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Research Article

INVESTIGATION OF NATURAL FREQUENCY AND CRITICAL SPEED FOR JEFFCOTT ROTOR SYSTEM

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ABSTRACT:
In the rotating machinery, the critical speed is one of the most important parameters that must be evaluate. In general, it is very important to determine the critical speed to avoid the breakdown of rotating machine. In this study, the purpose is to investigate critical speed and natural frequency of the rotor bearing system. Firstly, the dynamic characteristics of the rotor are identified. And then, the critical speed of the rotor is evaluated. The finite element model of the rotor is modelled in COMSOL Multiphysics software. The experiment is conducted to determine the natural frequencies of rotor bearing system. In the comparison of natural frequency, the maximum discrepancy is 9.47%which occurs at third mode. The discrepancy between theoretical and numerical simulation results of critical speed is less than 6%. Critical speed analysis is important for stable operation of rotating machine.

Keywords: *Rotor, Critical speed, Dynamic characteristics, Experiment*

1. INTRODUCTION

Vibration is one of the most important factors in the operation of the rotating machine. The study of mechanical signature or the vibration spectrum of a rotating machine allows identifying operating problems even before they become dangerous [1]. The dynamic characteristics of rotor system, natural frequency and mode shape (whirling of shaft), are important to identify in the rotor system. The structure is given initial disturbance from its rest position and it is allowed to vibrate freely at natural frequency. A mode shape is a specific pattern of vibration at a specific frequency.

Junfeng Wang and Kang Sun (2012) investigate the critical speed based on one dimensional model of the rotor system [2]. The theoretical result of critical speed for one dimensional rotor model was calculated but neither numerical simulation result nor experimental result were interpreted. R. Tamrakara and N. D. Mittal (2016) discussed about the vibration response of crack rotor with the help of Campbell diagram [3]. The effect of crack on the natural frequency of the rotor system was studied. Numerical simulation was done using ANSYS software and experiment was conducted with the impact hammer test. Zang et al. (2017) presented the dynamic behavior of rotor bearing system with bearing inner-race detect and explained the effect of the system stability with the change in speed [4]. A.J. Muminovic et al. (2018) presented numerical analysis of natural frequency for elastic rotor system [5]. Natural frequency analysis was done for one dimensional rotor model and three types of rotor, fixed end, simply supported and cantilever, were considered. However, the critical speed evaluation was not done. S Huaitao et al. (2018) studied critical speed of high speed motor. The calculation results of critical speed using Riccati transfer matrix method was

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compared with the experimental analysis spindle rotor system [6]. In previous studies, both natural frequency analysis and evaluation of critical speed for rotor bearing system have been rarely investigated experimentally.

In this study, the main aim is to evaluate critical speeds of simple rotor model. Firstly, the natural frequency and mode shape (dynamic characteristics) of the rotor are investigated using COMSOL Multiphysics software package. Experiment of free vibration analysis of rotor is conducted to get natural frequencies. And then, the next is to identify the critical speed of the rotor. Finally, the numerical simulation results and experimental results were compared.

2. Mathematical Modelling

Rotor dynamics is used to analyze the structure behavior of engines, turbines, etc. In the rotor dynamic analysis, the whirling of shaft and Campbell diagram is the basic need to consider. Whirling of a shaft is a transverse direction movement to the axis of rotation. Whirling of shafts occurs rotational unbalance of a shaft which causes resonance at critical speeds. Whirling speed is the critical speed of a shaft. When the excitation frequency coincides with the natural frequency, the critical speed occurs. The resonance occurs at the critical speed [7].

Campbell diagram is the plot of the system frequency and excitation frequency of rotor. The x-axis of the diagram represents the rotational speeds of the rotor and the y-axis represents natural frequencies of rotor. It can be seen that most of the modes will increase or decrease due to the frequency changes along the rotational speed range. There are two circular whirling motion which occur at natural frequencies. Forward whirling is the same direction of the rotating motion and reverse or backward whirling is the opposite direction of the spin motion [8].

The Jeffcott rotor model is considered for model analysis because this model is the simple design among the rotor systems. It consists of a shaft, at the center of which, a fixed rigid circular disc is mounted, which is supported two ball bearings at each end. If the analysis of simple rotor model is clearly understood, the analysis of other complicated design rotor system could be done.

The equations of motion for the rotor is discussed below.

$$m\ddot{y} + c\dot{y} + ky = me\omega^2 \cos\omega t \tag{1}$$

$$m\ddot{z} + c\dot{z} + kz = me\omega^2 \sin\omega t \tag{2}$$

$$m\ddot{r} + c\dot{r} + kr = me\omega^2 e^{i\omega t} \tag{3}$$

where,
e = mass eccentricity, m
ω = shaft rotational speed, rpm
k = shaft lateral stiffness, N/m
y, z = coordinates of the shaft
r = whirl radius, m

$$I_P = m\left(\frac{d^2 - D_o^2}{8}\right) \tag{4}$$

$$I_d = m\left(\frac{d^2 + D_o^2}{16} + \frac{h^2}{12}\right) \tag{5}$$

Where,
I_P = Polar mass moment of inertia, kg-m^2
I_d = Diametral mass moment of inertia, kg-m^2
d = diameter of shaft, m
D_o = diameter of rotor disc, m
h = thickness of disc, m

The natural frequency and critical speed of the Jeffcott rotor model can be calculated by

$$\omega_n = \frac{\pi}{2}\left(n+\frac{1}{2}\right)^2 \times \sqrt{\frac{gEI}{ml^4}} \quad (6)$$

$$N_{cr} = \sqrt{\frac{KEI}{ml^3}} \quad (7)$$

Where,
ω_n = natural frequency, rad/s
E = modulus of elasticity, N/m2
m = mass of rotor, kg
I = moment of inertia, m^4
l = length of rotor, m
Ncr = critical speed, rad/s

3. Numerical Simulation

In this paper, the numerical simulation is done for simple rotor model. Firstly, Jeffcott rotor model, shown in Fig. 1, is simulated for eigenfrequency analysis by using COMSOL Multiphysics software.

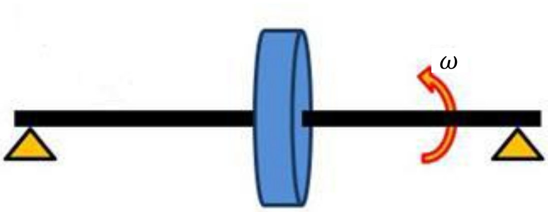

Fig. 1. Jeffcott rotor model.

In this analysis, the range of speed is considered from 0 to 25000 rpm in steps of 100 rpm. From this simulation, the dynamic characteristics are investigated. And then, the critical speed of the system is discussed as well. In this study, rotor shaft is made of mild steel and disc is made of PLA. The material properties of a simple rotor system are shown in Table 1. The geometry of rotor, as shown in Fig. 2, is drawn by using Bezier Polygon. Beam Rotor module in COMSOL Multiphysics FEM Package was used to simulate eigenfrequency analysis. The disc properties such as mass of disc and moment of inertia are set in the simulation. A modal analysis of rotor model for flexible bearing support is performed to obtain the mode shapes and corresponding natural frequencies. In the bearing boundary conditions, bearing stiffness values, identical undamped isotropic bearings stiffness, kyy = kzz = 5×10^5 N/m are added.

Table 1: Material properties of rotor.

Material	Steel
Shaft length	300 mm
Shaft diameter	8 mm
Disc diameter	100 mm
Disc thickness	5mm
Young's modulus	210 GPa
Density	7850 kg/m^3
Poisson's ratio	0.3

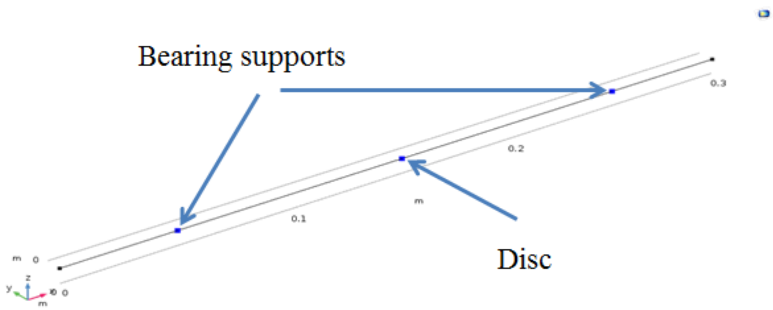

Fig. 2. Geometry of rotor

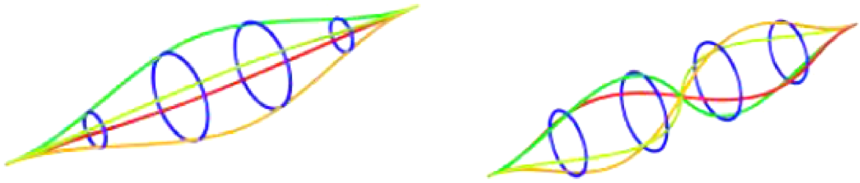

Fig. 3. Whirl Plots

The mode shapes are shown in Fig. 3. From the eigenfrequency analysis, first mode is at 195.79 Hz, second mode is at 457.85 Hz and third mode is at 855 Hz. In Fig. 4, the x-axis is the rotational speed of the rotor and the y-axis is the system frequencies. The Campbell diagrams are plotted for running the rotor at different speeds. Due to gyroscopic effect, the frequency line splits into two whirling motions. In the Campbell plot, the first mode across the excitation line at 11800 rpm is critical speed.

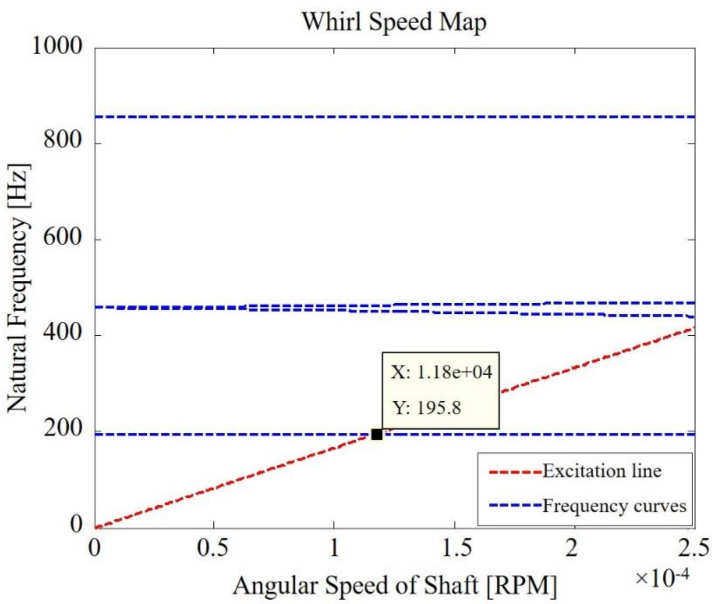

Fig. 4. Campbell plot

Campbell diagram is the plot of the natural frequency and excitation frequency of rotor. It is plotted to determine the critical speed of the rotor bearing system. In this analysis, the intersection points between frequency curves and excitation lines are calculated to find out the critical speed. Due to gyroscopic effect, the frequency line splits into two. In case of forward whirl, the natural frequency is increasing due to increase rotational speed. In backward whirl, natural frequency is decreasing as the rotational speed is increasing.

4. EXPERIMENTAL SET UP

The shaft, dimensions are shown in Table 1, is supported by two deep groove ball bearings at each end. A rigid disc, 100 mm in diameter and 5mm in thickness, is located at the half length of the shaft. The rotor is in stationary condition and free vibration analysis is done. The accelerometer 1 (Fujikura ARF – 500 A) is placed on right bearing block and the accelerometer 2 is placed on left bearing block to measure the acceleration responses in vertical (Z-axis) direction. The data from the sensor is collected by the computer which is connected to the data logger. The natural frequencies are identified from free vibration experiment. Impact hammer (Bruel & Kjaer 8204) is applied to the bearing block to get the initial disturbance in the rotor system. The acceleration responses are collected from accelerometer and time domain responses are converted to frequency domain responses by using data logger (Fujikura DC – 7004P).

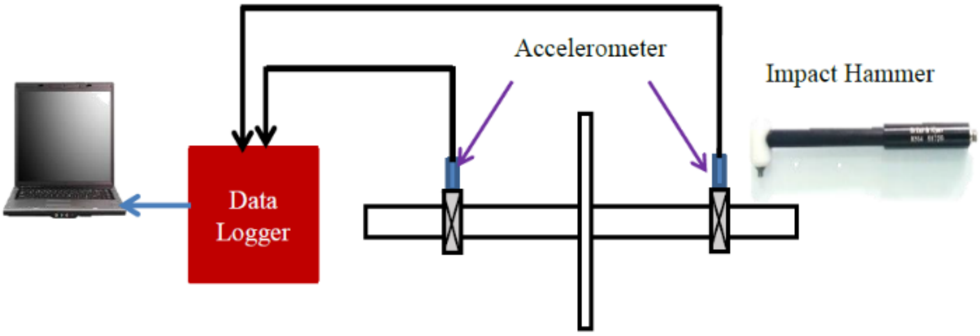

Fig. 5. Experimental set up diagram

The experimental set up diagram is shown in Fig. 5. The results are discussed in next section.

4. RESULTS AND DISCUSSION

The comparative results of natural frequencies between simulation and experiment for simple rotor system are shown in Table 2. From free vibration experiment, the natural frequencies are observed as shown in Fig. 6.

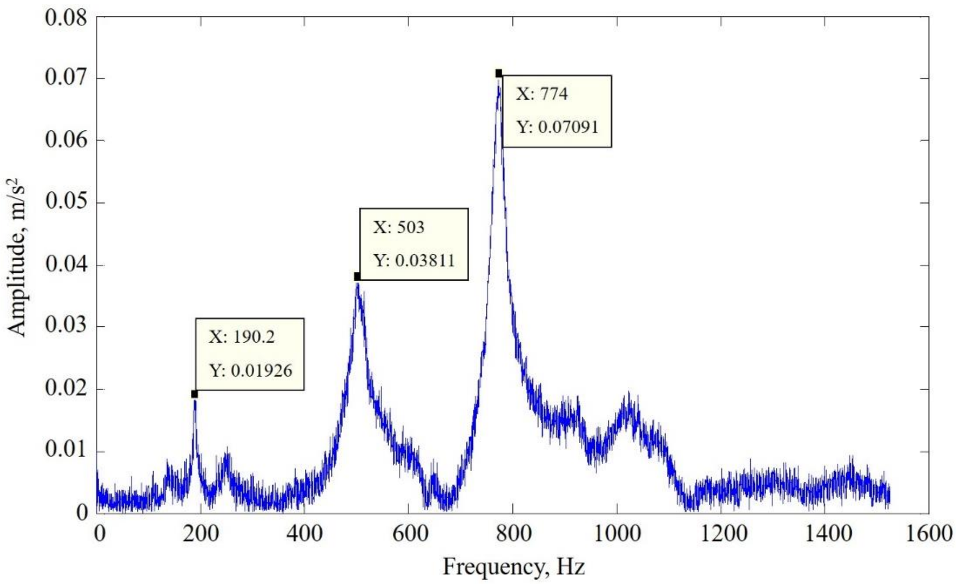

Fig. 6. Experimental results of natural frequency

In this study, the natural frequencies for first three modes are investigated. Although there is a little difference between theoretical and simulation results, these results are nearly the same. The maximum discrepancy is 9.47%which occurs at third mode. And then, the comparative results of critical speed are discussed as shown in Table 3. There is a good agreement in critical speed with maximum discrepancy of less than 6%.

Table 2: Comparison of natural frequency

Mode	Natural frequency (Hz)		Discrepancy (%)
	Simulation	Experiment	
1	195.79	190.2	2.86
2	457.85	503	8.97
3	855	774	9.47

Table 3: Comparison of critical speed

Description	Theoretical result	Simulation result	Discrepancy (%)
Critical Speed (rpm)	12500	11800	5.9

5. CONCLUSION

In this analysis, the critical speed of the Jeffcott rotor system has been evaluated. And then, experimental analysis of dynamic characteristic has been discussed. In the Campbell diagram, it is observed that the frequencies have been changed along the rotational speed range and also the whirl motion splits into two, forward and backward whirl, when the rotor is running above the zero. A forward whirling increases the natural frequency and a backward whirling decreases the natural frequency. It can be seen that the maximum discrepancy is under the acceptable limit. So, this type of analysis is important for stable operation of rotating machine.

NOMENCLATURE

k	shaft lateral stiffness, N/m
ω	shaft rotational speed, rpm
e	mass eccentricity, m
I_P	Polar mass moment of inertia, kg-m^2
I_d	Diametral mass moment of inertia, kg-m^2
D	diameter of shaft, m
D_o	diameter of rotor disc, m
H	thickness of disc, m
ω_n	natural frequency, rad/s
E	modulus of elasticity, N/m2
m	mass of rotor, kg

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