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THESIS

DEVELOPMENT OF WATER QUALITY MANAGEMENT MODEL FOR A TIDAL RIVER WITH APPLICATION TO THA CHIN RIVER

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A Thesis Submitted in Partial Fulfillment of
the Requirements for the Degree of
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Water quality management model for a tidal river was developed in this study. The main objective of the model was to determine the proper allocation of treatment levels at all wastewater treatment plants along the studied tidal river so that river water quality, measured in terms of BOD and DO, was still maintained within the specified standard. The formulated management model was in the form of linear programming. The objective function was to maximize total BOD discharge loading whereas the constraints included ranges of treatment plant efficiencies, and allowable BOD and DO concentrations in the river water. The BOD and DO constraints were formulated from unsteady-state BOD and DO dispersion models obtained from the two-dimensional vertically-averaged mass balance equations. The finite element method was used to develop the BOD and DO dispersion models. The BOD and DO dispersion models were developed such that the BOD and DO concentrations at any time were expressed in terms of the values at the initial time. By setting the BOD values less than or equal to the specified limits and the DO concentrations greater than or equal to the allowable values, the BOD and DO constraint inequalities were obtained. The degrees of treatment at various treatment plants in the study area were considered as model decision variables. The developed model was applied to the middle and lower sections of the Tha Chin River as a case study, so as to demonstrate applicability of the model and to test the model reliability. It was found that the model could be used for supporting water quality management of a tidal river. The results indicated the optimal degrees of BOD load removal of all treatment plants and showed the critical locations in the river with a risk of violating the BOD and DO standards.

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Nannapasorn Inyim

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LIST OF ABBREVIATIONS

B	=	BOD concentration
D	=	DO concentration
BOD	=	Biochemical Oxygen Demand
DO	=	Dissolved Oxygen
D_s	=	saturated concentration of dissolved oxygen in water
g/m^3	=	gram per cubic meter
g/m^3-s	=	gram per cubic meter per second
g/s	=	gram per second
k_1	=	BOD decaying rate
k_2	=	atmospheric reaeration coefficient
k_s	=	BOD removal rate by sedimentation
K_x	=	dispersion coefficient in the x-directions
K_y	=	dispersion coefficient in the y-directions
kg/d	=	kilogram per day
LP	=	linear programming
m/s	=	meter per second
m^2/s	=	square meter per second
mg/l	=	milligram per liter
P_c	=	ratio of remaining BOD load to generated BOD load
$P_{c,max}$	=	maximum P_c value
$P_{c,min}$	=	minimum P_c value
PCD	=	Pollution Control Department
R_{bc}	=	controllable BOD load
R_{bu}	=	uncontrollable BOD load
REO	=	Regional Environment Office
RHS	=	right-hand side term in the mathematical expression
RKM	=	the distance in kilometer from the river mount
s^{-1}	=	per second
u	=	vertically averaged flow velocity in x-direction
v	=	vertically averaged flow velocity in y-direction

DEVELOPMENT OF WATER QUALITY MANAGEMENT MODEL FOR A TIDAL RIVER WITH APPLICATION TO THA CHIN RIVER

INTRODUCTION

Human activities, such as domestic, industrial and agricultural activities, introduce significant amount of pollutants into rivers and streams. Biodegradable pollutant (measured as biochemical oxygen demand, BOD) has been paid considerable attention due to its influence on dissolved oxygen (DO) depletion in water body. The impacts of low dissolved oxygen concentrations are unbalanced ecosystem with fish mortality, odor and aesthetic nuisances (Kannel *et al*., 2007). Thus in order to maintain a good quality of the rivers, treatment of wastewaters to remove or stabilize biodegradable pollutants prior to disposal is necessary. In water quality management, particularly in developing countries, the treatment costs are as important as achievement of required water quality. Moreover, it is difficult to invest heavily to control non-point source pollution (Cho *et. al*, 2004). This study is focused on the control of discharge loads from controllable sources. Modeling technique with optimization method is used to find the optimal wastewater treatment policy.

Mathematical models are widely used in planning, management and design of environmental works nowadays. It is a rational and objective means for processing complex information to predict consequences of current decisions (Korfmacher, 1998). The most common application of the mathematical model in water quality management is to determine the impact of discharge on the receiving water. Such model is known as water quality simulation model. In control of point source pollution of a river, such as wastewater treatment plants and industries, the simulation model is applied to determine the amount of pollutant which can be discharged without deteriorating the quality of receiving water. However, when several treatment plants discharge wastewaters into the same water body, the policy of minimizing costs while keeping the desired levels of water quality are taken into account, thus the problem of management of these treatment systems arises. To solve this problem, optimization methods are very useful. The water quality model that incorporates optimization method is known as water quality management model.

Over the past decades, many water quality management models have been proposed. These models are more or less complex and different in terms of basic assumptions, water quality parameters, methods of modeling, etc. However, less attention had been paid to modeling unsteady river flow condition. In other words, most of the previous models were developed based on the steady flow condition which were not applicable to tidal rivers and estuaries where flow patterns change cyclically by tidal movement which consequently affect evolution of river water quality (Chapra, 1997).

In this study, a water quality management model for a tidal river is developed by embedding a water quality simulation model in the optimization scheme. Main function of the model is to determine the optimal solution of wastewater management. The finite element method with Galerkin's weighted residual technique is used to develop the water quality simulation model. The organic load is measured in terms of BOD, and the BOD and DO concentrations in river water are used as water quality indicators. Then, the linear programming is applied to develop the optimization model. The objective function is to maximize the total discharged BOD load subjected to a set of constraints related to the desired water quality in specified positions. The simplex method is used to solve the optimization problem. The developed model is applied to the middle and lower sections of the Tha Chin River in which water quality is so poor that a proper management policy for controlling pollution load is necessary.

OBJECTIVES

1. To develop water quality management model for a tidal river. The main concept of the model is to determine the degrees of BOD load removal, in such a way that the overall BOD load can be discharged at maximum rate, whereas the concentrations of BOD and DO at some specified points of the river at selected time can meet the desired values.

2. To apply the developed model to the middle and lower sections of the Tha Chin River. The model is used to find the appropriate degrees of BOD load removal of treatment plants in the study area. Results of this study are expected to provide useful information to support water quality management of this river.

LITERATURE REVIEW

1. Basic Concepts of Water Quality Management Model

Nowadays, mathematical models play an important role in water quality management. Main objectives of using models are: 1) to provide a better understanding of environmental processes associated with water quality behavior in a water system, and 2) to provide a more rational basis for making water quality control decisions (Thomann and Mueller, 1987; Korfmacher, 1998). There are two main types of models used in this field, i.e., simulation models and management models. Simulation models are used to simulate water quality in a water body under some given situations. This type of model is usually based on the principle of conservation of mass. Some numerical methods, such as finite difference method and finite element method, are used appreciably to solve the mass balance equations. The simulation models are used to predict impacts of discharging pollutant load on water quality. They can also be simulated to determine limits for pollutant discharges into water bodies. The management models are used to seek the most appropriate policy for controlling water pollution. This type of model usually relies on use of optimization techniques including deterministic algorithms (such as linear programming and dynamic programming) and heuristic algorithms (such as fuzzy programming and genetic algorithm) to fulfill the task.

2. Previous Works on Water Quality Management Modeling

According to Chapra (1997), water quality modeling has evolved appreciably in the early years of the twentieth century. Most of the early models were focused on urban waste load allocation problem. The seminal work was the model developed by Streeter and Phelps in 1925. However, due to the nonavailability of computers, model solutions were limited to linear kinetics, simple geometries, and steady-state receiving waters. In the 1960s, digital computers became widely available. It allowed analysts to address more complicated system geometries, kinetics, and time-variable simulations. During this period, the originally developed tools in the field of operation research were coupled with the models to generate cost-effective treatment alternatives. Since that time, a large number of models have been developed and used in water quality management. These models are more or less different based on their assumptions and modeling techniques; but their basic concepts in optimization were focused in the same point of view. That is to determine the optimal control of pollutants while achieving the desired quality of water bodies. For examples,

Brill *et al*. (1976) presented an approach to evaluate management programs in which discharges were divided into groups. In the programs, equal waste removal percentages were set within the groups, but the percentages were allowed to vary from one group to another. The non-linear constraints on effluent charge were established. However, the basic type of model was a linear programming formulation. In this work, the focus was only on proposing the algorithm of optimization; the development or application of water quality simulation model was not taken into account.

Jenq *et al*. (1983) developed a linear programming model for point and nonpoint source control decisions to control eutrophication in lakes. Total phosphorus was highlighted as water quality substance and the input to the lake were from tributaries flowing to the water body. In this study, the formulation of stream and lake water quality constraints was based on steady-state mass balance equation of phosphorus.

Burn and McBean (1987) applied nonlinear programming to water quality management. Their aim was to mitigate the adverse effects of waste discharges on DO concentration in rivers. Pollutant loading and transport in a water body were considered as random variables.

Li and Guangwen (1990) proposed a multi-objective programming to pollution control in rivers. The objectives on 1) minimizing total wastewater treatment cost and 2) minimizing BOD concentration in a water body were identified. However, in the solution of the problem the multi-objective programming was changed to a linear programming in which the first objective was taken as the basic objective and the second one was taken as the constraint of the problem.

Liengcharernsit *et al*. (1995) developed a water quality optimization model based on two-dimensional vertically averaged mass balance equations. Dissolved oxygen, and carbonaceous and nitrogenous biochemical oxygen demand were used as water quality parameters. Methods used in model formulation included the finite element method, the linear programming and the nonlinear programming. The model could be applied to surface waters with steady flow conditions.

Bikangaga and Nassehi (1995) applied modeling technique to test various discharge policies in a tidal river. The simulation of pollutant dispersion was based on one-dimensional hydrodynamic equations of the motion and continuity of water and pollutant transport. The method used to solve the equations was the Taylor-Galerkin method.

Cho *et at*. (2004) developed a water quality management model through the integration of Qual2e model (a steady-state one-dimensional water quality simulation model) and genetic algorithm. Water quality of a river was calculated by the Qual2e model and then the achievement of water quality goal was examined. The genetic algorithm was conducted to optimize pollution control cost. Pollution sources, land uses, geographic features and measured water quality of the river were incorporated in the ArcView geographical information system (GIS) database in the optimization.

Revelli and Ridolfi (2004) developed a stochastic dynamics of BOD in a stream with random inputs. The uncertainty involved the initial conditions and point inputs of BOD in the water body. Both nonlinear and linear decaying rates of BOD were examined. The stochastic differential approach was proposed and used in this work to determine the semi-analytical solution of the evolution of BOD probability distribution.

Ning and Chang (2006) also applied Qual2e model in their work. The model was used to evaluate pollution prevention projects in which BOD and ammonia nitrogen were of concern. This study was aimed to determine the total maximum daily load (TMDL) of waste into a water body.

Kachiashvili *et al*. (2006) developed a mathematical model to simulate the pollutant distribution in a water body with eutrophication problem. The model was developed based on the advection-diffusion of pollutant under various initial and boundary conditions. The finite difference scheme was applied for model solution.

Kuo *et al*. (2006) applied CE-QUAL-W2 model (a longitudinal-vertical hydro dynamics and water quality model, also called W2) to quantify the relationship between nutrient loading and water quality of reservoirs in Taiwan. The W2 model was based on finite difference approximation to the laterally averaged equations of fluid motion. In this work, various phosphorus reduction scenarios were established and tested by the W2 model. Water quality indicators in the water bodies included nutrients, DO, and algal biomass.

Aras *et al*. (2007) applied the genetic algorithm to develop a water quality management model. The simulation of BOD and DO concentration of a water body was based on the DO sag equation. The developed model was aimed to apply to rivers contaminated by several discharge sources.

Kannel *et al*. (2007) used QUAL2Kw model (a one-dimensional stream water quality model) to investigate various water quality management strategies during critical period. These strategies included wastewater treatment, flow augmentation and local oxygenation. The sensitivity analysis showed that the model was highly sensitive to water depth and moderately sensitive to point source flow, total nitrogen, carbonaceous BOD and nitrification rate.

Kuo *et al*. (2008) developed a water quality management model to determine the optimal nutrient removal rates and to identify the least-cost policies for lake eutrophication management by using dynamic programming. The Newton-iterative technique was used to solve the nonlinear equations for the steady-state lake eutrophication model. Standards of chlorophyll-a and BOD concentrations for lake were set as constraints of the optimization problem. The steady-state water quality model for lake eutrophication, the one-dimensional steady-state river model and the optimization model were combined to determine the minimal treatment cost by considering the total construction cost of treatment plants with annual interest rate and annual operation and maintenance costs of each treatment plant.

Qin *et al*. (2007) developed an interval-fuzzy nonlinear programming model for water quality management under uncertainty. The technique of piecewise linearization was developed to deal with nonlinearity of the objective function. The interval programming and the fuzzy programming were integrated within a general framework to address the uncertainty of nonlinear constraints. The Streeter-Phelps model was used to quantify water quality constraints related to BOD and DO discharges and their concentrations in the water body.

As can be seen in the above review, there have been various assumptions and techniques in modeling. However, most of these works were applied under steady-state conditions. Although some of recent models have been developed to predict water quality under uncertain inputs, the impact of tidal fluctuation on surface water quality was still ignored. According to author knowledge, water quality management models that are applicable to tidal rivers have seldom been found. In this study, in addition to developing a management model for tidal rivers, expressing a general form of the model with embedding unsteady-state simulation model in optimization one in order to facilitate model computation is conducted.

3. Basic Governing Equations of Water Quality Model

Mathematical model is the representation of processes in a considered system by means of mathematical equations (Jørgensen, 1988). In the development of a mathematical model, some assumptions and given conditions of the considered system must be defined and then the governing mathematical expressions are formulated to describe the system behavior (Kwon and Bang, 2000). Generally, water quality models are based on the conservation of mass; that is, within a finite volume of water, mass is neither created nor destroyed. In quantitative terms the principle is expressed as a mass balance equation that accounts for all transfers of matter across the system boundary and all transformations occurring within the system. In a volume of water body, the movement of matter through the volume along with water flow is termed transport. In addition to this flow, mass is gained or lost by transformations or reactions of substances within the volume. External loadings also increase mass of the substances. For a finite period of time this can be expressed as (Chapra, 1997)

$$\text{Accumulation} = \text{loadings} \pm \text{transport} \pm \text{reactions} \qquad (1)$$

The advection-dispersion process, biological process and interaction process of BOD and DO as well as source and sink terms of these substances are usually taken into account in the mass balance equations. When the BOD and DO concentrations distribute uniformly over water depth, the BOD and DO concentrations can be described by two-dimensional vertically averaged mass balance equations as shown below (Liengcharernsit *et al.*, 1995).

Mass balance equation of BOD:

$$\frac{\partial B}{\partial t} + u\frac{\partial B}{\partial x} + v\frac{\partial B}{\partial y} - \frac{1}{h}\left[\frac{\partial}{\partial x}(hK_x\frac{\partial B}{\partial x}) + \frac{\partial}{\partial y}(hK_y\frac{\partial B}{\partial y})\right] + k_1 B + k_s B - R_{bc} - R_{bu} = 0 \qquad (2)$$

Mass balance equation of DO:

$$\frac{\partial D}{\partial t} + u\frac{\partial D}{\partial x} + v\frac{\partial D}{\partial y} - \frac{1}{h}\left[\frac{\partial}{\partial x}(hK_x\frac{\partial D}{\partial x}) + \frac{\partial}{\partial y}(hK_y\frac{\partial D}{\partial y})\right] + k_1 B - k_2(D_s - D) - R_d = 0 \qquad (3)$$

where B is BOD concentration (g/m^3)

D is DO concentration (g/m^3)

u is vertically averaged flow velocity in the x-direction (m/s)

v is vertically averaged flow velocity in the y-direction (m/s)

h is water depth (m)

K_x is dispersion coefficient in the x-direction (m^2/s)

K_y is dispersion coefficient in the y-direction (m^2/s)

k_1 is BOD decaying rate (s^{-1})

k_2 is atmospheric reaeration coefficient (s^{-1})

k_s is BOD removal rate by sedimentation (s^{-1})

D_s is saturated concentration of dissolved oxygen in water (g/m^3)

R_{bc} is controllable BOD load ($g/m^3.s$)

R_{bu} is uncontrollable BOD load ($g/m^3.s$)

R_d is source/sink term for DO ($g/m^3.s$), e.g. DO content in the discharge waste, DO generated from photosynthesis of phytoplankton and other aquatic plants, DO consumed by benthic, respiration of aquatic organisms, etc.

The BOD loadings to a water body can be classified into two types. The first type, called controllable BOD load, is the BOD loads from those sources which can be collected and treated prior to discharging into the water body; whereas the second type, called uncontrollable BOD load, is the BOD loads from those sources which are difficult to control such as nonpoint sources.

4. Finite Element Method with Galerkin's Weighted Residual Technique

As presented in the previous article, the governing equations of water quality models consist of two differential equations that are difficult to solve for the solution. Nowadays, with the advent of high performance computers and the intensive development of various numerical techniques, it has become possible to solve such differential equations. There are various numerical solution techniques; however, the finite element method is used in this study. Major advantages of the finite element method for this study are that: 1) it is well suited for problems associated with complex domain, such as shape of rivers and estuaries; 2) it can handle problems with variables varying with position; and 3) conventional numerical techniques can be used to solve the equations resulting from the finite element analysis.

4.1 Basic concept

The finite element method is a numerical approach by which general differential equations are solved in an approximate manner. In this method, the primary unknown variables are approximated by a trial function, which is specified in terms of independent variables and undetermined parameters. The main idea of the method is that the distribution of the primary unknown quantity is represented based on the values at various points, usually being nodes on finite element grid, of the domain. The numerical solution corresponding to the values of the primary unknown variables at the nodes is obtained after solving a set of algebraic equations of the governing equations. To form the set of algebraic equations, the governing differential equations must first be converted to weighted-integral expressions. A method that is used to obtain a weighted-integral formulation, which is the one followed in this study, is a weighted residual method.

According to the above paragraph that in this method the primary unknown variables are approximated by a trial function, substitution of this function into the differential equation results in some error called residual. The method of weighted residuals seeks to find the undetermined parameters in the approximate solution in such a way that the residual over the entire domain is small. This is accomplished by multiplying the residual by a weighting function and then specifying the integral of this weighted-residual formulation to be zero over the entire domain. If the weighting functions are chosen from the same set of the interpolation functions, the weighted residual method is known as Galerkin method (Lewis and Ward, 1991; Ottosen and Petersson, 1992; Huebner *et al.*, 1995; Kwon and Bang, 2000; Polycarpou, 2006; Reddy, 2006).

Substituting the primary unknown variables of the governing equations by the trial function and applying the method of weighted residuals yields a set of algebraic equations of weighted-integral statements. In finite element analysis, the entire domain will be considered as the assemblage of subdomains, called finite elements. Thus, the set of weighted-integral equations can be expressed as the assemblage of finite number of elements equations.

4.2 Solution of finite element equations

The solution of a set of finite element equations to determine the distribution of unknown variables at nodes within the solution domain can be summarized as follow.

1) Consider the entire solution domain as a collection of finite number of elements.

2) Assign shapes and types of all elements. A variety of element shapes can be used, and different element shapes can be employed in the same solution domain.

3) Select interpolation function to represent the variation of the primary unknown variables over the elements.

4) Form matrix equations representing the distribution of the unknown variables at nodes of the elements.

5) Assemble all element matrices based on a concept that the value of a variable at a node where elements are interconnected is the same for each element sharing that node (Huebner *et al*., 1995). A matrix expressing the distribution of unknown variable at nodes of the entire domain is called system matrix.

6) Impose boundary conditions to the system matrix in order to obtain a unique solution of the problem.

7) Solve the system matrices by using matrix algebra to obtain a set of numerical solution, i.e., the values at nodes of the primary unknown variable, of the problem.

5. Linear Programming

Operations research is a scientific approach that seeks to best design and operate a system. Optimization is a dominant theme in the operations research. Tools for solving optimization problems are for examples, linear programming, nonlinear programming, dynamic programming, flow network programming, queuing theory, stochastic processes. In this study, the linear programming (LP) is applied in water quality management model formulation. The most efficient method to solve the LP model is the simplex method developed in 1947 by George Dantzig. The most powerful use of the simplex method is the ability to solve very large LP with a large amount of constraints and variables (Ecker and Kupferschmid, 1988; Winston, 2004).

5.1 Main components of LP

The LP consists of three main components (Jensen and Bard, 2003).

1) The objective function – expressed by the linear function of decision variables – that is required to be maximized or minimized.

2) Constraint equalities or inequalities, each being linear function, that are defined to restrict the values of decision variables.

3) Sign restriction on each decision variable. The variables are required to be nonnegative in most cases; however, sometimes variables are required to be nonpositive or may even be unrestricted.

5.2 Standard form of LP

For an LP containing n decision variables and m constraints in which n>m, the standard form of the model is written as (Jensen and Bard, 2003)

$$\text{Maximize (or Minimize)} \quad z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \tag{4}$$

Subject to

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ \vdots \qquad \vdots \qquad \vdots \quad & \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m \end{aligned}\right\} \tag{5}$$

$$x_j \geq 0;\ i = 1, 2,\ldots,m;\ j = 1, 2,\ldots,n$$

where all the right-hand side (RHS) parameters (b_i) are nonnegative; parameter a_{ij} is called a technological coefficient; and parameter b_i is called the RHS value of the i^{th} constraint. Constraints in Equation (5) are identified as structural constraints to distinguish them from the nonnegative restriction. The standard form of LP is necessary for solving the problem with the simplex method.

5.3 Preparation of LP model

Solution algorithms of LP require that the problem must be expressed in the standard form. Thus, an LP model must be put into the following characteristics (Jensen and Bard, 2003). First, the objective of the model must be to maximize. Second, the objective function must be linear in the variables and must not contain any constant terms. Third, all variables must be restricted to be nonnegative. Lastly, each structural constraint must be written as a linear equation with the variables on the left of the equal sign and a positive constant on the right.

If any of these characteristics are not presented in the original model, the following transformations can be used to put the model into the required form.

1) If the objective function is to minimize, it can be transformed into an equivalent maximization problem by changing the signs of all terms in the objective function. After optimization, the optimal solution of the original problem can be recovered by multiplying the optimal value of the new problem by -1.

2) If the objective function contains a constant term, it may be dropped from the model during optimization. To get the true objective function value, the constant must be reinstated after the optimal solution is found.

3) If one or more constraints are in inequality form, they must be converted into equations. For less than or equal to (≤) constraint, a slack variable is added into the converted equality constraint; whereas for greater than or equal to (≥) constraint, an excess variable is subtracting from the converted equality constraint. Also, the slack or excess variable must be restricted to be nonnegative.

6. Study Area: Tha Chin River

The Tha Chin River is a major branch of the Chao Phraya River, branching off in Chainat Province and flowing through the western part of the Central Plain of Thailand (see Figure 1). From Chainat Province, the river meanders on 325-km distance through Suphan Buri, Nakhon Pathom and Samut Sakhon Provinces, to drain its runoff into the Gulf of Thailand (Regional Environment Office 5 [REO5], 2008). According to Schaffner (2007), the Tha Chin River in the past was a complete natural water course; but nowadays it is one of several polluted rivers which receive wastewater from various sources located along the river course. At present, the flow of the Tha Chin River is controlled by four main regulators, namely, Pholathep Regulator (318 km from the river mount, RKM), Thabot Regulator (290 RKM), Chollamarkpijarn (also called Samchuk) Regulator (239 RKM), and Phophraya Regulator (202 RKM).

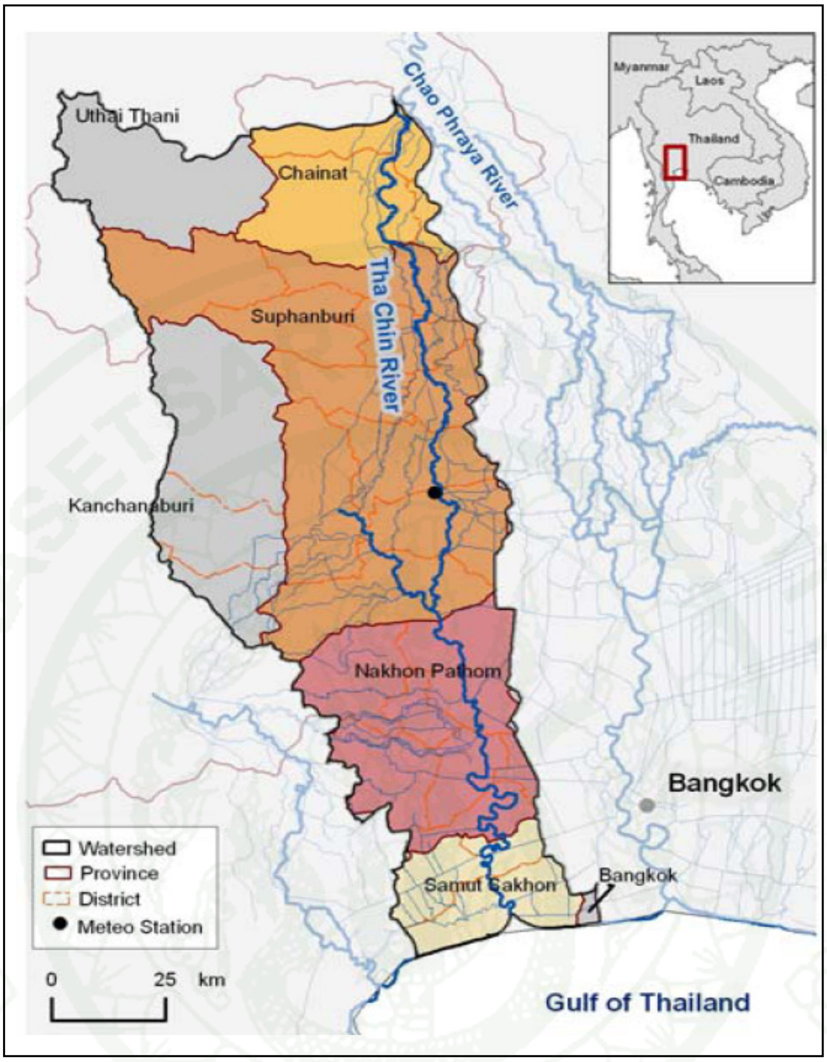

Figure 1 Tha Chin River and its watershed.

Source: Schaffner (2007)

Generally, the discharge of water at the upper-most regulator (Pholathep) varies depending on the availability of the water in the storage dams in northern Thailand. The variation increases along the river course, with a variability of 100% at the lowest regulator (Phophraya). The discharge patterns in the lower basin are a function of the water management rationales which focus on irrigation supply, flood retention, navigation purpose, as well as seawater intrusion prevention (Schaffner, 2007).

Tidal fluctuation at the river mount significantly influences flow pattern of the Tha Chin River. During high tide, the river flow is pushed backward which results in increasing water level and prolonged water retention, particularly during the dry season. With low river flow condition, seawater can intrude more strongly to the upstream part of the river. The impact of tide on flow pattern of the river extends about 180 km upstream during low flow period and about 120 km during high flow period (Simachaya and Healthcote, 1999).

6.1 Water quality of Tha Chin River

The surface water quality standards for major rivers in Thailand (enacted by the National Environment Board of Thailand) divide surface water into 5 water quality classes. For each water quality class, a set of water quality standards defining maximum levels of water quality parameters according to the designated use of the river are specified. The legislation includes basic physiochemical parameters (color, pH, temperature and dissolved oxygen, DO), inorganic nitrogen (ammonia and nitrate), as well as BOD, pathogens (coliform bacteria), heavy metals, pesticides, and other trace substances. According to the surface water quality legislation, the Tha Chin River is divided into three sections with water quality standards, for example for BOD and DO, and classification as listed in Table 1 (PCD, 2005).

Table 1 Water quality classes and standards for BOD and DO for Tha Chin River.

River section (km from river mouth)	Water Quality Class	Water Quality Standards (mg/l)	
		DO	BOD
0 – 82	4	≥ 2.0	≤ 4.0
82 – 202	3	≥ 4.0	≤ 2.0
202 – 325	2	≥ 6.0	≤ 1.5

Source: PCD (2005)

During the past decades, the Tha Chin River has faced water pollution problems due to increasing discharge of human-made contaminants. The crucial water quality indicators include BOD and DO. Figures 2 and 3 show the field data of BOD and DO concentrations along the river in the dry seasons during 2005-2009 compared to the standards. These field data show that water pollution occurs in the river, particularly in the middle and lower sections, indicated by BOD concentrations at several points violating the BOD standard. However, DO concentrations in 2009 mostly comply with the standard for DO. This improvement is the consequence of intense efforts to protect the river by government agencies during the past years.

According to REO5 (2009), sources of BOD loadings to the Tha Chin River include domestic and industrial activities, aquacultures and pig farms. Table 2 gives a compilation of BOD load from these sources in 2008. This indicates that domestic activity is a major source of BOD load in the upper and lower sections whereas pig farm is a major source of BOD load in the middle section. However, in this zone, domestic activity also produces a large amount of BOD load and can be considered as the major share of pollutant source.

Table 2 Sources of BOD load in Tha Chin River Basin in year 2008.

Source	BOD load generated in each section (kg/d)			Total BOD load (kg/d)
	Upper section	Middle section	Lower section	
Domestic	5,505	11,630	13,278	30,413
Industry	-	37	1,551	1,588
Aquaculture	184	6,540	7,250	13,974
Pig farm	1,106	20,243	3,646	24,995
Total	6,795	38,450	25,725	70,970

Source: REO5 (2009)

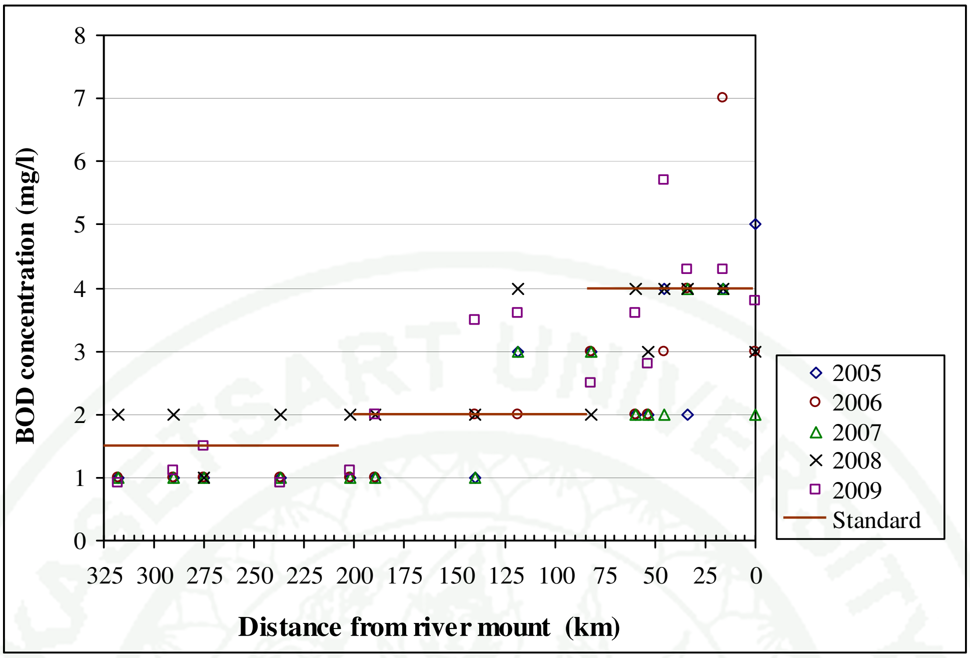

Figure 2 BOD concentrations of Tha Chin River in dry seasons during 2005-2009.

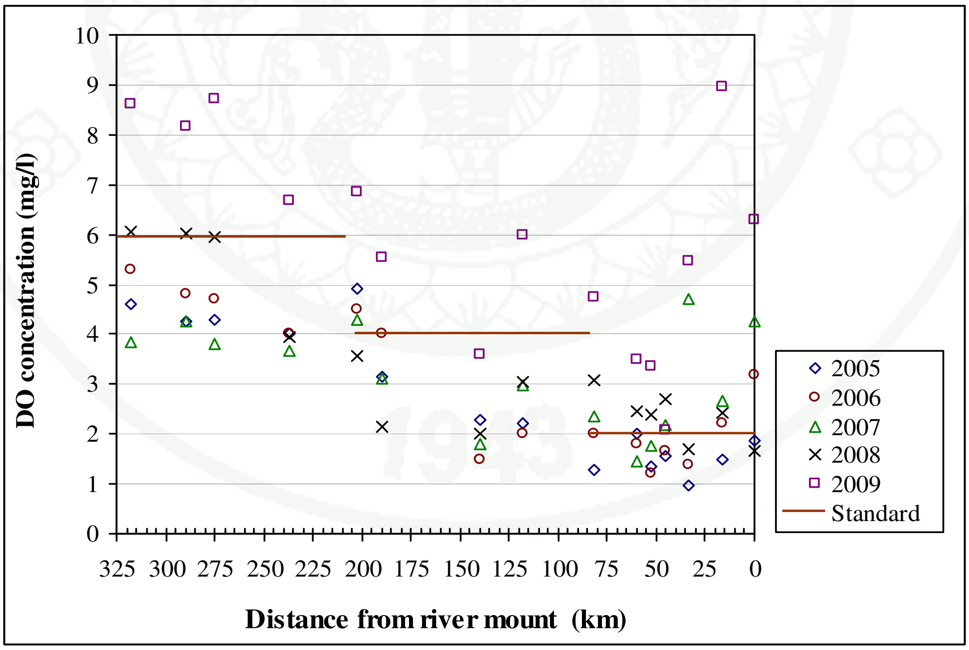

Figure 3 DO concentrations of Tha Chin River in dry seasons during 2005-2009.

6.2 Previous works on water quality management for Tha Chin River

Water quality problem from the excessive loadings of pollutants to water body is a major concern of the Tha Chin River. With respect to the problem, many works on water quality management for this river have been carried out during several past years. Most of them employed mathematical models as a tool to support their works. For examples:

Simachaya and Heathcote (1999) used WASP5 model (a water analysis simulation program) and the ArcView (a desktop geographical information system) to formulate the water quality management plan. In their work, these two systems were linked by the AVENUE language and then used to simulate water quality of the river under present and future situations. This work indicated that unless strict measures were conducted, DO concentrations in the river would dramatically decrease as far as 100 km from the river mouth over the next 15 years.

Kaewkrajang (2000) formulated a linear programming model in which an economic objective and constraints on environment and interactive features including industrial activity, agricultural production, land availability, and soil loss in the river basin were jointly considered. The model was used to investigate various environmental-economical scenarios to provide the basis for policy formulation regarding regional socio-economic development and environmental protection.

According to Schaffner (2007), water quality situation in the upper section of the Tha Chin River was investigated by PCD and Pro-En Technologies in 2002. The QUAL2E model, a type of simulation model, was used to determine the policy for BOD loading control based on the total maximum daily load (TMDL) concept. Their results showed that BOD loadings to this river section were too high with the major loadings from aquaculture (32%), community (25%), agriculture (22%) and livestock production (18%). Industries were found to play a minor role. With the TMDL concept, it was required that 75% of the current BOD load from domestic and industrial sources, 60% from livestock and agricultural sources, and 40% from non-point sources must be removed from the discharges.

Lekphet *et al*. (2004) combined the model of Simachaya and Healthcote (1999) with stakeholder analysis module to rank pollutant sources with respect to the perception of stakeholders. The analysis indicated that industrial waste was the most important while domestic and pig farming wastes ranked last. The highest score of DO improvement was attained by reducing 70% of waste from these three sources.

In the same year, Piyasatit (2004) developed a mathematical model based on two-dimensional vertically averaged mass balance equations of BOD and DO. The finite element method with Galerkin's weighted residual technique was used in the model formulation. The developed model was verified showing a good fit between model results and exact solutions. Model application to the Tha Chin River indicated that the decrease of BOD load from domestic wastewater in the middle section and from agricultural waste in the lower one of the river basin could provide the best result to water quality improvement.

Tungsubprayakorn (2004) worked on water quality management using the software package MIKE11 to simulate water quality of the river under various waste reduction scenarios. The BOD and DO concentrations in the water body during dry season in 2011 were predicted based on the discharge of treated and untreated wastewaters. This work indicated that the river would dramatically deteriorate with low DO condition by that time. However, water quality could be improved to meet the standard with the reduction of BOD load by 50 percent.

From the above works, it was found that their management policies were formulated mainly based on simulation model results. The incorporation of optimization techniques into their solution has been seldom conducted. Moreover, among a few works that employed optimization techniques, they were devoted to large-scale planning for pollution control without involving the impact of tide to the ecosystem, more particularly water quality, of the water body.

MATERIALS AND METHODS

Materials

1. Personal computer, Pentium 4, 2.4 GHz, 160 GB, RAM 1 GB
2. Tha Chin River map
3. Cross section data of Tha Chin River
4. Water quality data of Tha Chin River
5. Pollution source data in terms of BOD load

Methods

1. From the BOD and DO mass balance equations formulate water quality simulation models to predict BOD and DO concentrations in the river. The finite element method with Garlerkin's weighted residual technique is employed to transform the partial differential equations to a set of algebraic equations.

2. Define the decision variables, the objective function and the constraints of the optimization problem. In this study, the objective function is to maximize the total amount of BOD loadings to a river. The related constraints include the desired BOD and DO concentrations at some identified points and the practical range of degrees of BOD load removal of treatment plants.

3. Formulate the optimization model from the identified objective function and constraints. The BOD and DO constraints are obtained from the BOD and DO dispersion models. The optimization model is in the standard form of linear programming model

4. Verify the simulation model and the water quality management model.

5. Collect the necessary data of the study area, the middle and lower sections of the Tha Chin River.

6. Apply the developed water quality management model to the study area. The purpose of this application is to determine the proper wastewater management in order to maintain the BOD and DO concentrations of this river to meet the water quality standards. Computer program is written and used in model computation.

RESULTS AND DISCUSSION

1. Development of BOD and DO Dispersion Models

1.1 Basic governing equations

The two-dimensional vertically averaged mass balance equations for BOD and DO (presented in detail in literature review part) are the basic governing equations of dispersion models in this study. These equations are renumbered by Equations (6) and (7) as shown below.

$$\frac{\partial B}{\partial t} + u\frac{\partial B}{\partial x} + v\frac{\partial B}{\partial y} - \frac{1}{h}\left[\frac{\partial}{\partial x}(hK_x\frac{\partial B}{\partial x}) + \frac{\partial}{\partial y}(hK_y\frac{\partial B}{\partial y})\right] + k_1 B + k_s B - R_{bc} - R_{bu} = 0 \qquad (6)$$

$$\frac{\partial D}{\partial t} + u\frac{\partial D}{\partial x} + v\frac{\partial D}{\partial y} - \frac{1}{h}\left[\frac{\partial}{\partial x}(hK_x\frac{\partial D}{\partial x}) + \frac{\partial}{\partial y}(hK_y\frac{\partial D}{\partial y})\right] + k_1 B - k_2(D_s - D) - R_d = 0 \qquad (7)$$

1.2 Boundary conditions

For the uniqueness of solutions, appropriate boundary conditions must be specified. The boundary conditions of the two-dimensional mass balance equation can be separated into two types, i.e., 1) S_c boundary, where substance discharge flux is specified, and 2) S_o boundary, where substance concentration is specified (Figure 4).

On S_o boundary:

$$B = B_o^* \text{ on } S_o \qquad (8)$$

$$D = D_o^* \text{ on } S_o \qquad (9)$$

On S_c boundary:

$$Q_c = Q_c^* \text{ on } S_c \qquad (10)$$

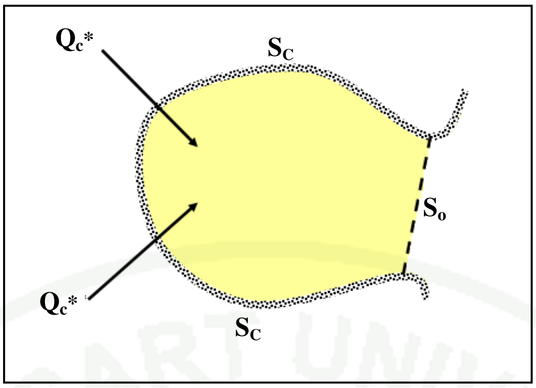

Figure 4 Boundary conditions.

1.3 Model formulation

The finite element method with Galerkin's weighted residual technique is used to solve the basic governing equations. In this method, the study domain (Ω) is divided into a finite number of subdomains, called elements; while unknown variables over each element are approximated by a trial function specified in terms of independent variables and undetermined parameters or in terms of variables at nodal points. Here, the unknown variables B and D are approximated by $\tilde{B}$ and $\tilde{D}$, respectively, which are a linear combination of their nodal values (B_i and D_i) and the approximating functions, sometimes called interpolation functions, (N_i). That is

$$B \approx \tilde{B} = \sum_{i=1}^{n} N_i B_i \tag{11}$$

$$D \approx \tilde{D} = \sum_{i=1}^{n} N_i D_i \tag{12}$$

where n is the total number of nodes.

Substituting the approximated values $\tilde{B}$ and $\tilde{D}$ into Equations (6) and (7), respectively, results in some error or residual. Expressing Equations (6) and (7) in more compact form by Equations (13) and (14), respectively.

$$F_B(u,v,h,B) = 0 \tag{13}$$

$$F_D(u,v,h,D,B) = 0 \tag{14}$$

The following residuals are obtained.

$$R_B = F_B(u,v,h,\tilde{B}) - F_B(u,v,h,B) = F_B(u,v,h,\tilde{B}) \tag{15}$$

$$R_D = F_D(u,v,h,\tilde{D},\tilde{B}) - F_D(u,v,h,D,B) = F_D(u,v,h,\tilde{D},\tilde{B}) \tag{16}$$

The notion in weighted residual technique is to force the residual to be zero in a weighted-residual sense. Then, we obtain weighted residual equations as

$$\iint_{\Omega} w_B F_B(u,v,h,\tilde{B})\,d\Omega = 0 \tag{17}$$

$$\iint_{\Omega} w_D F_D(u,v,h,\tilde{D},\tilde{B})\,d\Omega = 0 \tag{18}$$

where w_B and w_D are weighting functions.

In the Galerkin's method, the weighting functions are chosen to be the same as the interpolation functions; that is w = **N**. So, Equations (17) and (18) can be written by Equations (19) and (20), respectively.

$$\iint_{\Omega} \mathbf{N} F_B(u,v,h,\tilde{B})\,d\Omega = \mathbf{0} \tag{19}$$

$$\iint_{\Omega} \mathbf{N} F_D(u,v,h,\tilde{D},\tilde{B})\,d\Omega = \mathbf{0} \tag{20}$$

Replace $F_B(u,v,h,\tilde{B})$ and $F_D(u,v,h,\tilde{D},\tilde{B})$ by the extended expressions, we obtain

$$\iint_{\Omega} \mathbf{N} \left\{ \frac{\partial \tilde{B}}{\partial t} + u\frac{\partial \tilde{B}}{\partial x} + v\frac{\partial \tilde{B}}{\partial y} - \frac{1}{h}\left[\frac{\partial}{\partial x}(hK_x \frac{\partial \tilde{B}}{\partial x}) + \frac{\partial}{\partial y}(hK_y \frac{\partial \tilde{B}}{\partial y}) \right] + k_1\tilde{B} + k_s\tilde{B} - R_{bc} - R_{bu} \right\} d\Omega = \mathbf{0} \quad (21)$$

and

$$\iint_{\Omega} \mathbf{N} \left\{ \frac{\partial \tilde{D}}{\partial t} + u\frac{\partial \tilde{D}}{\partial x} + v\frac{\partial \tilde{D}}{\partial y} - \frac{1}{h}\left[\frac{\partial}{\partial x}(hK_x \frac{\partial \tilde{D}}{\partial x}) + \frac{\partial}{\partial y}(hK_y \frac{\partial \tilde{D}}{\partial y}) \right] + k_1\tilde{B} - k_2(D_s - \tilde{D}) - R_d \right\} d\Omega = \mathbf{0} \quad (22)$$

Apply partial differential and expand the above equations to yield

$$\iint_{\Omega} \mathbf{N} \left\{ \frac{\partial \tilde{B}}{\partial t} + u\frac{\partial \tilde{B}}{\partial x} + v\frac{\partial \tilde{B}}{\partial y} + k_1\tilde{B} + k_s\tilde{B} - R_{bc} - R_{bu} \right\} d\Omega - \iint_{\Omega} \frac{\mathbf{N}}{h} \left[K_x \frac{\partial h}{\partial x}\frac{\partial \tilde{B}}{\partial x} + K_y \frac{\partial h}{\partial y}\frac{\partial \tilde{B}}{\partial y} \right] d\Omega - \iint_{\Omega} \mathbf{N} \left[K_x \frac{\partial}{\partial x}(\frac{\partial \tilde{B}}{\partial x}) + K_y \frac{\partial}{\partial y}(\frac{\partial \tilde{B}}{\partial y}) \right] d\Omega = \mathbf{0} \quad (23)$$

and

$$\iint_{\Omega} \mathbf{N} \left\{ \frac{\partial \tilde{D}}{\partial t} + u\frac{\partial \tilde{D}}{\partial x} + v\frac{\partial \tilde{D}}{\partial y} + k_1\tilde{B} - k_2(D_s - \tilde{D}) - R_d \right\} d\Omega - \iint_{\Omega} \frac{\mathbf{N}}{h} \left[K_x \frac{\partial h}{\partial x}\frac{\partial \tilde{D}}{\partial x} + K_y \frac{\partial h}{\partial y}\frac{\partial \tilde{D}}{\partial y} \right] d\Omega - \iint_{\Omega} \mathbf{N} \left[K_x \frac{\partial}{\partial x}(\frac{\partial \tilde{D}}{\partial x}) + K_y \frac{\partial}{\partial y}(\frac{\partial \tilde{D}}{\partial y}) \right] d\Omega = \mathbf{0} \quad (24)$$

The third term of Equations (23) and (24) can be expanded to

$$\iint_{\Omega} \mathbf{N}\left[K_x \frac{\partial}{\partial x}\left(\frac{\partial \tilde{B}}{\partial x}\right) + K_y \frac{\partial}{\partial y}\left(\frac{\partial \tilde{B}}{\partial y}\right)\right] d\Omega = \iint_{\Omega}\left[K_x \frac{\partial}{\partial x}\left(\mathbf{N}\frac{\partial \tilde{B}}{\partial x}\right) + K_y \frac{\partial}{\partial y}\left(\mathbf{N}\frac{\partial \tilde{B}}{\partial y}\right)\right] d\Omega - \iint_{\Omega}\left[K_x \frac{\partial \mathbf{N}}{\partial x}\frac{\partial \tilde{B}}{\partial x} + K_y \frac{\partial \mathbf{N}}{\partial y}\frac{\partial \tilde{B}}{\partial y}\right] d\Omega \tag{25}$$

and

$$\iint_{\Omega} \mathbf{N}\left[K_x \frac{\partial}{\partial x}\left(\frac{\partial \tilde{D}}{\partial x}\right) + K_y \frac{\partial}{\partial y}\left(\frac{\partial \tilde{D}}{\partial y}\right)\right] d\Omega = \iint_{\Omega}\left[K_x \frac{\partial}{\partial x}\left(\mathbf{N}\frac{\partial \tilde{D}}{\partial x}\right) + K_y \frac{\partial}{\partial y}\left(\mathbf{N}\frac{\partial \tilde{D}}{\partial y}\right)\right] d\Omega - \iint_{\Omega}\left[K_x \frac{\partial \mathbf{N}}{\partial x}\frac{\partial \tilde{D}}{\partial x} + K_y \frac{\partial \mathbf{N}}{\partial y}\frac{\partial \tilde{D}}{\partial y}\right] d\Omega \tag{26}$$

So, Equations (23) and (24) become

$$\iint_{\Omega} \mathbf{N}\left\{\frac{\partial \tilde{B}}{\partial t} + u\frac{\partial \tilde{B}}{\partial x} + v\frac{\partial \tilde{B}}{\partial y} + k_1\tilde{B} + k_s\tilde{B} - R_{bc} - R_{bu}\right\} d\Omega - \iint_{\Omega}\frac{\mathbf{N}}{h}\left[K_x \frac{\partial h}{\partial x}\frac{\partial \tilde{B}}{\partial x} + K_y \frac{\partial h}{\partial y}\frac{\partial \tilde{B}}{\partial y}\right] d\Omega + \iint_{\Omega}\left[K_x \frac{\partial \mathbf{N}}{\partial x}\frac{\partial \tilde{B}}{\partial x} + K_y \frac{\partial \mathbf{N}}{\partial y}\frac{\partial \tilde{B}}{\partial y}\right] d\Omega - \iint_{\Omega}\left[K_x \frac{\partial}{\partial x}\left(\mathbf{N}\frac{\partial \tilde{B}}{\partial x}\right) + K_y \frac{\partial}{\partial y}\left(\mathbf{N}\frac{\partial \tilde{B}}{\partial y}\right)\right] d\Omega = \mathbf{0} \tag{27}$$

and

$$\iint_{\Omega} \mathbf{N}\left\{\frac{\partial \tilde{D}}{\partial t} + u\frac{\partial \tilde{D}}{\partial x} + v\frac{\partial \tilde{D}}{\partial y} + k_1\tilde{B} - k_2(D_s - \tilde{D}) - R_d\right\} d\Omega - \iint_{\Omega}\frac{\mathbf{N}}{h}\left[K_x \frac{\partial h}{\partial x}\frac{\partial \tilde{D}}{\partial x} + K_y \frac{\partial h}{\partial y}\frac{\partial \tilde{D}}{\partial y}\right] d\Omega + \iint_{\Omega}\left[K_x \frac{\partial \mathbf{N}}{\partial x}\frac{\partial \tilde{D}}{\partial x} + K_y \frac{\partial \mathbf{N}}{\partial y}\frac{\partial \tilde{D}}{\partial y}\right] d\Omega - \iint_{\Omega}\left[K_x \frac{\partial}{\partial x}\left(\mathbf{N}\frac{\partial \tilde{D}}{\partial x}\right) + K_y \frac{\partial}{\partial y}\left(\mathbf{N}\frac{\partial \tilde{D}}{\partial y}\right)\right] d\Omega = \mathbf{0} \tag{28}$$

Green's Theorem states that

$$\iint_{\Omega} \left(\frac{\partial m}{\partial x} + \frac{\partial n}{\partial y}\right) dx\,dy = \oint_{S} (m\,dy - n\,dx) \tag{29}$$

where m and n are functions of x and y, i.e., m(x,y) and n(x,y), respectively.

Apply Green's Theorem to the fourth term of Equations (27) and (28), we obtain

$$\iint_{\Omega} \left[K_x \frac{\partial}{\partial x}\left(\mathbf{N}\frac{\partial \tilde{B}}{\partial x}\right) + K_y \frac{\partial}{\partial y}\left(\mathbf{N}\frac{\partial \tilde{B}}{\partial y}\right) \right] d\Omega = \oint_{S} \mathbf{N}\left(K_x \frac{\partial \tilde{B}}{\partial x} dy - K_y \frac{\partial \tilde{B}}{\partial y} dx\right) \tag{30}$$

and

$$\iint_{\Omega} \left[K_x \frac{\partial}{\partial x}\left(\mathbf{N}\frac{\partial \tilde{D}}{\partial x}\right) + K_y \frac{\partial}{\partial y}\left(\mathbf{N}\frac{\partial \tilde{D}}{\partial y}\right) \right] d\Omega = \oint_{S} \mathbf{N}\left(K_x \frac{\partial \tilde{D}}{\partial x} dy - K_y \frac{\partial \tilde{D}}{\partial y} dx\right) \tag{31}$$

The terms $K_x \frac{\partial \tilde{B}}{\partial x} dy - K_y \frac{\partial \tilde{B}}{\partial y} dx$ and $K_x \frac{\partial \tilde{D}}{\partial x} dy - K_y \frac{\partial \tilde{D}}{\partial y} dx$ represent BOD and DO dispersive fluxes, respectively, per unit length of the boundary S_c. Write Equations (30) and (31) in more compact form, i.e.,

$$\oint_{S} \mathbf{N}\left(K_x \frac{\partial \tilde{B}}{\partial x} dy - K_y \frac{\partial \tilde{B}}{\partial y} dx\right) = \oint_{S} Q_b \mathbf{N}\, dS \tag{32}$$

and

$$\oint_{S} \mathbf{N}\left(K_x \frac{\partial \tilde{D}}{\partial x} dy - K_y \frac{\partial \tilde{D}}{\partial y} dx\right) = \oint_{S} Q_d \mathbf{N}\, dS \tag{33}$$

So, Equations (27) and (28) can be written as

$$\iint_{\Omega} \mathbf{N} \left\{ \frac{\partial \tilde{B}}{\partial t} + u\frac{\partial \tilde{B}}{\partial x} + v\frac{\partial \tilde{B}}{\partial y} + k_1\tilde{B} + k_s\tilde{B} - R_{bc} - R_{bu} \right\} d\Omega$$

$$- \iint_{\Omega} \frac{\mathbf{N}}{h} \left[K_x \frac{\partial h}{\partial x}\frac{\partial \tilde{B}}{\partial x} + K_y \frac{\partial h}{\partial y}\frac{\partial \tilde{B}}{\partial y} \right] d\Omega + \iint_{\Omega} \left[K_x \frac{\partial \mathbf{N}}{\partial x}\frac{\partial \tilde{B}}{\partial x} + K_y \frac{\partial \mathbf{N}}{\partial y}\frac{\partial \tilde{B}}{\partial y} \right] d\Omega \quad (34)$$

$$- \oint_{S} Q_b \mathbf{N} dS \;=\; \mathbf{0}$$

and

$$\iint_{\Omega} \mathbf{N} \left\{ \frac{\partial \tilde{D}}{\partial t} + u\frac{\partial \tilde{D}}{\partial x} + v\frac{\partial \tilde{D}}{\partial y} + k_1\tilde{B} - k_2(D_s - \tilde{D}) - R_d \right\} d\Omega$$

$$- \iint_{\Omega} \frac{\mathbf{N}}{h} \left[K_x \frac{\partial h}{\partial x}\frac{\partial \tilde{D}}{\partial x} + K_y \frac{\partial h}{\partial y}\frac{\partial \tilde{D}}{\partial y} \right] d\Omega + \iint_{\Omega} \left[K_x \frac{\partial \mathbf{N}}{\partial x}\frac{\partial \tilde{D}}{\partial x} + K_y \frac{\partial \mathbf{N}}{\partial y}\frac{\partial \tilde{D}}{\partial y} \right] d\Omega \quad (35)$$

$$- \oint_{S} Q_d \mathbf{N} dS \;=\; \mathbf{0}$$

As previously mentioned, in the finite element method the whole domain is divided into elements, and the unknown variables are replaced by the function of the nodal variables of each element and their interpolation functions. The integrals over the study domain are obtained by summation of integrals over each element.

Represent Equations (34) and (35) by

$$\mathbf{F_B} \;=\; \sum_{e=1}^{m} \mathbf{F_B^e} \;=\; \mathbf{0} \quad (36)$$

and

$$\mathbf{F_D} \;=\; \sum_{e=1}^{m} \mathbf{F_D^e} \;=\; \mathbf{0} \quad (37)$$

where $\mathbf{F_B^e}$ and $\mathbf{F_D^e}$ are the element weighted residual integrals for element e; m is the total number of elements in the whole domain.

The approximated solutions of B and D for element e, represented by B^e and D^e, are expressed by

$$B^e = \sum_{i=1}^{k} N_i B_i = \mathbf{N^e}^{\mathbf{T}} \mathbf{B^e} \quad (38)$$

$$D^e = \sum_{i=1}^{k} N_i D_i = \mathbf{N^e}^{\mathbf{T}} \mathbf{D^e} \quad (39)$$

where B_i and D_i are the BOD and DO values at nodal points of element e, and N_i is the interpolation function. The flow velocities u and v, water depth h, and DO saturation concentration D_s can also be expressed in the same manner, i.e.,

$$u^e = \sum_{i=1}^{k} N_i u_i = \mathbf{N^e}^{\mathbf{T}} \mathbf{U^e} \quad (40)$$

$$v^e = \sum_{i=1}^{k} N_i v_i = \mathbf{N^e}^{\mathbf{T}} \mathbf{V^e} \quad (41)$$

$$h^e = \sum_{i=1}^{k} N_i h_i = \mathbf{N^e}^{\mathbf{T}} \mathbf{H^e} \quad (42)$$

$$D_S^e = \sum_{i=1}^{k} N_i D_{S_i} = \mathbf{N^e}^{\mathbf{T}} \mathbf{D_S^e} \quad (43)$$

The weighted residual equations for BOD and DO dispersion models can be written in matrix form by Equations (44) and (45), respectively.

$$\mathbf{F_B} = \sum_{e=1}^{m} \mathbf{F_B^e}$$

$$= \sum_{e=1}^{m} [\iint_{A^e} \mathbf{N}^e \mathbf{N}^{e^T} \frac{\partial \mathbf{B}^e}{\partial t} dA + \iint_{A^e} \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{U}^e \frac{\partial \mathbf{N}^{e^T}}{\partial x} \mathbf{B}^e dA + \iint_{A^e} \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{V}^e \frac{\partial \mathbf{N}^{e^T}}{\partial y} \mathbf{B}^e dA$$

$$+ \iint_{A^e} k_1 \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{B}^e dA + \iint_{A^e} k_s \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{B}^e dA - \iint_{A^e} \mathbf{R}^e_{bc} \mathbf{N}^e dA - \iint_{A^e} \mathbf{R}^e_{bu} \mathbf{N}^e dA \tag{44}$$

$$- \iint_{A^e} \frac{K_x}{\mathbf{N}^{e^T} \mathbf{H}^e} \mathbf{N}^e \frac{\partial \mathbf{N}^{e^T}}{\partial x} \mathbf{H}^e \frac{\partial \mathbf{N}^{e^T}}{\partial x} \mathbf{B}^e dA - \iint_{A^e} \frac{K_y}{\mathbf{N}^{e^T} \mathbf{H}^e} \mathbf{N}^e \frac{\partial \mathbf{N}^{e^T}}{\partial y} \mathbf{H}^e \frac{\partial \mathbf{N}^{e^T}}{\partial y} \mathbf{B}^e dA$$

$$+ \iint_{A^e} K_x \frac{\partial \mathbf{N}^e}{\partial x} \frac{\partial \mathbf{N}^{e^T}}{\partial x} \mathbf{B}^e dA + \iint_{A^e} K_y \frac{\partial \mathbf{N}^e}{\partial y} \frac{\partial \mathbf{N}^{e^T}}{\partial y} \mathbf{B}^e dA - \oint_{S^e} \mathbf{Q}^e_b \mathbf{N}^e dL] = \mathbf{0}$$

$$\mathbf{F_D} = \sum_{e=1}^{m} \mathbf{F_D^e}$$

$$= \sum_{e=1}^{m} [\iint_{A^e} \mathbf{N}^e \mathbf{N}^{e^T} \frac{\partial \mathbf{D}^e}{\partial t} dA + \iint_{A^e} \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{U}^e \frac{\partial \mathbf{N}^{e^T}}{\partial x} \mathbf{D}^e dA + \iint_{A^e} \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{V}^e \frac{\partial \mathbf{N}^{e^T}}{\partial y} \mathbf{D}^e dA$$

$$+ \iint_{A^e} k_1 \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{B}^e dA - \iint_{A^e} k_2 \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{D}^e_s dA + \iint_{A^e} k_2 \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{D}^e dA - \iint_{A^e} \mathbf{R}^e_d \mathbf{N}^e dA \tag{45}$$

$$- \iint_{A^e} \frac{K_x}{\mathbf{N}^{e^T} \mathbf{H}^e} \mathbf{N}^e \frac{\partial \mathbf{N}^{e^T}}{\partial x} \mathbf{H}^e \frac{\partial \mathbf{N}^{e^T}}{\partial x} \mathbf{D}^e dA - \iint_{A^e} \frac{K_y}{\mathbf{N}^{e^T} \mathbf{H}^e} \mathbf{N}^e \frac{\partial \mathbf{N}^{e^T}}{\partial y} \mathbf{H}^e \frac{\partial \mathbf{N}^{e^T}}{\partial y} \mathbf{D}^e dA$$

$$+ \iint_{A^e} K_x \frac{\partial \mathbf{N}^e}{\partial x} \frac{\partial \mathbf{N}^{e^T}}{\partial x} \mathbf{D}^e dA + \iint_{A^e} K_y \frac{\partial \mathbf{N}^e}{\partial y} \frac{\partial \mathbf{N}^{e^T}}{\partial y} \mathbf{D}^e dA - \oint_{S^e} \mathbf{Q}^e_d \mathbf{N}^e dL] = \mathbf{0}$$

Equations (44) and (45) can be written in more compact form as

$$\mathbf{F_B} = \sum_{e=1}^{m} \mathbf{F_B^e} = \mathbf{M} \frac{d\mathbf{B}}{dt} + \mathbf{F}\,\mathbf{B} - \mathbf{M}_{Rc} - \mathbf{M}_{qb} = \mathbf{0} \tag{46}$$

and

$$\mathbf{F_D} = \sum_{e=1}^{m} \mathbf{F_D^e} = \mathbf{M} \frac{d\mathbf{D}}{dt} + \mathbf{G}\,\mathbf{D} + \mathbf{M}_{k1} \mathbf{B} - \mathbf{M}_{qd} = \mathbf{0} \tag{47}$$

where

$$\mathbf{M} = \sum_{e=1}^{m} [\iint_{A^e} \mathbf{N}^e \mathbf{N}^{e^T} dA] \tag{48}$$

$$\mathbf{F} = \sum_{e=1}^{m} [\iint_{A^e} \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{U}^e \frac{\partial \mathbf{N}^{e^T}}{\partial x} dA + \iint_{A^e} \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{V}^e \frac{\partial \mathbf{N}^{e^T}}{\partial y} dA + \iint_{A^e} k_1 \mathbf{N}^e \mathbf{N}^{e^T} dA + \iint_{A^e} k_s \mathbf{N}^e \mathbf{N}^{e^T} dA$$

$$- \iint_{A^e} \frac{K_x}{\mathbf{N}^{e^T} \mathbf{H}^e} \mathbf{N}^e \frac{\partial \mathbf{N}^{e^T}}{\partial x} \mathbf{H}^e \frac{\partial \mathbf{N}^{e^T}}{\partial x} dA - \iint_{A^e} \frac{K_y}{\mathbf{N}^{e^T} \mathbf{H}^e} \mathbf{N}^e \frac{\partial \mathbf{N}^{e^T}}{\partial y} \mathbf{H}^e \frac{\partial \mathbf{N}^{e^T}}{\partial y} dA \tag{49}$$

$$+ \iint_{A^e} K_x \frac{\partial \mathbf{N}^e}{\partial x} \frac{\partial \mathbf{N}^{e^T}}{\partial x} dA + \iint_{A^e} K_y \frac{\partial \mathbf{N}^e}{\partial y} \frac{\partial \mathbf{N}^{e^T}}{\partial y} dA]$$

$$\mathbf{G} = \sum_{e=1}^{m} [\iint_{A^e} \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{U}^e \frac{\partial \mathbf{N}^{e^T}}{\partial x} dA + \iint_{A^e} \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{V}^e \frac{\partial \mathbf{N}^{e^T}}{\partial y} dA + \iint_{A^e} k_2 \mathbf{N}^e \mathbf{N}^{e^T} dA$$

$$- \iint_{A^e} \frac{K_x}{\mathbf{N}^{e^T} \mathbf{H}^e} \mathbf{N}^e \frac{\partial \mathbf{N}^{e^T}}{\partial x} \mathbf{H}^e \frac{\partial \mathbf{N}^{e^T}}{\partial x} dA - \iint_{A^e} \frac{K_y}{\mathbf{N}^{e^T} \mathbf{H}^e} \mathbf{N}^e \frac{\partial \mathbf{N}^{e^T}}{\partial y} \mathbf{H}^e \frac{\partial \mathbf{N}^{e^T}}{\partial y} dA \tag{50}$$

$$+ \iint_{A^e} K_x \frac{\partial \mathbf{N}^e}{\partial x} \frac{\partial \mathbf{N}^{e^T}}{\partial x} dA + \iint_{A^e} K_y \frac{\partial \mathbf{N}^e}{\partial y} \frac{\partial \mathbf{N}^{e^T}}{\partial y} dA]$$

$$\mathbf{M}_{k1} = \sum_{e=1}^{m} [\iint_{A^e} k_1 \mathbf{N}^e \mathbf{N}^{e^T} dA] \tag{51}$$

$$\mathbf{M}_{Rc} = \sum_{e=1}^{m} [\iint_{A^e} \mathbf{R}^e_{bc} \mathbf{N}^e dA] \tag{52}$$

$$\mathbf{M}_{qb} = \sum_{e=1}^{m} [\iint_{A^e} \mathbf{R}^e_{bu} \mathbf{N}^e dA + \oint_{S^e} \mathbf{Q}^e_b \mathbf{N}^e dL] \tag{53}$$

$$\mathbf{M}_{qd} = \sum_{e=1}^{m} [\iint_{A^e} k_2 \mathbf{N}^e \mathbf{N}^{e^T} \mathbf{D}^e_s dA + \iint_{A^e} \mathbf{R}^e_d \mathbf{N}^e dA + \oint_{S^e} \mathbf{Q}^e_d \mathbf{N}^e dL] \tag{54}$$

2. Development of Water Quality Management Model (Optimization Model) Using Linear Programming

2.1 Objective function

In this study, the objective of water quality management is to maximize the total BOD load which can be discharged into a water body. Here, BOD loadings are divided into two types: 1) controllable BOD load, and 2) uncontrollable BOD load. The first type of BOD load is defined for wastewaters which are collected to municipal wastewater treatment plants. The amount of controllable BOD load in this sense is the remaining BOD load in the treated wastewaters. In contrast to the first type, the second type of BOD load is defined for BOD load of other discharges, including untreated domestic wastewaters, industrial effluents, and wastewaters from pig farms and aquacultures. The discharges of BOD load from the latter type are considered uncontrollable, though some wastewaters have been treated but it is not possible to force them to treat more if their effluents have already satisfied the effluent standards. Thus with this notion, the value of the objective function will vary only with the controllable BOD load discharged into the receiving water.

Let Z represent the objective function value. The expression of objective function can be written as

$$\text{Maximize } Z = \sum_{e=1}^{m}\left[R_{bc}^{e}\right] \qquad (55)$$

where R_{bc}^{e} is the total controllable BOD load that is discharged into element e.

Replace R_{bc}^{e} by the product of L_c^e and P_c^e, in which L_c^e is total controllable BOD load generated at element e, and P_c^e is the ratio of remaining BOD load to the generated BOD load, i.e., $P_c^e = R_{bc}^e / L_c^e$. Then, Equation (55) becomes

$$\text{Maximize } Z = \sum_{e=1}^{m} [L_c^e P_c^e] \tag{56}$$

Equation (56) shows that the objective function of the model is expressed by the linear function of L_c^e (which is known value and can be considered as the coefficient of the objective function) and P_c^e (which is the decision variable of the model). Determining a set of P_c^e values that maximize Z value will yield the optimal solution of this optimization problem.

2.2 Constraints

The constraints of this model are identified based on the desired water quality and the practical range of degree of BOD load removal by treatment plants. The constraints are expressed as follow.

2.2.1 Constraints on BOD and DO

Constraint on BOD states that at any time t_i, BOD concentrations at some identified nodal points (B_{t_i}) are not more than the specified values (B^*) for that point, i.e.,

$$B_{t_i} \leq B^* \tag{57}$$

Constraint on DO states that at any time t_i, DO concentrations at some identified nodal points (D_{t_i}) are not less than the specified values (D^*) for that point, i.e.,

$$D_{t_i} \geq D^* \tag{58}$$

2.2.2 Constraint on wastewater treatment

In this study, the values of decision variables (P_c^e) should be specified within the reasonable range. In fact, these values are related to the degree of wastewater treatment (Tr); that is, $P_c^e = 1 - \text{Tr}$. Thus, the lower bound of the range usually depends on the available technologies of wastewater treatment; whereas the upper bound should depend on effluent standards. The constraint on wastewater treatment can be expressed in terms of P_c^e as follow.

$$P_{c,min} \leq P_c^e \leq P_{c,max} \tag{59}$$

2.3 Formulation of BOD and DO constraint inequalities

Recalling Equation (46), we replace matrix $\mathbf{M}_{Rc}$ in this equation by the product of n×m matrix $\mathbf{M}_{rm}$ and m×1 matrix $\mathbf{R}_{bc}$, and then substitute matrix $\mathbf{R}_{bc}$ by the product of m×m matrix $\mathbf{L}_c$ and m×1 matrix $\mathbf{P}_c$ to obtain

$$\mathbf{M}\frac{d\mathbf{B}}{dt} + \mathbf{F}\mathbf{B} - \mathbf{M}_{rm}\mathbf{L}_c\mathbf{P}_c - \mathbf{M}_{qb} = \mathbf{0} \tag{60}$$

where

$$\mathbf{M}_{rm} = \sum_{e=1}^{m} [\iint_{A^e} \mathbf{N}^e dA] \tag{61}$$

Note that $\mathbf{L}_c$ is m×m matrix with L_c^e (e = 1, 2, m) on the diagonal whereas all other member are zero.

Applying Euler's method to Equations (60) and (47), we obtain

$$\frac{\mathbf{M}}{\Delta t}(\mathbf{B}_{t_2} - \mathbf{B}_{t_1}) + \mathbf{F}_{t_1}\mathbf{B}_{t_1} - \mathbf{M}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c - \mathbf{M}_{qb,t_1} = \mathbf{0} \tag{62}$$

and

$$\frac{\mathbf{M}}{\Delta t}(\mathbf{D}_{t_2} - \mathbf{D}_{t_1}) + \mathbf{G}_{t_1}\mathbf{D}_{t_1} + \mathbf{M}_{k1}\mathbf{B}_{t_1} - \mathbf{M}_{qd,t_1} = \mathbf{0} \tag{63}$$

where subscripts t_1 and t_2 indicate that the matrices are of the initial time and the next time, respectively, with a time step $\Delta t = t_2 - t_1$.

Substituting $\frac{\mathbf{M}}{\Delta t}$ in Equations (62) and (63) by $\mathbf{M}_t$ and rearranging the equations yields

$$\mathbf{M}_t\mathbf{B}_{t_2} = (\mathbf{M}_t - \mathbf{F}_{t_1})\mathbf{B}_{t_1} + \mathbf{M}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c + \mathbf{M}_{qb,t_1} \tag{64}$$

and

$$\mathbf{M}_t\mathbf{D}_{t_2} = (\mathbf{M}_t - \mathbf{G}_{t_1})\mathbf{D}_{t_1} - \mathbf{M}_{k1}\mathbf{B}_{t_1} + \mathbf{M}_{qd,t_1} \tag{65}$$

Let $\mathbf{M}_t - \mathbf{F}_{t_1} = \mathbf{M}_{f,t_1}$ and $\mathbf{M}_t - \mathbf{G}_{t_1} = \mathbf{M}_{g,t_1}$. Equations (64) and (65) can be written as

$$\mathbf{M}_t\mathbf{B}_{t_2} = \mathbf{M}_{f,t_1}\mathbf{B}_{t_1} + \mathbf{M}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c + \mathbf{M}_{qb,t_1} \tag{66}$$

and

$$\mathbf{M}_t\mathbf{D}_{t_2} = \mathbf{M}_{g,t_1}\mathbf{D}_{t_1} + \mathbf{M}_{k1}\mathbf{B}_{t_1} + \mathbf{M}_{qd,t_1} \tag{67}$$

To obtain the unique solution, Equations (66) and (67) must be modified such that the boundary conditions, i.e., BOD and DO concentrations at nodes on the open boundary of the domain, are satisfied. Applying the boundary conditions to Equations (66) and (67) results in the modified forms of $\mathbf{M}_t$, $\mathbf{M}_{f,t_1}$, $\mathbf{M}_{g,t_1}$, $\mathbf{M}_{rm}$, $\mathbf{M}_{k1}$, $\mathbf{M}_{qb,t_1}$ and $\mathbf{M}_{qd,t_1}$. We represent these modified matrices by $\hat{\mathbf{M}}_t$, $\hat{\mathbf{M}}_{f,t_1}$, $\hat{\mathbf{M}}_{g,t_1}$, $\hat{\mathbf{M}}_{rm}$, $\hat{\mathbf{M}}_{k1}$, $\hat{\mathbf{M}}_{qb,t_1}$ and $\hat{\mathbf{M}}_{qd,t_1}$, respectively. So, Equations (66) and (67) become

$$\hat{\mathbf{M}}_t \mathbf{B}_{t_2} = \hat{\mathbf{M}}_{f,t_1} \mathbf{B}_{t_1} + \hat{\mathbf{M}}_{rm} \mathbf{L}_{c,t_1} \mathbf{P}_c + \hat{\mathbf{M}}_{qb,t_1} \quad (68)$$

and

$$\hat{\mathbf{M}}_t \mathbf{D}_{t_2} = \hat{\mathbf{M}}_{g,t_1} \mathbf{D}_{t_1} - \hat{\mathbf{M}}_{k1} \mathbf{B}_{t_1} + \hat{\mathbf{M}}_{qd,t_1} \quad (69)$$

Note that $\mathbf{B}_{t_2}$ and $\mathbf{D}_{t_2}$ are the column vectors in which their members are the BOD and DO concentrations, respectively, at nodes at time t_2. From Equations (68) and (69), we can compute $\mathbf{B}_{t_2}$ and $\mathbf{D}_{t_2}$ by

$$\mathbf{B}_{t_2} = \hat{\mathbf{M}}_t^{-1} [\hat{\mathbf{M}}_{f,t_1} \mathbf{B}_{t_1} + \hat{\mathbf{M}}_{rm} \mathbf{L}_{c,t_1} \mathbf{P}_c + \hat{\mathbf{M}}_{qb,t_1}] \quad (70)$$

and

$$\mathbf{D}_{t_2} = \hat{\mathbf{M}}_t^{-1} [\hat{\mathbf{M}}_{g,t_1} \mathbf{D}_{t_1} - \hat{\mathbf{M}}_{k1} \mathbf{B}_{t_1} + \hat{\mathbf{M}}_{qd,t_1}] \quad (71)$$

which can be rewritten as

$$\mathbf{B}_{t_2} = \hat{\mathbf{M}}_t^{-1} \hat{\mathbf{M}}_{f,t_1} \mathbf{B}_{t_1} + \hat{\mathbf{M}}_t^{-1} \hat{\mathbf{M}}_{qb,t_1} + \hat{\mathbf{M}}_t^{-1} \hat{\mathbf{M}}_{rm} \mathbf{L}_{c,t_1} \mathbf{P}_c \quad (72)$$

and

$$\mathbf{D}_{t_2} = \hat{\mathbf{M}}_t^{-1} \hat{\mathbf{M}}_{g,t_1} \mathbf{D}_{t_1} - \hat{\mathbf{M}}_t^{-1} \hat{\mathbf{M}}_{k1} \mathbf{B}_{t_1} + \hat{\mathbf{M}}_t^{-1} \hat{\mathbf{M}}_{qd,t_1} \quad (73)$$

Replace $\mathbf{B}_{t_1}, \hat{\mathbf{M}}_{f,t_1}, \mathbf{L}_{c,t_1}, \hat{\mathbf{M}}_{qb,t_1}$ on the right hand side (RHS) of Equation (72) and $\mathbf{D}_{t_1}, \mathbf{B}_{t_1}, \hat{\mathbf{M}}_{g,t_1}, \hat{\mathbf{M}}_{qd,t_1}$ on the RHS of Equation (73) by $\mathbf{B}_{t_2}, \hat{\mathbf{M}}_{f,t_2}, \mathbf{L}_{c,t_2}, \hat{\mathbf{M}}_{qb,t_2}$, $\mathbf{D}_{t_2}, \mathbf{B}_{t_2}, \hat{\mathbf{M}}_{g,t_2}$, and $\hat{\mathbf{M}}_{qd,t_2}$, respectively. We obtain

$$\mathbf{B}_{t_3} = \hat{\mathbf{M}}_t^{-1} \hat{\mathbf{M}}_{f,t_2} \mathbf{B}_{t_2} + \hat{\mathbf{M}}_t^{-1} \hat{\mathbf{M}}_{qb,t_2} + \hat{\mathbf{M}}_t^{-1} \hat{\mathbf{M}}_{rm} \mathbf{L}_{c,t_2} \mathbf{P}_c \quad (74)$$

and

$$\mathbf{D}_{t_3} = \hat{\mathbf{M}}_t^{-1} \hat{\mathbf{M}}_{g,t_2} \mathbf{D}_{t_2} - \hat{\mathbf{M}}_t^{-1} \hat{\mathbf{M}}_{k1} \mathbf{B}_{t_2} + \hat{\mathbf{M}}_t^{-1} \hat{\mathbf{M}}_{qd,t_2} \quad (75)$$

Substituting $\mathbf{B}_{t_2}$ in Equations (74) and (75) by the expression in Equation (72) and $\mathbf{D}_{t_2}$ in Equation (75) by the expression in Equation (73) to obtain

$$\begin{aligned}\mathbf{B}_{t_3} = {} & \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\{\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1}\mathbf{B}_{t_1} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_1} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c\} \\ & + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_2} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_2}\mathbf{P}_c\end{aligned} \tag{76}$$

and

$$\begin{aligned}\mathbf{D}_{t_3} = {} & \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\{\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_1}\mathbf{D}_{t_1} - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{B}_{t_1} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_1}\} \\ & - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{B}_{t_2} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_2}\end{aligned} \tag{77}$$

Rearranging Equations (76) and (77), we can compute BOD and DO concentrations at time t_3 (i.e., $\mathbf{B}_{t_3}$ and $\mathbf{D}_{t_3}$, respectively) by

$$\begin{aligned}\mathbf{B}_{t_3} = {} & \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1}\mathbf{B}_{t_1} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_1} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_2} \\ & + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_2}\mathbf{P}_c\end{aligned} \tag{78}$$

and

$$\begin{aligned}\mathbf{D}_{t_3} = {} & \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_1}\mathbf{D}_{t_1} \\ & - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{B}_{t_1} - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1}\mathbf{B}_{t_1} \\ & - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_1} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_1} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_2} \\ & - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c\end{aligned} \tag{79}$$

Replace $\mathbf{B}_{t_2}$, $\hat{\mathbf{M}}_{f,t_2}$, $\mathbf{L}_{c,t_2}$, $\hat{\mathbf{M}}_{qb,t_2}$ on the RHS of Equation (74) and $\mathbf{D}_{t_2}$, $\mathbf{B}_{t_2}$, $\hat{\mathbf{M}}_{g,t_2}$, $\hat{\mathbf{M}}_{qd,t_2}$ on the RHS of Equation (75) by $\mathbf{B}_{t_3}$, $\hat{\mathbf{M}}_{f,t_3}$, $\mathbf{L}_{c,t_3}$, $\hat{\mathbf{M}}_{qb,t_3}$, $\mathbf{D}_{t_3}$, $\mathbf{B}_{t_3}$, $\hat{\mathbf{M}}_{g,t_3}$, and $\hat{\mathbf{M}}_{qd,t_3}$, respectively. We obtain

$$\mathbf{B}_{t_4} = \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\mathbf{B}_{t_3} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_3} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_3}\mathbf{P}_c \quad (80)$$

and

$$\mathbf{D}_{t_4} = \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\mathbf{D}_{t_3} - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{B}_{t_3} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_3} \quad (81)$$

Substituting $\mathbf{B}_{t_3}$ in Equation (80) and (81) by the expression in Equation (78) and $\mathbf{D}_{t_3}$ in Equation (81) by the expression in Equation (79), we obtain

$$\begin{aligned}\mathbf{B}_{t_4} = {} & \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\{\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1}\mathbf{B}_{t_1} \\ & + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_1} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_2} \\ & + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_2}\mathbf{P}_c\} \\ & + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_3} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_3}\mathbf{P}_c\end{aligned} \quad (82)$$

and

$$\begin{aligned}\mathbf{D}_{t_4} = {} & \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\{\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_1}\mathbf{D}_{t_1} \\ & - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{B}_{t_1} - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1}\mathbf{B}_{t_1} \\ & - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_1} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_1} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_2} \\ & - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c\} \\ & - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\{\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1}\mathbf{B}_{t_1} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_1} \\ & + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_2} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_2}\mathbf{P}_c\} \\ & + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_3}\end{aligned} \quad (83)$$

Rearranging Equations (82) and (83), we can compute BOD and DO concentrations at time t_4 (i.e., $\mathbf{B}_{t_4}$ and $\mathbf{D}_{t_4}$, respectively) by

$$
\begin{aligned}
\mathbf{B}_{t_4} = \; & \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1}\mathbf{B}_{t_1} \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_1} \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_2} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_3} \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_2}\mathbf{P}_c \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_3}\mathbf{P}_c
\end{aligned}
\tag{84}
$$

and

$$
\begin{aligned}
\mathbf{D}_{t_4} = \; & \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_1}\mathbf{D}_{t_1} \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{B}_{t_1} \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1}\mathbf{B}_{t_1} - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1}\mathbf{B}_{t_1} \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_1} \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_1} - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_2} \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_1} \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_2} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_3} \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_2}\mathbf{P}_c
\end{aligned}
\tag{85}
$$

Proceeding with a similar manner, we obtain the BOD and DO matrices at time t_i (denoted by $\mathbf{B}_{t_i}$ and $\mathbf{D}_{t_i}$) as presented in Equations (86) and (87), respectively.

$$\begin{aligned}
\mathbf{B}_{t_i} = \;& \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1}\mathbf{B}_{t_1} \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_1} \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_2} \\
& + \ldots\ldots + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_{i-2}} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_{i-1}} \qquad (86) \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_1}\mathbf{P}_c \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_2}\mathbf{P}_c \\
& + \ldots\ldots + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_{i-2}}\mathbf{P}_c + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_{i-1}}\mathbf{P}_c
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{D}_{t_i} = \;& \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_1}\mathbf{D}_{t_1} \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{B}_{t_1} \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_4}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{P}_{t_2}\mathbf{B}_{t_1} \\
& - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots - \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{P}_{t_{i-2}}\mathbf{B}_{t_1} - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{P}_{t_{i-1}}\mathbf{B}_{t_1} \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-2}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-3}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-4}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1}\mathbf{B}_{t_1} \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_1} \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_2} \\
& + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \qquad (87) \\
& + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_{i-2}} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_{i-1}} \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_4}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{Q}_{t_2} \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_4}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{Q}_{t_3} \\
& - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots - \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{Q}_{t_{i-2}} - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{Q}_{t_{i-1}} \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_4}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{R}_{t_2}\mathbf{P}_c \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\,\ldots\ldots\,\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_4}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{R}_{t_3}\mathbf{P}_c \\
& - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots - \\
& - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{R}_{t_{i-2}}\mathbf{P}_c - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{R}_{t_{i-1}}\mathbf{P}_c
\end{aligned}$$

Equations (86) and (87) can be written in more compact form as

$$\mathbf{B}_{t_i} = \mathbf{P}_{t_i}\mathbf{B}_{t_1} + \mathbf{Q}_{t_i} + \mathbf{R}_{t_i}\mathbf{P}_c \qquad (88)$$

and

$$\mathbf{D}_{t_i} = \mathbf{S}_{t_i}\mathbf{D}_{t_1} + \mathbf{X}_{t_i} - \mathbf{Y}_{t_i}\mathbf{B}_{t_1} - \mathbf{Z}_{t_i}\mathbf{P}_c \qquad (89)$$

where

$$\mathbf{P}_{t_i} = \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-2}}\ldots\ldots\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1} \qquad (90)$$

$$\begin{aligned}\mathbf{Q}_{t_i} = {} & \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-2}}\ldots\ldots\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_1}\\ & + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-2}}\ldots\ldots\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_2}\\ & + \ldots\ldots + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_{i-2}} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qb,t_{i-1}}\end{aligned} \qquad (91)$$

$$\begin{aligned}\mathbf{R}_{t_i} = {} & \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-2}}\ldots\ldots\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_1}\\ & + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-2}}\ldots\ldots\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_2}\\ & + \ldots\ldots + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_{i-2}} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{rm}\mathbf{L}_{c,t_{i-1}}\end{aligned} \qquad (92)$$

$$\mathbf{S}_{t_i} = \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\ldots\ldots\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_1} \qquad (93)$$

$$\begin{aligned}\mathbf{X}_{t_i} = {} & \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\ldots\ldots\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_1}\\ & + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\ldots\ldots\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_2}\\ & + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \\ & + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_{i-2}} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{qd,t_{i-1}}\\ & - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\ldots\ldots\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_4}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{Q}_{t_2}\\ & - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}}\ldots\ldots\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_4}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{Q}_{t_3}\\ & - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots - \\ & - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{Q}_{t_{i-2}} - \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{Q}_{t_{i-1}}\end{aligned} \qquad (94)$$

$$\mathbf{Y}_{t_i} = \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}} \ldots\ldots\ldots \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}$$
$$+\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}} \ldots\ldots\ldots \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_4}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{P}_{t_2}$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \qquad (95)$$
$$+\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{P}_{t_{i-2}} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{P}_{t_{i-1}}$$
$$+\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-2}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_{i-3}} \ldots\ldots\ldots \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_2}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{f,t_1}$$

$$\mathbf{Z}_{t_i} = \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}} \ldots\ldots\ldots \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_4}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_3}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{R}_{t_2}$$
$$+\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-2}} \ldots\ldots\ldots \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_4}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{R}_{t_3} \qquad (96)$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +$$
$$+\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{g,t_{i-1}}\hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{R}_{t_{i-2}} + \hat{\mathbf{M}}_t^{-1}\hat{\mathbf{M}}_{k1}\mathbf{R}_{t_{i-1}}$$

Note that the subscript t_i indicates that the matrices are of the time t_i. From the above equations, we obtain constraint inequalities for BOD and DO, describing the relationship between the predicted BOD and DO concentrations and their specified values at time t_i as follow:

$$\mathbf{P}_{t_i}\mathbf{B}_{t_1} + \mathbf{Q}_{t_i} + \mathbf{R}_{t_i}\mathbf{P}_c \leq \mathbf{B}^* \qquad (97)$$

and

$$\mathbf{S}_{t_i}\mathbf{D}_{t_1} + \mathbf{X}_{t_i} - \mathbf{Y}_{t_i}\mathbf{B}_{t_1} - \mathbf{Z}_{t_i}\mathbf{P}_c \geq \mathbf{D}^* \qquad (98)$$

At this point, we obtain the linear programming of the water quality management model expressed in the standard form of the linear programming as follow.

$$\text{Max } Z = \sum_{e=1}^{m}\left[L_c^e P_c^e \right] \qquad (99)$$

Subject to

$$\mathbf{R}_{t_i}\mathbf{P}_c \leq \mathbf{B}_{rhs} \qquad (100)$$

$$\mathbf{Z}_{t_i}\mathbf{P}_c \leq \mathbf{D}_{rhs} \tag{101}$$

$$\mathbf{P}_{c,min} \leq \mathbf{P}_c \leq \mathbf{P}_{c,max} \tag{102}$$

where

$$\mathbf{B}_{rhs} = \mathbf{B}^* - \mathbf{P}_{t_i}\mathbf{B}_{t_1} - \mathbf{Q}_{t_i} \tag{103}$$

$$\mathbf{D}_{rhs} = \mathbf{S}_{t_i}\mathbf{D}_{t_1} + \mathbf{X}_{t_i} - \mathbf{Y}_{t_i}\mathbf{B}_{t_1} - \mathbf{D}^* \tag{104}$$

3. Steps of Model Application and Computation

The main steps of model application and computation are presented by the diagram in Figure 5. It can be seen that at the last step of the diagram, the model will be solved by the simplex method. Thus, all matrices appearing in Figure 5 must be transformed to the required form of this method. The steps of transforming these matrices to the required form are shown in Figure 6. Once all matrices are in the required form, the model will be solved with the simplex algorithm as shown in Figure 7. In this study, computer is used to implement the computation. Source code for model computation in accordance with the steps shown in Figures 5–7 is presented in Appendix A.

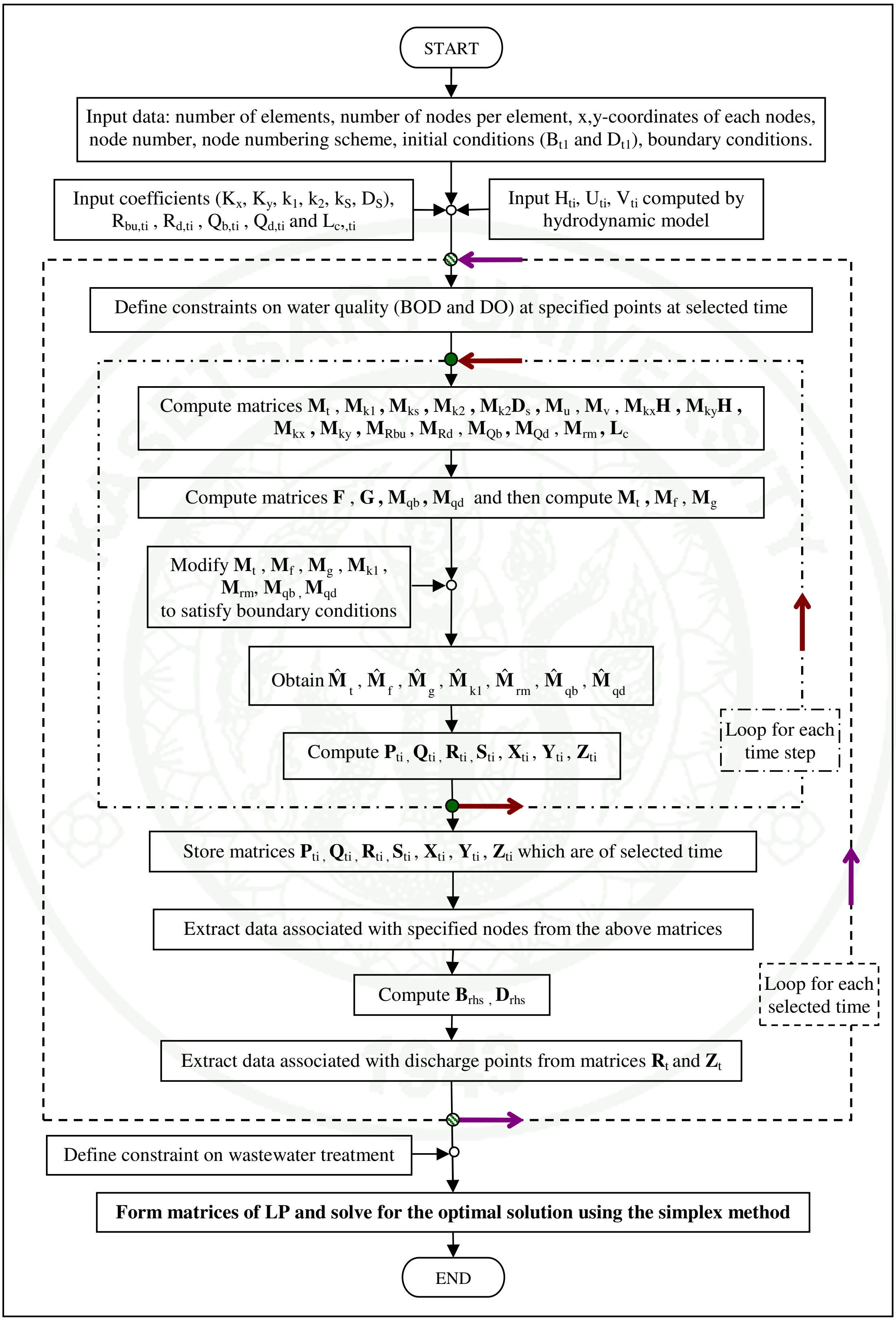

Figure 5 Principal steps of model computation.

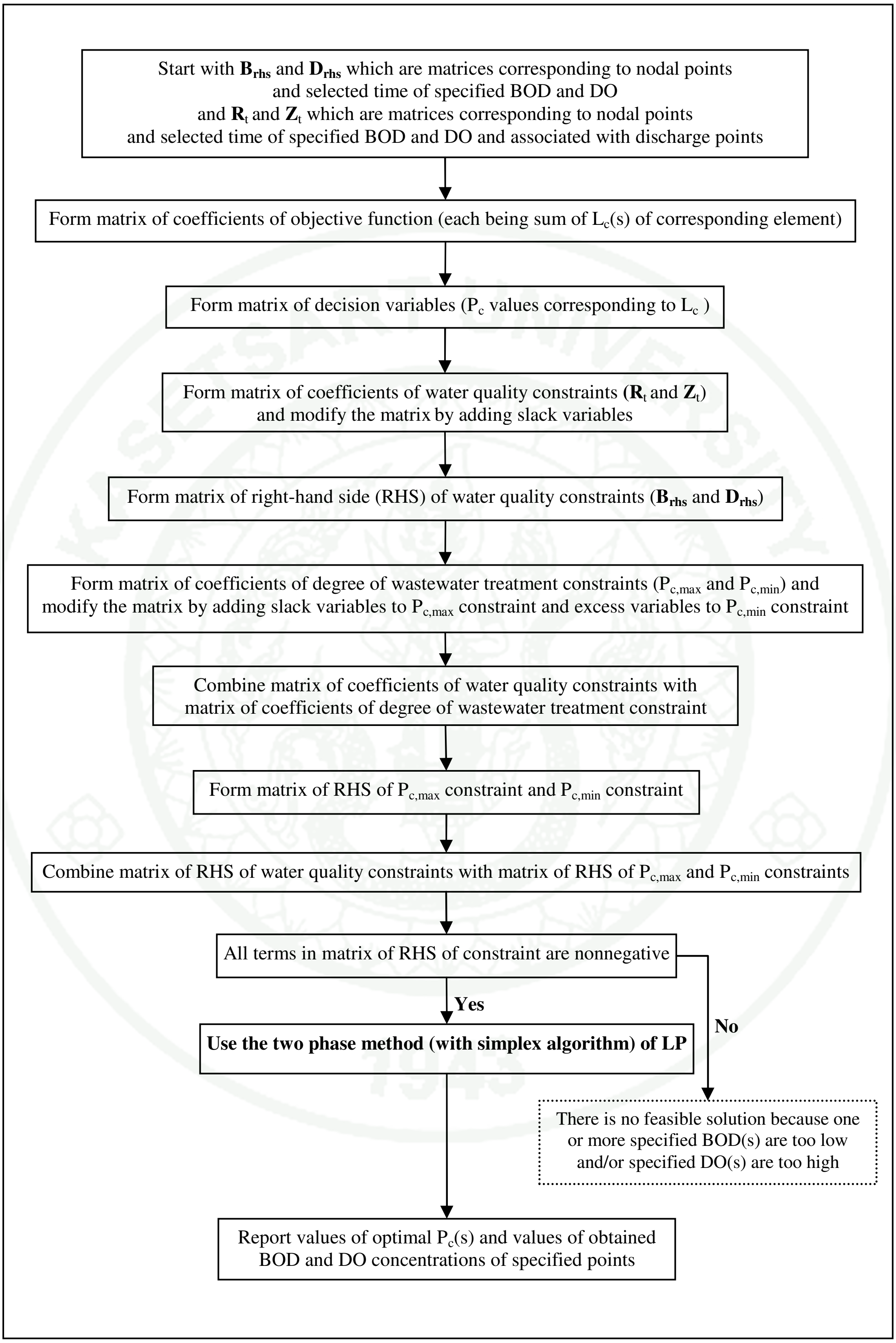

Figure 6 Steps of preparing data for model solving using the simplex method.

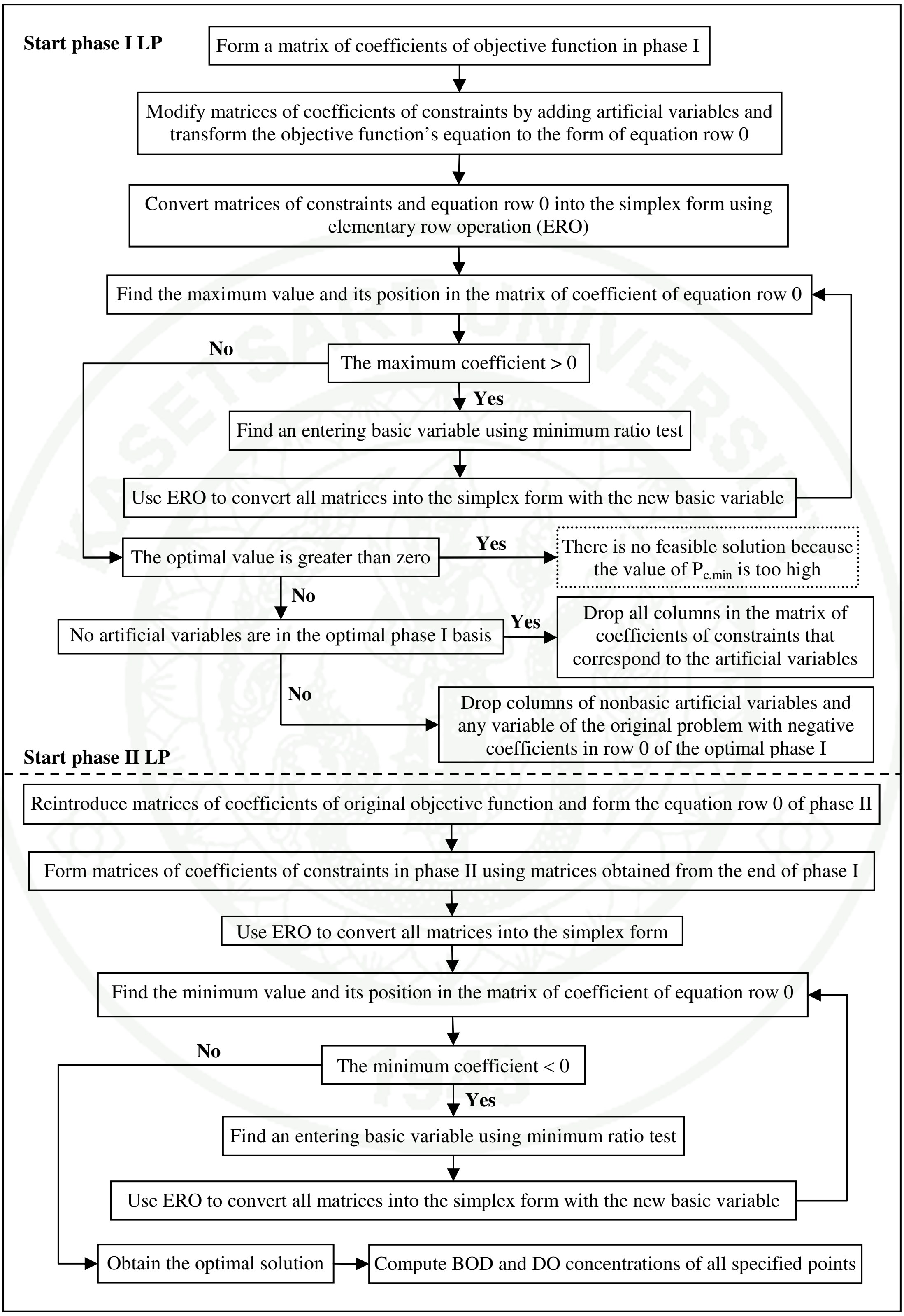

Figure 7 Steps of computing the optimal solution using the simplex method.

4. Model Verification

Verification is an important part in model development. It is conducted to test the internal logic of the model. Typical questions are whether the model acts as expected and whether the model is stable (Jørgensen, 1988). In this part, the developed dispersion model and water quality management model are tested to investigate the above behaviors.

4.1 Verification of dispersion model

Equation (105) expresses the one-dimensional mass balance equation for substance C. This equation will be used here as the basic equation to calculate analytical solution to compare with the numerical results of the developed dispersion model.

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} - K_x \frac{\partial^2 C}{\partial x^2} + KC = 0 \tag{105}$$

where C is the substance concentration,
U is the flow velocity,
K_x is the longitudinal dispersion coefficient, and
K is the decaying rate of the substance.

When the initial and boundary condition are specified, the solution of Equation (105) can be conducted as follow.

4.1.1 Uniform channel with specified substance concentration at the upper end

A uniform channel with specified substance concentration at the upper end is depicted in Figure 8.

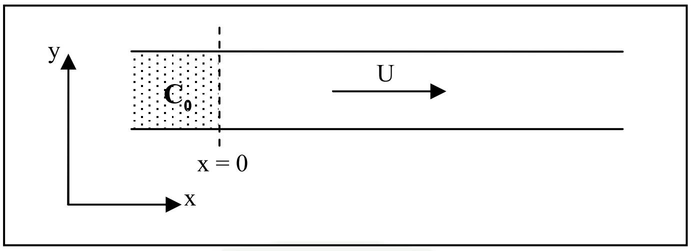

Figure 8 Uniform channel with specified substance concentration at the upper end.

The boundary and initial conditions are given by: $C(0,t) = C_0$ and $C(\infty,t) = 0$ (for $t \geq 0$), and $C(x,0) = 0$ (for $x \geq 0$). The solution for this case is (Ogata and Banks, 1961)

$$\frac{C}{C_0} = \frac{1}{2} e^{\left(\frac{xU}{2K_x}\right)} \left(\begin{array}{l} \exp\left(\frac{x}{2K_x}\sqrt{U^2 + 4K_x K}\right) \operatorname{erfc}\left(\frac{x + \sqrt{U^2 + 4K_x K}\; t}{\sqrt{4K_x t}}\right) \\ + \exp\left(-\frac{x}{2K_x}\sqrt{U^2 + 4K_x K}\right) \operatorname{erfc}\left(\frac{x - \sqrt{U^2 + 4K_x K}\; t}{\sqrt{4K_x t}}\right) \end{array} \right) \quad (106)$$

4.1.2 Uniform channel with specified substance concentration at the lower end

A uniform channel with specified substance concentration at the lower end is depicted in Figure 9.

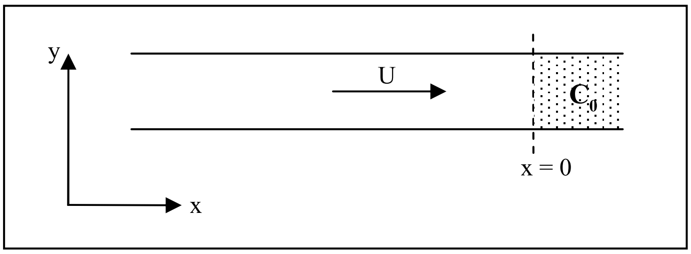

Figure 9 Uniform channel with specified substance concentration at the lower end.

With specified substance concentration at the lower end, the boundary and initial conditions are given by: $C(0,t) = C_0$ and $C(-\infty,t) = 0$ (for $t \geq 0$), and $C(x,0) = 0$ (for $-\infty < x < 0$). The solution for this case is by substituting x and U in Equation (106) by –x and –U, respectively, to yield

$$\frac{C}{C_0} = \frac{1}{2} e^{\left(\frac{xU}{2K_x}\right)} \left(\begin{array}{l} \exp\left(-\frac{x}{2K_x}\sqrt{U^2 + 4K_x K}\right) \operatorname{erfc}\left(\frac{-x + \sqrt{U^2 + 4K_x K \ t}}{\sqrt{4K_x \ t}}\right) \\ + \exp\left(\frac{x}{2K_x}\sqrt{U^2 + 4K_x K}\right) \operatorname{erfc}\left(\frac{-x - \sqrt{U^2 + 4K_x K \ t}}{\sqrt{4K_x \ t}}\right) \end{array} \right) \quad (107)$$

4.1.3 Results of model verification

Suppose that the model is applied to a channel with a uniform cross-section. This channel is 20-km long, 0.5-km wide, and 5-m deep. In applying the model, the channel is divided into 20 rectangular elements, each with a length of 1 km, as shown in Figure 10. Then, the substance dispersion patterns with specified substance concentration in both cases are computed using the developed dispersion model. At the same time, the analytical solutions are also computed by using Equations (106) and (107). The solutions of the developed model are compared with the analytical solutions as depicted in Figures 11 and 12. It can be seen that good agreement can be achieved in both cases, verifying that the developed dispersion model is reliable and can provide accurate results.

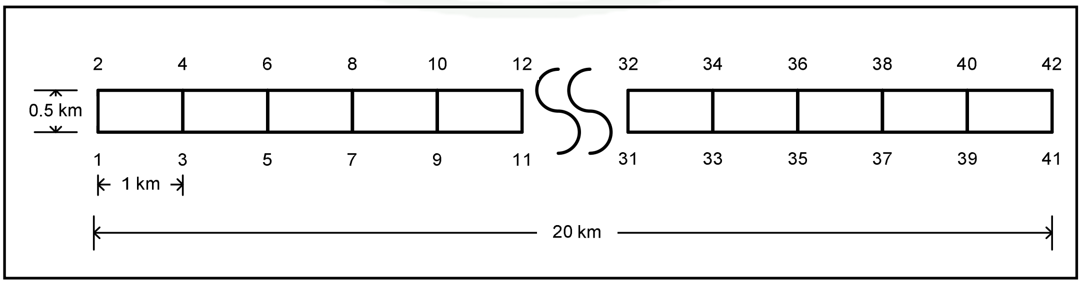

Figure 10 Finite element grid of a uniform channel.

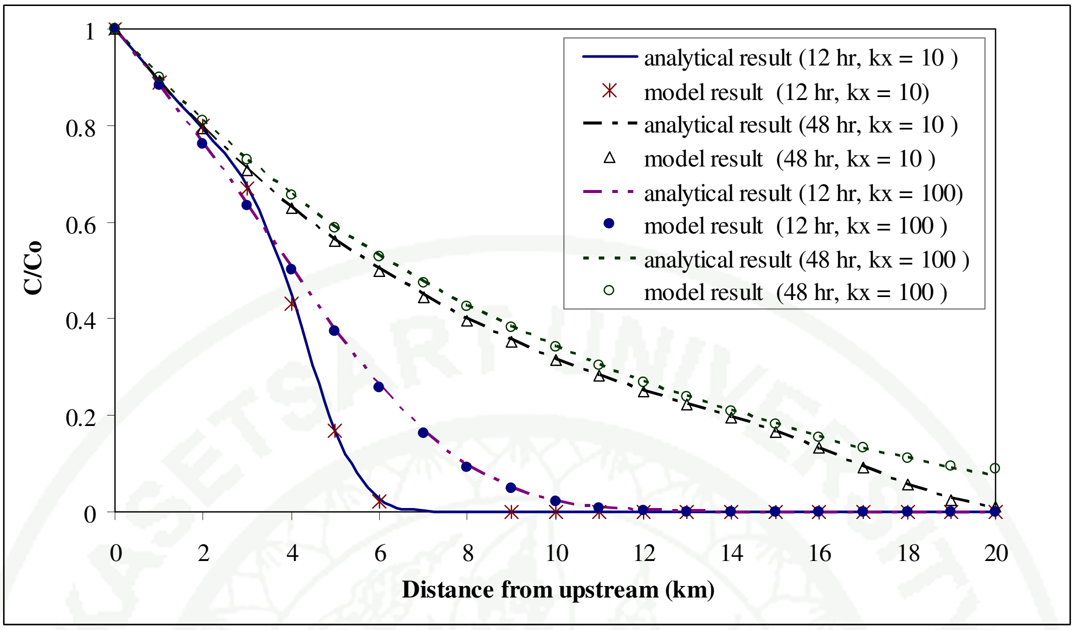

Figure 11 Distribution of substance in a uniform channel with specified substance concentration at the upper end (u = 0.1 m/s, k_1 = 1 day^{-1}).

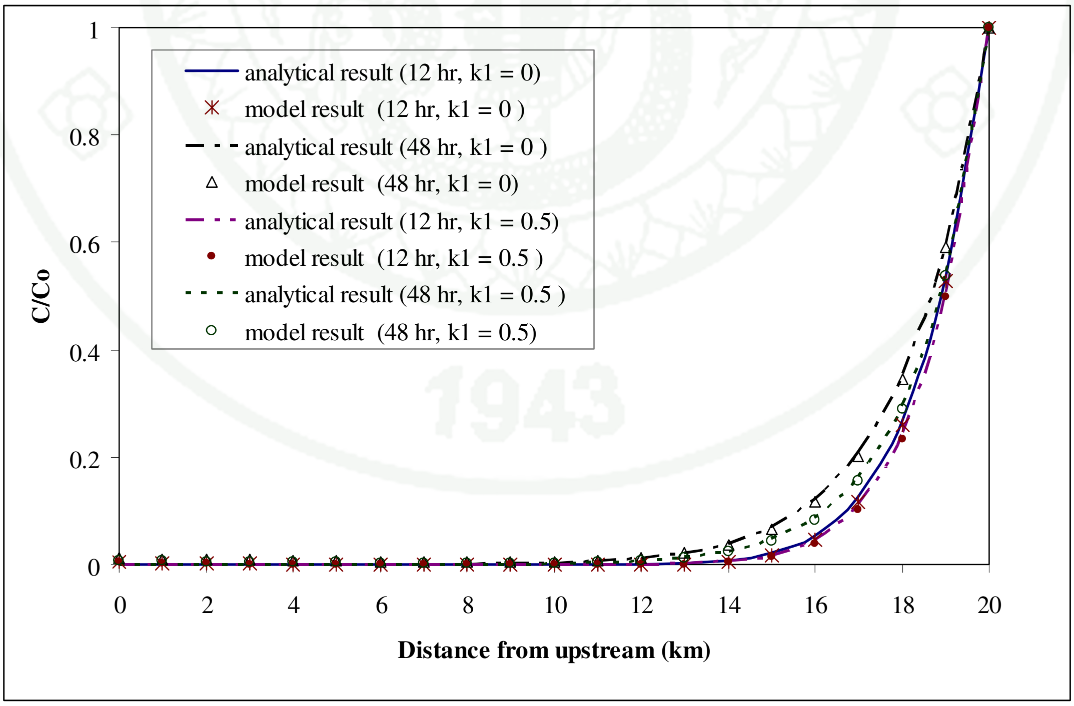

Figure 12 Distribution of substance in a uniform channel with specified substance concentration at the lower end (u = 0.05 m/s, k_x = 100 m^2/s).

4.2 Verification of water quality management model

Verification of the developed water quality management model is conducted with a hypothetical case of a tidal river with uniform cross section, 1 km in width, 40 km in length, and 5 m in depth. The river is divided into 40 rectangular elements with 82 nodal points as shown in Figure 13. It is assumed that wastewaters in this area are collected to three wastewater treatment plants before discharging into the river. Table 3 presents discharge points and BOD load of influent of each plant. Table 4 presents monitoring points and allowable BOD and DO concentrations. Here, the degree of BOD load removal is specified with the minimum degree of 50% ($P_{c,max}^{e}$ = 0.50) and the maximum degree of 80% ($P_{c,min}^{e}$ = 0.20).

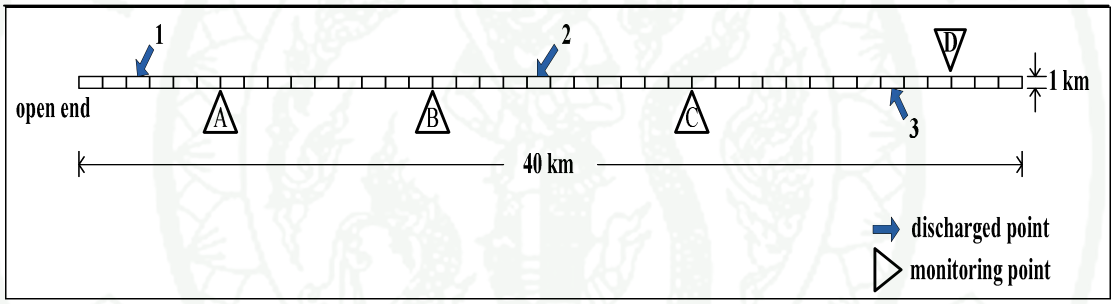

Figure 13 Finite element grid of a tidal river with uniform section.

Table 3 Discharge points and influent BOD loads.

Wastewater treatment plant	Position of discharge point (element number)	Influent BOD load (kg/d)
1	3	550
2	20	650
3	35	700

Table 4 Monitoring points and allowable BOD and DO concentrations.

Monitoring point	Distance from upstream (km)	Allowable concentrations (mg/l)	
		BOD	DO
A	6	≤ 1.5	≥ 6.0
B	15	≤ 2.0	≥ 4.0
C	26	≤ 2.0	≥ 4.0
D	37	≤ 4.0	≥ 2.0

As an example, the tidal fluctuation at the lower end is forced to follow the expression $\eta_0 = a_0 \sin \omega t$, with $a_0 = 0.25$ m and $\omega = 4\pi/(24\times3600)$ radian/s. The friction factor λ is constant at 0.0005 s^{-1}. The continuity equation and the momentum equation for one-dimensional flow of Ippen (1966) are used to determine water depth and flow velocity of each node at various time. These values are used as input data of the water quality management model. Suppose that $K_x = 50$ m^2/s, $K_y = 50$ m^2/s, $k_1 = 0.1$ d^{-1}, $k_2 = 0.2$ d^{-1}, $k_s = 0.01$ d^{-1}, and BOD and DO concentrations at the upstream end are 1.5 mg/l and 5.0 mg/l, respectively. The developed management model is used to compute for the optimal solution. The model results are shown in Table 5. From the model results, it is found that these values are within the specified range of degree of BOD load removal. So, they satisfy the constraint on wastewater treatment. According to the optimal P_c^e values, the maximum amount of overall BOD load which can be discharged to the river is 642.97 kg/d.

Table 5 Optimal solution (P_c^e) and objective function value (overall BOD loading).

Wastewater treatment plant	1	2	3
Optimal P_c^e	0.22852	0.45168	0.31956
Maximum BOD loading from each plant (kg/d)	125.686	293.592	223.692
Overall BOD loading (kg/d)		642.97	

To verify the model, the BOD and DO concentrations at monitoring points are calculated. The predicted values compared with the allowable BOD and DO concentrations, as shown in Table 6, indicate that a set of optimal P_c^e values also satisfy the constraints on BOD and DO.

Table 6 Predicted BOD and DO concentrations with the optimal P_c^e.

Monitoring point	BOD concentration (mg/l)		DO concentration (mg/l)	
	Predicted value	Allowable value	Predicted value	Allowable value
A	1.5000	≤ 1.5	6.2630	≥ 6.0
B	2.0000	≤ 2.0	6.1805	≥ 4.0
C	1.3123	≤ 2.0	6.4472	≥ 4.0
D	4.0000	≤ 4.0	5.2826	≥ 2.0

To verify that the value of overall BOD loading is the maximum amount, the BOD load is increased a little to be more than 642.97 kg/d by increasing one of the optimal P_c^e by 0.0001 (indicated by the italic numeric characters in Table 7). Then, the BOD and DO concentrations at monitoring points are recalculated. The results are presented in Table 7, which show that increasing the overall BOD loading causes violating the constraint on BOD (represented by the bold numeric characters).

The additional test to verify the optimal solution of the model is conducted by changing a couple of P_c^e values while holding the overall BOD loading at 642.97 kg/d, as presented by the italic numeric characters in Table 8. Then, the BOD and DO concentrations according to the new P_c^e values are recalculated. The results are also shown in Table 8, which show that the BOD concentrations at some monitoring points (represented by the bold numeric characters) cannot meet the specified values. This implies that other set of P_c^e values are not available.

Thus, it can be said that the verification of the developed water quality management model exists with the P_c^e values be the optimal solution and the overall BOD loading corresponding to the optimal P_c^e values be the maximum amount.

Table 7 Objective function value test.

New P_c^e value of treatment plant			New BOD and DO concentrations (mg/l) at monitoring point							
			A		B		C		D	
1	2	3	BOD	DO	BOD	DO	BOD	DO	BOD	DO
0.22862	0.45168	0.31956	**1.5004**	6.2628	2.0000	6.1804	1.3122	6.4472	4.0000	5.2826
0.22852	*0.45178*	0.31956	1.5000	6.2630	**2.0004**	6.1802	1.3125	6.4470	4.0000	5.2826
0.22852	0.45168	*0.31966*	1.5000	6.2630	2.0000	6.1805	1.3123	6.4471	**4.0012**	5.2819

Table 8 P_c^e value test.

New P_c^e value of treatment plant			New BOD and DO concentrations at monitoring point (mg/l)							
			A		B		C		D	
1	2	3	BOD	DO	BOD	DO	BOD	DO	BOD	DO
0.22862	*0.45160*	0.31956	**1.5004**	6.2628	1.9997	6.1807	1.3121	6.4473	4.0000	5.2826
0.22862	0.45168	*0.31948*	**1.5004**	6.2628	2.0000	6.1804	1.3122	6.4472	3.9999	5.2831
0.22842	*0.45176*	0.31956	1.4996	6.2632	**2.0003**	6.1803	1.3124	6.4470	4.0000	5.2826
0.22842	0.45168	*0.31964*	1.4996	6.2632	2.0000	6.1805	1.3123	6.4471	**4.0010**	5.2821
0.22840	*0.45178*	0.31956	1.4995	6.2632	**2.0004**	6.1802	1.3125	6.4470	4.0000	5.2826
0.22852	*0.45178*	*0.31947*	1.5000	6.2630	**2.0004**	6.1802	1.3124	6.4471	3.9988	5.2832
0.22864	*0.45158*	0.31956	**1.5005**	6.2628	1.9996	6.1807	1.3120	6.4474	4.0000	5.2826
0.22852	*0.45158*	*0.31965*	1.5000	6.2630	1.9996	6.1807	1.3121	6.4473	**4.0011**	5.2820
0.22839	0.45168	*0.31966*	1.4995	6.2633	1.9999	6.1805	1.3123	6.4471	**4.0012**	5.2819
0.22852	*0.45157*	*0.31966*	1.5000	6.2630	1.9995	6.1808	1.3121	6.4473	**4.0012**	5.2819
0.22865	0.45168	*0.31946*	**1.5006**	6.2627	2.0000	6.1804	1.3121	6.4473	3.9987	5.2833
0.22852	*0.45179*	*0.31946*	1.5000	6.2630	**2.0004**	6.1802	1.3124	6.4471	3.9987	5.2833

5. Application of Water Quality Management Model to the Middle and Lower Sections of Tha Chin River

The Tha Chin River is the second most important waterway in Thailand (Simachaya, 2003). This river is used for several purposes including water supply, aquaculture, transportation, and recreation as well as a sink for waste discharges. During the last decades, there has been considerable socio-economic development within the Tha Chin River Basin. As a result, excessive load of wastes has been discharged into the river, causing severe degradation of river water quality, especially along the lower and middle sections.

Waste discharge control is an important measure for preventing degradation of river water quality. Here, the developed water quality management model is applied to provide some information concerning wastewater management and water quality control for the Tha Chin River. The focus is on the middle and lower sections of this river which suffer from heavy pollution currently. The total length of these river sections is 202 km with the upper end at Phophraya Regulator in Suphan Buri Province and the lower end at the river mount in Sumut Sakhon Province.

Water regulators play a major role in variation of flow in the Tha Chin River. Most of inflowing water is discharged from Phophraya regulator (see Appendix Table B1). However, according to Simachaya and Healthcote (1999), variation of flow in the lower and some of the middle sections of this river are also affected by tidal fluctuation at the river mount. The impact of tide during high flow and low flow periods are about 120 and 180 km from the river mouth, respectively. This causes variation of flow velocity and depth at various points in the study area all the time with tidal movement. In this study, these values will be estimated by using an existing hydrodynamics model. The predicted flow velocity and depth will be used as input data of the water quality management model. Input data of the hydrodynamics model including water discharge rates from regulators, x- and y-coordinates and mean water depths at various points of the river, and heights of water at the river mount are listed in Appendix B.

For simplicity, the river banks are assumed to be symmetric on horizontal plane with centerline being parallel to x-axis as shown in Figure 14. To generate finite element grid, the study domain is divided at every 2 km along the centerline. Bilinear quadrilateral element is used and node number is assigned to each grid point. This results in the finite element grid as shown in Figure 15. The coordinates related to x- and y-axes on Figure 14 and average water depth of each node are listed in Appendix Table B3. Variables and parameters associated with the model are presented in Table 9. Here, the lateral dispersion and velocity are assumed to be zero, i.e., $K_y = 0$ and $V = 0$.

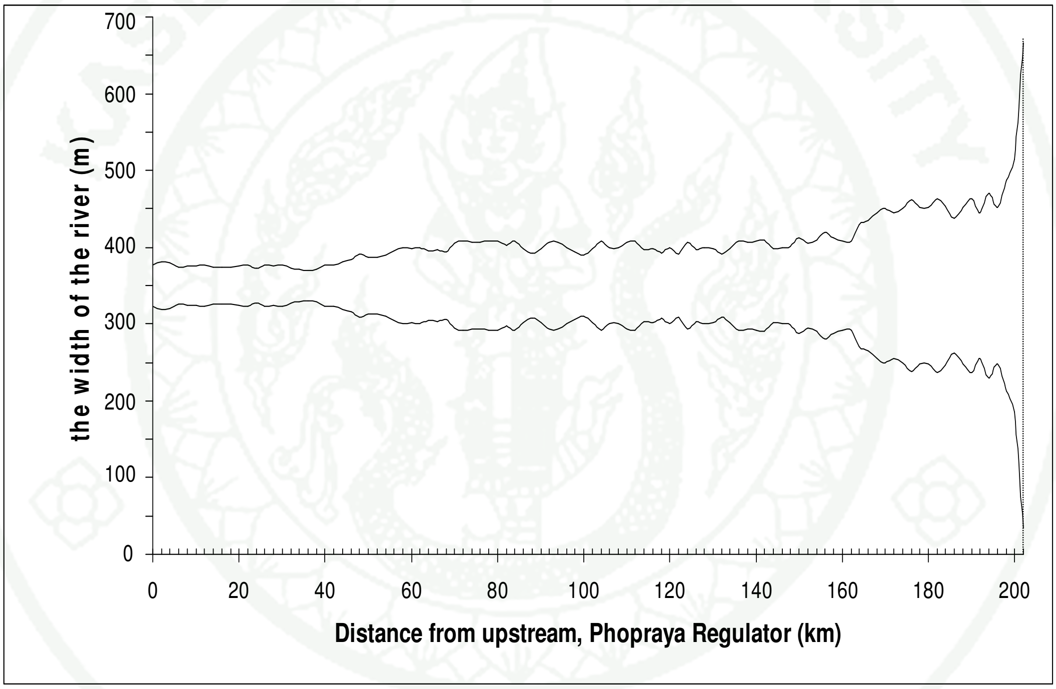

Figure 14 Tha Chin River with assumed symmetrical river banks.

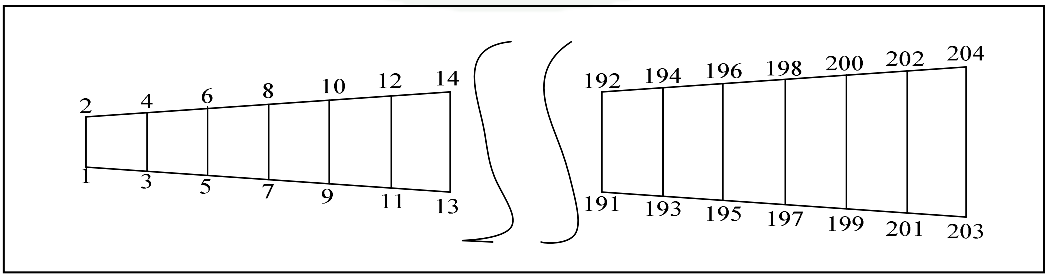

Figure 15 Finite element grid and node numbering.

Table 9 Variables and parameters associated with model.

Parameters	Values	Variable definition
k_1 (d^{-1}) [1]	0.1	
k_2 (d^{-1}) [2]	$5.01v^{0.969}H^{-1.673}$ for $H \leq 3.48$ $3.93v^{0.5}H^{-1.5}$ for $H > 3.48$	v = velocity (m/s), H = mean depth (m), B = width (m),
K_x (m^2/d) [1]	$0.05937\dfrac{Q}{SB}$	Q = mean flow (m^3/s) S = channel slope
k_s (d^{-1}) [1]	0.01	
D_s (mg/l) [2]	$\dfrac{468}{31.6+T}$	T = water temperature (°c)

Sources: [1] Chapra (1997); [2] Lung (2001)

The proper discharge control in terms of treatment plant operation for the present time, year 2020 and 2030 is investigated from various proposed scenarios (see Table 10). Two existing treatment plants in the study domain are included in all scenarios. These plants are in Muang Suphan Buri and Muang Nakhon Pathom Districts. In addition to these two plants, five locations of new plants (each being in Bang Pla Ma, Song Phi Nong, Bang Len, Nakhon Chai Si, and Sam Phran Districts) are proposed in different scenarios. Locations and discharge points of these plants are shown in Figure 16. Estimated BOD load from major pollutant sources including communities, industries, pig farms, and aquacultures are listed in Appendix C. Note that BOD load removal rates of 30% and 50% from pig farm wastewater along with 70% from aquaculture wastewater. Constraints on water quality and wastewater treatment for all scenarios are as follow.

For the first constraint, BOD and DO concentrations in the river are specified at every 4 km along its length; these values are specified based on the standards of that point. To ensure most of the water body meeting the standards all the time, the BOD and DO constraints are set for every two hours a day.

For the second constraint, based on treatment efficiency of conventional treatment plant usually used to treat domestic wastewater in Thailand, the maximum P_c value ($P_{c,max}$) is set at 0.5; while the minimum P_c ($P_{c,min}$) is set at 0.2 for the first investigation of each scenario. However, if there is no solution with the first $P_{c,min}$, the lower values (i.e., 0.15, 0.1, 0.05 and 0.01) will be tested later. Using the lower $P_{c,min}$ implies that a high efficiency in wastewater treatment is required to fulfill the treatment task. Model Results are presented in Tables 11-12.

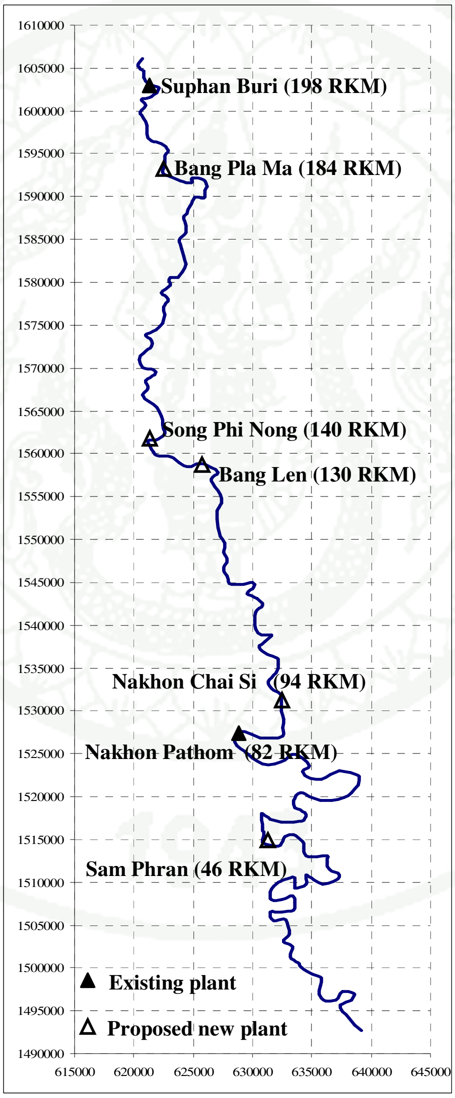

Figure 16 Locations and discharge points of wastewater treatment plants.

Table 10 Various scenarios for wastewater management and water quality control.

Scenario	Location of each treatment plant: District						
	Suphan Buri	Nakhon Pathom	Bang Pla Ma	Song Phi Nong	Bang Len	Nakhon Chai Si	Sam Phran
1	●	●					
2	●	●	●				
3	●	●		●			
4	●	●			●		
5	●	●				●	
6	●	●					●
7	●	●	●	●			
8	●	●	●		●		
9	●	●		●		●	
10	●	●		●			●
11	●	●		●	●		
12	●	●		●	●	●	
13	●	●		●	●	●	●
14	●	●	●	●	●		
15	●	●	●	●		●	
16	●	●	●	●			●
17	●	●	●	●	●	●	
18	●	●	●	●	●		●

● *indicates that location of the treatment plant is included in that scenario.*

Table 11 Optimal P_c values of various scenarios with BOD load removal of 30% from pig farm wastewater and 70% from aquaculture wastewater.

Year	Scenario	Location of each treatment plant: District							Critical point (RKM)
		Suphan Buri	Nakhon Pathom	Bang Pla Ma	Song Phi Nong	Bang Len	Nakhon Chai Si	Sam Phran	
		Optimal P_c for each plant							
2010	1	No solution is found; BOD concentrations at 86 RKM exceed BOD standard.							
	2	0.5	0.5	0.409					114
	3	0.307	0.5		0.5				158
	4	0.05	0.5			0.399			130
	5	No solution is found; BOD concentrations at 114 RKM exceed BOD standard.							
	6	No solution is found; BOD concentrations at 86 RKM exceed BOD standard.							
2020	2	0.298	0.5	0.2					90
	3	0.02	0.5		0.171				90
	4	No solution is found; BOD concentrations at 130-150 RKM exceed BOD standard.							
	7	0.5	0.5	0.309	0.5				182
2030	2	0.01	0.04	0.01					6
	7	0.15	0.15	0.15	0.279				6
	8	0.15	0.15	0.15		0.329			6
	9	0.1	0.174	0.1			0.5		6
	10	0.03	0.5	0.03				0.391	6, 118
	11	No solution is found; BOD concentrations at 154-162 RKM exceed BOD standard.							
	12	No solution is found; BOD concentrations at 154-162 RKM exceed BOD standard.							
	13	No solution is found; BOD concentrations at 154-162 RKM exceed BOD standard.							
	14	0.2	0.252	0.2	0.5	0.5			6
	15	0.2	0.264	0.2	0.5		0.5		6
	16	0.2	0.441	0.2	0.5			0.5	6, 90
	17	0.2	0.471	0.2	0.5	0.5	0.5		6, 182
	18	0.5	0.5	0.216	0.5	0.5		0.5	182

Table 12 Optimal P_c values of various scenarios with BOD load removal of 50% from pig farm wastewater and 70% from aquaculture wastewater.

Year	Scenario	Location of each treatment plant: District							Critical point (RKM)
		Suphan Buri	Nakhon Pathom	Bang Pla Ma	Song Phi Nong	Bang Len	Nakhon Chai Si	Sam Phran	
		Optimal P_c for each plant							
2010	1	No solution is found; BOD concentrations at 86 RKM exceed BOD standard.							
	2	0.5	0.5	0.5					-
	3	0.342	0.5		0.5				158
	4	0.05	0.5			0.467			130
	5	No solution is found; BOD concentrations at 114 RKM exceed BOD standard.							
	6	No solution is found; BOD concentrations at 86 RKM exceed BOD standard.							
2020	2	0.347	0.5	0.2					90
	3	0.05	0.5		0.26				90
	4	No solution is found; BOD concentrations at 130-150 RKM exceed BOD standard.							
	7	0.5	0.5	0.328	0.5				182
2030	2	0.01	0.209	0.01					6
	7	0.2	0.232	0.2	0.5				6
	8	0.2	0.239	0.2		0.5			6
	9	0.15	0.314	0.15			0.5		6
	10	0.191	0.5	0.05				0.5	90
	11	No solution is found; BOD concentrations at 154-162 RKM exceed BOD standard.							
	12	No solution is found; BOD concentrations at 154-162 RKM exceed BOD standard.							
	13	No solution is found; BOD concentrations at 154-162 RKM exceed BOD standard.							
	14	0.2	0.439	0.2	0.5	0.5			6
	15	0.2	0.451	0.2	0.5		0.5		6
	16	0.443	0.5	0.2	0.5			0.5	90
	17	0.5	0.5	0.233	0.5	0.5	0.5		182
	18	0.5	0.5	0.233	0.5	0.5		0.5	182

The model results indicate that with the above assumption of BOD load removal rates from pig farm and aquaculture the existing treatment plants at the present time are not sufficient to control BOD loading from domestic wastewater so that the water body could assimilate and meet the BOD standard. This insufficiency still exists although assuming that BOD load removal efficiency of each plant can be increased to 99% (i.e., $P_{c,min}$ = 0.01). From investigating other proposed scenarios, we find that the addition of at least one new treatment plant to treat some of the remaining amount of the domestic wastewater generated in the middle river basin is necessary. The first priority to be the proper location for the new plant is at Bang Pla Ma District whereas the second choice could be at Song Phi Nong. Location at Bang Len District should be ranked the third choice due to very less of P_c value required from the existing plant at Suphan Buri in this scenario. Location at Nakhon Chai Si and Sam Phran Districts can not be the choice because there is no feasible solution in these cases despite of lowering $P_{c,min}$ of all plants to 0.01.

The construction of a new plant at Bang Pla Ma or Song Phi Nong also prevents water quality of the river till year 2020; but with the optimal P_c required from the existing plant at Suphan Buri, selecting location at Bang Pla Ma would be better than that at Song Phi Nong. The suitability of the first location is confirmed by model results from the investigation in year 2030. The results indicate that, in addition to the two existing plants, at least two new plants should be constructed to adequately protect the river till the year 2030. One location should be at Bang Pla Ma while the remaining location could be at either Song Phi Nong or Bang Len.

In addition, the model results can indicate the critical point having a risk to violate the BOD standard and the optimal P_c value of each plant under certain amount of BOD load. These P_c values are directly related to BOD load removal efficiency which is required for each plant to control the overall BOD loading into the river. For example, the optimal P_c value of 0.347 implies that the BOD load removal efficiency of 65.3% is required for that plant. These values are useful in decision making when the optimal discharge control is of major concern.

CONCLUSION AND RECOMMENDATION

Conclusion

1. The water quality management model developed by using the finite element method with Galerkin's weighted residual technique and linear programming optimization is a reliable and effective tool for supporting wastewater management and water quality control of tidal rivers with several discharge points. From model verification, it is found that this model is very accurate and reliable in both prediction of pollutant dispersion in water body and determination of the optimal solution to the optimization problem. In addition to its applicability to deal with a tidal river, another advantage of the model is that it is expressed in a general form such that the optimal solution of any selected time can be determined from the initial value of BOD and DO concentrations. The simplex method can be effectively used to solve the model. In addition, although this model is focused on BOD and DO as water quality indicator, its mathematical expression is general enough to be applied to other water quality parameters.

2. The developed model is applicable to the Tha Chin River and can provide useful information for the decision making concerning wastewater management and water quality control for this river. With respect to the minimum investment for domestic wastewater treatment plants, and the assumption that 30% and 50% of BOD load in wastewater from pig farms along with 70% of BOD load in wastewater from aquacultures are removed, the model results indicate that the addition of a new treatment plant in the middle river basin is required to protect the river at the present time. Otherwise, BOD concentrations at some points in the middle section of this river cannot meet the standard. Recommended location for the new treatment plant should be in the middle zone of the river basin. The first priority for the proper location is at Bang Pla Ma District; the second choice should be at Song Phi Nong; whereas the location at Bang Len is the next choice. The construction of a new plant at Bang Pla Ma or Song Phi Nong can maintain water quality of the river till the year

2020. However, when considering constraint on wastewater treatment, location at Bang Pla Ma is much more suitable than that at Song Phi Nong. The suitability of the first location is confirmed by the requirement of wastewater treatment plants in the year 2030. Model results indicate that, in addition to the existing treatment plants at Muang Suphan Buri and Muang Nakhon Pathom Municipalities, at least two new plants should be constructed to adequately control BOD loading into the river, so that the river can assimilate and meet the BOD standard. One location is strongly suggested to be at Bang Pla Ma while the remaining location can be at either Song Phi Nong or Bang Len.

Recommendation

1. In actual situations of water pollution problem in the Tha Chin River, there are several pollutant sources, particularly wastewaters from farming and agriculture activities. Pollutant loadings of these wastewaters are difficult to control while they are considered as the major sources of BOD loads discharged into this river. Therefore, the use of information from the model without controlling BOD loads of these sources can cause failure in water quality management to achieve the target.

2. The developed model is intended to be an alternative tool to help wastewater treatment plant managers and decision makers in making the decision with the rational information. As mentioned above, the model informs us the optimal degree of BOD load removal for each treatment plant. This can be the important information for designing and operating these wastewater treatment plants. However, the capital cost, and the operation and maintenance costs of wastewater treatment plants are not considered in the modeling. Thus, the optimal solution of this model cannot be considered in terms of the treatment cost optimization, particularly in a case of employing the tertiary wastewater treatment systems. The model that can be solved for the latter objective could be developed by using nonlinear programming optimization.

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APPENDICES

Appendix A

Source code for model computation

Source Code for Model Computation

```
nC = xlsread('coordinate.xls');
n = size(nC,1); m = (n-2)/2;
for e = 1:m
   nS(e,[1,2,3,4]) = [2*e-1 2*e+1 2*e+2 2*e];
end
for e = 1:m
   x(e,:) = nC(nS(e,:),1);
   y(e,:) = nC(nS(e,:),2);
end
nB = [1; 2]; bB = [1.1; 1.1]; bD = [6.8; 6.8];
PcMax = input(['maximum Pc  =    ']);
PcMin = input(['minimum Pc  =    ']);
B_D_Constraint = xlsread('WQ_constraint.xls');
startT = B_D_Constraint(1,1);
specNode = B_D_Constraint(:,2);
specB = B_D_Constraint(:,3);
specD = B_D_Constraint(:,4);
eachTimeCheck = 2;
nCheckT = 24/eachTimeCheck+1;
countT = 0;
selT = [];

for i = 1:nCheckT
   selT(i) = startT+countT;
   countT = countT + eachTimeCheck*3600;
end

Nodal_data = xlsread('nodal_data.xls');
Bt1 = Nodal_data(:,1); Dt1 = Nodal_data(:,2);
Ds = Nodal_data(:,3);
Rd = xlsread('Rd_data.xls');
Rbu = xlsread('Rbu_data.xls');
Qbe = xlsread('discharge_flux.xls','BOD');
Qde = xlsread('discharge_flux.xls','DO');
B_load = xlsread('BOD_load.xls');

Qt1 = Bt1; Rt1 = zeros(n,m);
SXt1 = Dt1; Yt1 = zeros(n,1); Zt1 = zeros(n,m);
b_B = zeros(0); b_D = zeros(0);
A_B = zeros(0); A_D = zeros(0);

XI = [-1/sqrt(3); 1/sqrt(3); 1/sqrt(3); -1/sqrt(3)];
ETA = [-1/sqrt(3); -1/sqrt(3); 1/sqrt(3); 1/sqrt(3)];
wg = 1;
```

```
% SYSTEM MATRICES

M = zeros(n);
for e=1:m
   me = zeros(4);

for NG = 1:4
   xi = XI(NG);  eta = ETA(NG);
   N = [(1-xi)*(1-eta)/4; (1+xi)*(1-eta)/4; (1+xi)*(1+eta)/4; (1-xi)*(1+eta)/4];
   B = [-(1-eta)/4 (1-eta)/4 (1+eta)/4 -(1+eta)/4; -(1-xi)/4 -(1+xi)/4 (1+xi)/4 (1-xi)/4];
   J = B*[x(e,:)' y(e,:)'];
   me = me + wg*N*N'*det(J);
end
for i=1:4
   ii = nS(e,i);
for j=1:4
   jj = nS(e,j);
   M(ii,jj) = M(ii,jj) + me(i,j);
end
end

end
k1 = 0.1/86400; Mk1 = k1*M;
ks = 0.01/86400; Mks = ks*M;
deltt = 300;
data_Thachin = textread('data_thachin.txt');
U = data_Thachin(:,3); H = data_Thachin(:,2);
tiStart = 1; r0 = 207; rf = r0+n-1;
for nT = 1:size(selT,2)
   ntime = selT(nT)/deltt;

for ti = tiStart:ntime
   Mu = zeros(n); Mv = zeros(n);
   MHx = zeros(n); MHy = zeros(n);
   MKx = zeros(n);  MKy = zeros(n);
   Mk2 = zeros(n);
   MRbu = zeros(n,1);
   MRd = zeros(n,1);
   U_ti = U(r0:rf,1);  H_ti = H(r0:rf,1);

for e = 1:m
   mu = zeros(4); mv = zeros(4);
   mHx = zeros(4);  mHy = zeros(4);
   mKx = zeros(4);  mKy = zeros(4);
   mk2 = zeros(4);
   mrbu = zeros(4,1); mrd = zeros(4,1);
   ue = U_ti(nS(e,:));  ve = zeros(4,1);
   He = H_ti(nS(e,:));
```

```
    Xe = x(e,:);  Ye = y(e,:);
    Hevg = (He(1) + He(2) + He(3) + He(4))/4;
    a = sqrt((Xe(2)-Xe(1))^2 + (Ye(2)-Ye(1))^2);
    b = sqrt((Xe(3)-Xe(2))^2 + (Ye(3)-Ye(2))^2);
    c = sqrt((Xe(3)-Xe(4))^2 + (Ye(3)-Ye(4))^2);
    d = sqrt((Xe(4)-Xe(1))^2 + (Ye(4)-Ye(1))^2);
    p = sqrt((Xe(4)-Xe(2))^2 + (Ye(4)-Ye(2))^2);
    q = sqrt((Xe(3)-Xe(1))^2 + (Ye(3)-Ye(1))^2);
    Ae = (sqrt(4*p^2*q^2 - (b^2 + d^2 - a^2 - c^2)^2))/4;
    Ve = Ae*Hevg; Vol(e) = Ve;
    wid_e = (abs(Ye(1) - Ye(4)) + abs(Ye(2) - Ye(3)))/2;
    velo_e = (abs(ue(1))+abs(ue(2))+abs(ue(3))+abs(ue(4)))/4;
    Kx = 0.05937*velo_e*(wid_e*Hevg)/((0.061/100)*wid_e);
    Ky = 0;

if Hevg <= 3.48
    k2 = 5.01*(velo_e)^0.969*Hevg^(-1.673)/86400;
else
    k2 = 3.93*(velo_e)^0.5*Hevg^(-1.5)/86400;
end
    Rbue = Rbu(e)/Ve; Rde = Rd(e)/Ve;

for NG = 1:4
    xi = XI(NG);  eta = ETA(NG);
    N = [(1-xi)*(1-eta)/4; (1+xi)*(1-eta)/4; (1+xi)*(1+eta)/4; (1-xi)*(1+eta)/4];
    B = [-(1-eta)/4 (1-eta)/4 (1+eta)/4 -(1+eta)/4; -(1-xi)/4 -(1+xi)/4 (1+xi)/4 (1-xi)/4];
    J = B*[x(e,:)' y(e,:)'];
    dNT = inv(J)*B;
    Bx = dNT(1,:);  By = dNT(2,:);
    mu = mu + wg*N*(N'*ue)*dNT(1,:)*det(J);
    mv = mv + wg*N*(N'*ve)*dNT(2,:)*det(J);
    mHx = mHx + wg*(Kx/(N'*He))*N*dNT(1,:)*He*dNT(1,:)*det(J);
    mHy = mHy + wg*(Ky/(N'*He))*N*dNT(2,:)*He*dNT(2,:)*det(J);
    mKx = mKx + wg*Kx*dNT(1,:)'*dNT(1,:)*det(J);
    mKy = mKy + wg*Ky*dNT(2,:)'*dNT(2,:)*det(J);
    mk2 = mk2 + k2*wg*N*N'*det(J);
    mrbu = mrbu + wg*N*det(J);
    mrd = mrd + wg*N*det(J);
end
mRbu = Rbue*mrbu;
mRd = Rde*mrd;

for i=1:4
     ii = nS(e,i);
for j = 1:4
     jj = nS(e,j);
     Mu(ii,jj) = Mu(ii,jj) + mu(i,j);
     Mv(ii,jj) = Mv(ii,jj) + mv(i,j);
```

```
    MHx(ii,jj) = MHx(ii,jj) + mHx(i,j);
    MHy(ii,jj) = MHy(ii,jj) + mHy(i,j);
    MKx(ii,jj) = MKx(ii,jj) + mKx(i,j);
    MKy(ii,jj) = MKy(ii,jj) + mKy(i,j);
    Mk2(ii,jj) = Mk2(ii,jj) + mk2(i,j);
end
end

for i=1:4
    ii = nS(e,i);
    MRbu(ii) = MRbu(ii) + mRbu(i);
    MRd(ii) = MRd(ii) + mRd(i);
end

end

r0 = r0+n+1; rf = r0+n-1;
Mk2Ds = Mk2*Ds;
MQb = zeros(n,1);  MQd = zeros(n,1);
eQ = Qbe(:,1);

for e = 1:size(eQ,1)
    el = eQ(e);
    nQe = Qbe(e,3:4);
    xQ(1) = nC(nS(el,nQe(1)),1);
    xQ(2) = nC(nS(el,nQe(2)),1);
    yQ(1) = nC(nS(el,nQe(1)),2);
    yQ(2) = nC(nS(el,nQe(2)),2);
    hQ(1) = H_ti(nS(el,nQe(1)));
    hQ(2) = H_ti(nS(el,nQe(2)));
    lQ = sqrt((xQ(2)-xQ(1))^2 + (yQ(2)-yQ(1))^2);
    areaQ = 0.5*lQ*(hQ(1)+hQ(2));
    dl = zeros(4,1);

for ii = 1:4
for jj = 1:2
if ii == nQe(jj)
  dl(ii) = dl(ii) + 0.5;
else
  dl(ii) = dl(ii) + 0;
end
end
end

dL = lQ*dl;
mQb = Qbe(e,2)/areaQ*dL; mQd = Qde(e,2)/areaQ*dL;
```

```
for i=1:4
     ii = nS(el,i);
     MQb(ii) = MQb(ii) + mQb(i);
     MQd(ii) = MQd(ii) + mQd(i);
end

end

Mrm = zeros(n,m);
Lc = zeros(m);
for e = 1:size(B_load,1)
    el = B_load(e,1);
    Lce = B_load(e,2)/Vol(el)/86400;
    Lc(el,el) = Lc(el,el) + Lce;
    mrm = zeros(4,1);

for NG = 1:4
    xi = XI(NG); eta = ETA(NG);
    N = [(1-xi)*(1-eta)/4; (1+xi)*(1-eta)/4; (1+xi)*(1+eta)/4; (1-xi)*(1+eta)/4];
    B = [-(1-eta)/4 (1-eta)/4 (1+eta)/4 -(1+eta)/4; -(1-xi)/4 -(1+xi)/4 (1+xi)/4 (1-xi)/4];
    J = B*[x(el,:)' y(el,:)'];
    mrm = mrm + wg*N*det(J);
end

for i=1:4
    ii = nS(el,i);
    Mrm(ii,el) = Mrm(ii,el) + mrm(i);
end

end

sumLc = zeros(1,m);
for e = 1:size(B_load,1)
    el = B_load(e,1);
    sumLc(1,el) = sumLc(1,el) + B_load(e,2);
end

F = Mu + Mv + Mk1 + Mks - MHx - MKy + MKx + MKy;
G = Mu + Mv + Mk2 - MHx - MKy + MKx + MKy;
Mqb = MRbu + MQb;
Mqd = Mk2Ds + MRd + MQd;
Mt = M/deltt;
Mf = Mt - F;
Mg = Mt - G;
for i = 1:size(nB,1)
     Mt(nB(i),:) = 0;
     Mt(nB(i),nB(i)) = 1;
     Mf(nB(i),:) = 0;
```

```
    Mg(nB(i),:) = 0;
    Mrm(nB(i),:) = 0;
    Mk1(nB(i),:) = 0;
    Mqb(nB(i)) = bB(i);
    Mqd(nB(i,1)) = bD(i);
end
IMt = inv(Mt);
MMrm = IMt*Mrm;
MMk = IMt*Mk1;
Qt2 = IMt*Mf*Qt1 + IMt*Mqb;
Rt2 = IMt*Mf*Rt1 + MMrm*Lc;
SXt2 = IMt*Mg*SXt1 + IMt*Mqd;
Yt2 = IMt*Mg*Yt1 + MMk*Qt1;
Zt2 = IMt*Mg*Zt1 + MMk*Rt1;
Qt1 = Qt2; Rt1 = Rt2;
SXt1 = SXt2; Yt1 = Yt2; Zt1 = Zt2;

end
Qt(:,:,nT) = Qt2; Rt(:,:,nT) = Rt2;
SXt(:,:,nT) = SXt2; Yt(:,:,nT) = Yt2; Zt(:,:,nT) = Zt2;

for i = 1:size(specNode,1)
    QtNode(i,1) = Qt2(specNode(i));
    SXtNode(i,1) = SXt2(specNode(i));
    YtNode(i,1) = Yt2(specNode(i));
    RtNode(i,:) = Rt2(specNode(i),:);
    ZtNode(i,:) = Zt2(specNode(i),:);
end
Qti(:,nT) = QtNode; Rti(:,:,nT) = RtNode;
SXti(:,nT) = SXtNode; Yti(:,nT) = YtNode;  Zti(:,:,nT) = ZtNode;
specNodeT(:,nT) = specNode;
B_r = specB - QtNode;
D_r = SXtNode - YtNode - specD;

for i = 1:size(B_load,1)
    B_coef(:,i) = RtNode(:,B_load(i,1));
    D_coef(:,i) = ZtNode(:,B_load(i,1));
end
b_B = [b_B; B_r]; b_D = [b_D; D_r];
A_B = [A_B; B_coef]; A_D = [A_D; D_coef];

tiStart = ntime+1;
end

% START LINEAR PROGRAMMING

countLc = 0;
for i = 1:m
```

```
if sumLc(i) ~= 0
   countLc = countLc + 1;
   c0(countLc) = sumLc(i);
end

end
Pc = zeros(m,1);
A = [A_B; A_D];
slackVar = [eye(size(A,1)) zeros(size(A,1),size(B_load,1)*2)];
A = [A slackVar];
A_Pcmax = zeros(size(B_load,1),size(A,2));
A_Pcmin = zeros(size(B_load,1),size(A,2));
countMax = size(A,1) + size(B_load,1);
countMin = size(A,1) + size(B_load,1)*2;

for i = 1:size(B_load,1)
   A_Pcmax(i,i) = 1;
   A_Pcmin(i,i) = 1;
   countMax = 1 + countMax;
   A_Pcmax(i,countMax) = 1;
   countMin = 1 + countMin;
   A_Pcmin(i,countMin) = -1;
end
A = [A; A_Pcmax; A_Pcmin];
nBV = size(A,1);
b_Pcmax = diag(eye(size(B_load,1)))*PcMax;
b_Pcmin = diag(eye(size(B_load,1)))*PcMin;
b = [b_B; b_D; b_Pcmax; b_Pcmin];
count01 = 0;

for i = 1:size(b,1)
if b(i) < 0
    count01 = count01 + 1;
    bNeg(count01) = i;
end
end

if count01 = = 0
   nArf = size(B_load,1);
   Marf = zeros(size(A,1),nArf);
   A = [A Marf];
   indexi = size(A,1);  indexj = size(A,2);
   w = zeros(1,size(A,2));
   wSol = 0;
for i = 1:nArf
   A(indexi,indexj) = 1;
   w(1,indexj) = -1;
   w = w + A(indexi,:);
```

```
   wSol = wSol + PcMin;
   indexi = indexi - 1;
   indexj = indexj - 1;
end
[maxw,s] = max(w);

while maxw > 0

for i = 1:nBV
if A(i,s) > 0;
   ratio(i) = b(i)/A(i,s);
else
   ratio(i) = 0;
end

end
[minratio,r] = min(ratio);

while minratio == 0
   ratio(r) = max(ratio) + 1;
   [minratio,r] = min(ratio);
end
pivot_A = A(r,:)/A(r,s); pivot_b = b(r)/A(r,s);
wNew = w - w(s)*pivot_A;
wSolNew = wSol - w(s)*pivot_b;

for i = 1:nBV
if i ~= r
   ANew(i,:) = A(i,:) - A(i,s)*pivot_A;
   bNew(i,1) = b(i) - A(i,s)*pivot_b;
else
   ANew(r,:) = pivot_A; bNew(r,1) = pivot_b;
end
end

A = ANew; b = bNew; w = wNew; wSol = wSolNew;
[maxw,s] = max(w);

end

if wSol < 1e-15
   count02 = 0;
for i = 1:size(A,2)
   count1 = 0;  count2 = 0;
for j = 1:size(A,1)
if A(j,i) == 0
   count1 = count1 + 1;
end
```

```
if A(j,i) == 1
   count2 = count2 + 1;
end

if count1 == size(A,1)-1
if count2 == 1
   count02 = count02 + 1;
   bs(count02)= i;
end
end
end
end

count03 = 0;
for i = 1:size(bs,2)
if bs(i) > nBV
   count03 = count03 + 1;
end
end

if count03 > 0
   disp(['The problem has no feasible solution - try again with the lower PcMin ']);
else

A2(:,:) = A(:,1:size(A,2)-nArf);
c2 = [-c0 zeros(1,size(A2,2)-size(c0,2))];
z = 0;
indexi = nBV;
indexj = size(B_load,1);

for i = 1:size(B_load,1)
   c2New = c2 - c2(indexj)*A2(indexi,:);
   zNew = z - c2(indexj)*b(indexi);
   indexi = indexi - 1; indexj = indexj - 1;
   c2 = c2New; z = zNew;
end
[minc2,s] = min(c2);

while minc2 < 0
for i = 1:nBV
if A2(i,s) > 0;
   ratio(i) = b(i)/A2(i,s);
else
   ratio(i) = 0;
end
end
minratio,r] = min(ratio);
```

```
while minratio = = 0
   ratio(r) = max(ratio) + 1;
   [minratio,r] = min(ratio);
end
[minratio,r] = min(ratio);
pivot_A = A2(r,:)/A2(r,s); pivot_b = b(r)/A2(r,s);
c2New = c2 - c2(s)*pivot_A; zNew = z - c2(s)*pivot_b;

for i = 1:nBV
if i ~= r
   A2New(i,:) = A2(i,:) - A2(i,s)*pivot_A;
   bNew(i,1) = b(i) - A2(i,s)*pivot_b;
else
   A2New(i,:) = pivot_A;
   bNew(i,1) = pivot_b;
end
end
A2 = A2New; b = bNew;
c2 = c2New; z = zNew;
[minc2,s] = min(c2);
end

for i = 1:size(B_load,1)
   count1 = 0;  count2 = 0;
for j = 1:nBV
if A(j,i) = = 0
   count1 = count1+1;
end
if A(j,i) = = 1
   count2 = count2+1;
   indexj = j;
end
end
if count1 = = nBV-1
if count2 = = 1
   Xsol(i) = b(indexj);
end
end
end

for i = 1:size(B_load,1)
   disp(['Pc for element ' num2str(B_load(i,1)) ' is ' num2str(Xsol(i))]);
end
for i = 1:size(B_load,1)
   el = B_load(i,1);
   Pc(el) = Xsol(i);
end
```

```
for i = 1:size(selT,2)
   BOD(:,:,i) = Qti(:,i) + Rti(:,:,i)*Pc;
   DO(:,:,i) = SXti(:,i) - Yti(:,i) - Zti(:,:,i)*Pc;
   tCheck = 24/(size(selT,2)-1)*i-24/(size(selT,2)-1);
   disp(['---- At time ' num2str(tCheck) ' O Clock ----']);

for ii = 1:size(specNode,1)
   disp(['BOD at node ' num2str(specNodeT(ii,i)) ' = ' num2str(BOD(ii,i)) ' mg/l']);
end

for ii = 1:size(specNode,1)
   disp(['DO at node  ' num2str(specNodeT(ii,i)) ' = ' num2str(DO(ii,i))  ' mg/l']);
end
end

for i = 1:size(selT,2)
   BODall(:,:,i) = Qt(:,:,i) + Rt(:,:,i)*Pc;
   DOall(:,:,i) = SXt(:,:,i) - Yt(:,:,i) - Zt(:,:,i)*Pc;
end
end

else
   disp(['The problem has no feasible solution - try again with lower PcMin ']);
end

else
   disp(['This problem has no feasible solution because;']);
   noSN = size(specNodeT,1)*size(specNodeT,2);

for i = 1:size(bNeg,2)
if bNeg(i) <= noSN
   disp(['Specified BOD at node ' num2str(specNodeT(bNeg(i))) ' is too low']);
else
   disp(['Specified DO at node ' num2str(specNodeT(bNeg(i)-noSN)) ' is too high']);
end
end
disp(['Plase try again by changing the value of recommended parameter(s)']);
end

Total_BOD_Load = Xsol*B_load(:,2)/1000
```

Appendix B

Input data of hydrodynamics model

Appendix Table B1 Discharges from water regulators.

Date	Water regulator				
	Phopraya	Bang Mae Mai	Phaothalai	Song Phi Nong	Bang Pla
	Discharge (m^3/s)				
01/05/2009	19.46	0.00	0.00	21.35	29.69
02/05/2009	10.99	1.50	18.07	19.54	29.06
03/05/2009	7.06	1.51	20.71	19.54	29.69
04/05/2009	6.85	1.35	20.05	19.17	32.06
05/05/2009	6.74	1.32	18.94	18.64	31.44
06/05/2009	6.66	1.30	18.01	12.87	28.70
07/05/2009	6.57	1.00	15.25	12.79	25.43
08/05/2009	6.56	1.00	14.75	12.57	21.00
09/05/2009	6.56	0.93	12.68	4.41	18.19
10/05/2009	6.51	1.08	13.81	0.00	19.55
11/05/2009	6.47	1.10	13.90	0.00	21.00
12/05/2009	6.64	1.00	22.87	0.00	37.35
13/05/2009	6.88	0.83	18.43	9.56	37.35
14/05/2009	23.38	0.74	20.01	9.88	42.79
15/05/2009	58.28	1.95	19.87	10.49	40.74
16/05/2009	79.58	0.00	19.03	11.05	38.43
17/05/2009	84.55	0.00	22.57	11.42	36.46
18/05/2009	84.95	0.00	22.26	11.82	37.13
19/05/2009	85.82	0.00	18.50	11.84	36.46
20/05/2009	84.20	0.00	18.56	11.90	35.08
21/05/2009	80.60	0.00	17.79	11.90	35.08
22/05/2009	70.94	0.00	20.46	11.82	35.08
23/05/2009	75.06	0.00	15.35	11.61	35.08
24/05/2009	74.35	0.00	21.04	11.45	33.49
25/05/2009	66.13	0.00	20.81	11.37	33.49
26/05/2009	47.92	0.00	22.16	11.29	29.19
27/05/2009	37.94	0.00	21.58	11.07	24.70
28/05/2009	49.35	12.99	20.99	10.86	24.70
29/05/2009	46.81	13.94	20.66	10.75	23.47
30/05/2009	46.88	16.07	21.61	10.73	23.47
31/05/2009	24.84	8.14	23.41	10.75	27.10

Source: Regional Irrigation Office 12 (2010)

Appendix Table B2 Heights of water at Tha Chin River Mount in May, 2009, predicted in meters above the lowest low water

Date	Hours											
	0	1	2	3	4	5	6	7	8	9	10	11
	Height of water (m)											
1	3.3	3.1	2.9	2.8	2.7	2.7	2.8	2.8	2.7	2.6	2.2	1.8
2	3.3	3.3	3.1	2.9	2.7	2.6	2.6	2.6	2.6	2.5	2.4	2.1
3	3.2	3.2	3.2	3.0	2.7	2.5	2.4	2.3	2.3	2.3	2.3	2.2
4	3.0	3.1	3.2	3.0	2.8	2.5	2.3	2.1	1.9	1.9	2.0	2.1
5	2.7	3.0	3.1	3.0	2.8	2.5	2.2	1.9	1.7	1.5	1.6	1.7
6	2.5	2.8	2.9	3.0	2.8	2.6	2.2	1.8	1.5	1.3	1.2	1.3
7	2.3	2.6	2.8	2.9	2.8	2.6	2.2	1.9	1.5	1.2	1.0	0.9
8	2.2	2.4	2.7	2.8	2.8	2.6	2.3	1.9	1.5	1.2	0.9	0.8
9	2.2	2.3	2.5	2.7	2.8	2.7	2.4	2.1	1.6	1.3	1.0	0.8
10	2.2	2.2	2.4	2.6	2.8	2.8	2.6	2.2	1.8	1.4	1.1	0.9
11	2.4	2.3	2.3	2.5	2.7	2.8	2.7	2.4	2.0	1.6	1.2	1.0
12	2.5	2.4	2.3	2.4	2.6	2.8	2.8	2.6	2.3	1.8	1.4	1.1
13	2.7	2.5	2.4	2.4	2.5	2.7	2.8	2.7	2.5	2.1	1.6	1.2
14	2.9	2.7	2.5	2.4	2.5	2.6	2.7	2.7	2.5	2.2	1.8	1.4
15	3.1	2.8	2.6	2.5	2.4	2.5	2.5	2.5	2.5	2.3	2.0	1.6
16	3.1	2.9	2.7	2.5	2.4	2.4	2.4	2.4	2.3	2.3	2.1	1.8
17	3.1	3.0	2.8	2.6	2.4	2.3	2.3	2.3	2.2	2.1	2.0	1.9
18	3.0	3.0	2.8	2.6	2.4	2.2	2.1	2.1	2.0	1.9	1.9	1.9
19	2.9	3.0	2.9	2.6	2.4	2.1	1.9	1.8	1.7	1.7	1.8	1.8
20	2.8	2.9	2.8	2.7	2.4	2.1	1.8	1.6	1.5	1.4	1.5	1.7
21	2.7	2.8	2.8	2.7	2.4	2.1	1.7	1.4	1.3	1.1	1.2	1.4
22	2.5	2.7	2.7	2.7	2.5	2.2	1.8	1.4	1.1	1.0	0.9	1.0
23	2.3	2.5	2.7	2.7	2.6	2.3	1.9	1.5	1.1	0.9	0.8	0.7
24	2.2	2.4	2.6	2.7	2.6	2.4	2.1	1.7	1.2	0.9	0.8	0.7
25	2.3	2.3	2.4	2.6	2.7	2.6	2.3	1.9	1.4	1.1	0.8	0.7
26	2.5	2.4	2.4	2.5	2.7	2.7	2.5	2.2	1.7	1.3	1.0	0.8
27	2.8	2.6	2.5	2.5	2.6	2.7	2.6	2.4	2.0	1.6	1.2	0.9
28	3.1	2.8	2.7	2.5	2.6	2.6	2.7	2.6	2.3	1.9	1.5	1.1
29	3.3	3.0	2.8	2.6	2.5	2.5	2.6	2.6	2.5	2.2	1.8	1.4
30	3.3	3.1	2.8	2.6	2.4	2.4	2.4	2.4	2.4	2.4	2.1	1.7
31	3.3	3.2	2.9	2.6	2.3	2.2	2.1	2.2	2.2	2.3	2.2	2.0

Appendix Table B2 (Continued)

Date	Hours											
	0	1	2	3	4	5	6	7	8	9	10	11
	Height of water (m)											
1	1.4	1.1	0.9	0.7	0.7	0.7	1.1	1.6	2.1	2.6	3.0	3.2
2	1.7	1.4	1.1	1.0	0.8	0.8	0.9	1.3	1.7	2.2	2.7	3.0
3	2.0	1.8	1.5	1.3	1.1	1.0	1.0	1.2	1.5	1.9	2.3	2.7
4	2.1	2.0	2.0	1.8	1.6	1.5	1.4	1.4	1.5	1.7	2.0	2.4
5	1.9	2.1	2.2	2.2	2.2	2.0	1.9	1.8	1.7	1.7	1.9	2.2
6	1.6	1.9	2.2	2.5	2.6	2.6	2.4	2.3	2.1	2.0	1.9	2.1
7	1.1	1.5	2.0	2.5	2.8	3.0	3.0	2.8	2.6	2.3	2.1	2.1
8	0.8	1.1	1.6	2.3	2.8	3.1	3.3	3.2	3.0	2.7	2.4	2.2
9	0.7	0.8	1.2	1.9	2.6	3.1	3.4	3.5	3.4	3.1	2.7	2.4
10	0.7	0.7	0.9	1.5	2.2	2.8	3.3	3.5	3.5	3.3	3.0	2.6
11	0.8	0.7	0.7	1.1	1.8	2.5	3.0	3.4	3.5	3.5	3.2	2.9
12	0.9	0.8	0.7	0.8	1.3	2.0	2.7	3.1	3.4	3.5	3.3	3.1
13	1.0	0.9	0.7	0.7	1.0	1.6	2.2	2.8	3.1	3.3	3.3	3.2
14	1.1	0.9	0.8	0.7	0.8	1.2	1.8	2.4	2.9	3.2	3.3	3.2
15	1.3	1.0	0.9	0.8	0.7	1.0	1.5	2.0	2.5	2.9	3.1	3.2
16	1.4	1.2	1.0	0.9	0.8	0.9	1.3	1.8	2.2	2.6	3.0	3.1
17	1.6	1.4	1.2	1.1	1.0	1.0	1.2	1.6	2.0	2.4	2.7	3.0
18	1.7	1.6	1.5	1.3	1.3	1.2	1.3	1.6	1.9	2.2	2.6	2.8
19	1.8	1.8	1.8	1.7	1.6	1.5	1.6	1.7	1.9	2.1	2.4	2.6
20	1.8	2.0	2.0	2.1	2.0	2.0	1.9	1.9	1.9	2.0	2.2	2.5
21	1.7	2.0	2.2	2.4	2.5	2.5	2.4	2.2	2.1	2.1	2.1	2.3
22	1.3	1.8	2.2	2.6	2.8	2.9	2.8	2.7	2.5	2.3	2.1	2.1
23	0.9	1.3	1.9	2.5	2.9	3.2	3.2	3.1	2.9	2.6	2.3	2.2
24	0.7	0.9	1.4	2.1	2.7	3.1	3.4	3.4	3.3	3.0	2.7	2.4
25	0.6	0.6	0.8	1.5	2.2	2.8	3.3	3.5	3.6	3.4	3.1	2.7
26	0.7	0.6	0.5	0.8	1.5	2.3	2.9	3.3	3.5	3.6	3.4	3.1
27	0.8	0.6	0.5	0.5	0.9	1.6	2.3	2.9	3.3	3.5	3.5	3.4
28	0.9	0.7	0.6	0.4	0.6	1.1	1.8	2.4	3.0	3.3	3.5	3.5
29	1.1	0.9	0.7	0.5	0.5	0.8	1.3	2.0	2.6	3.0	3.3	3.4
30	1.4	1.1	0.9	0.7	0.6	0.7	1.1	1.7	2.2	2.7	3.1	3.3
31	1.7	1.5	1.2	1.0	0.9	0.9	1.2	1.5	2.0	2.5	2.9	3.1

Source: Hydrographic Department Royal Thai Navy (2009)

Appendix Table B3 x- and y-coordinates, mean depth soundings in meters reduced to lowest low water and mean sea level (M.S.L.) compared with datum[1] of nodes in finite element grid

Node number	Coordinates		Average S.W.L (m)	M.S.L. (m)	Node number	Coordinates		Average S.W.L (m)	M.S.L. (m)
	X	Y				X	Y		
1	0	322.5	2.46	-0.765	27	26000	322.5	1.338	-0.32
2	0	377.5	2.46	-0.765	28	26000	377.5	1.338	-0.32
3	2000	318.75	1.617	-0.765	29	28000	325	1.543	-0.28
4	2000	381.25	1.617	-0.765	30	28000	375	1.543	-0.28
5	4000	309.375	2.15	-0.716	31	30000	323.75	2.038	-0.27
6	4000	390.625	2.15	-0.716	32	30000	376.25	2.038	-0.27
7	6000	326.25	1.35	-0.64	33	32000	326.875	1.843	-0.26
8	6000	373.75	1.35	-0.64	34	32000	373.125	1.843	-0.26
9	8000	325	1.317	-0.62	35	34000	328.75	1.967	-0.23
10	8000	375	1.317	-0.62	36	34000	371.25	1.967	-0.23
11	10000	325	1.371	-0.57	37	36000	330	2.529	-0.21
12	10000	375	1.371	-0.57	38	36000	370	2.529	-0.21
13	12000	322.5	1.143	-0.53	39	38000	328.75	2.014	-0.19
14	12000	377.5	1.143	-0.53	40	38000	371.25	2.014	-0.19
15	14000	325.625	1.788	-0.50	41	40000	323.125	2.05	-0.18
16	14000	374.375	1.788	-0.50	42	40000	376.875	2.05	-0.18
17	16000	326.25	1.786	-0.48	43	42000	322.5	2.7	-0.17
18	16000	373.75	1.786	-0.48	44	42000	377.5	2.7	-0.17
19	18000	326.25	1.786	-0.44	45	44000	319.375	2.325	-0.14
20	18000	373.75	1.786	-0.44	46	44000	380.625	2.325	-0.14
21	20000	325	1.25	-0.42	47	46000	316.25	2.83	-0.13
22	20000	375	1.25	-0.42	48	46000	383.75	2.83	-0.13
23	22000	323.75	1.7	-0.38	49	48000	309.375	2.767	-0.12
24	22000	376.25	1.7	-0.38	50	48000	390.625	2.767	-0.12
25	24000	327.5	1.65	-0.34	51	50000	313.75	3.214	-0.10
26	24000	372.5	1.65	-0.34	52	50000	386.25	3.214	-0.10

Appendix Table B3 (Continued)

Node number	Coordinates X	Coordinates Y	Average S.W.L (m)	M.S.L. (m)	Node number	Coordinates X	Coordinates Y	Average S.W.L (m)	M.S.L. (m)
53	52000	312.5	1.814	-0.06	81	80000	291.25	2.544	0.28
54	52000	387.5	1.814	-0.06	82	80000	408.75	2.544	0.28
55	54000	310	1.743	-0.04	83	82000	297.5	3.444	0.29
56	54000	390	1.743	-0.04	84	82000	402.5	3.444	0.29
57	56000	305	1.97	-0.02	85	84000	291.25	2.533	0.32
58	56000	390	1.97	-0.02	86	84000	408.75	2.533	0.32
59	58000	300.625	1.767	-0.01	87	86000	301.25	2.922	0.35
60	58000	399.375	1.767	-0.01	88	86000	398.75	2.922	0.35
61	60000	301.25	1.667	0.00	89	88000	307.5	3.213	0.39
62	60000	398.75	1.667	0.00	90	88000	392.5	3.213	0.39
63	62000	280	1.5	0.03	91	90000	301.875	3.5	0.42
64	62000	420	1.5	0.03	92	90000	398.125	3.5	0.42
65	64000	305	2.478	0.05	93	92000	293.75	3.03	0.44
66	64000	395	2.478	0.05	94	92000	406.25	3.03	0.44
67	66000	303.75	2.689	0.09	95	94000	293.75	3.57	0.49
68	66000	396.25	2.689	0.09	96	94000	406.25	3.57	0.49
69	68000	306.25	2.614	0.09	97	96000	300.625	4.5	0.49
70	68000	393.75	2.614	0.09	98	96000	399.375	4.5	0.49
71	70000	273.125	1.464	0.14	99	98000	306.25	4.325	0.49
72	70000	426.875	1.464	0.14	100	98000	393.75	4.325	0.49
73	72000	287.5	1.475	0.18	101	100000	310	5.029	0.53
74	72000	412.5	1.475	0.18	102	100000	390	5.029	0.53
75	74000	293.75	2.7	0.19	103	102000	302.5	4.345	0.53
76	74000	406.25	2.7	0.19	104	102000	397.5	4.345	0.53
77	76000	288.75	2.364	0.23	105	104000	291.875	4.08	0.54
78	76000	411.25	2.364	0.23	106	104000	408.125	4.08	0.54
79	78000	292.5	2.19	0.25	107	106000	300	5	0.54
80	78000	407.5	2.19	0.25	108	106000	400	5	0.54

Appendix Table B3 (Continued)

Node number	Coordinates X	Coordinates Y	Average S.W.L (m)	M.S.L. (m)	Node number	Coordinates X	Coordinates Y	Average S.W.L (m)	M.S.L. (m)
109	108000	300.625	4.189	0.54	137	136000	287.5	5.2	0.80
110	108000	399.375	4.189	0.54	138	136000	412.5	5.2	0.80
111	110000	278.75	3.708	0.66	139	138000	293.75	5.1	0.84
112	110000	421.25	3.708	0.66	140	138000	406.25	5.1	0.84
113	112000	282.5	3.42	0.66	141	140000	292.5	5.067	0.91
114	112000	417.5	3.42	0.66	142	140000	407.5	5.067	0.91
115	114000	303.75	4.8	0.84	143	142000	290	5.967	0.91
116	114000	396.25	4.8	0.84	144	142000	410	5.967	0.91
117	116000	301.25	4.222	0.84	145	144000	302.5	4.783	0.93
118	116000	398.75	4.222	0.84	146	144000	397.5	4.783	0.93
119	118000	306.875	5.063	0.84	147	146000	280	4.85	0.93
120	118000	393.125	5.063	0.84	148	146000	420	4.85	0.93
121	120000	300	5.644	0.83	149	148000	298.75	5.2	0.95
122	120000	400	5.644	0.83	150	148000	401.25	5.2	0.95
123	122000	308.75	7.333	0.83	151	150000	287.5	4.1	0.99
124	122000	391.25	7.333	0.83	152	150000	412.5	4.1	0.99
125	124000	293.75	4.93	0.83	153	152000	295	5.2	0.99
126	124000	406.25	4.93	0.83	154	152000	405	5.2	0.99
127	126000	303.75	6.413	0.77	155	154000	290	5.483	0.99
128	126000	396.25	6.413	0.77	156	154000	410	5.483	0.99
129	128000	300	6.189	0.77	157	156000	280	4.383	1.01
130	128000	400	6.189	0.77	158	156000	420	4.383	1.01
131	130000	301.25	6.275	0.74	159	158000	287.5	4.567	1.03
132	130000	398.75	6.275	0.74	160	158000	412.5	4.567	1.03
133	132000	308.75	7.343	0.77	161	160000	292.5	6.725	1.03
134	132000	391.25	7.343	0.77	162	160000	407.5	6.725	1.03
135	134000	301.25	5.88	0.80	163	162000	292.5	6.02	1.03
136	134000	398.75	5.88	0.80	164	162000	407.5	6.02	1.03

Appendix Table B3 (Continued)

Node number	Coordinates		Average S.W.L (m)	M.S.L. (m)	Node number	Coordinates		Average S.W.L (m)	M.S.L. (m)
	X	Y				X	Y		
165	164000	270	3.38	1.03	185	184000	217	2.993	1.42
166	164000	430	3.38	1.03	186	184000	483	2.993	1.42
167	166000	265	3.7	1.10	187	186000	262	6.092	1.43
168	166000	435	3.7	1.10	188	186000	438	6.092	1.43
169	168000	255	3.311	1.10	189	188000	248	5.009	1.43
170	168000	445	3.311	1.10	190	188000	452	5.009	1.43
171	170000	250	2.833	1.18	191	190000	216	2.925	1.46
172	170000	450	2.833	1.18	192	190000	484	2.925	1.46
173	172000	255	4.29	1.18	193	192000	255	5.285	1.49
174	172000	445	4.29	1.18	194	192000	445	5.285	1.49
175	174000	250	4.6	1.32	195	194000	230	4.246	-1.51
176	174000	450	4.6	1.32	196	194000	470	4.246	-1.51
177	176000	228	3.392	1.32	197	196000	248	5.782	-1.60
178	176000	472	3.392	1.32	198	196000	452	5.782	-1.60
179	178000	248	3.291	1.32	199	198000	212	5.48	-1.54
180	178000	452	3.291	1.32	200	198000	488	5.48	-1.54
181	180000	248	4.036	1.39	201	200000	186	3.912	-1.60
182	180000	452	4.036	1.39	202	200000	514	3.912	-1.60
183	182000	222	2.729	1.39	203	202000	34	1.694	-1.65
184	182000	478	2.729	1.39	204	202000	666	1.694	-1.65

Source: [1] Marine Department (n.d.)

Appendix C

Estimated BOD load in 2010

Appendix Table C1 Estimated BOD load based on municipal population

Province	District	Subdistrict	Population (persons)	BOD load (kg/d)
Suphan Buri	Muang Suphan Buri	Phihan Daeng	5,029	181.044
		Sanam Chai	10,530	379.080
		Tha Phi Liang	18,449	664.164
		Rua Yai	16,245	584.820
		Phai Kwang	7,509	270.324
		Tha Rahat	9,947	358.092
		Don Kamyan	9,817	353.412
		Thap Ti Lek	3,667	132.012
	Bang Pla Ma	Bang Pla Ma	8,695	313.020
		Khok Khram	7,873	283.428
		Chorakhe Yai	5,004	180.144
		Ban Laem	4,666	167.976
		Wat Dao	6,212	223.632
		Takha	5,822	209.592
		Ongkharak	5,551	199.836
		Phai Kong Din	6,961	250.596
		Bang Yai	4,531	163.116
		Kritsana	3,892	140.112
		Sali	7,181	258.516
		Makham Lom	5,110	183.960
	Song Phi Nong	Ban Kum	3,625	130.500
		Bang Phlap	5,213	187.668
		Ban Chang	3,115	112.140
		Ton Tan	3,537	127.332
		Bang Takhian	5,680	204.480
		Bang Ta Then	15,455	556.380
		Song Phi Nong	12,809	461.124
		Noen Phra Prang	4,342	156.312
		Bang Len	6,987	251.532
		Don Manao	5,034	181.224
Nakhon Pathom	Bang Len	Bang Luang	9,654	347.544
		Hin Mun	5,973	215.028
		Sai Ngam	5,278	190.008
		Bang Sai Pa	5,929	213.444
		Bang Phasi	9,564	344.304
		Bang Len	11,605	417.780
		Don Tum	4,815	173.340
		Khlong Nok Krathung	3,781	136.116
		Bang Pla	7,012	252.432
		Lam Phaya	4,430	159.480

Appendix Table C1 (Continued)

Province	District	Subdistrict	Population (persons)	BOD load (kg/d)
Nakhon Pathom	Bang Len	Bang Rakam	4,607	165.852
		Bua Pak Tha	4,562	164.232
		Nin Phet	4,397	158.292
		Phai Hu Chang	4,166	419.976
		Nara Phirom	4,847	174.492
	Nakhon Chai Si	Bang Phra	2,725	98.100
		Bang Kaeo Fa	3,089	111.204
		Wat Lamut	5,221	187.956
		Huai Phlu	4,451	160.236
		Si Maha Pho	3,822	137.592
		Don Faek	2,905	104.580
		Wat Samrong	1,843	66.3480
		Lan Tak Fa	7,218	259.848
		Ngio Rai	2,932	105.552
		Sampathuan	3,901	140.436
		Thaiyawat	2,964	106.704
		Sisa Thong	7,080	254.880
		Wat Khae	2,636	94.896
		Nakhon Chai Si	4,311	155.196
		Bang Krabao	5,949	176.904
		Khun Kaeo	6,998	251.928
		Tha Tamnak	7,523	270.828
		Phaniat	3,814	137.304
		Tha Phraya	3,520	126.72
		Khok Phra Chedi	4,568	164.448
		Laem Bua	9,730	350.281
		Tha Krachap	3,930	141.48
		Bang Kaeo	3,164	113.904
	Sam Phran	Hom Kret	8,088	291.168
		Song Khanong	4,811	173.196
		Bang Toei	4,014	144.504
		Bang Krathuek	9,848	354.528
		Rai Khing	23,585	849.060
		Tha Talat	15,105	543.780
		Yai Cha	7,066	254.376
		Tha Kham	9,952	358.272
		Sam Phran	12,468	448.848
		Bang Chang	7,744	278.784
		Ban Mai	9,454	340.344
		Om Yai	16,071	578.556
		Khlong Mai	11,417	411.012
		Khlong Chinda	11,590	417.240

Appendix Table C1 (Continued)

Province	District	Subdistrict	Population (persons)	BOD load (kg/d)
Nakhon Pathom	Sam Phran	Talat Chinda	7,492	269.712
		Krathum Lom	17,552	631.872
	Muang Nakhon Pathom		162,950	5,866.200
		Nakhon Pathom		
		Phra Pathom Chedi	33,182	1194.552
		Nakhon Pathom	20,734	746.424
		Bo Phlap	15,095	543.420
		Phra Prathon	11,685	420.660
		Thammasala	12,888	200.988
		Sam Khwai Phueak	8,904	320.544
		Huai Chorakhe	13,771	495.756
		Thung Noi	5,291	190.476
	Phutthamonthon	Salaya	18,499	665.964
		Khlong Yong	7,738	278.568
		Maha Sawat	6,946	250.056
Samut Sakhon	Kratumban	Tha Mai	9,720	349.920
		Bang Yang	5,110	183.960
		Krathum Baen	18,342	660.312
		Suan Luang	25,725	926.100
		Tha Sao	6,270	225.720
		Don Kai Di	6,799	244.764
		Om Noi	50,687	1824.732
		Khlong Maduea	18,094	651.384
		Khae Rai	5,014	180.504
		Nong Nok Khai	3,440	123.840
	Ban Phaeo	Ban Phaeo	10,446	376.056
		Lak Sam	13,675	492.300
		Lak Song	4,459	160.524
		Chet Rio	3,796	136.656
		Khlong Tan	4,566	164.376
		Suan Som	5,049	181.764
		Kaset Phatthana	4,599	165.564
		Amphaeng	6,245	224.820
	Muang Samut Sakhon	Ban Ko	13,029	469.044
		Tha Chin	9,958	358.488
		Chai Mongkhon	3,922	141.192
		Bang Krachao	8,185	294.660
		Ban Bo	7,708	277.488

Appendix Table C1 (Continued)

Province	District	Subdistrict	Population (persons)	BOD load (kg/d)
Samut Sakhon	Muang Samut Sakhon	Bang Tho Rat	8,024	288.864
		Tha Sai	24,833	893.988
		Maha Chai	40,250	1449.000
		Na Di	19,667	708.012
		Bang Ya Phraek	23,031	829.116
		Khok Kham	17,967	646.812
		Khok Krabue	9,056	326.016
		Bang Nam Chuet	11,719	421.884
		Phan Thai Norasing	15,846	570.456
		Krok Krak	5,262	189.432
		Tha Chalom	9,865	355.140

Source: Department of Provincial Administration (n.d.)

Appendix Table C2 Estimated BOD load based on the amount of pigs

Province	District	Subdistrict	The amount of pigs	BOD load (kg/d)
Suphan Buri	Muang Suphan Buri	Sanam Chai	125	4.687
		Tha Phi Liang	21	0.787
		Rua Yai	3,464	129.900
		Phai Kwang	173	64.875
		Tha Rahat	2,522	94.575
		Don Kamyan	16,175	606.562
		Thap Ti Lek	67	2.515
	Bang Pla Ma	Bang Pla Ma	1,225	45.937
		Khok Khram	42	1.575
		Chorakhe Yai	48	1.800
		Ban Laem	1,732	64.950
		Wat Dao	24	0.900
		Phai Kong Din	100	3.750
		Bang Yai	47	1.762
		Kritsana	63	2.362
		Sali	146	5.475
	Song Phi Nong	Bang Phlap	223	8.362
		Ton Tan	410	15.375
		Bang Ta Then	30	1.125
		Song Phi Nong	24	0.900
		Noen Phra Prang	35	1.312
		Bang Len	447	16.762
		Don Manao	219	8.212
Nakhon Pathom	Bang Len	Bang Luang	1,523	57.112
		Bang Len	790	29.625
		Don Tum	3,046	114.225
		Khlong Nok	203	7.612
		Bang Pla	1,713	64.237
		Bang Rakam	1,511	56.662
	Nakhon Chai Si	Wat Lamut	3,351	125.662
		Si Maha Pho	3,416	128.100
		Lan Tak Fa	1,466	54.975
		Sampathuan	1,358	50.925
		Khun Kaeo	579	21.712
		Tha Tamnak	60	2.250
		Tha Phraya	4,525	169.688
		Khok Phra Chedi	513	19.237
		Laem Bua	9,730	363.862

Appendix Table C2 (Continued)

Province	District	Subdistrict	The amount of pigs	BOD load (kg/d)
	Nakhon Chai Si	Tha Krachap	4,646	
		Bang Kaeo	5,493	
	Dontoom	Sam Ngam	19,032	713.700
	Sam Phran	Song Khanong	104	3.900
		Tha Kham	62,391	2,339.662
		Sam Phran	11,892	445.950
		Bang Chang	1,212	45.450
		Ban Mai	6,057	227.137
		Om Yai	190	7.125
	Muang Nakhon Pathom	Phra Pathom Chedi	2,070	77.625
		Nakhon Pathom	640	24.000
		Bo Phlap	17,894	671.025
		Phra Prathon	16,281	610.538
		Thammasala	24,323	912.112
		Sam Khwai Phueak	92,921	3,484.538

Source: PCD and Thailand Environmental Technique Limited Company (2008)

Appendix Table C3 Estimated BOD load from industries.

Province	District	Subdistrict	BOD load (g/d)
Suphan Buri	Muang Suphan Buri	Tha Phi Liang	2.7
Nakhon Pathom	Bang Len	Bang Sai Pa	249
		Bang Len	7,000
	Nakhon Chai Si	Don Faek	1,660
		Sampathuan	13,067
		Nakhon Chai Si	35,475
		Khun Kaeo	19,057.5
	Dontoom	Sam Ngam	400
		Lam Luk Bua	25,700
	Sam Phran	Hom Kret	8,783
		Bang Krathuek	3,500
		Rai Khing	108,218
		Tha Talat	3,300
		Yai Cha	61,720
		Tha Kham	33,939
		Sam Phran	2,247
		Ban Mai	29,486
		Om Yai	58,217
		Khlong Mai	13,360
		Khlong Chinda	9,240
		Talat Chinda	7,000
		Krathum Lom	7,113
Samut Sakhon	Kratumban	Tha Mai	30,218
		Suan Luang	3,024
		Tha Sao	20,750
		Don Kai Di	342
		Om Noi	335,080
		Khlong Maduea	7,604
		Khae Rai	410
	Ban Phaeo	Lak Sam	57
		Suan Som	1,440
	Muang Samut Sakhon	Ban Ko	82,747
		Ban Bo	2,912
		Bang Tho Rat	35,534
		Maha Chai	16,805
		Khok Krabue	10,225

Source: Industrial Department Works (2010)

Appendix Table C4 BOD loading into each element

Element	BOD loading from various pollutant sources (kg/d)		
	Pig farms and aquacultures	Industries	Communities
1	-	-	0
2	40.041	-	186.708
3	40.041	-	186.708
4	74.860	-	749.442
5	40.238	0.001	332.082
6	40.238	0.001	332.082
7	100.031	0.178	537.462
8	64.313	0.178	245.052
9	44.792	-	119.290
10	44.792	-	119.290
11	44.792	-	119.290
12	44.792	-	119.290
13	46.630	-	299.434
14	46.536	-	68.527
15	46.536	-	68.527
16	46.686	-	143.071
17	46.686	-	143.071
18	46.686	-	143.071
19	40.728	-	136.008
20	40.728	-	101.076
21	44.403	-	359.592
22	40.041	-	56.070
23	40.041	-	99.570
24	40.111	-	113.048
25	46.139	-	270.548
26	46.139	-	267.944
27	40.111	-	110.444
28	40.111	-	110.444
29	40.111	-	110.444
30	45.408	-	697.334
31	44.302	-	359.546
32	49.560	-	115.848
33	49.560	-	115.848
34	49.560	-	115.848
35	40.041	-	71.676
36	40.041	-	71.676
37	40.041	-	150.822
38	40.041	-	95.004
39	40.041	-	245.298
40	40.041	-	146.136

Appendix Table C4 (Continued)

Element	BOD loading from various pollutant sources (kg/d)		
	Pig farms and aquacultures	Industries	Communities
41	40.041	-	318.288
42	47.447	3.500	208.890
43	47.447	3.500	208.890
44	486.122	26.1	425.772
45	41.944	-	68.058
46	41.944	-	240.210
47	40.041	-	79.740
48	40.041	-	79.740
49	54.206	-	82.926
50	54.206	-	82.926
51	42.647	-	285.150
52	42.647	-	111.316
53	42.647	-	215.950
54	42.647	0.830	170.656
55	42.647	0.830	170.656
56	56.391	-	218.812
57	56.391	-	1,114.302
58	55.378	6.534	105.402
59	55.378	6.534	423.966
60	1,965.096	17.817	10,320.354
61	42.647	17.738	104.274
62	53.503	19.058	278.604
63	42.647	2.928	123.732
64	42.647	2.928	97.056
65	42.647	2.928	97.056
66	44.597	-	173.196
67	42.647	-	144.504
68	42.647	7.113	953.484
69	42.647	108.218	598.956
70	44.822	1.650	554.910
71	44.822	1.650	271.890
72	42.647	44.220	127.188
73	42.647	30.860	127.188
74	672.158	16.970	268.906
75	87.242	-	295.276
76	104.436	-	969.892
77	94.817	-	182.698
78	679.733	16.970	361.834
79	80.503	9.829	113.448
80	80.503	9.829	113.448

Appendix Table C4 (Continued)

Element	BOD loading from various pollutant sources (kg/d)		
	Pig farms and aquacultures	Industries	Communities
81	84.066	68.046	2,516.736
82	42.647	10.094	116.640
83	42.647	10.094	116.640
84	42.647	345.174	147.300
85	42.647	-	30.660
86	42.647	-	30.660
87	42.647	-	154.500
88	42.647	-	975.219
89	42.647	13.105	1,861.836
90	42.647	5.908	192.753
91	42.647	5.908	274.923
92	42.647	5.188	168.840
93	42.647	5.188	214.281
94	42.647	41.374	234.522
95	42.647	41.431	726.822
96	42.647	101.410	446.994
97	42.647	101.410	446.994
98	42.647	159.805	1,360.692
99	42.647	136.512	2,858.958
100	42.647	8.429	1,104.444
101	42.647	125.511	1,532.466

Appendix D

Results from investigating various wastewater treatment scenarios

Appendix Table D1 Results from investigating various scenarios with BOD load removal of 30% from pig farm wastewater and 70% from aquaculture wastewater

Year	Scenario	Treatment plant site (Districts)	BOD load to each plant (kg/d)	Optimal P_c of each plant	Total R_{bc} (kg/d)	Total R_{bu} (kg/d)	Overall BOD load (kg/d)
2010	1	Suphan Buri	2,437.52	No feasible solution can be found;			
		Nakhon Pathom	8,861.90	BOD concentrations at 86 RKM exceed BOD.			
	2	Suphan Buri	2,437.52	0.5	6,553.53	43,121.07	49,674.60
		Nakhon Pathom	8,861.90	0.5			
		Bang Pla Ma	2,209.82	0.409			
	3	Suphan Buri	2,437.52	0.307	6,543.77	42,849	49,392.77
		Nakhon Pathom	8,861.90	0.5			
		Song Phi Nong	2,729.01	0.5			
	4	Suphan Buri	2,437.52	0.05	5,854.49	41,793	47,647.49
		Nakhon Pathom	8,861.90	0.5			
		Bang Len	3,262.32	0.399			
	5	Suphan Buri	2,437.52	No feasible solution can be found;			
		Nakhon Pathom	8,861.90	BOD concentrations at 114 RKM exceed BOD			
		Nakhon Chai Si	3,654.07	standard.			
	6	Suphan Buri	2,437.52	No feasible solution can be found.;			
		Nakhon Pathom	8,861.90	BOD concentrations at 86 RKM exceed BOD			
		Sam Phran	6,075.54	standard.			
2020	2	Suphan Buri	2,708.36	0.298	6,246.00	51,903	58,149.00
		Nakhon Pathom	9,846.56	0.5			
		Bang Pla Ma	2,578.12	0.2			
	3	Suphan Buri	2,708.36	0.02	5,521.89	51,586	57,107.89
		Nakhon Pathom	9,846.56	0.5			
		Song Phi Nong	3,183.85	0.171			
	4	Suphan Buri	2,708.36	No feasible solution can be found;			
		Nakhon Pathom	9,846.56	BOD concentrations at 130-150 RKM exceed			
		Bang Len	3,987.28	BOD standard.			
	7	Suphan Buri	2,708.36	0.5	8,666.02	49,323	57,989.02
		Nakhon Pathom	9,846.56	0.5			
		Bang Pla Ma	2,578.12	0.309			
		Song Phi Nong	3,183.85	0.5			

Appendix Table D1 (Continued)

Year	Scenario	Treatment plant site (Districts)	BOD load to each plant (kg/d)	Optimal P_c of each plant	Total R_{bc} (kg/d)	Total R_{bu} (kg/d)	Overall BOD load (kg/d)
2030	2	Suphan Buri	2,979.19	0.01	492.75	61,262	61,754.75
		Nakhon Pathom	10,831.21	0.04			
		Bang Pla Ma	2,970.98	0.01			
	7	Suphan Buri	2,979.19	0.15	3,540.86	58,531	62,071.86
		Nakhon Pathom	10,831.21	0.15			
		Bang Pla Ma	2,970.98	0.15			
		Song Phi Nong	3,669.01	0.279			
	8	Suphan Buri	2,979.19	0.15	4,091.38	56,974	61,065.38
		Nakhon Pathom	10,831.21	0.15			
		Bang Pla Ma	2,970.98	0.15			
		Bang Len	4,784.73	0.329			
	9	Suphan Buri	2,979.19	0.1	5,159.30	55,903	61,062.30
		Nakhon Pathom	10,831.21	0.174			
		Bang Pla Ma	2,970.98	0.1			
		Nakhon Chai Si	5,359.30	0.5			
	10	Suphan Buri	2,979.19	0.03	9,078.23	52,352	61,430.23
		Nakhon Pathom	10,831.21	0.5			
		Bang Pla Ma	2,970.98	0.03			
		Sam Phran	8,910.79	0.391			
	11	Suphan Buri	2,979.19	No feasible solution can be found;			
		Nakhon Pathom	10,831.21	BOD concentrations at 154-162 RKM exceed			
		Song Phi Nong	3,669.01	BOD standard.			
		Bang Len	4,784.73				
	12	Suphan Buri	2,979.19	No feasible solution can be found;			
		Nakhon Pathom	10,831.21	BOD concentrations at 154-162 RKM exceed			
		Song Phi Nong	3,669.01	BOD standard.			
		Bang Len	4,784.73				
		Nakhon Chai Si	5,359.30				

Appendix Table D1 (Continued)

Year	Scenario	Treatment plant site (Districts)	BOD load to each plant (kg/d)	Optimal P_c of each plant	Total R_{bc} (kg/d)	Total R_{bu} (kg/d)	Overall BOD load (kg/d)
2030	13	Suphan Buri	2,979.19	No feasible solution can be found;			
		Nakhon Pathom	10,831.21	BOD concentrations at 154-162 RKM exceed			
		Song Phi Nong	3,669.01	BOD standard.			
		Bang Len	4,784.73				
		Nakhon Chai Si	5,359.30				
		Sam Phran	8,910.79				
	14	Suphan Buri	2,979.19	0.2	8,146.37	54,023	62,169.37
		Nakhon Pathom	10,831.21	0.252			
		Bang Pla Ma	2,970.98	0.2			
		Song Phi Nong	3,669.01	0.5			
		Bang Len	4,784.73	0.5			
	15	Suphan Buri	2,979.19	0.2	8,563.63	53,172	61,735.63
		Nakhon Pathom	10,831.21	0.264			
		Bang Pla Ma	2,970.98	0.2			
		Song Phi Nong	3,669.01	0.5			
		Nakhon Chai Si	5,359.30	0.5			
	16	Suphan Buri	2,979.19	0.2	12,256.50	49,621	61,877.50
		Nakhon Pathom	10,831.21	0.441			
		Bang Pla Ma	2,970.98	0.2			
		Song Phi Nong	3,669.01	0.5			
		Sam Phran	8,910.79	0.5			
	17	Suphan Buri	2,979.19	0.2	13,198.05	48,663	61,861.05
		Nakhon Pathom	10,831.21	0.471			
		Bang Pla Ma	2,970.98	0.2			
		Song Phi Nong	3,669.01	0.5			
		Bang Len	4,784.73	0.5			
		Nakhon Chai Si	5,359.30	0.5			
	18	Suphan Buri	2,979.19	0.5	16,229.20	45,112	61,341.20
		Nakhon Pathom	10,831.21	0.5			
		Bang Pla Ma	2,970.98	0.216			
		Song Phi Nong	3,669.01	0.5			
		Bang Len	4,784.73	0.5			
		Sam Phran	8,910.79	0.5			

Appendix Table D2 Results from investigating various scenarios with BOD load removal of 50% from pig farm wastewater and 70% from aquaculture wastewater

Year	Scenario	Treatment plant site (Districts)	BOD load to each plant (kg/d)	Optimal P_c of each plant	Total R_{bc} (kg/d)	Total R_{bu} (kg/d)	Overall BOD load (kg/d)
2010	1	Suphan Buri	2,437.52	No feasible solution can be found;			
		Nakhon Pathom	8,861.90	BOD concentrations at 86 RKM exceed BOD.			
	2	Suphan Buri	2,437.52	0.5	6,754.62	41,416	48,170.62
		Nakhon Pathom	8,861.90	0.5			
		Bang Pla Ma	2,209.82	0.5			
	3	Suphan Buri	2,437.52	0.342	6,629.09	41,144	47,773.09
		Nakhon Pathom	8,861.90	0.5			
		Song Phi Nong	2,729.01	0.5			
	4	Suphan Buri	2,437.52	0.05	6,076.33	40,088	46,164.33
		Nakhon Pathom	8,861.90	0.5			
		Bang Len	3,262.32	0.467			
	5	Suphan Buri	2,437.52	No feasible solution can be found;			
		Nakhon Pathom	8,861.90	BOD concentrations at 114 RKM exceed BOD			
		Nakhon Chai Si	3,654.07	standard.			
	6	Suphan Buri	2,437.52	No feasible solution can be found;			
		Nakhon Pathom	8,861.90	BOD concentrations at 86 RKM exceed BOD			
		Sam Phran	6,075.54	standard.			
2020	2	Suphan Buri	2,708.36	0.347	6,378.70	50,032	56,410.7
		Nakhon Pathom	9,846.56	0.5			
		Bang Pla Ma	2,578.12	0.2			
	3	Suphan Buri	2,708.36	0.05	5,886.50	49,715	55,601.5
		Nakhon Pathom	9,846.56	0.5			
		Song Phi Nong	3,183.85	0.26			
	4	Suphan Buri	2,708.36	No feasible solution can be found;			
		Nakhon Pathom	9,846.56	BOD concentrations at 130-150 RKM exceed			
		Bang Len	3,987.28	BOD standard.			
	7	Suphan Buri	2,708.36	0.5	8,715.01	47,452	56,167.01
		Nakhon Pathom	9,846.56	0.5			
		Bang Pla Ma	2,578.12	0.328			
		Song Phi Nong	3,183.85	0.5			

Appendix Table D2 (Continued)

Year	Scenario	Treatment plant site (Districts)	BOD load to each plant (kg/d)	Optimal P_c of each plant	Total R_{bc} (kg/d)	Total R_{bu} (kg/d)	Overall BOD load (kg/d)
2030	2	Suphan Buri	2,979.19	0.01	2,323.22	59,225	61,548.22
		Nakhon Pathom	10,831.21	0.209			
		Bang Pla Ma	2,970.98	0.01			
	7	Suphan Buri	2,979.19	0.2	5,537.38	56,494	62,031.38
		Nakhon Pathom	10,831.21	0.232			
		Bang Pla Ma	2,970.98	0.2			
		Song Phi Nong	3,669.01	0.5			
	8	Suphan Buri	2,979.19	0.2	6,171.06	54,937	61,108.06
		Nakhon Pathom	10,831.21	0.239			
		Bang Pla Ma	2,970.98	0.2			
		Bang Len	4,784.73	0.5			
	9	Suphan Buri	2,979.19	0.15	6,973.18	53,865	60,838.18
		Nakhon Pathom	10,831.21	0.314			
		Bang Pla Ma	2,970.98	0.15			
		Nakhon Chai Si	5,359.30	0.5			
	10	Suphan Buri	2,979.19	0.191	10,588.59	50,314	60,902.57
		Nakhon Pathom	10,831.21	0.5			
		Bang Pla Ma	2,970.98	0.05			
		Sam Phran	8,910.79	0.5			
	11	Suphan Buri	2,979.19	No feasible solution can be found;			
		Nakhon Pathom	10,831.21	BOD concentrations at 154-162 RKM exceed			
		Song Phi Nong	3,669.01	BOD standard.			
		Bang Len	4,784.73				
	12	Suphan Buri	2,979.19	No feasible solution can be found;			
		Nakhon Pathom	10,831.21	BOD concentrations at 154-162 RKM exceed			
		Song Phi Nong	3,669.01	BOD standard.			
		Bang Len	4,784.73				
		Nakhon Chai Si	5,359.30				

Appendix Table D2 (Continued)

Year	Scenario	Treatment plant site (Districts)	BOD load to each plant (kg/d)	Optimal P_c of each plant	Total R_{bc} (kg/d)	Total R_{bu} (kg/d)	Overall BOD load (kg/d)
2030	13	Suphan Buri	2,979.19	No feasible solution can be found; BOD concentrations at 154-162 RKM exceed BOD standard.			
		Nakhon Pathom	10,831.21				
		Song Phi Nong	3,669.01				
		Bang Len	4,784.73				
		Nakhon Chai Si	5,359.30				
		Sam Phran	8,910.79				
	14	Suphan Buri	2,979.19	0.2	10,171.81	51,985	62,156.81
		Nakhon Pathom	10,831.21	0.439			
		Bang Pla Ma	2,970.98	0.2			
		Song Phi Nong	3,669.01	0.5			
		Bang Len	4,784.73	0.5			
	15	Suphan Buri	2,979.19	0.2	10,589.06	51,135	61,724.06
		Nakhon Pathom	10,831.21	0.451			
		Bang Pla Ma	2,970.98	0.2			
		Song Phi Nong	3,669.01	0.5			
		Nakhon Chai Si	5,359.30	0.5			
	16	Suphan Buri	2,979.19	0.443	13,619.48	47,583	61,202.48
		Nakhon Pathom	10,831.21	0.5			
		Bang Pla Ma	2,970.98	0.2			
		Song Phi Nong	3,669.01	0.5			
		Sam Phran	8,910.79	0.5			
	17	Suphan Buri	2,979.19	0.5	14,503.96	46,626	61,129.96
		Nakhon Pathom	10,831.21	0.5			
		Bang Pla Ma	2,970.98	0.233			
		Song Phi Nong	3,669.01	0.5			
		Bang Len	4,784.73	0.5			
		Nakhon Chai Si	5,359.30	0.5			
	18	Suphan Buri	2,979.19	0.5	16,279.7	43,074	59,353.7
		Nakhon Pathom	10,831.21	0.5			
		Bang Pla Ma	2,970.98	0.233			
		Song Phi Nong	3,669.01	0.5			
		Bang Len	4,784.73	0.5			
		Sam Phran	8,910.79	0.5			

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