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Some fuzzy congruences on Γ-semigroups

สุภาวิณี สัตยาภรณ์

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Supavinee Sattayaporn *

Abstract

In this research, we investigate this concepts intuitionistic (α, β)-fuzzy subsemigroup and intuitionistic (α, β)-fuzzy interior ideal on Γ-semigroup where α, β denote any one of $\epsilon, q, \epsilon \vee q$ or $\epsilon \wedge q$ unless otherwise specified. Finally, we find some fuzzy congruence on Γ-semigroup.

Keywords fuzzy congruence, intuitionistic fuzzy congruence , fuzzy band congruence
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1 Introduction

The fundamental concept of a fuzzy set, introduced by Zadeh [17] in 1965, has been applied by many authors to generalize some of the basic notions of algebra. Since then, there has been a tremendous interest in the subject due to its diverse applications ranging from engineering and computer science to social behavior studies. Subsequencetly, Goguen[3] and Sanchez[14] studied fuzzy relations in various contexts. In Nemitz [11] discussed fuzzy equivalence relations, fuzzy functions as fuzzy relations, and fuzzy partitions. Murali [10] developed some properties of fuzzy equivalence relations and certain lattice theoretic properties of fuzzy

*Faculty of Science and Technology, Uttaradit Rajabhat University, Uttaradit 53000 Thailand
E-mail : supavinee_uru@windowslive.com

equivalence relations. Samhan [13] characterized the fuzzy congruences generated by fuzzy relations on a semigroup and studied the lattice of fuzzy congruences an a semigroup. Kuroki [6] introduced the notion of a quotient semigroup S/ρ induced by a fuzzy congruence relation ρ on a semigroup S, and give homomorphism theorem with respect to the fuzzy congruence and showed that an idempotent separating fuzzy congruence, a group fuzzy congruence on inverse semigroups. Tan [15] proved that the set of all fuzzy congruences on a regular semigroup contained in δ_H forms a modular lattice, where δ_H is the characteristic function of H and H is the H-equivalent relation on the semigroup. Next, they showed that the idempotent separating fuzzy congruences on a regular semigroup form a modular lattice and proved that the lattice of fuzzy congruence on a regular semigroup is a disjoint union of some modular sublattices of the lattice. In 2006, Hur, Jang and Kang [5] introduced the concept of intuitionistic fuzzy group congruence, intuitionistic fuzzy normal congruence and proved that an intuitionistic fuzzy normal congruence on idempotent of an inverse semigroup is complete lattice.

A fuzzy relation $\rho : S \times S \to [0,1]$ on a Γ-semigroup S is called a fuzzy congruence on S if (1) $\rho(a,a) = 1$ (2) $\rho(a,b) = \rho(b,a)$ (3) $\rho(a,c) \geqslant \rho(a,b) \wedge \rho(b,c)$ for any $a,b,c \in S$ and (4) $\rho(a\alpha b, c\alpha d) \geqslant \rho(a,c) \wedge \rho(b,d)$ for any $a,b,c,d \in S, \alpha \in \Gamma$. A fuzzy congruence ρ on a Γ-semigroup is called group fuzzy congruence on S if S/ρ is a group. A fuzzy congruence μ on S is called an idempotent separating fuzzy congruence on S is for any idempotent $e, f \in S$ if $\mu_e = \mu_f$ implies $e = f$.

In this research, we investigate this concepts intuitionistic (α,β)-fuzzy subsemigroup and intuitionistic (α,β)-fuzzy interior ideal on Γ-semigroups where α, β denote any one of $\epsilon, q, \epsilon \vee q$ or $\epsilon \wedge q$ unless otherwise specified. Finally, we find some fuzzy congruence on Γ-semigroup.

2 Preliminaries

Let S be a Γ-semigroup. A function α from $S \times S$ to the unit interval $[0,1]$ is called a *fuzzy relation* on S. Let α and β be two fuzzy relations on S. The product $\alpha \circ \beta$ of α and β is defined by

$$(\alpha \circ \beta)(a,b) = \sup_{x \in S}[\min\{\alpha(a,x), \beta(x,b)\}]$$

For all $a, b \in S$, and $\alpha \subseteq \beta$ is defined by $\alpha(x) \leqslant \beta(x)$ for all $x \in S$.

A fuzzy relation μ on a Γ-semigroup S is called a *fuzzy equivalence relation* on S if

(1) $\mu(a, a) = 1$ for all $a \in S$,

(2) $\mu(a, b) = \mu(b, a)$ for all $a, b \in S$,

(3) $\mu \circ \mu \subseteq \mu$.

A fuzzy equivalence relation μ on S is a *fuzzy congruence relation* on S if

$$\mu(a\alpha x, b\alpha x) \geqslant \mu(a, b) \quad \text{and} \quad \mu(x\alpha a, x\alpha b) \geqslant \mu(a, b)$$

for all $a, b, x \in S, \alpha \in \Gamma$.

Let μ be a fuzzy equivalence relation on S. For each $a \in S$ we define a fuzzy subset μ_a of S as follows :

$$\mu_a(x) = \mu(a, x)$$

for all $x \in S$.

Let μ be fuzzy congruence relation on a semigroup S. μ is called *idempotent separating* if for all $e, f \in E_\alpha(S)$ the equality $\mu_e = \mu_f$ implies $e = f$. Let μ be a fuzzy equivalence relation on S and let $a, b \in S$. Then $\mu_a = \mu_b$ if and only if $\mu(a, b) = 1$. Let α and β be fuzzy subset of S. Then the product $\alpha \circ \beta$ of α and β is defined by

$$(\alpha \circ \beta)(x) = \begin{cases} \sup\limits_{x=yz} [\min\{\alpha(y), \beta(z)\}] & \\ 0 & \text{if } x \text{ is not expressible as } x = yz. \end{cases}$$

Let μ be a fuzzy equivalence relation on a Γ-semigroup S. Then

$$\mu^{-1}(1) = \{(a, b) \in S \times S \mid \mu(a, b) = 1\}$$

is a congruence relation on S.

2.1 Theorem. *The following are equivalent for a regular semigroup S, μ is a fuzzy congruence relation on S and $a \in S, \alpha \in \Gamma$:*

(1) $\mu_a \in E_\alpha(S/\mu)$.

(2) $\mu_a = \mu_e$ for some $e \in E_\alpha(S)$ such that $S\alpha e \subseteq S\alpha a$ and $e\alpha S \subseteq a\alpha S$.

(3) $\mu_a = \mu_e$ for some $e \in E_\alpha(S)$.

3 Main Results

In this section, we introduced and some properties are investigated. We characterize a Γ-semigroup by means of intuitionistic (α, β)-fuzzy ideals and intuitionistic (α, β)–fuzzy interior ideals.

Let S be a Γ-semigroup and α, β denote any one of $\epsilon, q, \epsilon \vee q$ or $\epsilon \wedge q$ unless otherwise specified.

3.1 Definition. An intuitionistic fuzzy Γ-subsemigroup $A = (\mu_A, \gamma_A)$ of a semigroup S is called an *intuitionistic* (α, β)*-fuzzy* Γ*-subsemigroup* of S, where $\alpha \neq \epsilon \wedge q$ if

(1) $x_{t_1} \alpha \mu_A$ and $y_{t_2} \alpha \mu_A \Rightarrow (xy)_{\min\{t_1,t_2\}} \beta \mu_A$

(2) $x_{t_3} \alpha \gamma_A$ and $y_{t_4} \alpha \gamma_A \Rightarrow (xy)_{\min\{t_3,t_4\}} \beta \gamma_A$

for all $x, y \in S, t_1, t_2, t_3, t_4 \in [0, 1]$.

3.2 Definition. An intuitionistic fuzzy subsemigroup $A = (\mu_A, \gamma_A)$ of a semigroup S is called an *intuitionistic* (α, β)*-fuzzy interior ideal* of S if

(1) $a_t \alpha \mu_A \Rightarrow (xay)_t \beta \mu_A$

(2) $(xay)_t \alpha \gamma_A \Rightarrow a_t \beta \gamma_A$

for all $x, y, a \in S, t \in [0, 1]$.

3.1 Lemma. *An intuitionistic fuzzy subset $A = (\mu_A, \gamma_A)$ of a semigroup S is an intuitionistic fuzzy (ϵ, ϵ)-subsemigroup of S if and only if it satisfies for all $x, y \in S$ and $t_1, t_2, t_3, t_4 \in (0, 1]$,*

$$x_{t_1} \epsilon \mu_A, y_{t_2} \epsilon \mu_A \Rightarrow (xy)_{\min\{t_1,t_2\}} \epsilon \mu_A,$$

and

$$x_{t_3} \epsilon \gamma_A, y_{t_4} \epsilon \gamma_A \Rightarrow (xy)_{\min\{t_3,t_4\}} \epsilon \gamma_A.$$

Proof. Suppose $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy (ϵ, ϵ)-subsemigroup of a semigroup S. Let $x, y \in S$ and $t_1, t_2, t_3, t_4 \in (0, 1]$ be such that $x_{t_1} \epsilon \mu_A, y_{t_2} \epsilon \mu_A, x_{t_3} \epsilon \gamma_A$ and $y_{t_4} \epsilon \gamma_A$. Then $\mu_A(x) \geqslant t_1$ and $\mu_A(y) \geqslant t_2$. Since $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy (ϵ, ϵ)-subsemigroup of S, then

$$\mu_A(xy) \geqslant \min\{\mu_A(x), \mu_A(y)\} \geqslant \min\{t_1, t_2\}.$$

Thus $(xy)_{\min\{t_1,t_2\}} \epsilon \mu_A$.

Since $x_{t_3} \epsilon \gamma_A$ and $y_{t_4} \epsilon \gamma_A$, so $\gamma_A(x) \leqslant t_3$ and $\gamma_A(y) \leqslant t_4$ which implies that

$$\gamma_A(xy) \leqslant \max\{\gamma_A(x), \gamma_A(y)\} \leqslant \max\{t_3, t_4\}.$$

Hence $(xy)_{\max\{t_3,t_4\}}\epsilon\gamma_A$.

Conversely, assume that $A = (\mu_A, \gamma_A)$ satisfies the given condition. We will show that $\mu_A(xy) \geqslant \mu_A(x) \wedge \mu_A(y)$ and $\gamma_A(xy) \geqslant \gamma_A(x) \vee \gamma_A(y)$ for all $x, y \in S$.

On the contrary assume that there exist $x, y \in S$ such that

$$\mu_A(xy) < \mu_A(x) \wedge \mu_A(y)$$

and

$$\gamma_A(xy) > \gamma_A(x) \vee \gamma_A(y).$$

Let $t \in (0, 1]$ be such that

$$\mu_A(xy) < t < \mu_A(x) \wedge \mu_A(y)$$

and

$$\gamma_A(xy) > t > \gamma_A(x) \vee \gamma_A(y).$$

Then $x_t\epsilon\mu_A$ and $y_t\epsilon\mu_A$ but $(xy)_t\bar{\epsilon}\mu_A$. This contradicts our hypothesis. Similarly, we have $x_t\epsilon\gamma_A$ and $y_t\epsilon\gamma_A$ but $(xy)_t\bar{\epsilon}\gamma_A$.

Hence $\mu_A(xy) \geqslant \mu_A(x) \wedge \mu_A(y)$ and $\gamma_A(xy) \leqslant \gamma_A(x) \vee \gamma_A(y)$ this implies that A is an intuitionistic fuzzy (ϵ, ϵ)-subsemigroup of S. □

3.1 Theorem. *Let $A = (\mu_A, \gamma_A)$ be a nonzero intuitionistic (α, β)-fuzzy subsemigroup of S. Then the set $\mu_0 = \{x \in S \mid \mu > 0\}$ and $\gamma_1 = \{x \in S \mid \gamma(x) < 1\}$ are subsemigroup of S.*

Proof. Let $x, y \in \mu_0$. Then $\mu(x) > 0$ and $\mu(y) > 0$. Let $\mu(xy) = 0$. If $\alpha \in \{\epsilon, \epsilon \vee q\}$, then $x_{\mu(x)\alpha\mu}$ and $y_{\mu(x)}\alpha\mu$ but $\mu(xy) = 0 < \min\{\mu(x), \mu(y)\}$ and $\mu(xy) + \min\{\mu(x), \mu(z)\} \leqslant 0 + 1 = 1$. So $(xy)_{\min\{\mu(x),\mu(y)\}}\overline{\beta}\mu$ for every $\beta \in \{\epsilon, q, \epsilon \vee q, \epsilon \wedge q\}$, a contradiction. Hence $\mu(xy) > 0$ that is $xy \in \mu_0$.

Let $x, y \in \gamma_1$. Then $\gamma(x) < 1$ and $\gamma(y) < 1$. Let $\gamma(xy) = 1$. If $\alpha \in \{\epsilon, \epsilon \vee q\}$, then $x_{\gamma(x)}\alpha\gamma$ and $y_{\gamma(x)}\alpha\gamma$ but $\gamma(xy) = 1 > \max\{\gamma(x), \gamma(y)\}$ and $\gamma(xy) + \max\{\gamma(x), \gamma(y)\} \geqslant 1 + 1 = 2$.
So $(xy)_{\max\{\gamma(x),\gamma(y)\}}\overline{\beta}\gamma$ for every $\beta \in \{\epsilon, q, \epsilon \vee q, \epsilon \wedge q\}$ a contradiction.
Hence $\gamma(xy) < 1$, that is $(xy)\epsilon\gamma_1$. Thus μ_0 and γ_1 are subsemigroup of S. □

3.2 Theorem. *If $\{A_i\}_{i\in\Lambda}$ is a family of intuitionistic (α,β)-fuzzy interior ideals of S, then $\cap A_i$ is an intuitionistic (α,β)-fuzzy interior ideal of S where $\cap A_i = (\wedge\mu_{A_i}, \vee\gamma_{A_i})$ and $\wedge\mu_{A_i}$ and $\vee\gamma_{A_i}$ are defined as follows :*

$$\wedge\mu_{A_i}(x) = \inf\{\mu_{A_i}(x) \mid i \in \Lambda, x \in S\}$$

$$\vee\gamma_{A_i}(x) = \sup\{\gamma_{A_i}(x) \mid i \in \Lambda, x \in S\}.$$

Proof. Let $x, y, a \in S, t_1, t_2 \in (0, 1]$.

(1) We will show that $\wedge\mu_{A_i}$ is intuitionistic (α,β)-fuzzy subsemigroup of S. Let $x_{t_1}\alpha\wedge\mu_{A_i}$ and $y_{t_2}\alpha\wedge\mu_{A_i}$. Then $\wedge\mu_{A_i}(x) \geqslant t_1$ and $\wedge\mu_{A_i}(y) \geqslant t_2$ which implies that

$$\wedge\mu_{A_i}(xy) \geqslant \min\{\wedge\mu_{A_i}(x), \wedge\mu_{A_i}(y)\} \geqslant \min\{t_1, t_2\}.$$

Thus $(xy)_{\min\{t_1,t_2\}}\beta\wedge\mu_{A_i}$.

Suppose that $a_t\alpha\wedge\mu_{A_i}$. Then $\wedge\mu_{A_i}(a) \geqslant t$ for all $t \in (0, 1]$. Since A_i is an intuitionistic (α,β)-fuzzy interior ideal of S, we get that $\mu_{A_i}(xay) \geqslant \mu_{A_i}(a)$ for all $i \in \Lambda$. Then $\wedge\mu_{A_i}(xay) \geqslant t$, so $(xay)_t\beta\wedge\mu_{A_i}$.

(2) We will show that $\vee\gamma_{A_i}$ is intuitionistic (α,β)-fuzzy subsemigroup of S. Let $x_{t_1}\alpha\vee\gamma_{A_i}$ and $y{-}t_2\alpha\vee\gamma_{A_i}$. Then $\vee\gamma_{A_i}(x) \leqslant t_1$ and $\vee\gamma_{A_i}(y) \leqslant t_2$ which implies that $\vee\gamma_{A_i}(xy) \leqslant \max\{\vee\gamma_{A_i}(x), \vee\gamma_{A_i}(y)\} \leqslant \max\{t_1, t_2\}$. Thus $(xy)_{\max\{t_1,t_2\}}\beta\vee\gamma_{A_i}$.

Suppose that $a_t\alpha\vee\gamma_{A_i}$. Then $\vee\gamma_{A_i}(a) \leqslant t$ for all $t \in (0, 1]$. Since A_i is an intuitionistic (α,β)-fuzzy intuitionistic of S, we get that $\gamma_{A_i}(xay) \leqslant \gamma_{A_i}(a)$ for all $i \in \Lambda$. Then $\vee\gamma_{A_i}(xay) \leqslant t$ implies that $(xay)_t\beta\vee\gamma_{A_i}$.

Hence $\cap A_i$ is an intuitionistic (α,β)-fuzzy interior ideal of S. □

A fuzzy congruence ρ on a semigroup S is called a *group fuzzy congruence* if S/ρ is a group under the binary operation $*$ defined by

$$\rho_a * \rho_b = \rho_{ab}$$

for any $\rho_a, \rho_b \in S/\rho$.

Clifford and Preston [2] showed that let S be a regular semigroup. Define

$$\rho = \{(a, b) \in S \times S \mid (sat, sbt) \in \mathcal{H} \text{ for all } s, t \in S^1\}$$

the maximum idempotent separating congruence on S.

3.3 Proposition. *Let S be a regular semigroup and σ be intuitionistic fuzzy congruence on S. If σ_a is an idempotent element of S/σ, then there exists $e \in E(S)$ such that $\sigma_e = \sigma_a$.*

Koroki[8] investigated the maximum idempotent separating fuzzy congruence and group fuzzy congruence on an inverse semigroup.

3.2 Lemma. *(Howie [4], p.141, Theorem 3.2) If S is an inverse semigroup with semilattice of idempotents $E(S)$, then the relation*

$$\eta = \{(a, b) \in S \times S \mid a^{-1}ea = b^{-1}eb \text{ for all } e \in E(S)\}$$

is the maximum idempotent separating congruence on S.

3.3 Lemma. *(Al-thu Kair [1], Proposition 2.7) Let μ be a fuzzy congruence relation on an inverse semigroup S. Then S/μ is an inverse semigroup, and $\mu(a^{-1}, b^{-1}) = \mu(a, b)$ for all $a, b \in S$.*

3.4 Theorem. *[8] Let S be an inverse semigroup. Then the fuzzy relation λ_η is an idempotent separating fuzzy congruence on S.*

Notation λ_R is the characteristic function of a binary relation R on a semigroup S.

3.5 Theorem. *[8] Let μ be a fuzzy congruence relation on an inversive semigroup S. Then μ is idempotent separating congruence if and only if $\mu^{-1}(1) \subseteq \eta$.*

3.4 Lemma. *(Howie [4], p.139, Theorem 3.1) If S is an inverse semigroup with semilattice of idempotent $E(S)$, then the relation*

$$\sigma = \{(a, b) \in S \times S \mid ea = eb \text{ for all } e \in E(S)\}$$

is the least group congruence relation on S.

3.6 Theorem. *[8] Let S be an inverse semigroup. Then λ_σ is a group fuzzy congruence relation on S.*

3.7 Theorem. *[8] Let S be an inverse semigroup and μ be a fuzzy congruence relation on S. Then μ is a group fuzzy congruence if and only if $\sigma \subseteq \mu^{-1}(1)$.*

The second, Zhang [18] investigated fuzzy group congruences on a regular semigroup.

3.8 Theorem. *[18] Let S be a regular semigroup. Then fuzzy congruence ρ on S is a fuzzy group congruence if and only if $\rho(e, f) = 1$ for any $e, f \in E(S)$.*

In 2001, Tan [15] investigated idempotent separating fuzzy congruence on a regular semigroup.

3.9 Theorem. *[15] Let S be a regular semigroup. Then a fuzzy congruence ρ on S is idempotent separating fuzzy congruence on S if and only if $\rho \in \Sigma(_H)$. [$\rho \in \lambda_H$]*

3.10 Theorem. *[16] Let S be an E-inversive semigroup and ρ be a t-fuzzy congruence on S. Then ρ is a group t-fuzzy congruence if and only if*

(1) $\rho(e, f) = t$ for all $e, f \in E(S)$
(2) $\rho(aa'a, a) = t$ for all $a \in S, a' \in W(a)$.

The first section, we investigated a group fuzzy congruence on an orthodox semigroup inwhich as a group congruence on orthodox semigroup and as in Zhang [18].

3.11 Theorem. *(Mills, [9], Theorem 2.2) If S is an orthodox semigroup. Then ρ is the least group congruence on S if and only if*

$$\rho = \{(a, b) \in S \times S \mid eae = ebe \text{ for some } e \in E(S)\}$$

3.5 Lemma. *(Kuroki, [6], Theorem 2.4) Let S be a semigroup and R be a binary relation on S. Then R is a congruence on S if and only if λ_R is a fuzzy congruence on S.*

The following Theorem 3.12 can be easily seen.

3.12 Theorem. *Let S be an orthodox semigroup. Then λ_ρ is a group fuzzy congruence on S.*

Proof. By Lemma 3.5. □

3.6 Lemma. *[16] Let ρ be a t-fuzzy congruence on a semigroup S. For any $a \in S$, define a fuzzy subset ρ_a in S as follows : $\rho_a(x) = \rho(a, x)$ for all $x \in S$, and*

(1) $\rho_a = \rho_b$ if and only if $\rho(a, b) = t$ for all $a, b \in S$.

(2) $S/\rho = \{\rho_a \mid a \in S\}$ is a semigroup with the multiplication $\rho_a\rho_b = \rho_{ab}$ for all $a, b \in S$.

Zhang [18], studied a group fuzzy congruence on regular semigroup in term $\rho(e, f) = 1$ for any $e, f \in E(S)$, we investigated a group t-fuzzy congruence on an orthodox semigroup as in [18].

3.13 Theorem. *Let S be an orthodox semigroup. Then a fuzzy congruence ρ on S is a group t-fuzzy congruence on S if and only if $\rho(e, f) = 1$ for any $e, f \in E(S)$.*

Proof. Suppose that ρ is a group t-fuzzy congruence on S. Then for any $t \in [0, 1]$, S/ρ is a group and ρ_e is an identity of S/ρ for all $e \in E(S)$.

Its implies that for any $e, f \in E(S), \rho_e = \rho_f$ and $\rho(e, f) = t$.

Conversely, let $a \in S, a' \in V(a)$ and $e \in E(S)$. For any $t \in [0, 1]$, we shall show that S/ρ is a group. Suppose $\rho(e, f) = t$ for all $e, f \in E(S), t \in [0, 1]$ and so $(e, f) \in \rho$ its implies that $\rho_e = \rho_f$ for all $e, f \in E(S)$ then $\rho_{(aa')} = \rho_{(a'a)} = \rho_e$. By Lemma 3.6,

$$\rho_a = \rho_{aa'a} = \rho_a\rho_{a'a} = \rho_a\rho_e$$

and

$$\rho_a = \rho_{(aa')a} = \rho_{aa'}\rho_a = \rho_e\rho_a.$$

Thus ρ_e is an identity of S/ρ.
Let $a \in S, a' \in V(a)$ and $a\rho \in S/\rho$, we have $\rho_a\rho_{a'} = \rho_{aa'} = \rho_e$ and $\rho_{a'}\rho_a = \rho_{aa'} = \rho_e$, so $\rho_{a'}$ is the inverse of $\rho_a \in S/\rho$.

S/ρ is a group t-fuzzy congruence for any $t \in [0, 1]$. □

3.7 Lemma. *Let μ be a fuzzy congruence on an orthodox semigroup S. Then S/μ is an orthodox semigroup.*

Proof. Since S is an orthodox semigroup, S is a regular and μ is a fuzzy congruence on S, then S/μ is a regular semigroup. Let $e, f \in E(S)$ with $ef \in E(S)$. Since $\mu_{ef} \in S/\mu$. For any $\mu_e, \mu_f \in E(S/\mu)$,

$$\mu_e\mu_f = \mu_{ef} \quad \text{and} \quad \mu_{ef}\mu_{ef} = \mu_{(ef)^2} = \mu_{ef}.$$

Thus $\mu_{ef} \in E(S/\mu)$ and so S/μ is an orthodox semigroup. $\square$

3.14 Theorem. *Let S be an orthodox semigroup and μ be a fuzzy congruence on S. If μ is a group fuzzy congruence on S implies $\rho \subseteq \mu^{-1}(1)$ [ρ is in Theorem 3.11]*

Proof. Suppose that, μ is a group fuzzy congruence on S and let $(a, b) \in \rho$, then $eae = ebe$ for some $e \in E(S)$. Since S/μ is a group, we have μ_e is an identity of S/μ and for any $a \in S$,

$$\mu_a = \mu_e\mu_a\mu_e = \mu eae = \mu ebe = \mu_e\mu_b\mu_e = \mu_b$$

and so $\mu(a, b) = 1$.

Hence $(a, b) \in \mu^{-1}(1)$, we obtain $\rho \in \mu^{-1}(1)$. $\square$

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คำสำคัญ สมภาควิภัชนัย, แกมมากึ่งกรุป, สมภาควิภัชนัยกรุปน้อยที่สุด

บทคัดย่อ

การวิจัยเรื่องสมภาควิภัชนัยบางอย่างบนแกมมากึ่งกรุป มีวัตถุประสงค์เพื่อศึกษาเงื่อนไขที่จำเป็นและเพียงพอในการอธิบายความสัมพันธ์สมภาควิภัชนัยในปริภูมิแกมมากึ่งกรุป โดยเริ่มจากการศึกษาลักษณะเฉพาะและสมบัติพื้นฐานของสมภาควิภัชนัย และสมภาควิภัชนัยกรุปน้อยที่สุด