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Short Communication

Analytical formulas for pricing discretely-sampled skewness and kurtosis swaps based on Schwartz's one-factor model

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Abstract

In this paper, analytical formulas for pricing discretely-sampled skewness and kurtosis swaps based on the Schwartz's one-factor model is derived by applying the results of the conditional moments proposed by Chumpong, Mekchay, and Rujivan (2019). The results would be beneficial for market practitioners to describe commodity prices. The analytical pricing formulas for the skewness and kurtosis swaps of commodity will be useful for hedging against price volatility risks in commodity markets.

Keywords: skewness swaps, kurtosis swaps, discrete sampling, Schwartz's model

1. Introduction

Skewness swaps (third order moment swaps) and kurtosis swaps (fourth order moment swaps) are the two special types of moment swaps nowadays traded in the markets. Schoutens (2005) and Rompolis and Tzavalis (2017) stated that using these two types of moment swaps including variance swaps to hedge European options has better performance compared with using traditional delta hedging strategies. Consequently, tremendous growth in studying the skewness and kurtosis risks has been witnessed in recent years, see for example in Neuberger (2012), Kozhan, Neuberger, and Schneider (2013), Zhao, Zhang, and Chang (2013), Rompolis and Tzavalis (2017), and Zhang, Zhen, Sun, and Zhao (2017).

A literature review for analytically pricing various types of variance swaps reveals that many researchers mainly focus on equity stocks, such as in Zhu and Lian (2011, 2012), Rujivan and Zhu (2012, 2014), Swishchuk (2013), Zheng and Kwok (2014), and Rujivan (2016). For the case when the underlying asset is a commodity, there is still a little work dealing with the pricing problem, such as in Chunhawiksit and Rujivan (2016) and Weraprasertsakun and Rujivan (2017) that presented analytical formulas for pricing discretely-sampled variance swaps. Therefore, the contribution of this paper is to provide analytical formulas for pricing discretely-sampled skewness and kurtosis swaps when the underlying asset is a commodity, in which its price process is assumed to follow a continuous-time stochastic process introduced by Schwartz (1997).

In this paper we consider a probability space (Ω, F, Q) with a filtration $(F_t)_{t\geq 0}$ where Q is a risk-neutral probability measure. The conditional expectation of a random variable X with respect to a filtration F_t is denoted by $E^Q[X \mid F_t] = E_t^Q[X]$. The Schwartz's one-factor model describes the commodity price S_t by using the stochastic differential equation (SDE):

$$dS_t = \kappa(\mu - \ln S_t)S_t dt + \sigma S_t dz_t, \quad S_0 > 0, \tag{1.1}$$

where $\mu > 0$ is the long-run mean, $\kappa > 0$ is the speed of the

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reversion, σ is the volatility of the commodity prices, and Z_t is a standard Brownian motion under the probability space (Ω, F, Q) with a filtration $(F_t)_{t\geq 0}$ generated by $(S_t)_{t\geq 0}$. Note that in general the risk-neutral measure in (1.1) and the probability measure are not necessary the same; however, one can transform (1.1) to have the same risk-neutral measure as the probability measure, where the difference in (1.1) is absorbed in the parameter μ; see Schwartz (1997) for detail. According to the assumption imposed by Schwartz (1997) concerning the theory of storage, we assume that the convenience yield δ_t satisfies the following relation:

$$\delta_t = \kappa \ln S_t, \tag{1.2}$$

where $\kappa > 0$ is the speed of the reversion in (1.1). Equation (1.2) implies that δ_t is positively correlated to a logarithmic commodity price defined by $X_t := \ln S_t$.

The paper is organized as follows. In Section 2, we adopt the method presented by Rujivan and Zhu (2014) together with the result obtained by Chumpong *et al.* (2019), who obtained a closed-form formula for the conditional moments of the Ornstein-Uhlenbeck (O-U) process, in order to derive an analytical formula for pricing discretely-sampled moment swaps based on the Schwartz's one-factor model (1.1). Finally, we conclude the results in Section 3 by providing analytical formulas for pricing discretely-sampled skewness and kurtosis swaps.

2. Main Results

Schoutens (2005) introduced the annualized realized m-moment for a positive integer $m \geq 2$, in terms of discrete sampling over the contract life $[0,T]$ for a maturity time $T > 0,$ on an underlying asset as

$$MOMS^{(m)} = N' \sum_{i=1}^{N} \ln^m \left(\frac{S_{t_i}}{S_{t_{i-1}}} \right),$$

where S_{t_i} is the closing price of the underlying asset at the observation time t_i for $i = 0, 1, \ldots, N$ and there are altogether N observations, and N' is the nominal amount. For simplicity, we let $N' = \frac{AF}{N}$, where AF is the annualized factor converting this expression to an annualized higher moment. If the sampling frequency is every trading day, then $AF = 252$, assuming that there are 252 trading days in one year; if every week, then $AF = 52$; if every month, then $AF = 12$; and so on. Typically, we set $T = \frac{N}{AF}$ and assume equally-spaced discrete observations, where $\Delta t = t_i - t_{i-1} > 0$ for all $i = 1, 2, \ldots, N.$ Hence, the typical formula for measure of realized m-moment can be written as

$$MOMS^{(m)} = \frac{1}{T} \sum_{i=1}^{N} \ln^m \left(\frac{S_{t_i}}{S_{t_{i-1}}} \right) = \frac{1}{T} \sum_{i=1}^{N} \left(X_{t_i} - X_{t_{i-1}} \right)^m, \tag{2.1}$$

where $X_t = \ln S_t$ is a log price process.

In a risk-neutral world Q, the value of a m-moment swap at time t, denoted by V_t, is the expected present value of the future payoff:

$$V_t = E_t^Q \left[e^{-r(T-t)} \left(MOMS^{(m)} - K^m \right) L \right],$$

where K^m is the annualized delivery price for the m-moment swap and L is the notional amount of the contract. The value of V_t should be zero at the beginning of contract since there is no cost for either party to enter into a forward contract. Therefore, the fair delivery price of m-moment swap can be defined as $K^m = E_0^Q \left[MOMS^{(m)} \right]$, after setting the value of $V_0 = 0$ initially. The valuation problem for m-moment swaps is reduced to calculating the conditional expectation of the realized m-moment (2.1) in the risk-neutral world.

From (2.1), the fair strike price for discretely sampled moment swaps can be written as

$$\begin{aligned} K^m = E_0^Q \left[MOMS^{(m)} \right] &= E_0^Q \left[\frac{1}{T} \sum_{i=1}^{N} \left(X_{t_i} - X_{t_{i-1}} \right)^m \right] \\ &= \frac{1}{T} \sum_{i=1}^{N} E_0^Q \left[\left(X_{t_i} - X_{t_{i-1}} \right)^m \right]. \end{aligned} \tag{2.2}$$

Therefore, the problem of pricing moment swaps is reduced to calculating N conditional expectations in the form

$$E_0^Q \left[\left(X_{t_i} - X_{t_{i-1}} \right)^m \right]. \tag{2.3}$$

Then, by binomial expansion of $\left(X_{t_i} - X_{t_{i-1}} \right)^m$, the tower property for the conditional expectation and $F_{t_{i-1}}$-measurability of $X_{t_{i-1}}$,

$$\begin{aligned} E_0^Q \left[\left(X_{t_i} - X_{t_{i-1}} \right)^m \right] &= E_0^Q \left[\sum_{k=0}^{m} \binom{m}{k} (-1)^k X_{t_{i-1}}^k X_{t_i}^{m-k} \right] && \text{(binomial theorem)} \\ &= E_0^Q \left[E_{t_{i-1}}^Q \left[\sum_{k=0}^{m} \binom{m}{k} (-1)^k X_{t_{i-1}}^k X_{t_i}^{m-k} \right] \right] && \text{(tower property)} \\ &= E_0^Q \left[\sum_{k=0}^{m} \binom{m}{k} (-1)^k X_{t_{i-1}}^k E_{t_{i-1}}^Q \left[X_{t_i}^{m-k} \right] \right]. && (X_{t_{i-1}} \text{ is } F_{t_{i-1}}\text{-measurable}) \end{aligned} \tag{2.4}$$

Note that by setting $X_t = \ln S_t$, where S_t is defined in (1.1), and by Ito's lemma we obtain the O-U process for X_t, $dX_t = \kappa(\alpha - X_t)dt + \sigma dz_t$ where $\alpha = \mu - \frac{\sigma^2}{2\kappa}$.

To obtain the result (2.2) via (2.4), we applied the conditional expectation $E^Q_{t_{i-1}}\left[X^{m-k}_{t_i}\right]$ which is derived by Chumpong *et al.* (2019) stated as follow.

Theorem 2.1 Suppose that S_t follows the dynamics described in (1.1) and $n \in$ N. Let $X_t = \ln S_t$ and $\alpha = \mu - \frac{\sigma^2}{2\kappa}$. Then

$$E^Q_{t_{i-1}}\left[X^n_t\right] = E^Q\left[X^n_t \mid X_{t_{i-1}} = x\right] = \left(\sum_{j=0}^{n} A^{(n)}_j(\tau)x^j\right)e^{-n\kappa\tau} \tag{2.5}$$

for all $t \in [t_{i-1}, t_i]$ and $x \in \mathbf{R}$, where $\tau = t - t_{i-1} \geq 0$ and $A^{(n)}_j(\tau),\ j = 1,2,\ldots,n$, can be written in the form

$$A^{(n)}_j(\tau) = \left(\prod_{r=0}^{n-j-1}(n-r)\right)\sum_{l=0}^{\left\lfloor \frac{n-j}{2} \right\rfloor} \frac{1}{\kappa^l}\alpha^{n-j-2l}\sigma^{2l}\left(e^{\kappa\tau}-1\right)^{n-j-l}\left(e^{\kappa\tau}+1\right)^l c^{(l)}_{n,j}, \tag{2.6}$$

when $c^{(l)}_{n,j}$ is defined using $j = n-k$ as an index in

$$\bar{C}_{n,k} = \begin{bmatrix} c^{(0)}_{n,n-k} \\ c^{(1)}_{n,n-k} \\ \vdots \\ c^{\left(\left\lfloor \frac{k}{2} \right\rfloor\right)}_{n,n-k} \end{bmatrix} \in \mathrm{R}^{\left\lfloor \frac{k}{2} \right\rfloor + 1}, \tag{2.7}$$

which is defined recursively on k as follows:

$$\bar{C}_{n,1} = \left[c^{(0)}_{n,n-1}\right] = [1], \bar{C}_{n,2} = \begin{bmatrix} c^{(0)}_{n,n-2} \\ c^{(1)}_{n,n-2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}, \tag{2.8}$$

for odd $k \geq 3$,

$$\bar{C}_{n,k} = \frac{1}{k}\left(\bar{C}_{n,k-1} + \frac{1}{2}\begin{bmatrix} 0 \\ \bar{C}_{n,k-2} \end{bmatrix}\right), \tag{2.9}$$

and for even $k \geq 4$,

$$\bar{C}_{n,k} = \frac{1}{k}\left(\begin{bmatrix} \bar{C}_{n,k-1} \\ 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 \\ \bar{C}_{n,k-2} \end{bmatrix}\right). \tag{2.10}$$

Theorem 2.2 Suppose that S_t follows the dynamics described in (1.1). Let $\Delta t = t - t_{i-1}$ for all $t \in (t_{i-1}, t_i)$. The conditional expectation in (2.4) can be written as

$$E^Q_0\left[\left(X_{t_i} - X_{t_{i-1}}\right)^m\right] = \sum_{j=0}^{m} A_{m,j}(\Delta t, t_{i-1})X^j_0 \tag{2.11}$$

for all $i = 1,2,\ldots,N$ and $X_0 > 0$, where $\Delta t = t_i - t_{i-1}$ and

$$A_{m,j}(\Delta t, t_{i-1}) = \sum_{l=j}^{m}\sum_{k=0}^{l}\binom{m}{k}(-1)^k A^{(m-k)}_{l-k}(\Delta t)A^{(l)}_j(t_{i-1})e^{-(m-k)\kappa\Delta t}e^{-l\kappa t_{i-1}}.$$

Proof. From (2.4), we have

$$E_0^Q\left[\left(X_{t_i}-X_{t_{i-1}}\right)^m\right]=E_0^Q\left[\sum_{k=0}^{m}\binom{m}{k}(-1)^k X_{t_{i-1}}^k E_{t_{i-1}}^Q\left[X_{t_i}^{m-k}\right]\right]. \tag{2.12}$$

Utilizing Theorem 2.1 when $n=m-k$, the conditional expectations with respect to $F_{t_{i-1}}$ on the right-hand side of (2.12) can be written as

$$E_{t_{i-1}}^Q\left[X_{t_i}^{m-k}\right]=\left(\sum_{j=0}^{m-k}A_j^{(m-k)}(\Delta t)X_{t_{i-1}}^j\right)e^{-(m-k)\kappa\Delta t},$$

where $A_j^{(m-k)}(\Delta t), j=0,1,\ldots,m-k,$ are defined in (2.6). This implies

$$\begin{aligned}&E_0^Q\left[\sum_{k=0}^{m}\binom{m}{k}(-1)^k X_{t_{i-1}}^k E_{t_{i-1}}^Q\left[X_{t_i}^{m-k}\right]\right]\\&=E_0^Q\left[\sum_{k=0}^{m}\binom{m}{k}(-1)^k X_{t_{i-1}}^k\left(\sum_{j=0}^{m-k}A_j^{(m-k)}(\Delta t)X_{t_{i-1}}^j\right)e^{-(m-k)\kappa\Delta t}\right].\end{aligned} \tag{2.13}$$

Next, we rearrange the terms in the summations on the right-hand side of (2.13),

$$\begin{aligned}&E_0^Q\left[\sum_{k=0}^{m}\binom{m}{k}(-1)^k X_{t_{i-1}}^k\left(\sum_{j=0}^{m-k}A_j^{(m-k)}(\Delta t)X_{t_{i-1}}^j\right)e^{-(m-k)\kappa\Delta t}\right]\\&=E_0^Q\left[\sum_{l=0}^{m}\sum_{k=0}^{l}\binom{m}{k}(-1)^k A_{l-k}^{(m-k)}(\Delta t)e^{-(m-k)\kappa\Delta t}X_{t_{i-1}}^l\right]\\&=\sum_{l=0}^{m}\sum_{k=0}^{l}\binom{m}{k}(-1)^k A_{l-k}^{(m-k)}(\Delta t)e^{-(m-k)\kappa\Delta t}E_0^Q\left[X_{t_{i-1}}^l\right].\end{aligned} \tag{2.14}$$

Applying Theorem 2.1 when $n=l$ in (2.14), we obtain

$$\begin{aligned}&\sum_{l=0}^{m}\sum_{k=0}^{l}\binom{m}{k}(-1)^k A_{l-k}^{(m-k)}(\Delta t)e^{-(m-k)\kappa\Delta t}E_0^Q\left[X_{t_{i-1}}^l\right]\\&=\sum_{l=0}^{m}\sum_{k=0}^{l}\binom{m}{k}(-1)^k A_{l-k}^{(m-k)}(\Delta t)e^{-(m-k)\kappa\Delta t}\left(\sum_{j=0}^{l}A_j^{(l)}(t_{i-1})X_0^j\right)e^{-l\kappa t_{i-1}},\end{aligned} \tag{2.15}$$

where $A_j^{(l)}(t_{i-1}), j=0,1,\ldots,m,$ are defined in (2.6). Again, rearranging the terms in the summations on the right-hand side of (2.15), we complete the proof with

$$\begin{aligned}&\sum_{l=0}^{m}\sum_{k=0}^{l}\binom{m}{k}(-1)^k A_{l-k}^{(m-k)}(\Delta t)e^{-(m-k)\kappa\Delta t}\left(\sum_{j=0}^{l}A_j^{(l)}(t_{i-1})X_0^j\right)e^{-l\kappa t_{i-1}}\\&=\sum_{j=0}^{m}\left[\sum_{l=j}^{m}\sum_{k=0}^{l}\binom{m}{k}(-1)^k A_{l-k}^{(m-k)}(\Delta t)A_j^{(l)}(t_{i-1})e^{-(m-k)\kappa\Delta t}e^{-l\kappa t_{i-1}}\right]X_0^j.\end{aligned}$$

Applying Theorems 2.1 and 2.2, the fair delivery price of moment swaps under the Schwartz model (1.1) can be deduced as follows.

Theorem 2.3 Suppose that S_t follows the Schwartz model (1.1) and $m\geq 2$ is an integer. Then, the fair delivery price of the m-moment swap can be expressed as

$$K^m(T,\Delta t,\delta_0)=\frac{1}{T}\sum_{j=0}^{m}\sum_{i=1}^{N}\tilde{A}_{m,j}\left(\Delta t,t_{i-1}\right)\left(\frac{\delta_0}{\kappa}\right)^j, \tag{2.16}$$

where $\Delta t=\dfrac{T}{N}, t_i=i\Delta t, i=0,1,\ldots,N,$ and $\delta_0=\kappa\ln S_0$.

Proof. The proof follows directly from the definition (2.2) of K^m and Theorem 2.2 with $X_0=\ln S_0=\dfrac{\delta_0}{\kappa}$, namely,

$$K^m(T,\Delta t,\delta_0)=\frac{1}{T}\sum_{i=1}^{N}E_0^Q\left[\left(X_{t_i}-X_{t_{i-1}}\right)^m\right]$$
$$=\frac{1}{T}\sum_{i=1}^{N}\sum_{j=0}^{m}A_{m,j}(\Delta t,t_{i-1})X_0^j \qquad \text{(Theorem 2.2)}$$
$$=\frac{1}{T}\sum_{i=1}^{N}\sum_{j=0}^{m}A_{m,j}(\Delta t,t_{i-1})\left(\frac{\delta_0}{\kappa}\right)^j.$$

3. Our Analytical Pricing Formulas

In this section, we apply Theorem 2.3 to derive the two special types of moment swaps when $m=3,4$ based on the Schwartz model (1.1).

3.1 Skewness swaps

Skewness swaps (third order moment swaps) provide market practitioners who require to protect against price volatility risk in asymmetry of the underlying commodity price distribution. By applying Theorem 3.3 with $m=3$, the fair price of a skewness swap can be expressed as

$$K^3(T,\Delta t,\delta_0)=\frac{1}{T}\sum_{j=0}^{3}\sum_{i=1}^{N}\tilde{A}_{3,j}\left(\Delta t,t_{i-1}\right)\left(\frac{\delta_0}{\kappa}\right)^j, \qquad (3.1)$$

where

$$A_{3,0}(\Delta t,t_{i-1})=\frac{\alpha}{2\kappa}e^{-3\kappa(t_{i-1}+\Delta t)}\left(e^{\kappa\Delta t}-1\right)^2\left(6\sigma^2e^{\kappa(2t_{i-1}+\Delta t)}+\left(e^{\kappa\Delta t}-1\right)\left(2\alpha^2\kappa-3\sigma^2\right)\right),$$
$$A_{3,1}(\Delta t,t_{i-1})=\frac{3}{2\kappa}e^{-3\kappa(t_{i-1}+\Delta t)}\left(e^{\kappa\Delta t}-1\right)^2\left(-2\sigma^2e^{\kappa(2t_{i-1}+\Delta t)}+\left(e^{\kappa\Delta t}-1\right)\left(\sigma^2-2\alpha^2\kappa\right)\right),$$
$$A_{3,2}(\Delta t,t_{i-1})=3\alpha e^{-3\kappa(t_{i-1}+\Delta t)}\left(e^{\kappa\Delta t}-1\right)^3,$$
$$A_{3,3}(\Delta t,t_{i-1})=-e^{-3\kappa(t_{i-1}+\Delta t)}\left(e^{\kappa\Delta t}-1\right)^3.$$

3.2 Kurtosis swaps

Kurtosis swaps (fourth order moment swaps) provide market practitioners who need to protect against price volatility risk from unexpected occurrences of very large jumps or changes in tail behavior of the underlying commodity price distribution. Utilizing Theorem 3.3 with $m=4$, the fair price of a kurtosis swap can be expressed as

$$K^4(T,\Delta t,\delta_0)=\frac{1}{T}\sum_{j=0}^{4}\sum_{i=1}^{N}\tilde{A}_{4,j}\left(\Delta t,t_{i-1}\right)\left(\frac{\delta_0}{\kappa}\right)^j, \qquad (3.2)$$

where

$$A_{4,0}(\Delta t,t_{i-1})=\frac{1}{4\kappa^2}e^{-4\kappa(t_{i-1}+\Delta t)}\left(e^{\kappa\Delta t}-1\right)^2\begin{pmatrix}4\alpha^4\kappa^2\left(e^{\kappa\Delta t}-1\right)^2-12\alpha^2\kappa\sigma^2\left(e^{\kappa\Delta t}-1\right)\left(-1+e^{\kappa\Delta t}-2e^{\kappa(2t_{i-1}+\Delta t)}\right)\\+3\sigma^4\left(1+e^{\kappa\Delta t}\left(-1+2e^{2\kappa t_{i-1}}\right)\right)\end{pmatrix},$$
$$A_{4,1}(\Delta t,t_{i-1})=\frac{2\alpha}{\kappa}e^{-4\kappa(t_{i-1}+\Delta t)}\left(e^{\kappa\Delta t}-1\right)^3\left(-6e^{\kappa(2t_{i-1}+\Delta t)}\sigma^2+\left(e^{\kappa\Delta t}-1\right)\left(-2\alpha^2\kappa+3\sigma^2\right)\right),$$
$$A_{4,2}(\Delta t,t_{i-1})=\frac{3}{\kappa}e^{-4\kappa(t_{i-1}+\Delta t)}\left(e^{\kappa\Delta t}-1\right)^3\left(2\sigma^2e^{\kappa(2t_{i-1}+\Delta t)}+\left(e^{\kappa\Delta t}-1\right)\left(2\alpha^2\kappa-\sigma^2\right)\right),$$
$$A_{4,3}(\Delta t,t_{i-1})=-4\alpha e^{-4\kappa(t_{i-1}+\Delta t)}\left(e^{\kappa\Delta t}-1\right)^4,$$
$$A_{4,4}(\Delta t,t_{i-1})=e^{-4\kappa(t_{i-1}+\Delta t)}\left(e^{\kappa\Delta t}-1\right)^4.$$

3.3 Example

This section we present an example of computations for the fair prices of skewness and kurtosis swaps according to the formulas (3.1) and (3.2). The parameters of the Schwartz's model (1.1) used in the formulas are given as follows: $\mu = 2.857$, $\sigma = 0.129$, $\kappa = 0.099$, $N = 252$, $T = 1$, and for various values of $\delta_0 = -1.0, -0.8, -0.6, \ldots, 1.0$. The fair prices of skewness and kurtosis swaps for various values of convenience yields δ_0 are shown in Table 1.

Table 1 shows that the fair prices of the skewness and kurtosis swaps depending on the current convenience yields δ_0, which is referred to as the storage of products or physical goods that is related to the current commodity price via $S_0 = e^{\frac{\delta_0}{\kappa}}$. For this example for the studied range of δ_0 from -1.0 to 1.0, the prices for both derivatives increase as δ_0 increases and the price of the skewness is higher than that of the kurtosis. Note that the computation in this example for the prices using the formulas (3.1) and (3.2) is very fast, which is the advantage of this study, compared to the computation using (2.2) where the Monte Carlo simulation is needed for obtaining the expectations.

Table 1. Fair prices of skewness and kurtosis swaps for various values of δ_0.

δ_0	K^3	K^4
-1.0	0.0007593	0.0000165
-0.8	0.0006925	0.0000148
-0.6	0.0006300	0.0000133
-0.4	0.0005716	0.0000121
-0.2	0.0005172	0.0000108
0	0.0004664	0.0000097
0.2	0.0004190	0.0000087
0.4	0.0003747	0.0000078
0.6	0.0003334	0.0000070
0.8	0.0002948	0.0000063
1.0	0.0002585	0.0000057

4. Conclusions

In this work we obtain the closed-form formulas for the fair prices of discretely-sampled skewness and kurtosis swaps with underlying assets described by Schwartz's one-factor model. The formulas are derived based on the known result of the conditional moments by Chumpong *et al.* (2019) together with some combinatorial techniques. An example is given to demonstrate the computation of the fair prices using the formulas (3.1) and (3.2), which is very fast, thus, more suitable for practical usage than standard methods such as Monte Carlo simulation.

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