x,y)\right)\right], \left[\cap\left(\cup I_{-X\times Y}(x,y)\right), \cup\left(\cap I_{+X\times Y}(x,y)\right)\right], \left[\cup\left(\cap F_{-X\times Y}(x,y)\right), \cup\left(\cap F_{+X\times Y}(x,y)\right)\right]\right\}/u_t$$

b) Interval neutrosophic soft lower value of the tp-ivns-game is denoted by V_* and it is defined as

$$V_* = \left\{\cup\left(\cap \nu_{X\times Y}(x,y)\right)\right\}/u_s$$

$$= \left\{\left[\cup\left(\cap T_{-X\times Y}(x,y)\right), \cup\left(\cap T_{+X\times Y}(x,y)\right)\right], \left[\cup\left(\cap I_{-X\times Y}(x,y)\right), \cap\left(\cup I_{+X\times Y}(x,y)\right)\right], \left[\cap\left(\cup F_{-X\times Y}(x,y)\right), \cap\left(\cup F_{+X\times Y}(x,y)\right)\right]\right\}/u_s$$

c) If interval neutrosophic soft upper (ivns-upper) and interval neutrosophic soft lower (ivns-lower) value of a tp-ivns-game are equal, then they are called the value of the tp-ivns-game and it is denoted by V. So $V=V^*=V_*$.

Example 3.10 For getting the value of the tp-ivns-game we have the following example:
Let $U=\{u_1,u_2\}$ be a set of alternatives, $X=\{x_1,x_2,x_3,x_4,x_5\}$ and $Y=\{y_1,y_2\}$ be the strategies of player 1 and player 2 respectively. Then, tp-ivns-game of player 1 is given in Table 6 as shown below;

Table 6. Two person interval neutrosophic soft game of player 1

$\Gamma^1_{X\times Y}$	y_1	y_2
x_1	$\langle[0.6,0.8],[0.8,0.9],[0.1,0.5]\rangle/u_1$, $\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle/u_3$, $\langle[0.5,0.7],[0.2,0.8],[0.1,0.6]\rangle/u_4$	$\langle[0.1,0.4],[0.5,0.8],[0.3,0.7]\rangle/u_1$, $\langle[0.1,0.9],[0.6,0.9],[0.2,0.3]\rangle/u_2$, $\langle[0.6,0.8],[0.4,0.9],[0.1,0.7]\rangle/u_3$
x_2	$\langle[0.2,0.9],[0.1,0.5],[0.7,0.8]\rangle/u_1$, $\langle[0.4,0.9],[0.1,0.6],[0.5,0.7]\rangle/u_2$, $\langle[0.2,0.5],[0.6,0.9],[0.4,0.7]\rangle/u_3$	$\langle[0.6,0.9],[0.6,0.9],[0.6,0.9]\rangle/u_1$, $\langle[0.5,0.9],[0.6,0.8],[0.1,0.8]\rangle/u_3$
x_3	$\langle[0.2,0.9],[0.1,0.8],[0.3,0.6]\rangle/u_1$, $\langle[0.2,0.9],[0.3,0.8],[0.2,0.5]\rangle/u_2$, $\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle/u_4$	$\langle[0.1,0.9],[0.1,0.8],[0.2,0.6]\rangle/u_1$, $\langle[0.3,0.9],[0.5,0.7],[0.2,0.5]\rangle/u_2$, $\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle/u_3$, $\langle[0.6,0.8],[0.3,0.9],[0.4,0.7]\rangle/u_4$

$$V^* = \begin{Bmatrix} \left(\left[\cap\{0.7,0.6\},\cap\{0.9,0.9\}\right]\left[\cap\{0.8,0.6\},\cup\{0.8,0.5,0.8,0.4,0.2\}\right]\left[\cup\{0.1,0.6,0.2,0.2,0.3\},\cup\{0.5,0.8,0.6,0.6,0.6\}\right]\right)/u_1, \\ \left(\left[\cap\{0.6,0.5\},\cap\{0.9,0.9\}\right]\left[\cap\{0.3,0.6\},\cup\{0.9,0.6,0.7,0.4,0.4\}\right]\left[\cup\{0.1,0.1,0.2,0.5,0.1\},\cup\{0.3,0.7,0.5,0.9,0.8\}\right]\right)/u_2 \end{Bmatrix}$$

$$= \{([0.6,0.9][0.6,0.8][0.6,0.8])/u_1, ([0.5,0.9][0.3,0.9][0.5,0.9])/u_2\}$$

$$V_* = \begin{Bmatrix} \left(\left[\cup\{0.1,0.2,0.1,0.6,0.1\},\cup\{0.4,0.9,0.9,0.9,0.5\}\right]\left[\cup\{0.6,0.1,0.1,0.1,0.1\},\cap\{0.9,0.8\}\right]\left[\cap\{0.7,0.6\},\cap\{0.8,0.9\}\right]\right)/u_1, \\ \left(\left[\cup\{0.1,0.5,0.2,0.1,0.1\},\cup\{0.8,0.9,0.9,0.8,0.3\}\right]\left[\cup\{0.2,0.1,0.3,0.1,0.1\},\cap\{0.9,0.9\}\right]\left[\cap\{0.6,0.5\},\cap\{0.9,0.9\}\right]\right)/u_2 \end{Bmatrix}$$

$$= \{([0.6,0.9][0.6,0.8][0.6,0.8])/u_1, ([0.5,0.9][0.3,0.9][0.5,0.9])/u_2\}$$

Here, $V^*=V_*$

Therefore, there exists a value of the tp-ivns-game . If the value is denoted by V then $V= V^*=V_*$ But it is not necessary that in all the tp-ivns-game such value exist. To justify this we consider the following. In Table 6, if we replace the entries $\langle[0.1,0.4],[0.6,0.8],[0.3,0.7]\rangle/u_1$ of x_1y_2 by $\langle[0.1,0.4],[0.5,0.8],[0.3,0.7]\rangle/u_1$, $\langle[0.6,0.9],[0.6,0.8],[0.6,0.9]\rangle/u_1$ of x_2y_2 by $\langle[0.6,0.9],[0.6,0.9],[0.6,0.9]\rangle/u_1$ and $\langle[0.5,0.9],[0.1,0.6],[0.5,0.7]\rangle/u_2$ of x_2y_1 by $\langle[0.4,0.9],[0.1,0.6],[0.5,0.7]\rangle/u_2$ then we have $V^* = \{([0.6,0.9][0.6,0.8][0.6,0.8])/u_1, ([0.5,0.9][0.3,0.9][0.5,0.9])/u_2\}$

and $V_* = \{([0.6,0.9][0.5,0.9][0.6,0.8])/u_1, ([0.4,0.9][0.3,0.9][0.5,0.9])/u_2\}$

Thus, $V^* \neq V_*$. Therefore, in this case the tp-ivns-game has no value and it justifies the above statement appropriately.

Theorem 3.11 If V_* and V^* be a ivns-lower and ivns-upper value of a tp-ivns-game respectively then, $V_* \subseteq V^*$.

Proof. Proof is obvious.

Theorem 3.12 Let $\nu_{X\times Y}(x,y)$ be an ivns-saddle point value, V^* and V_* be ivns-upper and ivns-lower value of a tp-ivns-game respectively then $V_* \subseteq \nu_{X\times Y}(x,y) \subseteq V^*$.

Proof. proof is straight forward.

4. n-Person Interval Neutrosophic Soft Games

The interval neutrosophic soft games can be often played between more than two players. Therefore, tp-ivns-games can be generalized to n-person interval neutrosophic soft games in which each player has a finite set of pure strategies and in which a definite set of ivns-payoffs to the n players corresponds to each n-tuple of pure strategies and one strategy being taken for each player. For n-person game, all possible available ivn-actions denoted by X_n^* and it stands for $X_1 \times X_2 \times .... \times X_n$.

Definition 4.1 Let U be a set of alternatives $IN^*(U)$ be all interval neutrosophic sets over U, E be a set of n-pure strategies such as $X_1 \times X_2 \times .... \times X_n$ and X_k is the set of strategies of player k,(k=1,2,……,n).Then for each player k, an n-person interval neutrosophic soft game can be defined by a neutrosophic soft set over U as

$$\Gamma^k_{X^*_n} = \left\{ \left((x_1, x_2, ..., x_n), \left[T^k_{-X^*_n}(x_1, x_2, ..., x_n), T^k_{+X^*_n}(x_1, x_2, ..., x_n) \right], \left[I^k_{-X^*_n}(x_1, x_2, ..., x_n), I^k_{+X^*_n}(x_1, x_2, ..., x_n) \right], \left[F^k_{-X^*_n}(x_1, x_2, ..., x_n), F^k_{+X^*_n}(x_1, x_2, ..., x_n) \right] \right) : (x_1, x_2, ..., x_n) \in X^*_n \right\}$$

Where $T^k_{X^*_n} = \left[T^k_{-X^*_n}(x_1, x_2, ..., x_n), T^k_{+X^*_n}(x_1, x_2, ..., x_n) \right]$, $I^k_{X^*_n} = \left[I^k_{-X^*_n}(x_1, x_2, ..., x_n), I^k_{+X^*_n}(x_1, x_2, ..., x_n) \right]$ and $F^k_{X^*_n} = \left[F^k_{-X^*_n}(x_1, x_2, ..., x_n), F^k_{+X^*_n}(x_1, x_2, ..., x_n) \right]$ are called interval neutrosophic soft payoff functions for truth-membership, inderminacy-membership and falsity-membership respectively and $\sup T^k_{X^*_n} + \sup I^k_{X^*_n} + \sup F^k_{X^*_n} \leq 3$.

The n-person ivns-game is played as follows:
At a certain time player 1 chooses a strategy $x_1 \in X_1$ and simultaneously each player k (k=2, 3,….., n) chooses a strategy $x_k \in X_k$ and once this is done each player k receives the interval neutrosophic soft payoffs .

Definition 4.2 Let $T^k_{X^*_n}$, $I^k_{X^*_n}$ and $F^k_{X^*_n}$ be ivns-payoff functions for truth-membership, indeterminacy-membership and falsity-membership respectively of a np-ivns-game $G^k_{X^*_n}$. If for each player k(k=1,2,…..,n) the following properties hold;

$$T^k_{X^*_n}(x_1, x_2, ... x_{k-1}, x_k, x_{k+1} ..... x_n) \supseteq T^k_{X^*_n}(x_1, x_2, ... x_{k-1}, x, x_{k+1} ..... x_n),$$

$$I^k_{X^*_n}(x_1, x_2, ... x_{k-1}, x_k, x_{k+1} ..... x_n) \supseteq I^k_{X^*_n}(x_1, x_2, ... x_{k-1}, x, x_{k+1} ..... x_n) \text{ and}$$

$$F^k_{X^*_n}(x_1, x_2, ... x_{k-1}, x_k, x_{k+1} ..... x_n) \subseteq F^k_{X^*_n}(x_1, x_2, ... x_{k-1}, x, x_{k+1} ..... x_n)$$

For each $x \in X_k$, then $(x_1, x_1, ..., x_1) \in X^*_n$ is called an ivns-Nash equilibrium for np-ivns-game.

5. Application of tp-ivns-game in a decision-making problem

Suppose there are two teams denoted by team A and team B and they are going to clash with each other in a football match. Both the teams have to consider the best strategy for winning the game. Let U= {u_1,u_2} be a set of alternatives and E={x_i:i=1,2,…..,7} be a set of strategies where x_i stands for goal scoring opportunity through free kick, conversion of half chance to full chance, ball possession ability, home advantage, chance of winning the toss, weakness and team formation respectively. Let the set of strategies of team A and team B be denoted by the sets X={x_1,x_2,x_3,x_4,x_5} and Y={y_1,y_2} respectively. Then the all possible action pairs of the ivns-game is denoted by $X \times Y$. If we represent the tp-ivns-game by the Table 6 of example 3.10, then the value of the game exist and it is denoted by $\left\{ \left([0.6, 0.9][0.6, 0.8][0.6, 0.8] \right) / u_1, \left([0.5, 0.9][0.3, 0.9][0.5, 0.9] \right) / u_2 \right\}$. It is the value of the game which is minimum along the row and maximum along the column i.e. by using the min-max principle we have the value of the game and it is considered as the best strategy of winning the game of one team and losing the game of other team.

6. A Comparative Study of Fuzzy Soft Game, Intuitionistic Fuzzy Soft Game, Neutrosophic Soft Game and Interval Valued Neutrosophic Soft Game Theory

We illustrate the comparative study of different types of game theory by using Table 7, given below.

7. Conclusions

In this work we deal with game theory in neutrosophic environment and introduce a new theory of games which is called interval neutrosophic soft game. We actually extend the concept defined by Anjan et al. in [25,26]. Then we have defined interval neutrosophic soft saddle point in tp-ivns-game. We also define ivns-lower value and ivns-upper value and establish a relation between them with the help of an example which actually gives the value of the game. Main objective of preparing this paper was to extend the notion of interval valued intuitionistic fuzzy soft game theory so that the range of the domain will be wider and help us to explain the problems based on uncertainty in an appropriate way. Finally we give an application of tp-ivns-game to show its practical use in the real world. We also illustrate the comparative study of different types of game theory. Apart from this, by combining the concept of neutrosophic set and hypersoft set we obtain neutrosophic hypersoft set and by using the concept of neutrosophic hypersoft set. In future there is scope for extending the earlier notions of game theory and we can use it for determining different strategies in solving real-world problems.

Table 7. Comparative study of fuzzy soft game, intuitionistic fuzzy soft game, neutrosophic soft game and interval valued neutrosophic soft game theory

	Advantages	Limitations
Fuzzy soft game theory	It is used for dealing with uncertainties that is based on both soft sets and fuzzy sets. In this game we use fuzzy soft payoff functions. Payoff functions of the fuzzy soft game are set valued function. We obtain the solution of the game by using the operations of soft sets and fuzzy sets that boost this game very comprehensive and easy approach in practice. It is an extension of soft game where corresponding to each players strategy there corresponds to the membership value which meas-ures the degree of belongingness and the value belongs to the standard unit interval [0, 1].	As we know that in most of the real world problems, for every membership value there corresponds a non membership value and it gives a better shape to describe the uncertainty in a balanced manner. But in this game payoffs only take the membership value.
Intuitio-nistic fuzzy soft game theory	Intuitionistic fuzzy soft game has more applicability in uncertainty and imprecision. It is used for dealing with uncertainties and is based on both intuitionistic fuzzy set and soft set. In this game we find the degree of membership of payoffs and the degree of non membership of payoffs. Intuitionistic fuzzy soft payoffs can be converted to fuzzy soft payoffs under some conditions. Here both membership and non-membership values belong to the closed interval [0, 1] . In this game to obtain the solution we use the operations of intuitionistic fuzzy set and soft set. It is the extension of soft set, fuzzy soft set.	In this game, the degree of membership of the payoffs and the degree of non-membership of the payoffs depend on each other and their sum cannot exceed one.
Neutros-ophic soft game theory	It is an extension of intuitionistic fuzzy soft game. In intuitionistic fuzzy soft game we are mainly concerned with uncertainty and incompleteness of data. But in various scientific problem we have to deal with the concept of indeterminacy, and the concept of indeterminacy can be handled appropriately by using neutrosophic set. In neutrosophic soft game we find the degree of truth-membership of payoffs, degree of false-membership of payoffs and degree of indeterminacy-membership of payoffs. All the membership functions are independent of each other and each belongs to the unit closed interval [0, 1] (as we use the word 'neutrosophy' in scientific point of view).	Sometimes it is difficult to measure the membership value or non-membership value or indeterminacy value of payoffs by a single real value and it occurs due to the more complexity involved in the real world problem. To find the solution of the game, calculation is tedious.
Interval valued neutros-ophic soft game theory	It is an extension of neutrosophic soft game, where the truth members-hip payoffs, falsity-membership payoffs and indeterminacy members-hip payoffs are all the subset of the unit closed interval [0, 1]. We use this concept when it is difficult to measure the indeterminacy by a single point. It is the more generalized form which we use for modeling human behaviour. It is an amalgamation of interval valued fuzzy set, neutrosophic set and soft set. So there is a wide range of domain to handle the concept of indeterminacy.	Calculation is tedious in finding the solution of the game.

References

Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20*, 87-96.

Atanassov, K., & Gargov, G. (1989). Interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 31*, 343-349.

Broumi, S., Smarandache, F., & Dhar, M. (2014). Rough neutrosophic sets. *Neutrosophic Sets and Systems, 3*, 60-66.

Broumi, S., & Smarandache, F. (2015). Interval neutrosophic rough sets. *Neutrosophic Sets and Systems, 7*, 23-31.

Broumi, S., Son, L.H., Bakali. A., Talea, M., Smarandache, F., & Selvachandran, G. (2017). Computing operational matrices in neutrosophic environments: A matlab toolbox. *Neutrosophic Sets and Systems, 18,* 58-66.

Broumi, S., Bakali. A., Talea, M., & Smarandache, F. (2017). A matlab toolbox for interval valued neutrosophic matrices for computer applications. *Uluslararasi Yonetim Bilisim Sistemlerive Bilgisayar Bilimleri Dergisi, 1,* 1-21.

Broumi, S., Talea, M., Bakali. A., Singh, P. K., & Smarandache, F. (2019). Energy and spectrum analysis of interval valued neutrosophic graph using matlab. *Neutrosophic Sets and Systems, 24,* 46-60.

Broumi, S., Nagarajan, D., Bakali. A., Talea, M., Smarandache, F., & Lathamaheswari, M. (2019). The shortest path problem in interval valued trapezoidal and triangular neutrosophic environment. *Complex and Intelligent Systems, 5,* 391-402. doi:10.1007/s40747-019-0092-5

Cagman, N., & Deli, I. (2013). Soft games. Retreived from http://arxiv.org/abs/1302.4568 2013.

Cagman, N., & Deli, I. (2015). Fuzzy soft games. *Filomat, 29,* 1901-1917.

Cagman, N., & Deli, I. (2016). Probabilistic equilibrium solution of soft games. *Journal of Intelligent and Fuzzy Systems, 30*, 2237-2244.

Cagman, N., & Deli, I. (2016). Application of soft sets in decision making based on game theory. *Annals of Fuzzy Mathematics and Informatics, 11*, 425-438.

Cagman, N., & Enginoglu, S. (2010). Soft set theory and uni-int decision making. *European Journal of Operational Research, 207,* 848-855.

Chetia, B., & Das, P. K. (2010). An application of interval-val ued fuzzy soft sets in medical diagnosis. *Internation al Journal of Contemporary Mathematical Sciences, 5*, 1887 – 1894.

Deli, I. (2013). Applications of soft and fuzzy soft sets to Gam e Theory (in Turkish) (Doctoral thesis, *Gaziosmanpasa University, Tokat, Turkey)*.

Deli, I. (2014). Interval-valued neutrosophic soft sets and its decision making. Retrieved from arXiv:1402.3130[math.GM]

Edalatpanah, S. A. (2020). A direct model for triangular neutrosophic linear programming. *International Journal of Neutrosophic Science, 1,* 19-28.

Feng, F. (2011). Soft rough sets applied to multicriteria group decision making. *Annals of Fuzzy Mathematics and Informatics, 2,* 69-80.

Feng, F., Liu, X., Leoreanu, V., & Young, J. B. (2011). Soft sets and soft rough sets. *Information Sciences, 181*, 1125-1137.

Gau, W. L., & Buehrer, D. J. (1993). Vague sets. *IEEE Transactions on Systems, Man and Cybernetics, 23*, 610-614.

Gorzalczany, M. (1987). A method of inference in approximate reasoning based on interval-valued fuzzy sets. *Fuzzy Sets and Systems, 21,* 1-17.

Jiang, Y., Tang, Y., Chen, Q., Liu, H., & Tung, J. (2010). Interval-valued intuitionistic fuzzy soft sets and their properties. *Computers and Mathematics with Applications, 60*, 906-918, 2010.

Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers and Mathematics with Applications, 45*, 555-562.

Maji, P. K., Biswas, R., & Roy, A. R. (2001). Fuzzy soft sets. *Journal of Fuzzy Mathematics, 9*, 589-602.

Maji, P. K., Biswas, R., & Roy, A. R. (2004). Intuitionistic fuzzy soft sets. *Journal of Fuzzy Mathematics, 12*, 669-683.

Maji, P. K. (2013). Neutrosophic soft set. *Annals of Fuzzy Mathematics and Informatics, 5*, 157-168.

Maji, P.K. (2012). A neutrosophic soft set approach to a decision making problem. *Annals of Fuzzy mathematics and Informatics, 3*, 313-319.

Moinuddin, K. (2017). Rough soft sets. *International Journal of Computational and Applied Mathematics, 12,* 537-543.

Molodtsov, D. (1999). Soft set theory-first results. Computers and Mathematics with Application, 37, 19-31.

Mukherjee, A., & Debnath, S. (2016). Generalized fuzzy soft sets in decision making based on game theory and their applications. *Bulletin of Calcutta Mathematical Society,108*, 431-448.

Mukherjee, A., & Debnath, S. (2018). Intuitionistic fuzzy soft game theory. *Songlanakarin Journal of Science and Technology, 40*, 409-417.

Mullai, M., Sangeetha, K., Surya, R., Madhan Kumar, G., Jeyabalan, R., & Broumi, S. (2020). A single valued neutrosophic inventory model with neutrosophic random variable. *International Journal of Neutrosophic Science, 1,* 52-63.

Neumann, J. V., & Morgenstern, O. (1944). *The theory of games and economic behaviour*. Princeton, NJ: Princeton University Press.

Pawlak, Z. (1982). Rough sets. *International Journal of Computing and Information Sciences, 11*, 341-356.

Rattana, A., & Chinram, R. (2020). Applications of neutrosophic N-structures in ternary semigroups. *Journal of Mathematical and Computational Science, 10,* 1437-1455.

Rattana, A., & Chinram, R. (2020). Applications of neutrosophic N-structures in n-ary groupoids. *European Journal of Pure and Applied Mathematics, 13,* 200-215.

Roy, A. R., & Maji, P.K. (2007). A fuzzy soft set theoretic approach to decision making problems. *Journal of Computational and Applied Mathematics, 203,* 412–418.

Sahin, M., Alkhazaleh, S., & Ulucay, V. (2015). Neutrosophic soft expert sets. Applied Mathematics, 6, 116-127.

Salama, A. A., Bondok Henawy, M.S., & Alhabib, R. (2020). Online analytic processing operations via neutrosophic systems. *International Journal of Neutrosophic Science, 8,* 87-109.

Shyamal, A. K., & Pal, M. (2004). Notes on intuitionistic fuzzy sets. *Journal of Applied Mathematics and Computing, 15*, 91-107.

Smarandache, F. (1995).Neutrosophic logic and set. Retrieved from http://fs.gallup.unm.edu/neutrosophy.htm,1995

Smarandache, F. (1998). Neutrosophy, neutrosophic probability, set, and logic. American Research Press, Rehoboth, USA (4th ed.), 105, 1998. Retrieved from http://fs.gallup.unm.edu/eBook-neutrosophics4.pdf.

Smarandache, F. (2005). Neutrosophic set-a generalization of the intuitionistic fuzzy sets. *International Journal of Pure and Applied Mathematics, 24*, 287–297.

Smarandache, F., & Broumi, S. (2015). Single valued neutrosophic soft expert sets and their application in decision making. *Journal of New Theory, 3*, 67-88.

Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to pilthogenic hypersoft set. *Neutrosophic Sets and Systems, 22,* 168-170.

Wang, H., Smarandache, F., Zhang, Y.Q., & Sunderraman, R. (2005). Interval neutrosophic sets and logic theory and applications in computing Hexis. Neutrosophic book series, No 5, 2005.

Yang, X. B., Lin, T. Y., Yang, J. Y., Li, Y., & Yu, D. (2009). Combination of interval-valued fuzzy set and soft set. *Computers and Mathematics with Applications, 58*, 521-527.

Yao, B., Liu, J., & Yan, R. (2008). Fuzzy soft set and soft fuzzy set. *Fourth International Conference on Natural Computation, 6*, 2008.

Zadeh, L. A. (1965). Fuzzy set. *Information and Control, 8*, 338-353.

Zhenhua, Z., Jingyu, Y., Youpei, Y., & Qian Sheng, Z. (2011). Interval-valued intuitionistic fuzzy sets. *Procedia Engineering, 15*, 2037-2041.