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THESIS

DEVELOPMENT OF PHYTOPLANKTON AND NUTRIENT DISPERSION MODELS FOR SONGKHLA LAKE

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A Thesis Submitted in Partial Fulfillment of
the Requirements for the Degree of
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In this study, the analysis of water quality was conducted at 16 stations in canals drained from the Songkhla Lake Basin and 25 stations inside the songkhla Lake. It was found that inorganic nutrient loading from various sub-basins were higher than organic nutrients. The result obtained from water sample collected at the Songkhla Lake was founded that most nutrients were at highest concentrations at the Lower Lake which received maximum waste loading from communities located in the nearby sub-basin.

The mathematical models were developed to simulate dispersion of phytoplankton and essential nutrients, including organic nitrogen, ammonia nitrogen, nitrite nitrogen, nitrate nitrogen, organic phosphorus, and inorganic phosphorus. Two-dimensional vertically averaged mass balance equations are basic governing equations for the dispersion models. The finite element method with Galerkin's weighted residual technique was used to solve these mass balance equations.

The developed models were applied to the Songkhla Lake. The results obtained from model computation showed that phytoplankton population was at higher concentration in the Upper Lake and the Middle Lake where water depth was rather shallow (about 1-2 m.), while concentrations of various forms of nitrogen and phosphorus in the Lower Lake were higher due to heavier loads from the cities located in its catchment area. These results were found to agree with data from field sampling and analyses.

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Student's signature Thesis Advisor's signature

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Sirirat Suwanidcharoen
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LIST OF ABBREVIATIONS

Chl-a	=	Chlorophyll-a
D_j	=	Depth of Segment j to Segment i
FEM	=	Finite Element Method
F(I)	=	Effect of Light Intensity
F(NU)	=	Nutrient Limitation
F(T)	=	Effect of Water Temperature
Gr_{PhyN}	=	Maximum Growth Rate of Phytoplankton Nitrogen
I_0	=	Incident Light Intensity at Water Surface
I_s	=	Optimum Light Intensity
IP	=	Inorganic Phosphorus
k_{mN}	=	Michaelis-Menton constant for Nitrogen
k_{mNH3}	=	Michaelis-Menton constant for Ammonia Nitrogen
k_{mNO3}	=	Michaelis-Menton constant for Nitrate Nitrogen
k_{mP}	=	Michaelis-Menton constant for Phosphorus
kT	=	Temperature Coefficient
M_{ON}	=	Mineralization/Hydrolysis Rate of Organic Nitrogen to ammonia
Mr_{NH4}	=	Oxidation Rate for Ammonia Nitrogen to Nitrite Nitrogen
Mr_{NO2}	=	Oxidation Rate for Nitrite Nitrogen to Nitrate Nitrogen
NH4	=	Ammonia Nitrogen
NO2	=	Nitrite Nitrogen
NO3	=	Nitrate Nitrogen
ON	=	Organic Nitrogen
OP	=	Organic Phosphorus
PhyN	=	Phytoplankton Nitrogen
Rr_{PhyN}	=	Respiration Rate of Phytoplankton Nitrogen
Sal	=	Salinity
SKL	=	Songkhla Lake Basin
S_{PhyN}	=	Settlling Rate of Phytoplankton Nitrogen

DEVELOPMENT OF PHYTOPLANKTON AND NUTRIENT DISPERSION MODELS FOR SONGKHLA LAKE

INTRODUCTION

The Songkhla Lake is a large natural water resource which has a diversity ecosystems, consisting of fresh water, brackish water and saline water ecosystems. The watershed area covers 8,754 km^2, whereas the lake area is about 1,042 km^2. Disposal of organic pollutants from various activities around the lake have affected water quality of the lake. One severe problem is the occurrence of eutrophication which adversely affected water quality and ecological balance in the lake. (La-ongsiriwong, 2004; Sompongchaiyakul, 2004)

One of the tools used for providing necessary information for effective eutrophication restoration is the development of water quality model. Most of water quality models were developed to clarify the effects of various factors on eutrophication, especially the influence of nutrient loading on phytoplankton growth (Chao *et al*., 2006; Gonanone, 2004; Sarawuth *et al*., 2008; Umgiesser *et al*., 2003). Some studies developed models for predicting the effects of salinity on plankton community (Dube *et al*., 2010; Ferreira *et al*., 2005; Gasiūnaitè *et al*., 2005). Some researchers also considered the effects of salinity on biochemical processes such as nitrification rate (Zheng *et al*., 2004).

In this study, two-dimensional models were developed to simulate dispersion patterns of phytoplankton and some essential nutrients, i.e., nitrogen and phosphorus, in the Songkhla Lake. The finite element method is used in solving the two-dimensional mass balance equations, with data on current velocities and water depths at various nodes identified in the lake are obtained from a hydrodynamic model. In order to cope with the effects of salinity, the phytoplankton growth rate is described in terms of salinity by using an expression in the form of the Monod's equation. These models were developed for better understanding on phytoplankton and nutrients dispersions in the diverse ecosystems of the Songkhla Lake.

OBJECTIVES

1. To develop two dimensional water quality model to simulate the temporal and spatial dispersion of phytoplankton nitrogen. The parameters to be considered includes phytoplankton nitrogen (PhyN), organic nitrogen (ON), ammonia nitrogen (NH_4^+), nitrite nitrogen (NO_2^-), nitrate nitrogen (NO_3^-), organic phosphorus (OP) and dissolved inorganic phosphorus (PO_4^{-3}).

2. To apply the developed model for prediction of phytoplankton and nutrient dispersion in the Songkhla Lake.

LITERATURE REVIEW

1. Factor influencing phytoplankton growth

The growth of phytoplankton is a complex process, involving interaction between physiological processes (photosynthesis, respiration, etc.), physical variables (light intensity, water temperature), chemical variables (macro- and micro-nutrients), and ecological factors (competition, grazing). As such those environmental parameters could influence aquatic plant photosynthesis respiration and mortality.

1.1 Effect of light intensity on phytoplankton growth

The effect of light on phytoplankton growth is complicated by the fact that several factors have to be integrated to come up with the total effect. These factors are diurnal surface-light variation, light attenuation with depth, and effect of light intensity on the growth rate.

1.1.1 Diurnal of light intensity variation

The temporal variation in light intensity can be characterized by a half-sinusoid wave and can be computed as average light intensity in two cases; (Chapra, 1997)

1) The average daily light intensity

$$I = I_m \left(\frac{2f}{\pi} \right) \quad (1)$$

where I = average daily light intensity (average over day)

I_m = maximum light intensity

f = fraction of day subject to sunlight (photoperiod)

2) The average light intensity over the daylight hours. (during the shining period)

$$I_a = I_m \left(\frac{2}{\pi} \right) \tag{2}$$

where I_a = average daylight intensity

I_m = maximum light intensity

1.1.2 Light attenuation with depth

The spatial variation of light intensity down through the water column can be modeled by the Beer-Lambert law. The intensity at any depth (z) measured from water surface can be expressed as; (Chapra, 1997)

$$I(z) = I_o e^{-k_e z} \tag{3}$$

where I_o = light intensity at water surface

k_e = light extinction coefficient

The light extinction coefficient, k_e, is the rate at which light is attenuated per unit water depth. The values of light extinction coefficient can be express as the function of water attenuation, planktonic chlorophyll concentration, and macrophyte biomss (Herb and Stefan, 2003).

$$k_e = k_{ew} + k_{eChl} + k_{eM} \tag{4}$$

where k_{ew} = light extinction coefficient for clear water

k_{eChl} = specific phytoplankton attenuation coefficient; expressed as a function of phytoplankton concentration measured as μ g

Chlorophyll *a* per liter (P_{Chl}) (Riley, 1956);

$$= 0.0088\ P_{Chl} + 0.054\ P_{Chl}^{\ 2/3}$$

k_{eM} = specific macrophyte attenuation coefficient, (m^2/g DW)

The light extinction coefficient due to specific macrophyte attenuation of 6 species macrophytes reported by Herb and Stefan, 2003, are in the range of 0.01 – 0.02 m^2/ g DW, as shown in Table 1.

Table 1 Light extinction coefficient due to 6 species of macrophyte

Species of macrophyte	k_{eM} (m^2 / g dry weight)
Potamogeton pectinatus L.	0.02
Potamogeton praelongus W.	0.01
Myriophyllum specicatum L.	0.006
Hydrilla verticillata R.	0.01
Ceratophyllum demersum L.	0.016
Elodea Canadensis M.	0.018

Source: Herb and Stefan (2003)

1.1.3 Effect of light intensity on phytoplankton growth rate

There are several possible formulations for estimating light intensity effect on photosynthesis. The commonly available functions are the Michaelis-Menton function, the hyperbolic tangent function, and Steel's light function. Among these functions, the two former have been used to describe photosynthesis-irradiance relationship in linearly proportional to light intensity at low light levels and constant at high levels, as the absence of photo inhibition (Carre *et al*, 1997; Chapra, 1997; Herb and Stefan, 2003). While the latter function proposed by Steele (1965), has been developed to fit light dependence in accordance with the presence of photo inhibition effect on growth at high light level, i.e. photosynthesis rates increase with increasing levels of light to the maximum level and the light level above that limit, the photosynthesis rate decreases, (Chapra, 1997; Muhammetoğlu and Soyupak, 2000).

The expressions of available functions are given below;

1) The Michaelis-Menton function:

$$P = P_{max} \frac{PAR}{K_m + PAR} \tag{5}$$

where P = gross photosynthesis rates

P_{max} = maximum gross photosynthesis rates

K_m = half-saturation constant for light

2) The hyperbolic tangent function:

$$P = P_{max} \tanh(PAR / I_k) \tag{6}$$

where I_k = light-saturation threshold for photosynthesis

3) Steele's light function:

$$F(I) = \frac{I}{I_s} \exp\left(-\frac{I}{I_s} + 1\right) \tag{7}$$

where; I = light limitation factor at a defined depth

I_s = optimum light intensity for growth

The Steele's function can be applied to compute the mean light imitation for a well-mixed layer. By substitution Eq. 3 into Eq. 7, an equation for the light dependent fraction at various depth, z, is obtained, i.e.

$$F(I) = \frac{I_a e^{-k_e z}}{I_s} \exp\left(-\frac{I_a e^{-k_e z}}{I_s} + 1\right) \qquad (8)$$

This function can be integrated over depth and time to develop the mean value, i.e.

$$\overline{F}(I) = \frac{1}{H}\int_0^H \frac{1}{T_p}\int_0^{fT_p} \frac{I_a e^{-k_e z}}{I_s} e^{-\frac{I_a e^{-k_e z}}{I_s} + 1} dt.dz \qquad (9)$$

The obtained result is;

$$\overline{F}(I) = \frac{2.718 f}{k_e H}\left[\exp\left\{\frac{I_a}{I_s} e^{-k_e H}\right\} - \exp\left\{-\frac{I_a}{I_s}\right\}\right] \qquad (10)$$

The factor $\overline{F}(I)$ in Eq. 10 is used to characterize the growth and nutrient uptake rates when the steady state condition is considered (Liengcharernsit, 1979; Chapra,1997).

In this study, the Steele's light function is used to represent the light intensity effect on photosynthesis, and the expression in term of average function, Eq. 10, is used in phytoplankton growth model.

1.2 Effect of water temperature on phytoplankton growth

Water temperature is one of the most important factors influencing the growth of phytoplankton by controlling the rate of chemical and biological reactions occurring in water. Most biological processes function have a maximum rate at some optimal temperature or range of temperatures, with declining rates as temperature departs from the optimum (Carr *et al*, 1997). The mathematical expressions representing the effect of temperature on phytoplankton growth have been proposed in several forms, eq. linear model, Arrhenius equation, Q_{10} equation, and formulation based on the normal or bell-shaped distribution.

linear model The simplest formulation is a linear model with some minimum temperature below which growth does not occur (Chapra, 1997);

$$k_T = 0 \qquad T \le T_{\min}$$

$$k_T = k_{opt} \frac{T - T_{\min}}{T_{opt} - T_{\min}} \qquad T_{\min} \le T \le T_{opt}$$

$$k_T = k_{opt} \frac{T_{\max} - T}{T_{\max} - T_{opt}} \qquad T > T_{opt} \tag{11}$$

where k_T = growth rate (day $^{-1}$) at temperature T (^{o}C)

k_{opt} = growth rate (day $^{-1}$) at the optimal temperature T_{opt} (^{o}C)

$T_{\min}$ = minimum temperature, which growth ceases

$T_{\max}$ = maximum temperature, which growth rate decreases

Arrhenius equation One of the available formulation used to quantify the temperature dependence of aquatic plant growth is proposed by Goldman and Carpenter which expresses metabolic responses to temperature as Arrhenius equation (Chapra, 1997).

$$k(T_a) = Ae^{-\frac{E}{RT_a}} \tag{12}$$

where T_a = absolute temperature (K)

A = pre exponential or frequency factor

E = activation energy (J mole^{-1})

R = gas constant (3.314 J mole $^{-1}$ K^{-1})

Equation 12 is often used to compare the reaction rate constant at two different temperatures. This can be done by expressing the ratio of the rates as:

$$\frac{k(T_{a2})}{k(T_{a1})} = e^{\frac{E(T_{a2} - T_{a1})}{RT_{a2}T_{a1}}} \tag{13}$$

Equation 13 can be simplified by defining the following term as a constant;

$$\theta \equiv e^{\frac{E}{RT_{a2}T_{a1}}}$$

and can be expressed as;

$$\frac{k(T_2)}{k(T_1)} = \theta^{T_2 - T_1}$$

or

$$k(T_2) = k(T_1)\theta^{T_2 - T_1} \tag{14}$$

where the temperature is expressed in ^{O}C

In water-quality modeling, many reactions are reported at 20 ^{O}C (Chapra, 1997). Therefore, Eq.14 is usually expressed as

$$k(T) = k(20\,^{o}C)\theta^{T - 20} \tag{15}$$

Q_{10} equation The temperature dependence of biological mediated reactions is often expressed in terms of temperature coefficient, Q_{10}, which estimates that biological rates are double for every 10 ^{O}C increase in temperature. This

temperature formulation is used to capture the temperature dependence within a moderate range (Carr *et al*, 1997). The quantity Q_{10} is specified as:

$$Q_{10} = \left(\frac{k(T_2)}{k(T_1)} \right)^{\frac{10}{T_2 - T_1}}$$

$$k(T_2) = k(T_1).Q_{10}^{(T_2 - T_1)/10} \qquad (16)$$

normal or bell-shaped distribution The normal or bell-shaped distribution has been proposed to represent a temperature dependence of aquatic plant growth rate; that is zero at a minimum temperature, increases to a peak growth rate at an optimal temperature, and then decreases at higher temperatures. Several investigators have suggested various functions to fit such a shape smoothly.

Lammana proposed an expression for temperature effect on algal production as follows (Liengcharernsit, 1979):

$$R_T = R_{opt} \left(\frac{T}{T_{opt}} \right)^n \exp \left[1 - \left(\frac{T}{T_{opt}} \right)^n \right] \qquad 0 < T < T_{opt}$$

$$= R_{opt} \left[1 - \left(\frac{T - T_{opt}}{T_{max} - T_{opt}} \right)^m \right] \qquad T_{opt} < T < T_{max} \qquad (17)$$

where R_T = production rate at temperature T

R_{opt} = production rate at optimal temperature, T_{opt}

T_{max} = maximum possible temperature at which the production can occur.

m and n = constants depending on species and some other environmental conditions. The typical values of m and n are 2.0 and 2.5, respectively.

Cerco and Cole proposed the expression for temperature effect as the following formulation (Chapra, 1997);

$$k_T = k_{opt} \exp\left[-k_1 (T - T_{opt})^2\right] \qquad T \leq T_{opt}$$

$$k_T = k_{opt} \cdot \exp\left[-k_2 (T_{opt} - T)^2\right] \qquad T > T_{opt} \tag{18}$$

where k_1 and k_2 are parameters that determine the shape of the relationship of growth to temperature below and above the optimal temperature, respectively.

In this study, the Arrehenius formulation as expresses in Eq. 15 is used to represent the effect of temperature on phytoplankton growth.

1.3 Effect of nutrients limitation on phytoplankton growth

The general approach to model phytoplankton productivity with respect to nutrients uses the Michaelis-Menton formulation which consider amount of nitrogen and phosphorus from both the water-body and bottom sediments (Carr *et al*., 1997). The Michaelis-Menton formulation is the empirical equation describing the relationship between the growth rate and the concentration of substrate as follow;

$$\mu = \mu_{\max} \frac{S}{k_s + S} \tag{19}$$

where μ = specific growth rate (day^{-1})

$\mu_{\max}$ = maximum specific growth rate (day^{-1})

S = limiting nutrient concentration (mg/l)

k_s = Michaelis-Menton constant / a half-saturation constant

The above expression, Eq.19, is obtained when only one limiting nutrient is considered while all other nutrients are in excessive amount. The half-saturation constant may vary depending on the form of the nutrient that is limiting (e.g., ammonium, nitrate and total inorganic nitrogen for nitrogen; soluble reactive phosphorus for phosphorus).

Multiple nutrients In case of more than one nutrient are limited, there are several ways in which the nutrient limitation term can be refined. The general approach has been suggested to determine the combined effect of the multiple nutrients comprises of minimum, summation, multiplicative, and harmonic mean.

1) Minimum: According to *Liebig's law of the minimum,* the nutrient in shortest supply controls growth. This approach is the most commonly accepted formulation (Chapra, 1997). The expression in minimum approach is written as;

$$\mu = \min \{\phi_p , \phi_n\} \qquad (20)$$

where ϕ_n = attenuation for nitrogen limitation

$$= \frac{n}{k_{sn} + n}$$

ϕ_p = attenuation for phosphorus limitation

$$= \frac{p}{k_{sp} + p}$$

2) Summation: Most species of phytoplankton can utilize inorganic nitrogen in both forms of ammonia nitrogen and nitrate nitrogen. On the lack of one form, another form can be used as source of nitrogen (Liengcharernsit, 1979). Several investigators have considered the summation of ammonia and nitrate nitrogen as the limiting nutrient (Liengcharernsit, 1979; Plus *et al*., 2003; Giusti and Libelli, 2005). This approach can be expressed as;

$$\mu = \mu_{\max} \frac{[NH_4^+] + [NO_3^-]}{k_{s,N} + [NH_4^+] + [NO_3^-]} \tag{21}$$

where $k_{s,N}$ = Michaelis-Menton constant for growth limited by inorganic nitrogen

$[NH_4^+]$ = ammonia nitrogen concentration

$[NO_3^-]$ = nitrate nitrogen concentration

3) Multiplicative: In this approach, it is assumed that the nutrients have a synergistic effect (Chapra, 1997). Then the two limitation terms are multiplied, as

$$\phi_{Nu} = \phi_n . \phi_p \tag{22}$$

where ϕ_{Nu} = attenuation for multiple nutrients limitation

ϕ_n = attenuation for nitrogen limitation

$= \frac{n}{k_{sn} + n}$

ϕ_p = attenuation for phosphorus limitation

$= \frac{p}{k_{sp} + p}$

4) Harmonic mean: This formulation was developed to allow some interaction among multiple limiting nutrients while not being as serve as the multiplicative approach (Chapra, 1997). The reciprocals of the limitation terms are combined as

$$\phi_{Nu} = \frac{m}{\sum_{j=1}^{m} \frac{1}{\phi_j}} \tag{23}$$

In this study, ammonia nitrogen, nitrate nitrogen, and dissolved inorganic phosphorus are considered as limiting nutrients. To combine the effect of nitrogen and phosphorus, the specific growth rate limited by nitrogen and phosphorus is described by the minimum approach as in Eq. 20. While the summation formulation, Eq. 21, is used to represent the growth rate limited by inorganic nitrogen. Thus, the effect of limiting nutrients (both nitrogen and phosphorus) on specific growth rate of phytoplankton is expresses by the following equation;

$$\mu = \mu_{max} \left\{ \min \frac{\left[NH_4^+\right] + \left[NO_3^-\right]}{k_{s,N} + \left[NH_4^+\right] + \left[NO_3^-\right]} \right\}, \left\{ \frac{\left[PO_4^-\right]}{k_{s,P} + \left[PO_4^-\right]} \right\} \qquad (24)$$

1.4 Effect of temperature on phytoplankton respiration

Respiration is the most important process that should be accounted for by a non-predator mortality term. Respiration rate in aquatic plants is influenced by several environmental parameters, including water temperature, dissolved oxygen, and tissue nutrient concentrations (Carr *et al*, 1997).

The most often used to modify respiration rates in phytoplankton productivity models is water temperature. Estimate of maximum specific respiration or respiration at a standard temperature (usually 20^{O}C) are used as a point of reference for temperature modified rate (Carr *et al*., 1997).

Plus *et al.* (2003) has suggested that respiration rates of several algal species decrease when the concentration of dissolved oxygen is low. Therefore, a limitation function for the respiration has been added to the model. The calculation of phytoplankton respiration is proposed as

$$\mathrm{Resp} = \left\{ \mathrm{Resp}_0 \times \theta_r^{(T - T_b)} \right\} \times \left\{ \frac{[O_2]}{k_{sO} + [O_2]} \right\} \times B \qquad (25)$$

where $Resp_0$ = base respiration rate

θ_r = 1.072

T_b = base temperature for nominal growth rate

B = phytoplankton biomass

In this study, the relation of temperature on phytoplankton respiration rate as a function of Arrhenius equation proposed by Muhammetoğlu and Soyuk (2000) is used in phytoplankton growth model.

1.5 Effect of temperature on phytoplankton mortality rate

Phytoplankton mortality rate in this study has been defined as a cumulative loss term, which includes mortality and excretion (non-predator mortality). Several investigators have proposed formulation to quantify mortality rate in various equations. Most of them calculated phytoplankton mortality rate as a function of temperature as follow.

Muhammetoğlu and Soyupak (2000) have proposed mortality rate formulation based on Arrhenius equation, in the same approach to quantify photosynthesis and respiration rate. It has been applied to simulate the growth of two species, i.e. *Najas marina* L. and *Potamogeton pectinatus* L.

$$M = \left[M_0 \times \theta_m^{(T - T_b)}\right] + \left[M_E \times \theta_E^{(T - T_b)}\right] \tag{26}$$

where M_0 = mortality rate (= 0.03 day^{-1})

θ_m = temperature coefficient for mortality rate =1.05

M_E = excretion rate (non-predator mortality rate)

= 0.012 day^{-1}

θ_E = temperature coefficient for excretion rate (=1.05)

T_b = base temperature for nominal mortality rate (=20^{O} C)

Giusti and Liblli (2005) has also proposed mortality rate relation to temperature dependence. The proposed formulation is expressed as exponential form, as follow;

$$\Omega_R = SR \times \left(0.098 + e^{-6.59 + 0.2217\,(T)}\right) \qquad (27)$$

where Ω_R = decaying rate

SR = mortality rate

= 0.041 day^{-1} for *Ruppia maritime*

Plus *et al*. (2003) has suggested the mortality formulation depending on production and respiration processes. The expression was derived from respiration with the following equation:

$$M = 0.027 \times \frac{R}{PQ \times CN} \qquad (28)$$

where M = mortality rate (mol N m^{-3} d^{-1})

R = respiration rate (g O_2 m^{-3} d^{-1})

PQ = photosynthetic quotient (oxygen moles produced per mole of fixed carbon)

CN = annual men carbon-nitrogen ratio

The expression introduced by Asaeda and Van Bon (1997) are enlarged due to the increasing shortage of resources as the following formulation;

$$\gamma = r_{\max} \times \frac{k_s}{k_s + Ph - R + C_d . De} \qquad (29)$$

where γ = mortality rate (day^{-1})

$r_{\max}$ = maximum mortality rate (day^{-1})

k_s = half saturation constant for the mortality rate ($g\ m^{-2}\ d^{-1}$)

Ph = gross photosynthesis rate ($g\ m^{-2}\ d^{-1}$)

R = respiration rate ($g\ m^{-2}\ d^{-1}$)

C_d = fraction of the dead biomass used for growth

De = mortality rate of the previous day ($g\ m^{-2}\ d^{-1}$)

In this study, the formulation describing the effect of temperature on mortality rate and the temperature coefficient as expresses in Eq. 26 is used in phytoplankton model.

2. Finite Element Method

The Finite Element Method (FEM) is a numerical analysis technique for obtaining approximate solutions to a physical phenomenon which having complex domains subjected to general boundary conditions. As the name implies, the basis of FEM relies on the decomposition of the domain into a finite number of sub domains (elements) connected at specific node points. Within each element, the systematic approximate solution is constructed by applying the variational or weighted residual methods. The solution of entire domain is then obtained by assemblage the properties of these elements.

One of the major advantages of the finite element methods is the flexibility of the finite-element grid, which allows a close spatial approximation of irregular domains. In addition, the finite element method also has many advantages over most other numerical analysis methods, such as (Cook *et al.*, 2001);

- Boundary conditions and loading are not restricted.
- Components that have different behaviors and different mathematical descriptions can be combined.
- It is applicable to steady-state and time dependent problems.
- It is applicable to linear and nonlinear problems.

As many importance advantages, the finite element method has become an practical analysis tools for the design and modeling of a physical phenomenon in various engineering disciplines including water quality modeling.

2.1 General step of the finite element method

The finite element method always follows an orderly step-by-step process as follows.

2.1.1 Discretization: The problem domain is discretized into a collection of simple shapes, or elements.

2.1.2 Select Interpolation function: This step is to assign nodes to each element and then choose the interpolation function to represent the variation of the field variable over the element.

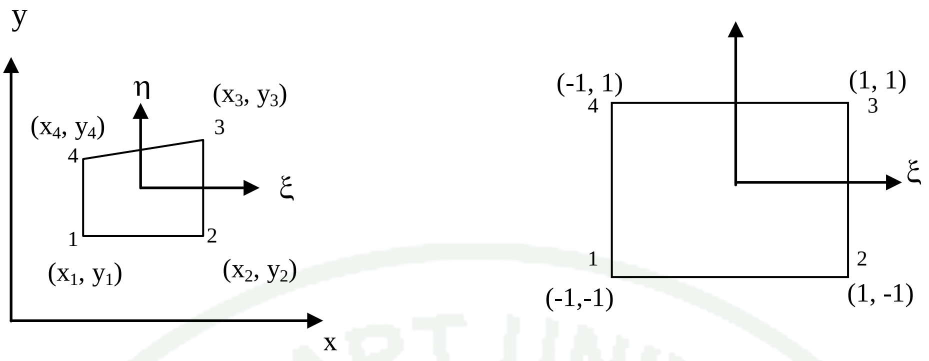
2.1.3 Assembly of the discrete elements: The element equations for each element in the FEM mesh are assembled into a set of global equations that model the properties of the entire systems

2.1.4 Application of boundary conditions: Solution cannot be obtained unless boundary conditions are applied. At this step known values of certain nodal variables are imposed into a set of the equations.

2.1.5 Find the solution of a set of simultaneous equation: In this final step, the unknown nodal values of the problem are computed using matrix algebra.

2.2 Iso parametric element: interpolation function

Iso parametric element is the element type which the value of coordinate and variable can be expressed in term of their nodal values with the same function. To simplify the obtained substance balance equation in iso parametric element form, so that the local coordinate (x, y) are transformed in natural coordinate (ξ, η). The length of 4 nodal in natural coordinate which obtained number as 1, 2, 3 and 4 are range from -1 to +1 both ξ axis and η axis as shown in Figure 1



local coordinate (global domain) natural coordinate (parent domain)

Figure 1 Transformation from local coordinate (x,y) to natural coordinate (ξ, η)

In the transformation between these two domains, it is required that one point in the parent domain should correspond to one point in the global domain and vice versa. To find this relationship, let x, y coordinates are the function of parameter a_i and b_i as follows.

$$x = a_1 + a_2\xi + a_3\eta + a_4\xi\eta \tag{30}$$

$$y = a_1 + a_2\xi + a_3\eta + a_4\xi\eta \tag{31}$$

From Eq. 30, substitution x_i (i = 1, 2, 3, 4) in the x-y coordinate system and ξ_i *and* η_i in natural coordinate system, and then solving for a_1, a_2, a_3 and a_4. The results are

$$a_1 = \frac{1}{4}\left(x_1 + x_2 + x_3 + x_4\right) \tag{32}$$

$$a_2 = \frac{1}{4}\left(-x_1 + x_2 + x_3 - x_4\right) \tag{33}$$

$$a_3 = \frac{1}{4}\left(-x_1 - x_2 + x_3 + x_4\right) \tag{34}$$

$$a_4 = \frac{1}{4}\left(x_1 - x_2 + x_3 - x_4\right) \tag{35}$$

Substitution of equation (32) through (35) and rearrangement to yield

$$x = \frac{1}{4}(\xi - 1)(\eta - 1)x_1 - \frac{1}{4}(\xi + 1)(\eta - 1)x_2 + \frac{1}{4}(\xi + 1)(\eta + 1)x_3 - \frac{1}{4}(\xi - 1)(\eta + 1)x_4 \qquad (36)$$

In a similar manner, the y coordinate can be written in terms of its nodal values as

$$y = \frac{1}{4}(\xi - 1)(\eta - 1)y_1 - \frac{1}{4}(\xi + 1)(\eta - 1)y_2 + \frac{1}{4}(\xi + 1)(\eta + 1)y_3 - \frac{1}{4}(\xi - 1)(\eta + 1)y_4 \qquad (37)$$

As a result, coordinate of x and y can be expressed in terms of their nodal values as

$$x = \sum_{i=1}^{4} \phi_i^e x_i^e = \phi^{eT} x^e \qquad (38)$$

$$y = \sum_{i=1}^{4} \phi_i^e y_i^e = \phi^{eT} y^e \qquad (39)$$

Where x^e, y^e are matrix of x_i, y_i respectively, and matrix of interpolation function ϕ^e for bilinear quadrilateral elements in terms of natural coordinates involves

$$\begin{aligned} \phi_1 &= \frac{1}{4}(\xi - 1)(\eta - 1) \\ \phi_2 &= -\frac{1}{4}(\xi + 1)(\eta - 1) \\ \phi_3 &= \frac{1}{4}(\xi + 1)(\eta + 1) \\ \phi_4 &= -\frac{1}{4}(\xi - 1)(\eta + 1) \end{aligned} \qquad (40)$$

Derivatives of a variable *u* expressed in terms of its nodal value matrix $\underline{u}$ can be expressed as:

$$\begin{Bmatrix} \dfrac{\partial u}{\partial \xi} \\ \dfrac{\partial u}{\partial \eta} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \phi^T}{\partial \xi} \\ \dfrac{\partial \phi^T}{\partial \eta} \end{Bmatrix} \underline{u} \qquad (41)$$

which can be expanded to:

$$\begin{Bmatrix} \dfrac{\partial u}{\partial \xi} \\ \dfrac{\partial u}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial \phi_1}{\partial \xi} & \dfrac{\partial \phi_2}{\partial \xi} & \dfrac{\partial \phi_3}{\partial \xi} & \dfrac{\partial \phi_4}{\partial \xi} \\ \dfrac{\partial \phi_1}{\partial \eta} & \dfrac{\partial \phi_2}{\partial \eta} & \dfrac{\partial \phi_3}{\partial \eta} & \dfrac{\partial \phi_4}{\partial \eta} \end{bmatrix} \begin{Bmatrix} u_{n1} \\ u_{n2} \\ u_{n3} \\ u_{n4} \end{Bmatrix} \qquad (42)$$

$$= \begin{bmatrix} -\frac{1}{4}(1-\eta) & \frac{1}{4}(1-\eta) & \frac{1}{4}(1+\eta) & -\frac{1}{4}(1+\eta) \\ -\frac{1}{4}(1-\xi) & -\frac{1}{4}(1+\xi) & \frac{1}{4}(1+\xi) & \frac{1}{4}(1-\xi) \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$= \underline{B}^* \underline{u}^n$$

Coordinate x, y can be written in term of curvilinear (local coordinate) as follows

$$x = x(\xi, \eta) \qquad (43)$$

$$y = y(\xi, \eta) \qquad (44)$$

The derivative in cartesian coordinate of element matrix equation has to be transformed between local coordinates and cartesian coordinates which the chain rule is applied to yield

$$\frac{\partial \phi}{\partial \xi} = \frac{\partial \phi}{\partial x} \cdot \frac{\partial x}{\partial \xi} + \frac{\partial \phi}{\partial y} \cdot \frac{\partial y}{\partial \xi} \qquad (45)$$

$$\frac{\partial \phi}{\partial \eta} = \frac{\partial \phi}{\partial x} \cdot \frac{\partial x}{\partial \eta} + \frac{\partial \phi}{\partial y} \cdot \frac{\partial y}{\partial \eta} \tag{46}$$

The value of ξ, η can be computed from the derivative of ϕ in the function of x and y as

$$\begin{Bmatrix} \frac{\partial \phi}{\partial \xi} \\ \frac{\partial \phi}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \frac{\partial \phi}{\partial x} \\ \frac{\partial \phi}{\partial y} \end{Bmatrix} = J \begin{Bmatrix} \frac{\partial \phi}{\partial x} \\ \frac{\partial \phi}{\partial y} \end{Bmatrix} \tag{47}$$

which can be obtained as

$$\begin{Bmatrix} \frac{\partial \phi}{\partial x} \\ \frac{\partial \phi}{\partial y} \end{Bmatrix} = J^{-1} \begin{Bmatrix} \frac{\partial \phi}{\partial \xi} \\ \frac{\partial \phi}{\partial \eta} \end{Bmatrix} \tag{48}$$

where

$$\frac{\partial \phi}{\partial x} = \frac{1}{|J|} \left(\frac{\partial y}{\partial \eta} \frac{\partial \phi}{\partial \xi} - \frac{\partial y}{\partial \xi} \frac{\partial \phi}{\partial \eta} \right) \tag{49}$$

$$\frac{\partial \phi}{\partial y} = \frac{1}{|J|} \left(- \frac{\partial x}{\partial \eta} \frac{\partial \phi}{\partial \xi} + \frac{\partial x}{\partial \xi} \frac{\partial \phi}{\partial \eta} \right) \tag{50}$$

Derivative of $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$ can be computed from

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{Bmatrix} = J^{-1} \begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{Bmatrix} = J^{-1} \underline{B}^{*} \begin{Bmatrix} u_{n1} \\ u_{n2} \\ u_{n3} \\ u_{n4} \end{Bmatrix} \tag{51}$$

J (Jacobian matrix) can be computed from

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \underline{B}^{*} \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix} = \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix} \quad (52)$$

where $|J|$ is determinant of J which can be expressed as

$$|J| = \frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta} - \frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \xi} \quad (53)$$

The value of differential area is written in the term of $d\xi,\ d\eta$ as

$$dA = (\text{absolute value of } |J|)\ d\xi\ d\eta \quad (54)$$

3. Previous Study

A number of mathematical models have been developed to simulate the dispersion of water quality. In general, these models are complex and need field data for both initial concentrations and boundary condition which vary from point to point throughout the irregular geography. As a practical tool to deal with these constraints, the finite element method has gained acceptance in previous studies as follows. Liengcharernsit (1979) used finite element method with Galerkin weighted residual method in the formulation of 2-dimentional hydrodynamic model and dispersion model. The developed models were applied to some uniform areas which analytical solutions are available. The results obtained from the verifications are very satisfactory. The finite element method (triangle element) with Galerkin's weighted residual techniques was also used in several studies to develop the dispersion water quality models. The calibration of those developed models shown very satisfactory results (Banjongraksa, 2004; Gonanone, 2004; Sodsai, 2004). In addition, Umgiesser *et al*. (2003) set up a Finite Element Ecological Model (FEEM) by fully coupling a primitive equation finite element hydrodynamic model (FEM) to an ecological submodel contained in the water quality model WASP. It presents some advantages in respect with WASP, since the transport phenomena are resolved by means of a finite element primitive equation model, which is a parameterization that is much more flexible and powerful of those adopted in the hydrodynamic module of the WASP SYSTEM, both in term of spatial resolution and capability to cope with complex dynamics.

In order to understand the dispersion of phytoplankton in the lake, most of previous studies were developed mathematical models to clarify the effects of various factors on phytoplankton community (Chao *et al*., 2006; Gonanone, 2004; Chesoh *et al*., 2008; Umgiesser *et al*., 2003). One of the most importance factors influencing phytoplankton growth rate is nutrient loading (Saiyachot, 2001; Wattanamongkol, 2001). While phytoplankton dynamic in brackish water bodies depend on salinity (Dube *et al*., 2010; Ferreira *et al*., 2005; Gasiūnaitė *et al*., 2005). Some studies also considered the effects of salinity on biochemical processes such as nitrification rate (Zheng *et al*., 2004).

4. The Songkhla Lake

The Songkhla Lake is a large natural water resource which located along the eastern side of the southern Thai Peninsula. The Songkhla Lake Basin is located between latitude 6^{o} 27′ N to 8^{o} N, and longitude 99^{o} 44′ E to 100^{o} 41′ E. This basin encompasses three provinces, namely Phatthalung, Nakhon Si Thammarat, and Songkhla. The dimension of the lake has the widest part from east to west about 65 km. and the longest part from north to south about 150 km. The area of the Songkhla Lake Basin is 8,754 km^2 with water surface area of 1,042 km^2 (approximately 10 % of the whole area), (Sompongchaiyakul, 2004).

4.1 Characteristic of the Songkhla Lake

The Songkhla Lake connects to the Gulf of Thailand with the narrow strait at the south end of the lake. Due to the influence from salinity intrusion in relatively dry season, the Songkhla Lake has a diversity of ecosystems, which are fresh water, brackish water and salinity water. The Songkhla Lake can be divided into 3 main parts from north to south (or from inner/upper part to outer/lower part) which is inter-connected by a narrow chanel; that are:

4.1.1 *The Upper Lake (Thale Luang)*: This is the part next to Thale Noi, through Krasae Sin district, Songkhla. It is the largest part which has an area of 491 km^2. The average depth is 1.9 m. This part is a fresh water ecosystem throughout the year, but sometimes the salinity may increase due to the intrusion of sea water in dry season. The salinity of this part is in the range of 0 –11 psu.

4.1.2 *The Middle Lake (Thale Sap)*: This part is next to the Upper Lake. It is located between Pak Phayun district, Phatthalung and Krasae Sin District, Songkhla. The surface area is 336 km^2. It is the shallowest part of the lake which average depth is 1.1 m. There are many islands in this part, namely Ko Si, Ko Ha, Ko Mak and Ko Nang Kham. It is connected to the Lower Lake by Pak Ror Channel named Khlong Laung which has 7-8 m in depth. This part consists of the fresh water and brackish water ecosystems. The salinity is in the range of 0 – 32 psu.

4.1.3 *The Lower Lake (Thalesap Songkhla)*: This part is the lower or the outer part of the Songkhla Lake. The border of this part starts from Ban Pak Ro, Singhanakhon district, Songkhla, to its southern end which is connected to the Gulf of Thailand. The average depth is 1.5 m, except the strait area for navigation (depth > 7 m.). The salinity of this part is in the range of 0 – 33 psu which is the highest level and more variation than other part. It is the mixing between brackish water and saline water. Fresh water is found only short duration in the rainy season (Dec – Jan). There are many fishing gears placed in this part.

The main characteristics of the Songkhla Lake are summarized in Table 2.

Table 2 Characteristic of the Songkhla Lake.

Characteristic	Upper Lake	Middle Lake	Lower Lake
Surface area (km^2)	491	336	190
Volume (m^3)	$9.32x10^8$	$3.69\ x10^8$	$2.85\ x10^8$
Mean depth (m.)	1.9	1.1	1.5
Residence time (day)	55	28	15
Salinity (psu)	0 – 11.1	0 - 32.0	0 – 33.0

Source : La-ongsiriwong *et al* (2004)

4.2 The Songkhla Lakes Basin

The sources of inflow water drained into the Songkhla Lake come from sub-basins surrounding the lake. The catchment area is approximately more than 7,000 km^2. The north of the Songkhla lake basin is a large swamp, called Phru Khuan Khreng, having a small fresh water lake, namely Thale Noi (28 km^2). The east of the Songkhla Lake Basin is the coastal plain along the Gulf of Thailand. The main water sources of Songkhla Lakes Basin come from Banthat and San Kala Khiri mountain ranges along the west from north to south. Next to the mountains is the rolling plane parallel to Banthat mountain range and the large plain surrounding the lake.

The Royal Irrigation Department has divided the Songkhla Lake Basin into 12 sub-basins consisting of 8 sub-basins related to main channels along the west and 4 sub-basins along the east coast (figure 2); that are:

4.2.1 *Pa Phayom Channel Sub-Basin:* This sub-basin covers 805 km^2. The main channel of this sub-basin is Pa Phayom channel which has 33 km in length from Banthat mountain range to Phru Khuan Khreng. Pa Phayom channel has the effluent channel flow into Tha Nae channel.

4.2.2 *Tha Nae Channel Sub-Basin:* Tha Nae sub-basin covers 353 km^2. The main channel of this sub-basin is Tha Nae channel which has 38 km. in length from Banthat mountain range to Thale Noi at Ban Pra Neao.

4.2.3 *Na Thom Channel Sub-Basin:* The catchment area is 747 km^2. The main channel of this sub-basin is Na Thom channel which has 42 km in length from Banthat mountain range to the Upper Lake (Thale Luang) at Ban Lum Pam.

4.2.4 *Tha Chead Channel Sub-Basin:* This sub-basin covers about 759 km^2. The main channel is Tha Chead channel which has 42 km. in length from Banthat mountain range to The Middle Lake (Thale Sap) at Ban Pak Phon.

4.2.5 *Pa Bon Channel Sub-Basin:* This sub-basin covers 323 km^2. The main channel is Pa Bon channel which has 40 km. in length from Banthat mountain range to the Middle Lake (Thale Sap) at Ban Pra Kred.

4.2.6 *Phru Phor Channel Sub-Basin:* The catchment area of this sub-basin is 500 km^2. The main channel is Phru Phor channel which has 36 km. in length from Banthat mountain range to the Lower Lake at Ban Ta Yhee.

4.2.7 *Rattaphum Channel Sub-Basin:* The catchment area covers 617 km^2. The main channel is Rattaphum channel which has 63 km. in length from Banthat mountain range to the Lower Lake (Thalesap Songkhla) at Ban Bang Huk.

4.2.8 *U-Taphao Channel Sub-Basin:* This is the largest sub-basin which covers 2,383 km^2. The main channel of this sub-basin is U-Taphao channel which has 130 km in length from San Kala Khiri mountain range to the Lower Lake at Ban Lam Pho, Hatyai district, Songkhla.

4.2.9 The East Coast Sub-Basin 1 (Ranode) : This sub-basin covers 488 km^2. There are many channels in this sub-basin; e.g. Kok channel, Ranode channel, Rong channel, and Ta Khrea channel.

4.2.10 *The East Coast Sub-Basin 2:* This sub-basin covers 202 km^2. The important channels in this sub-basin are Ku Khud channel, and Ree channel.

4.2.11 *The East Coast Sub-Basin 3:* The catchment area covers 136 km^2. The important channel is Sating Mhoa channel.

4.2.12 *The East Coast Sub-Basin 4:* The catchment area covers 204 km^2. There are many channels; e.g. Sam Rong channel and Pa Wong channel.

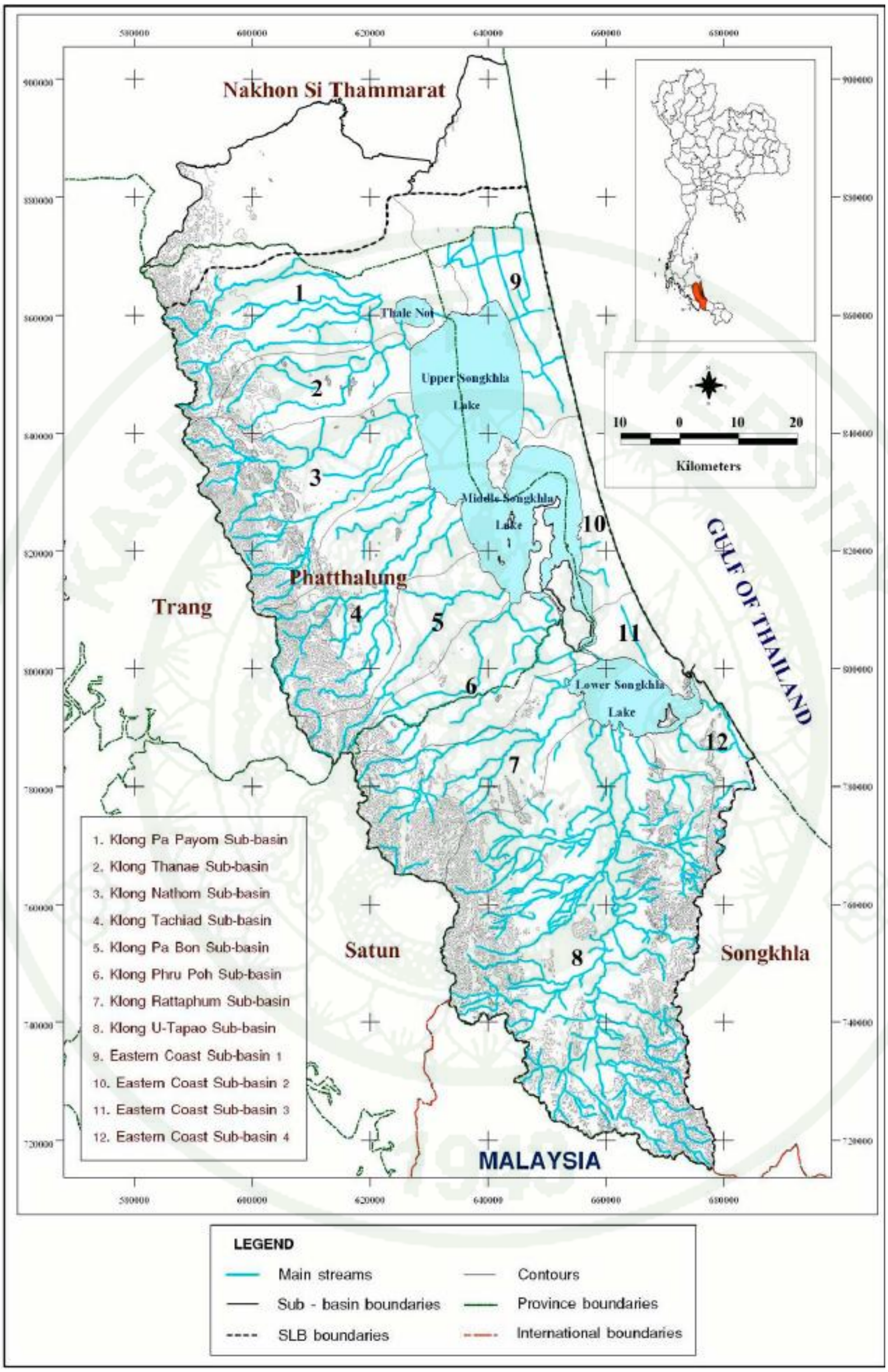


Figure 2 Channel and sub-basin of the Songkhla Lake Basin

Source: Sutiwipakorn and Ratanachai, 2004.

4.3 The Sources of Water Pollution in the Songkhla Lake Basin

The Songkhla Lake is the shallow lake system which has the mean depth of 2 m and has the hydraulic residence time in the range of 15 to 55 days. The disposed pollutants from various activities in the lake and sub-basins surrounding the lake have affected water quality of the Songkhla Lake. The dominant sources which cause severe pollution in the Songkhla Lake include municipal wastewater, industrial wastewater, swine farms, shrimp farms and non-point source such as agricultural runoff (Chaiprapat, *et al*, 2004).

4.3.1 Municipal wastewater: Large quantity of wastewater has been disposed into the Songkhla Lake, comprising untreated wastewater from Had Yai and Songkhla municipalities and some small communities surrounding the Songkhla Lake Basin. In addition, the municipal wastewater treatment plants of the two municipalities have been designed mainly for treatment of organic pollutants measured in terms of BOD_5. There is no treatment unit for nutrients, i.e. nitrogen and phosphorus, which are crucial factors leading to the occurrence of eutrophication.

According to Chaiprapat, *et al* (2004) the total quantity of municipal wastewater discharged from the Songkhla Lake Basin is approximately 105,423 m^3 / day, which has organic loading of 8,434 kg BOD_5 / day. Comparing among 12 sub-basins, the highest organic loading of 3,452 kg/day are discharged from U-Taphao Channel Sub-Basin, followed by the East Coast Sub-Basin 4 which has organic loading of 1,302 kg BOD_5 / day. Organic loading from both of two sub-basins are discharged into the Lower Lake.

4.3.2 Industrial wastewater: The types of industries which are the main industrial wastewater source discharged into the Songkhla Lake compose of seafood product, frozen sea food, rubber and rubber product. In terms of organic loading these can be divided into 3 groups. The first group consists of 7 factories with has organic loading more than 100 kg BOD_5/day. The second group consists of 16 factories with has organic loading in the range of 40 – 100 kg BOD_5/day. The last group is the remaining factories with has organic loading less than 40 kg BOD_5/day.

The total quantity of industrial wastewater discharged into the Songkhla Lake is 86,084 m^3/day with has total organic loading of 3,097 kg BOD_5/day. Comparing among 12 sub-basins, organic loading of industrial wastewater in each sub-basin is similar to organic loading of municipal wastewater. U-Taphao Channel Sub-basin has the highest organic loading of 2,336.5 kg BOD_5/day, followed by the East Coast Sub-Basin 1 and Rattaphum Channel Sub-Basin with has organic loading of 142.8 kg BOD_5/day and 130.4 kg BOD_5/day, respectively. Organic loading from U-Taphao Channel and Rattaphum Channel Sub-Basin are discharged into the Lower Lake, whereas organic loading from the East Coast Sub-Basin 1 is discharged into the Upper Lake.

4.3.3 Swine Farm: The quantity of wastewater from swine farm estimated by Chaiprapat, *et al* (2004) was about 667 m^3/day with has total organic loading of 1,179 kg BOD_5/day. The Lower Lake is the part that receives highest loading of wastewater from swine farm; about 279.2 kg BOD_5/day from U-Taphao Channel Sub-Basin and 167.9 kg BOD_5/day from Rattaphum Sub-Basin.

4.3.4 Shrimp Farm: Wastewater pollutants of shrimp farm come from 3 main sources, i.e. remaining of food feeding, the residual antibiotic and used chemical, and the excretion of shrimps. The shrimp farm area locates along the east coastal. However the number of operating always varies depending on the demand of international market. Because of the rapidly decreasing of export in 2002, about 10-30 % of the total shrimp farms are left without operating.

Referring to the report of Prince of Songkhla University (Chaiprapat, *et al*, 2004), the total area of shrimp farm is in the range of 8,800 to 11,300 rai (1 km^2 = 625 rai), which has organic loading of 1,625 to 2,090 kg BOD_5/day. Comparing among various part of the Songkhla Lake, the Lower Lake has the highest organic loading from shrimp farm which receiving 450 – 579 kg BOD_5/day from the East Coast Sub-Basin 3, 157 –202 kg BOD_5/day from the East Coast Sub-Basin 4, and 257 - 330 kg BOD_5/day from U-Taphao Channel Sub-Basin.

Besides shrimp farm, seabass aquaculture in floating basket is an important activity affecting water quality of the Lower Lake. The areas which have high aquaculture activity are Laung Channel (Pak Ror), Hua Khao Dang community and Ko Yo community.

4.3.5 Agricultural runoff: The characteristic of agricultural runoff is nonpoint source which scaterring over large area and be discharged into the lake without treatment. The agricultural area can be divided into 3 main sorts; including rubber plantation about 2,125,775 rai, rice field about 1,209,552 rai, and orchard about 161,193 rai (Chaiprapat, *et al*, 2004). However, there is no observed data on quantity of agricultural runoff drained into the Songkhla Lake.

4.4 Water Quality Related to Eutrophication in the Songkhla Lake

Eutrophication has occurred in all area of the Songkhla Lake which may be rather different depending on water depth and water circulation in each part of the lake. The Upper Lake is the deepest part, therefore the type of algae that grows in this part is phytoplankton. It is not suitable for the growth of macrophyte which must attach on water bottom. The phytoplankton found in this part is the type which can grow in freshwater ecosystem. The environmental condition in the Middle Lake is suitable for the growth of macrophyte, because it is shallow lake with optimum salinity, and has long hydraulic retention time. In addition, water circulation in this part is obstructed by a lot of fishery activities in the lake, mainly shrimp farm and aquaculture. The Lower Lake has the shortest hydraulic retention time due to tidal effect, therefore the effect from eutrophication in this part is less than other parts.

Referring to the water quality monitoring by National Institute of Coastal Aquaculture (NICA) and Faculty of Environmental Management, Prince of Songkhla University, eutrophication has manifested in Songkhla Lake which has nutrient enrichment, and the occurrence of algae bloom. The crucial parameters including nutrient, chlorophyll *a*, and macrophyte distribution in large area have reflected the occurrence of eutrophication in Songkhla Lake as follow:

4.4.1 *nutrient enrichment in the Songkhla Lake*. The over enrichment of nutrient (nitrogen and phosphorus) can result in the occurrence of eutrophication. There are more than 4,000 tons/yr of nitrogen and 600 tons/yr of phosphorus has disposed into the Songkhla Lake. The part of the lake which has the highest nutrient loading is the Lower Lake which has high population communities and factories around. The Lower Lake mainly receives inflow inorganic nitrogen and phosphorus from U-Taphao channel and Pak Phawong channel particularly in the rainy season (La-ongsiriwong et al, 2004). Water quality of the Songkhla Lake analysed by NICA is displayed in Table 3.

Table 3 Water quality in the Songkhla Lake during 1992 – 2000.

Parameter	Average concentration (concentration range)		
	The Upper Lake	The Middle Lake	The Lower Lake
pH	8.0 (7.6 – 8.4)	7.5 (7.4 – 7.6)	7.7 (7.6 – 7.8)
Secchi depth (cm.)	32.2 (21 – 41)	58.0 (34 – 86)	61.1 (53 – 73)
PO_4^{3-} (mg-P /l)	0.006 (0.002 – 0.011)	0.006 (0.001 – 0.011)	0.023 (0.005 – 0.040)
NO_3^- + NO_2^- (mg-N/l)	0.028 (0.020 – 0.045)	0.046 (0.022 – 0.078)	0.121 (0.038 – 0.224)
NH_3 (mg-N/l)	0.028 (0.020 – 0.045)	0.044 (0.027 – 0.060)	0.155 (0.118 – 0.263)
DO (mg/l)	7.5 (6.3 – 8.7)	6.4 (6.1 - 6.7)	5.9 (5.3 – 6.3)
Chlorophyll *a* (µg /l)	38.4 (14.5 – 56.5)	11.9 (5.6 – 20.2)	10.3 (6.8 – 13.9)

Source: Sompongchaiyakul (2004)

From the observed nutrient concentration data during 1992 – 2000, it was found that the nutrient enrichment in the Songkhla Lake has increased, especially in U-Ta-Phao Channel and Pa Wong Channel. Comparing among various parts of the lake, the concentration of nitrogen in the Middle Lake and the Lower Lake are higher than in the Upper Lake. However the nutrient concentration data in Table 3 is focused on inorganic form of nutrient which is available for aquatic plants, not including organic form in water body and sediment which can be decomposed into inorganic form. The concentration of nutrients in the form of total nitrogen and total phosphorus have been investigated since 1999 by NICA which can displayed in Table 4.

Table 4 Annual average concentrations of total nitrogen and total phosphorus (mg/l) in the Songkhla Lake during 1999 – 2003

Year	Total nitrogen (mg/l)			Total phosphorus (mg/l)		
	The Upper Lake	The Middle Lake	The Lower Lake	The Upper Lake	The Middle Lake	The Lower Lake
1999	0.54*	0.49*	0.40*	0.12	0.07	0.06
2000	0.60	0.43	0.46	0.10	0.09	0.07
2001	0.72	0.60	0.55	0.09	0.08	0.07
2002	0.72**	0.73**	1.09**	0.07	0.05	0.06
2003	0.70	0.42	0.38	0.06	0.05	0.06

Notation: * average on May. – Dec.

** average on Jan. – Apr.

Source : La-ongsiriwong et al (2004)

4.4.2 *Chlorophyll a in the Songkhla Lake*. Chlorophyll *a* is the photosynthesis pigment in live phytoplankton. It is the main variable that frequently used as an indicator of phytoplankton biomass and eutrophication. The annual average concentration of chlorophyll *a* in the Songkhla Lake observed by NICA rather varies as shown in Table 5. In 1992, there were similar concentrations of chlorophyll *a* in all parts of the Songkhla Lake with the range of 8.9 – 11.0 μg/l. Later, chlorophyll *a* has increased sharply in the Upper Lake while remained unchanged in the other parts of the lake (the Middle Lake and the Lower Lake).

Table 5 Annual average of chlorophyll a (μg/l) in Songkhla Lake in 1992-2003

Year	Annual average concentration of chlorophyll a (μg/l)		
	The Upper Lake	The Middle Lake	The Lower Lake
1992	11.0	8.9	10.9
1993	18.8	5.8	5.7
1994	52.4	20.7	12.0
1995	49.3	16.1	10.5
1996	49.8	10.0	6.9
1997	57.0	18.9	9.3
1998	36.5	6.9	5.5
1999	34.3	8.0	6.7
2000	36.0	14.3	10.2
2001	37.0	13.1	12.9
2002	44.5	6.0	7.7
2003	21.3	5.4	7.1

Source: La-ongsiriwong et al (2004)

The NICA has also monitored the seasonal variations of chlorophyll *a* in Songkhla lake. As displayed in Table 6, the concentration of chlorophyll *a* in the Upper Lake often reached the peak in summer (May – September) and decreased in the rainy season. The Middle Lake has peak concentration of chlorophyll *a* (17 μg/l) in rainy season (October – December) while the concentration in January – April duration and May – September duration are about 8 μg/l and 11 μg/l, respectively. However, there was no difference in the average concentration of chlorophyll *a* among various seasons in the Lower Lake.

Table 6 Seasonal variation of chlorophyll *a* in the Songkhla Lake

Time duration	Average concentration of chlorophyll *a* (µg/l) in 1992-2003		
	The Upper Lake	The Middle Lake	The Lower Lake*
Jan. – Apr.	26	8	10
May – Sep.	46	11	10
Oct. –Dec.	41	17	11

Notation: * including U-Taphao channel and Pak Phawong channel

Source : National Institute of Castal Aquaculture (2004)

4.4.3 *Phytoplankton species in the Songkhla Lake*. List of spicies of phytoplankton in the Songkhla Lake is shown in Table 7. They are composed of 5 divisions, namely, Bacillariophyta (diatom), Cyanophyta, Chlorophyta, Pyrrophyta, and Euglenophyta. The average density of phytoplankton was 25,067 cell/l. The abundant species of phytoplankton were *Trichodesmium, Nitzschia, Oscillatoria, Spirulina, and Skeletonema* (Predalumpabud and La-ongsiriwong, 1997).

Table 9 Phytoplankton in the Songkhla Lake

Division	Species
Chromophyta	*Bacillariophyta* (Diatom) : *Achnanthes , Asterionella ,Amphora, Amphiprora, Bacteriastrum, Biddulphia, Campylodiscus, Chlorobotrys, Chaetoceros, Coscinodiscus,Diatoma ,Ditylum, Climacodium,Fragilaria, Frustulia, Eucampia, Guinardia, Gyrosigma, Grammatophora, Hemiaulus, Leptocylindrus, Melosira,Navicula, Nitzschai, Rhizosolinia , Planktonella , Pinnularia, Pluerosigma, Skeletoma, Streptotheca, Surirella Synedra, Thalassiosira, Triceratium*
Chlorophyta	*Chlorophyceae* : *Chlorella , Closterium, Chlamydomonas, Dictyosphaerium, Euglena, Micrasterias, Mougeonia, Oocystis, Palmella, Phacus, Pediastrum, Scenedesmus, Schizogonium, Spirogyra, Staurastrum, Stigeoclonium, Xanthidium* *Eugenophyta* : *Euglena, Phacus*
Cyanophyta	*Cyanophyta* (Blue Green algae) : *Anabaena, Aphanizomenon, Aphanocapsa, Aulosira, Chroococcus, Lyngbya, Merismopedia, Microcystis, Nodularia, Nostoc, Oscillatoria, Polycystis, Richelia, Schizomeris, Spirulina* *Dinophyceae* (Dinoflagellate) : *Ceratium, Dinophysis , Diplosalis , Gonyaulax , Gymnodinium, Noctiluca, Peridinium, Triposolenia*

Source: Predalumpabud and La-ongsiriwong (1997)

WATER QUALITY ANALYSIS

The development of phytoplankton blooms is closely related to available nutrients (Chan, T, 2006). Therefore, the observing of available nutrients in study area are needed in order to simulate the dispersion of phytoplankton as close to the existing situation as possible. In this study, the analysis of water quality was conducted to examine nutrients loading through inflow canals drained from the Songkhla Lake Basin and the variation of available nutrients at the Songkhla Lake.

Water Sampling Station

Water samples for nutrients analysis were collected at 41 stations which include 14 stations at inflow canals drained from the Songkhla Lake Basins and 25 stations at the Songkhla Lake.

1. Water Sampling at Inflow Canals Drained from the Songkhla Lake Basin

Most available nutrients from the Songkhla Lake Basin are discharged into the Songkhla Lake through several inflow canals. In order to examine the variation of nutrients loading, the collection of water samples from 14 inflow canals drained from the Songkhla Lake Basin were conducted monthly from April 2008 to March 2009. Considering where these inflow canals enter the Songkhla Lake, these water sampling stations were categorized as representative of inflow canals enter the Upper Lake, the Middle Lake and the Lower Lake, respectively. There are 6 inflow canals enter the Upper Lake, including Pa Phayom canal, Pak Pra canal, Lum Pam canal, Ka Ram canal, Mahakan canal and Ra Node canal. The inflow canals enter the Middle Lake were collected at 5 canals including Tha Chead canal, Pa Bon canal, Phru Phor canal, Pruan canal and Sa Nam Chai canal. The remaining 3 water sampling stations comprise of Rattaphum canal, Bang Klam canal and U-Taphao canal were represented as inflow canal enter the Lower Lake.

The water samples were analyzed for salinity and concentrations of following nutrients; dissolved ammonia nitrogen (NH_3^+), dissolved nitrite nitrogen (NO_2^-), dissolved nitrate nitrogen (NO_3^-), dissolved organic nitrogen (OrgN), dissolved inorganic phosphorus (PO_4^{3-}) and dissolved organic phosphorus (TP).

2. Water Sampling at the Songkhla Lake

With supporting by National Institute of Aquaculture (NICA), water samples collection were taken at the same stations as those undertaken by NICA which consisting of 6 stations at the Upper Lake, 8 stations at the Middle Lake and 7 stations at the Lower Lake. The water samples at the Songkhla Lake were collected on seasonal basis, i.e. in the dry season (February-April), during early rainy season (May-October) and during heavy rainy season (November-January) (La-ongsiriwong, 2006). The water samples were analyzed for pH, salinity, chlorophyll-*a* (Chl-*a*) and concentration of following nutrients; dissolved ammonia nitrogen (NH_3^+), dissolved nitrite nitrogen (NO_2^-), dissolved nitrate nitrogen (NO_3^-), dissolved total nitrogen (TN), dissolved inorganic phosphorus (PO_4^{3-}) and dissolved total phosphorus (TN).

The location of water sampling stations at inflow canals drained from the Songkhla Lake Basin and inside the Songkhla Lake are shown in figure 3.

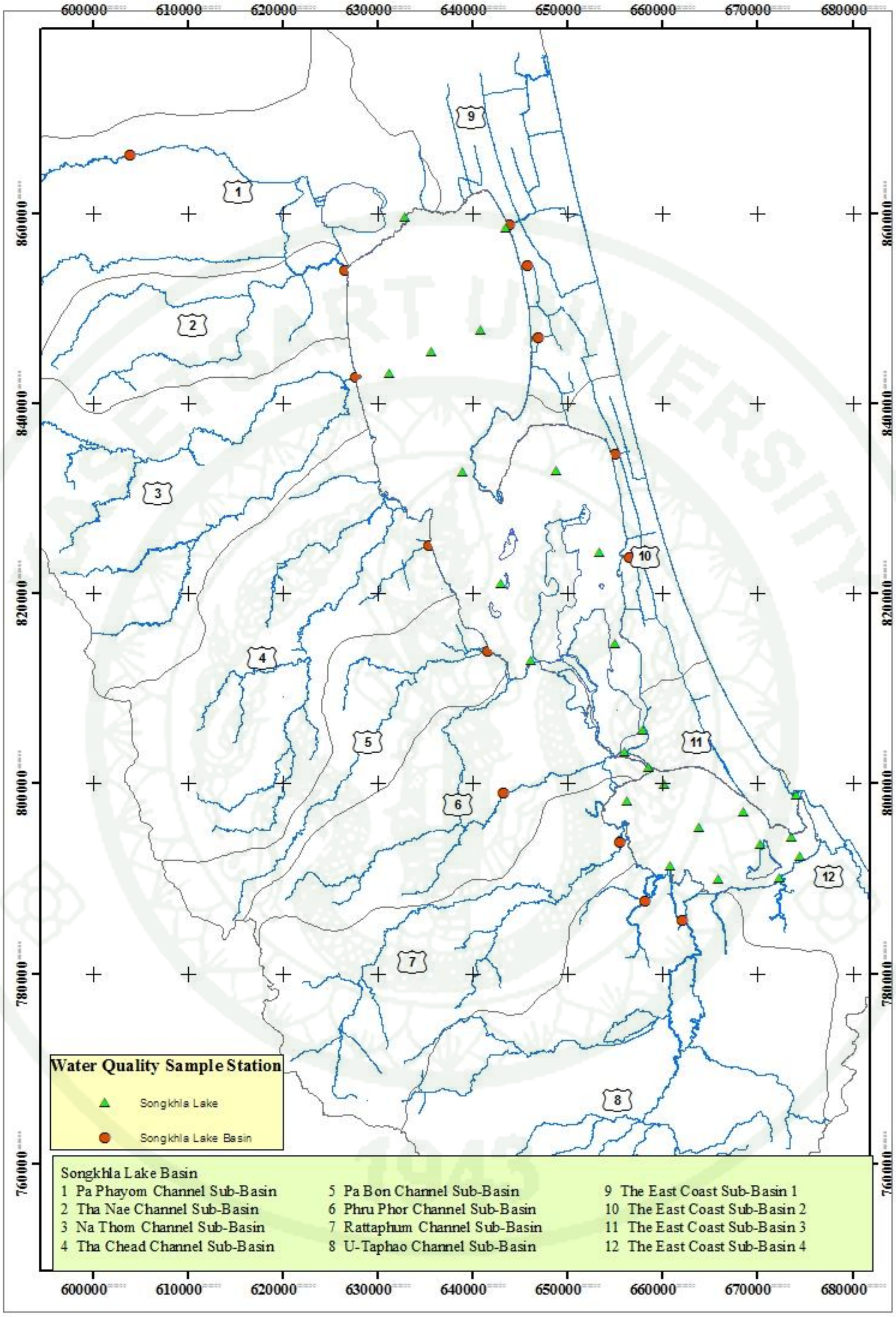


Figure 3 Water sampling station at the Songkhla Lake Basin and the Songkhla Lake

Materials and Methods

Water samples were collected with Kemmerer water sampler at 1 m depth from water surface. Collected water samples were contained in one-liter polyethylene bottles and were kept on ice in coolers during transportation to perform water quality analysis in laboratory at Faculty of Environmental Management, Prince of Songkhla University. All nutrients analysis was performed within 48 hours of sample collection. Nutrient concentrations (*i.e.*, NH_3^+, NO_2^-, NO_3^-, TN, PO_4^{3-} and TP) were first filter through glass fiber filter (0.45 µm) before being analyzed using colorimetric method. Organic nutrient concentrations (Org N, Org P) were obtained by subtracting inorganic nutrient concentrations from total nutrient concentrations (TN, TP).

The analytical methods for water quality analysis are summarized in table 5.

Table 5 Water quality analysis methodology

Parameter	Analytical method	Reference
Salinity	Electrical Conductivity method	Conductivity and TDS meter (WTW)
Chl-*a*	spectrophotometric	Stickland and Parsons, 1973
NH_3^+	phenol-hypochlorite	Stickland and Parsons, 1973
NO_2^-	cadmium reduction	Stickland and Parsons, 1973
NO_3^-	cadmium reduction	Stickland and Parsons, 1973
PO_4^{3-}	ascorbic acid	APHA, AWWA and WPCD, 1995
TN	persulfate oxidation	APHA, AWWA and WPCD, 1995
TP	persulfate oxidation	APHA, AWWA and WPCD, 1995

Results and Discussion

1. Water quality of inflow canals drained from the Songkhla Lake Basin

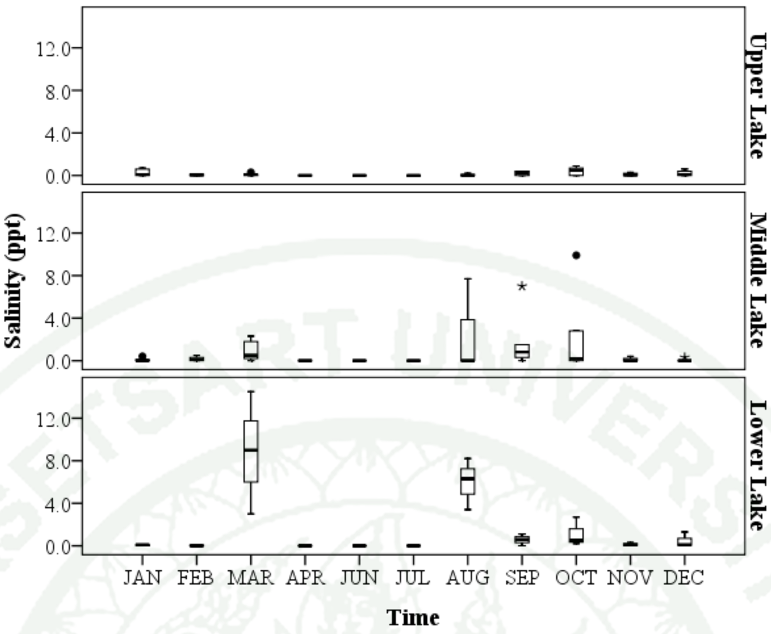
The result of water quality analysis of inflow canals drained from the Songkhla Lake Basin was summarized in table 6. It was found that the highest concentration of most nutrients were observed at inflow canals enter the Lower Lake (2.436 mg-N/L of TN and 0.223 mg-P/l of TP), follow by the Upper Lake (0.615 mg-N/L of TN and 0.1 mg-P/L of TP) and the Middle Lake (0.54 mg-N/L of TN and 0.076 mg-P/L of TP), respectively. This was due to the affected of heavy nutrients loading from industries and communities located along the U-Tapao canals (La-ongsiriwong, 2004; Ratanachai et al., 2004).

In respect of nutrient compositions, the inorganic nutrients consist of inorganic nitrogen (NH_3^+, NO_2^- and NO_3^-) and inorganic phosphorus (PO_4^{3-}) are the important parameters concerning as available nutrients for phytoplankton. The result obtained from all collected inflow canals was found that the concentration of inorganic nutrients i.e., inorganic nitrogen (0.634 mg N/l) and inorganic phosphorus (0.521 mg P/l) were higher than the concentrations of organic nutrients (organic nitrogen 0.330 mg N/l and organic phosphorus 0.043 mg P/l). Therefore, it may conclude that nutrient loading from inflow canals drained from the Songkhla Lake Basin was mainly in the form of available inorganic nutrients more than organic nutrients.

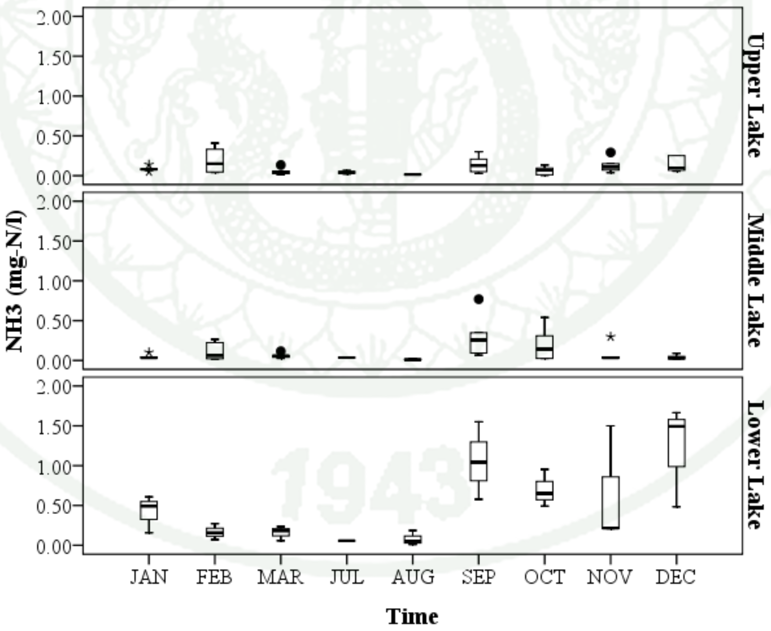
Table 6 Nutrients concentration of inflow canals drained from the Songkhla Lake Basin

Area		NH_3^+ (mg-N/L)	NO_2^- (mg-N/L)	NO_3^- (mg-N/L)	OrgN (mg-N/L)	PO_4^{3-} (mg-P/L)	OrgP (mg-P/L)
The Upper Lake	Mean	0.103	0.036	0.185	0.291	0.059	0.041
	S.D.	0.098	0.048	0.232	0.259	0.045	0.040
	Minimum	0.006	0.002	0.013	0.004	0.004	0.001
	Maximum	0.410	0.259	1.307	1.064	0.199	0.248
The Middle Lake	Mean	0.110	0.020	0.084	0.326	0.042	0.034
	S.D.	0.156	0.024	0.076	0.229	0.032	0.027
	Minimum	0.009	0.001	0.005	0.002	0.005	0.001
	Maximum	0.770	0.136	0.459	1.045	0.152	0.109
The Lower Lake	Mean	0.499	0.184	1.354	0.399	0.166	0.057
	S.D.	0.524	0.247	2.683	0.441	0.133	0.056
	Minimum	0.010	0.001	0.102	0.008	0.005	0.001
	Maximum	1.665	1.177	2.683	1.863	0.521	0.271
Total	Mean	0.198	0.066	0.430	0.330	0.079	0.043
	S.D.	0.319	0.141	0.643	0.306	0.088	0.042
	Minimum	0.006	0.001	0.005	0.002	0.004	0.001
	Maximum	1.665	1.178	2.683	1.863	0.521	0.271

According to the temporal variation, the concentration of salinity and nutrients collected monthly from inflow canals enter each part of the Songkhla Lake are shown in figure 4. It was found that inorganic nitrogen concentration of inflow canals enters the Upper Lake and the Middle Lake seemed to have no variation. Whereas the variation of most nutrients concentration of inflow canals enter the Lower Lake were higher during the rainy season (from September to December), owing to the nutrient loading from agriculture and municipal areas carried by surface runoff.

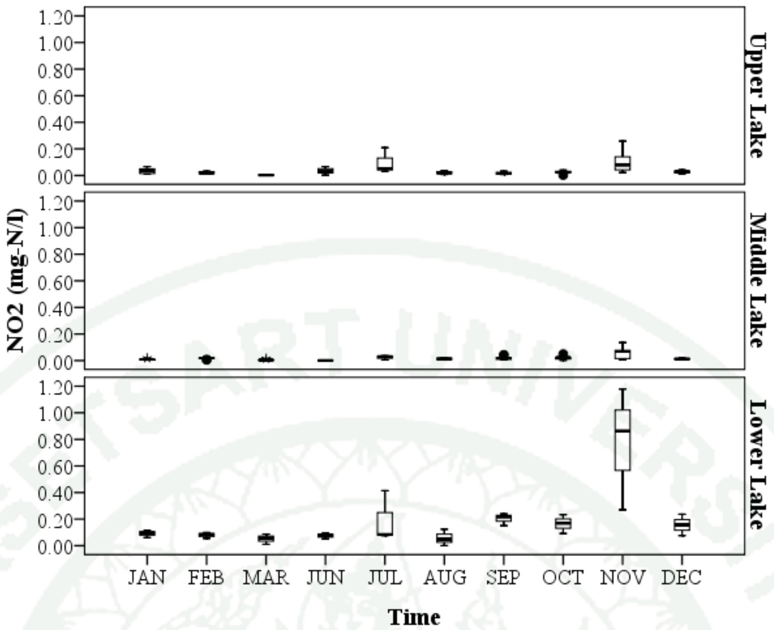


(a) salinity

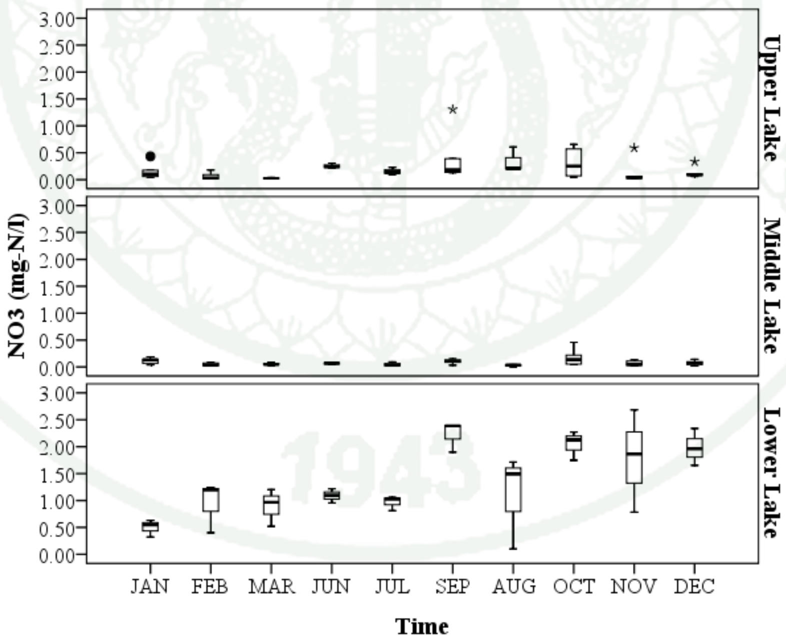


(b) ammonia nitrogen

Figure 4 Temporal variation of salinity and nutrients concentration of water samples collected from inflow canals drained into the Songkhla Lake.

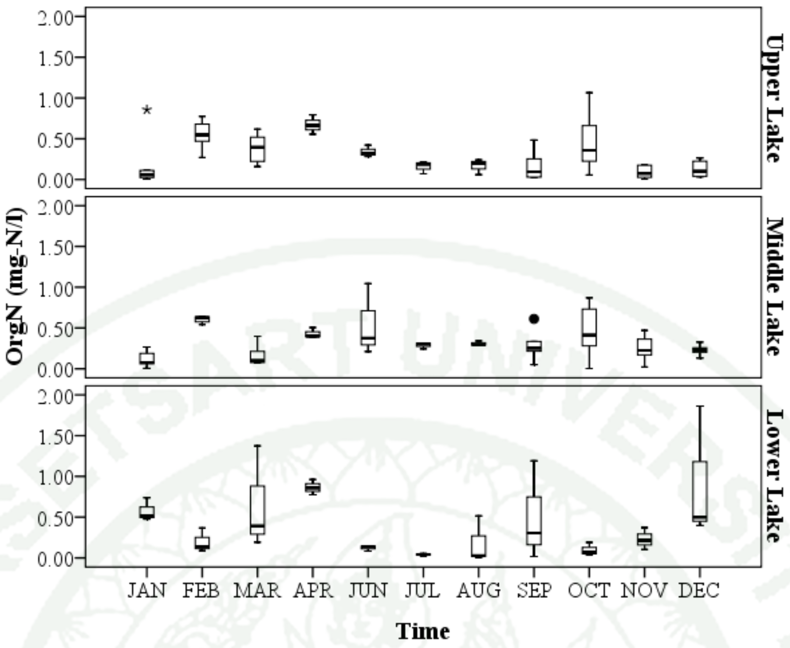


(c) nitrite nitrogen

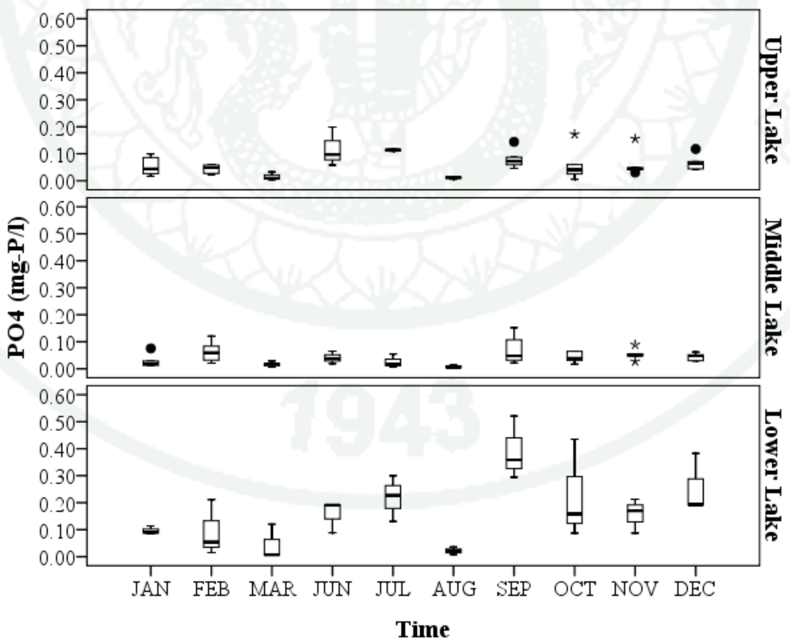


(d) nitrate nitrogen

Figure 4 (Continued)

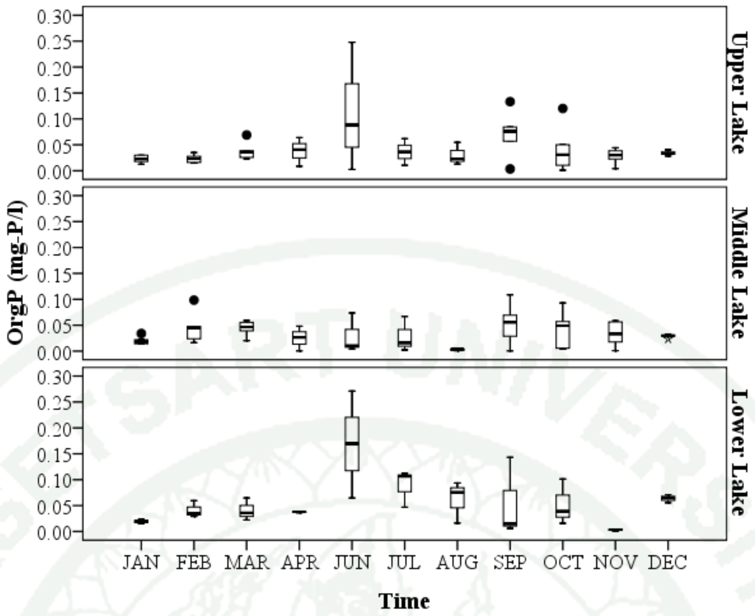


(e) Organic nitrogen



(f) Inorganic phosphorus

Figure 4 (Continued)



(g) organic phosphorus

Figure 4 (Continued)

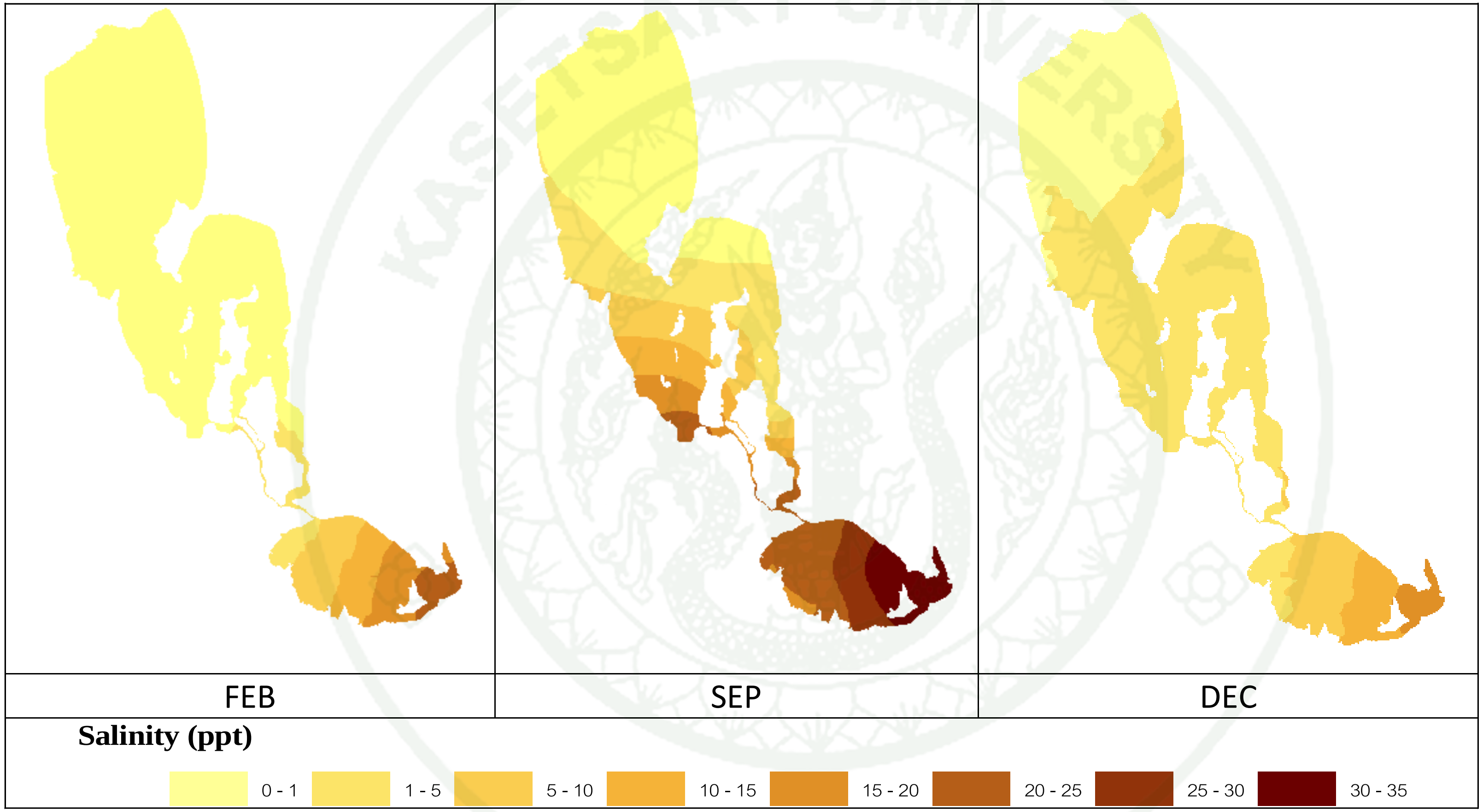
2. Water quality of the Songkhla Lake

The results obtained from water quality analysis of water sampled collected from each part of the Songkhla Lake are summarized in Table 7. It was found that most nutrients were at highest concentrations at the Lower Lake which received maximum waste loading (La-ongsiriwong, 2004; Ratanachai et al., 2004). The maximum salinity was found in the Lower Lake which connected to the sea at the southern end, while the minimum salinity was found in the Upper Lake which apart from the sea. On the opposite, the concentration of chlorophyll-*a* was highest at the Upper Lake and was lowest at the Lower Lake. Therefore salinity may be the limiting factor of some group of phytoplankton in the Lower Lake which cannot tolerant to high salinity condition.

Table 7 The concentration of salinity and nutrients of water samples collected from the Songkhla Lake

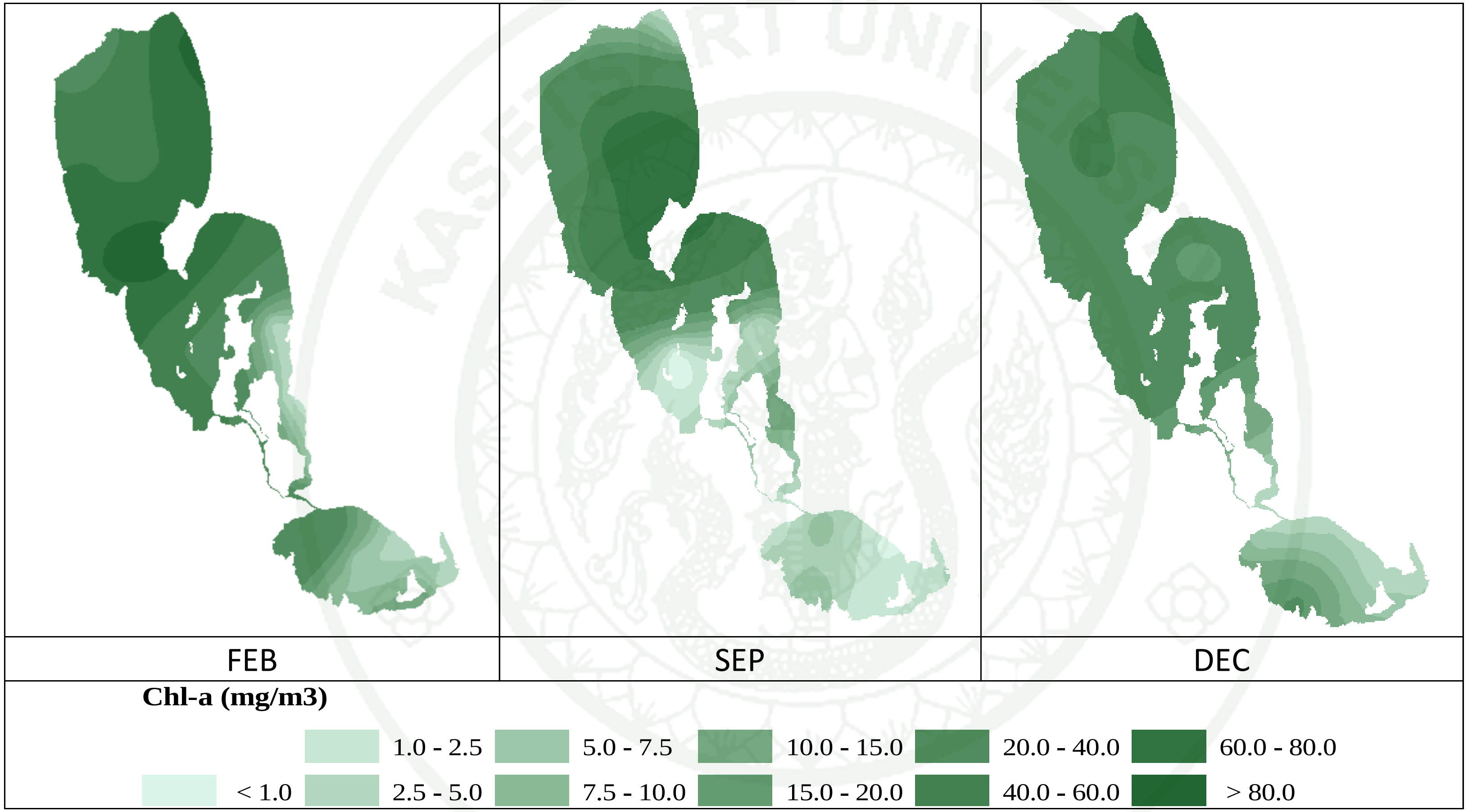
Area		Salinity (ppt)	NH_3^+ (mg-N/L)	NO_2^- (mg-N/L)	NO_3^- (mg-N/L)	OrgN (mg-N/L)	PO_4^{3-} (mg-P/L)	OrgP (mg-P/L)	Chl-*a* (mg/m^3)
The Upper Lake	mean	0.23	0.061	0.014	0.100	0.332	0.049	0.023	47.936
	S.D.	0.24	0.074	0.010	0.124	0.247	0.032	0.018	32.231
	min.	0.00	0.005	0.003	0.010	0.035	0.007	0.002	6.648
	max.	0.80	0.270	0.042	0.422	1.003	0.119	0.055	92.551
The Middle Lake	mean	5.33	0.078	0.025	0.113	0.260	0.073	0.048	16.949
	S.D.	8.26	0.059	0.024	0.069	0.195	0.058	0.138	18.140
	min.	0.00	0.002	0.001	0.024	0.016	0.015	0.001	0.509
	max.	23.10	0.291	0.094	0.246	0.699	0.258	0.688	56.159
The Lower Lake	mean	19.16	0.077	0.012	0.105	0.173	0.032	0.039	7.449
	S.D.	12.66	0.106	0.011	0.099	0.143	0.033	0.026	9.590
	min.	0.30	0.001	0.001	0.003	0.011	0.001	0.007	0.431
	max.	33.40	0.577	0.046	0.406	0.595	0.112	0.128	40.450
Total	mean	8.24	0.072	0.017	0.106	0.255	0.051	0.037	24.111
	S.D.	7.05	0.080	0.015	0.097	0.195	0.041	0.060	19.987
	min.	0.10	0.003	0.002	0.012	0.021	0.008	0.003	2.529
	max.	19.10	0.380	0.061	0.358	0.766	0.163	0.290	63.053

The spatial and temporal variation of salinity, chlorophyll-*a* and dissolved nutrients concentration of water sample collected from the Songkhla Lake are depicted as shown in figure 5. It was found that most forms of inorganic nutrients were at highest concentrations in September (during early rainy season). It might due to the influent of agricultural runoff from the Songkhla Lake Basin. Whereas the organic nutrients were at highest concentrations in February (during dry season) which correlated to chlorophyll *a*.



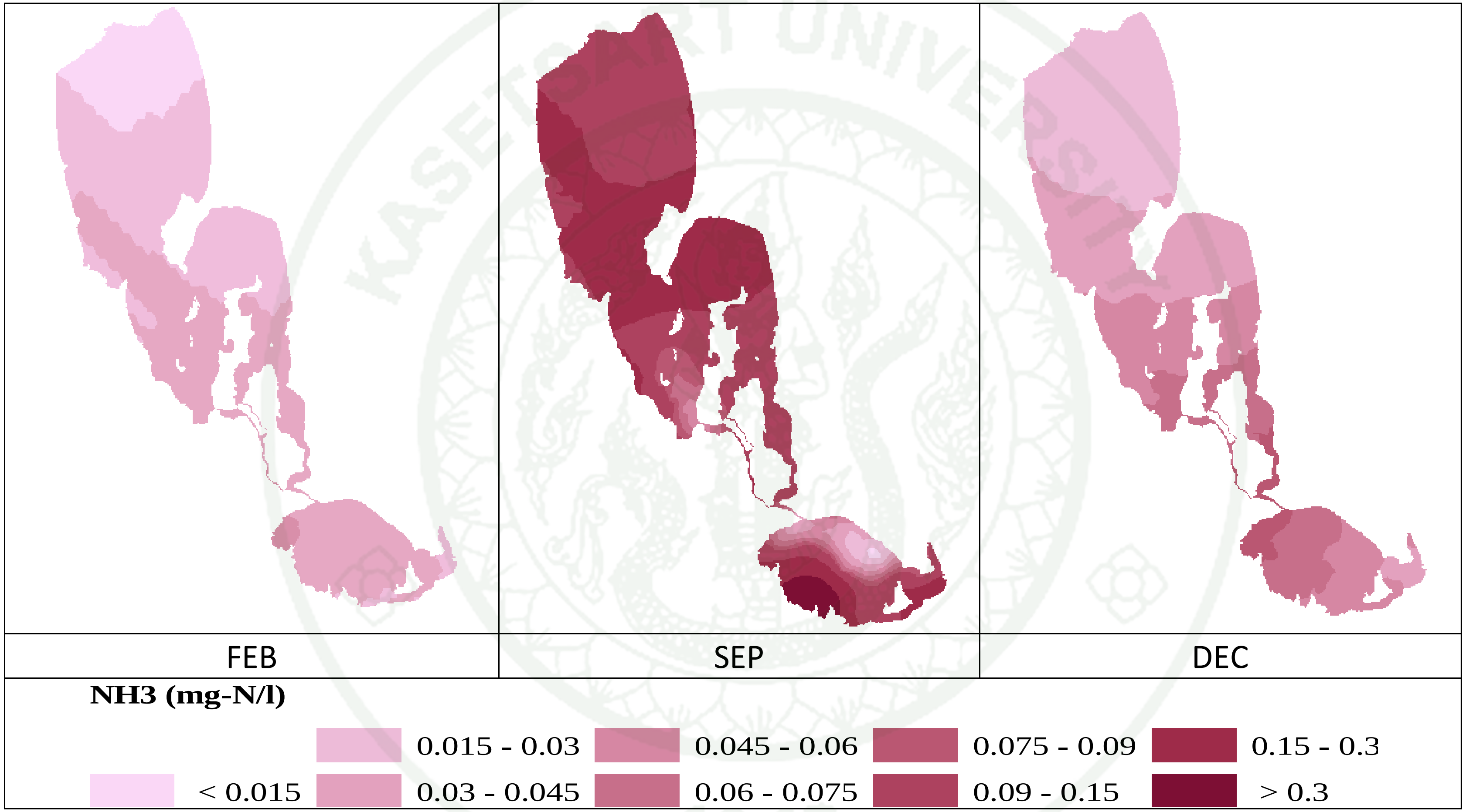
(a) Salinity

Figure 5 The spatial and temporal variation of water sample collected from Songkhla Lake.



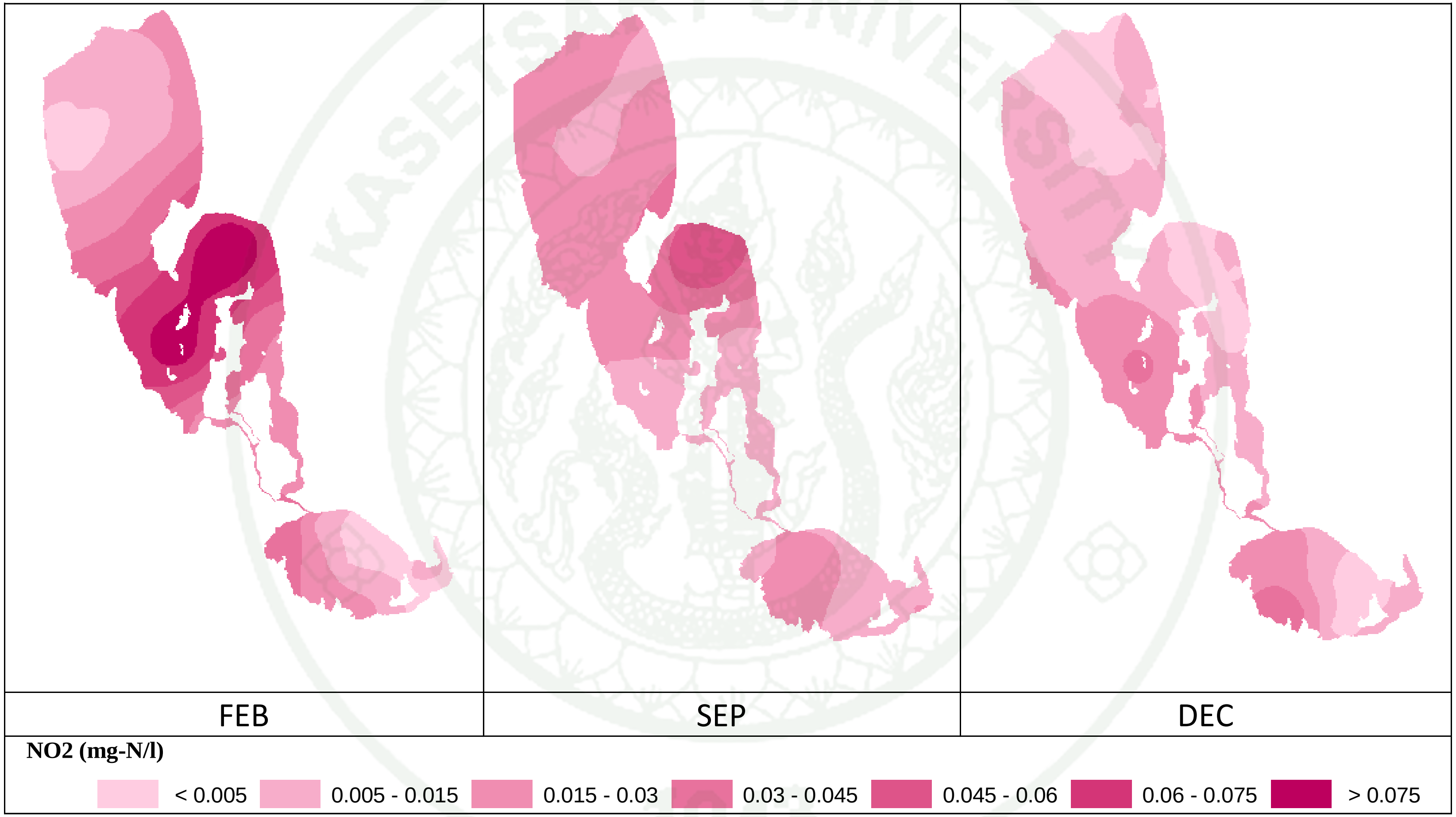
(b) Chlorophyll-*a*

Figure 5 (Continued)



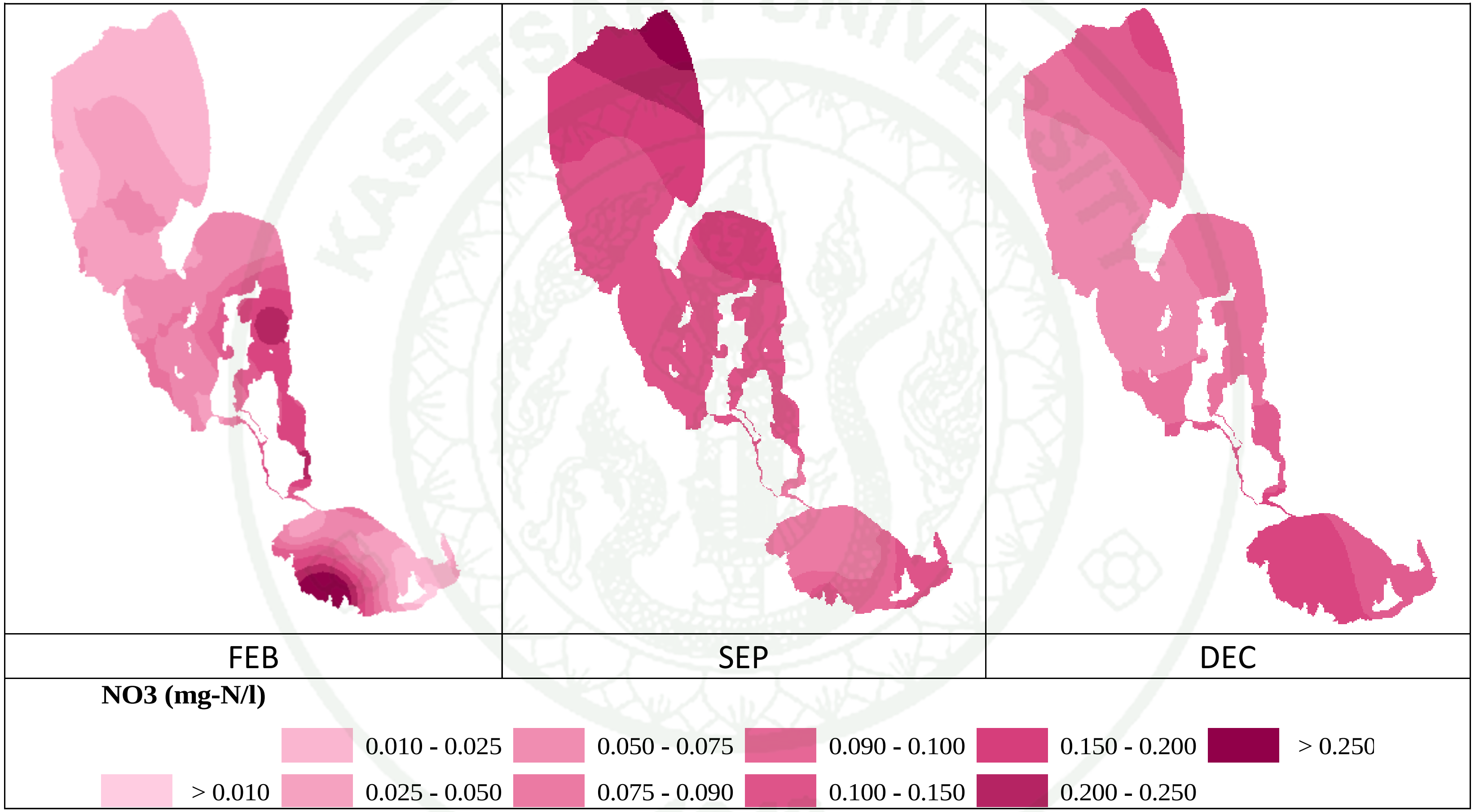
(c) Ammonia nitrogen

Figure 5 (Continued)



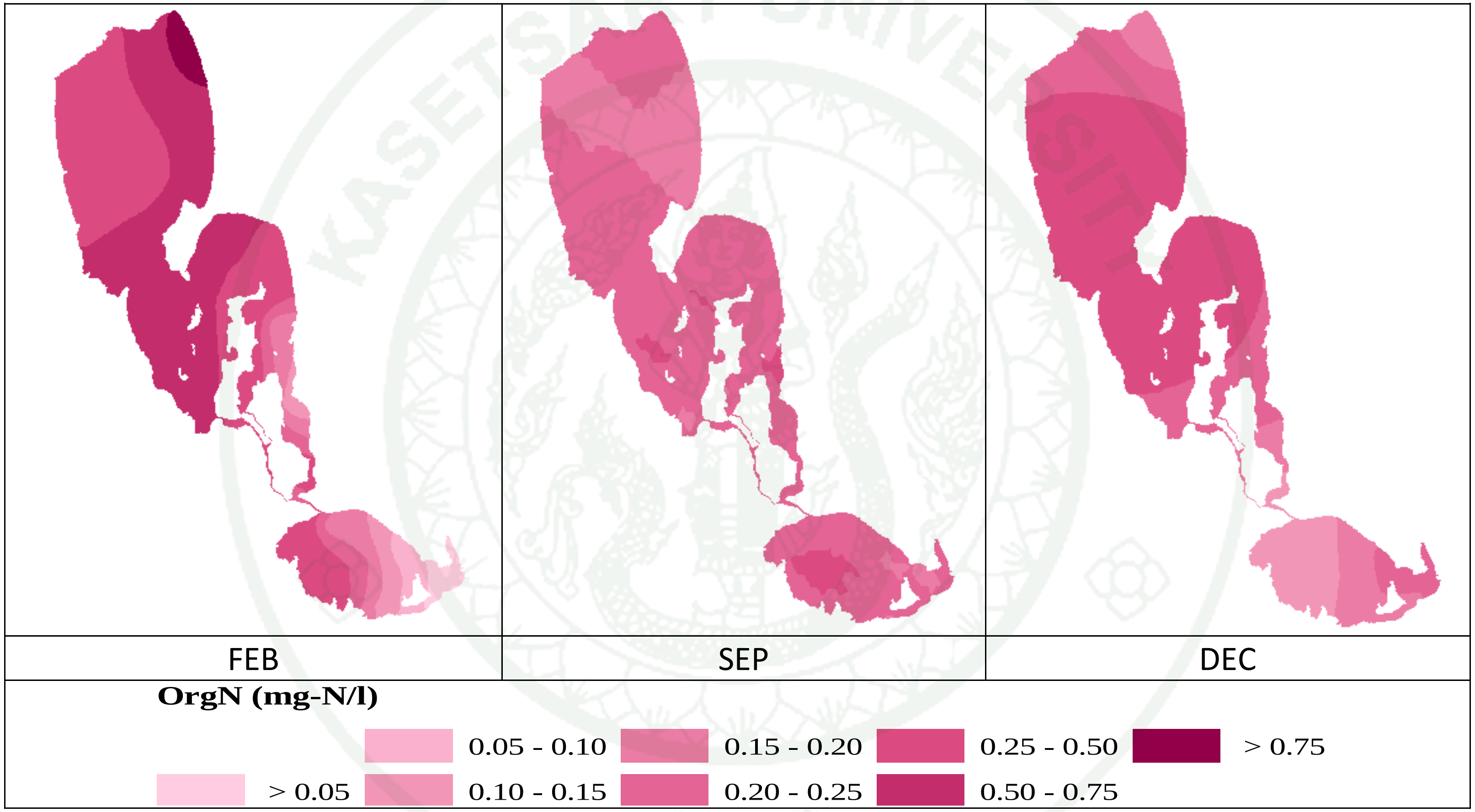
(d) Nitrite nitrogen

Figure 5 (Continued)



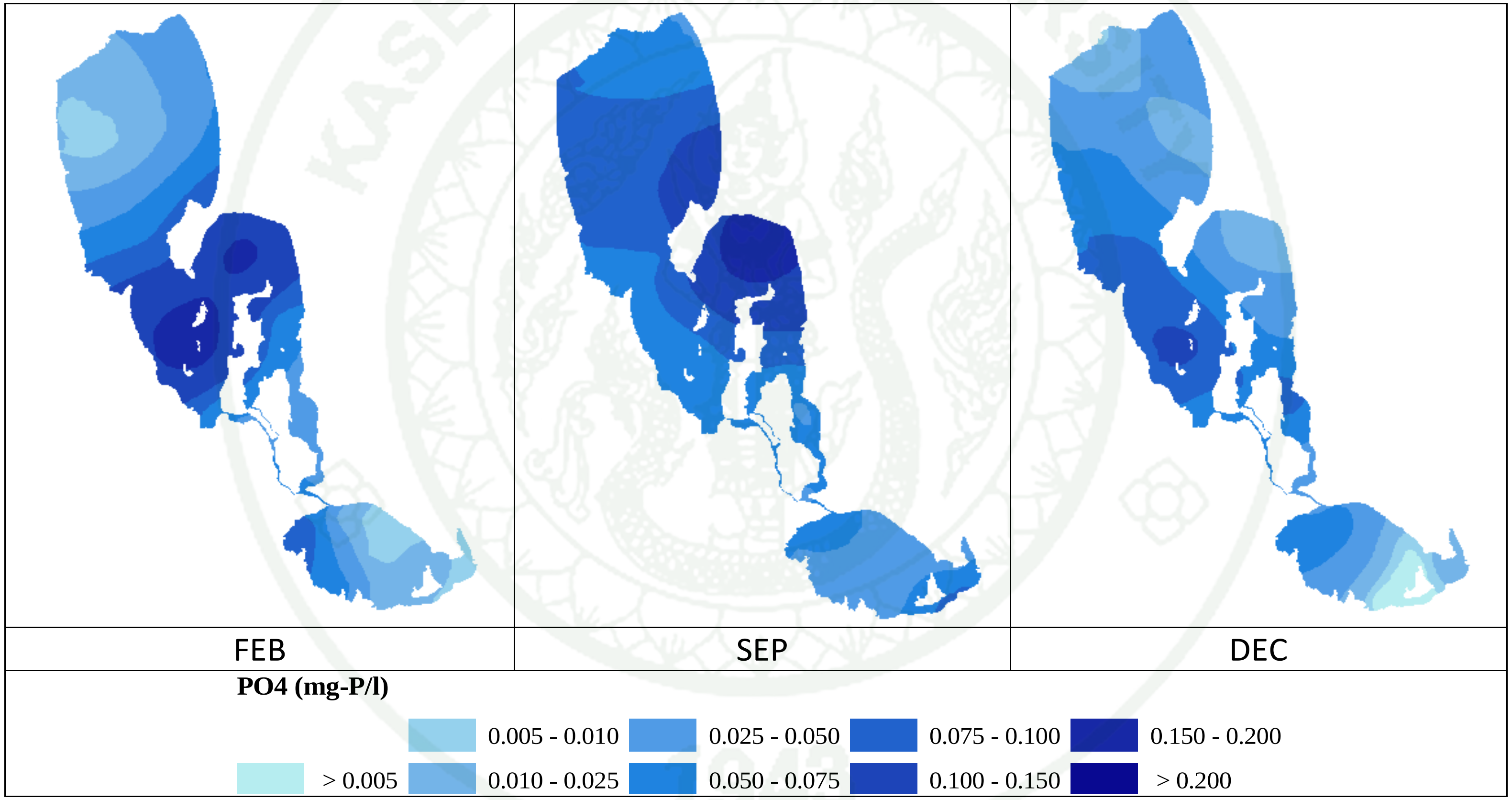
(e) Nitrate nitrogen

Figure 5 (Continued)



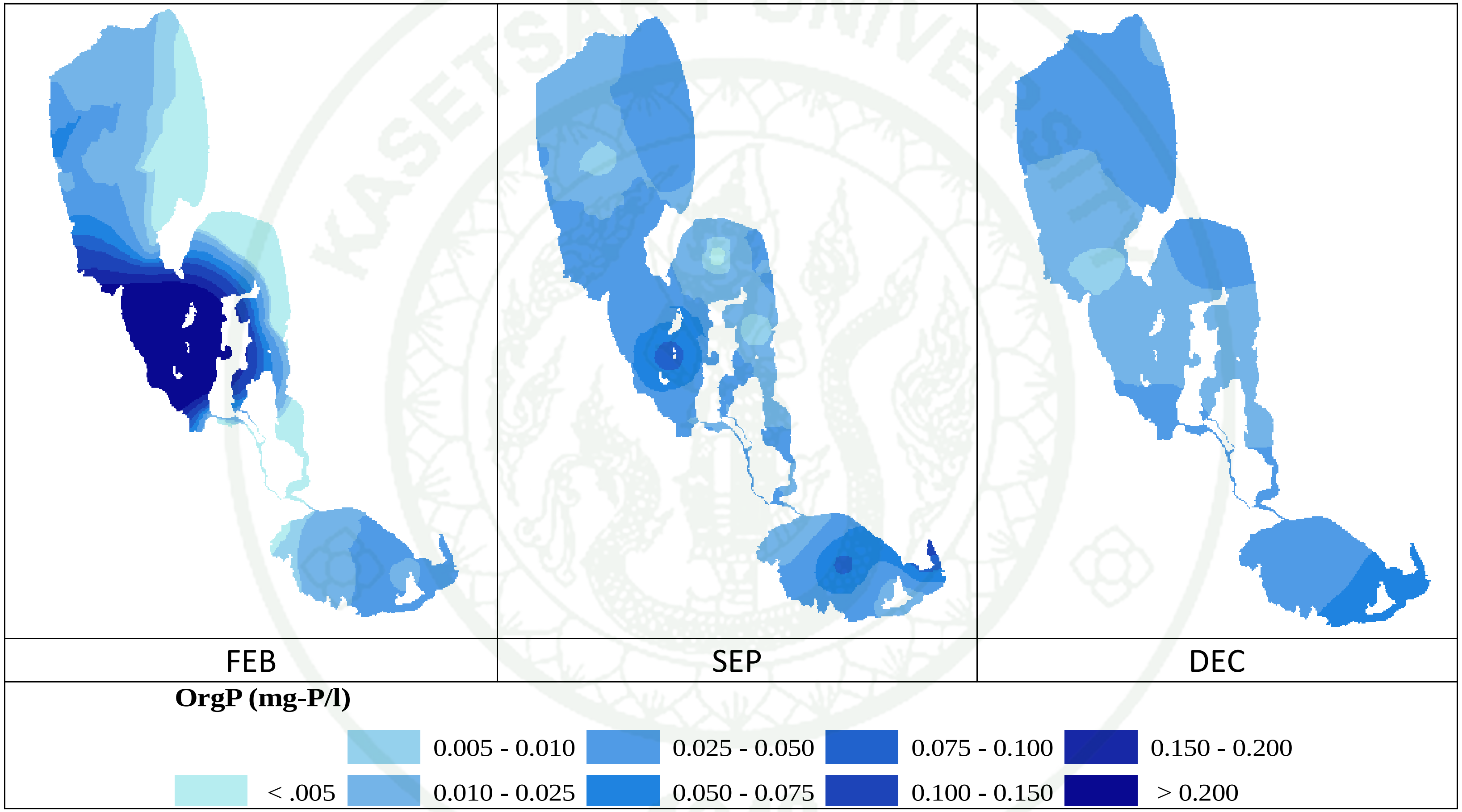
(f) Organic nitrogen

Figure 5 (Continued)



(g) Inorganic phosphorus

Figure 5 (Continued)



(h) Organic phosphorus

Figure 5 (Continued)

WATER QUALITY MODEL DEVELOPMENT

Materials

1. Computer note book, Intel (R) Core (TM)2 Duo CPU, P8600 @ 2.4 GHz, RAM 5 GB
2. Software Microsoft Window Vista 2007
3. Software Program MATLAB 7.5

Methods

1. The two-dimensional vertical averaged mass balance equation was used to simulate the dispersion of phytoplankton and essential nutrients, including organic nitrogen, ammonia nitrogen, nitrite nitrogen, nitrate nitrogen, organic phosphorus, and inorganic phosphorus. Therefore, these dispersion models composes of 7 sub-models (7 set of differential equations), each sub-models for each parameter.

2. The finite element method with Galerkin's weighted residual technique was used to solve these mass balance equations. Isoparametric elements with linear interpolation function are used in this study.

3. MATLAB program was used to develop computer program for solving the finite element equation. At first, the initial values were assumed from the average field concentration in each part of Songkhla Lake. Then, the computation was made for the distribution pattern of phytoplankton nitrogen based on these assumed values of nutrient concentrations. Once the distribution pattern of phytoplankton nitrogen was known, the distribution of organic nitrogen was computed and replaced its original value. The computation was continued for ammonia nitrogen, nitrite nitrogen, nitrate nitrogen, organic phosphorus and inorganic phosphorus. Then the computation of phytoplankton nitrogen distribution was repeated based on the new values of nutrient concentrations, and the computations of other substances were followed. Flow chart of computation of unsteady state model in this study is shown in Figure 5.

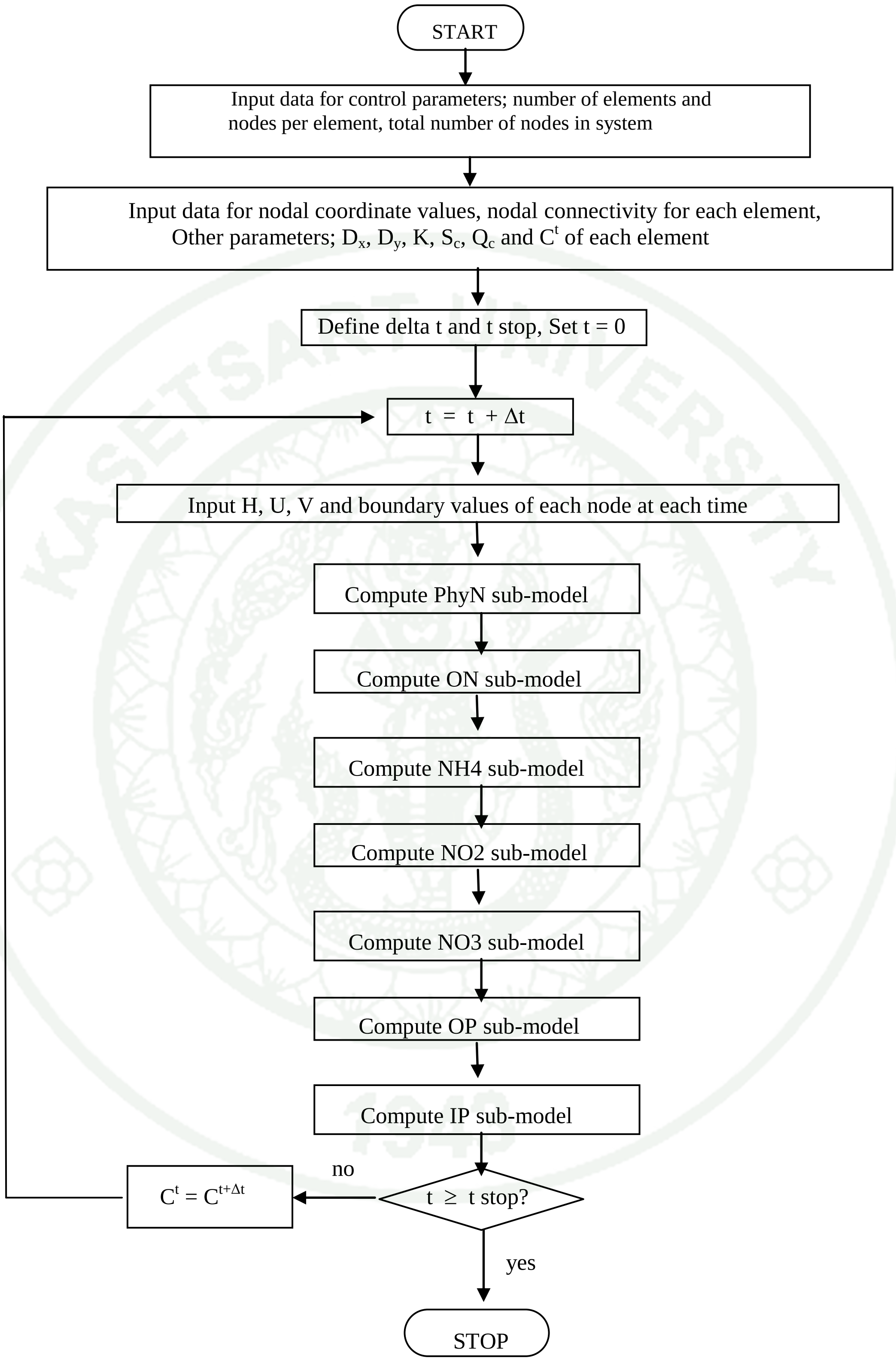


Figure 5 Flow chart of computation of unsteady state dispersion model

4. In order to ensure the validity, the developed model is applied to a simple problem in which analytical solution are available. The calibration of dispersion models were undertaken with measured field data.

5. The developed model is applied to predict the distribution pattern of phytoplankton and nutrients in the Songkhla Lake. With limitation of essential data in applied area, these models were developed with the following assumptions;

5.1 Considering as the shallow lake, the distribution of substances through any vertically cross section of the Songkhla Lake are considered uniformly.

5.2 The variation of phytoplankton and nutrients are simulated on the transport, dispersion and transformation process occurring within the water body.

5.3 In order to simplify for computation process, in this study phytoplankton nitrogen is used as a measure of phytoplankton biomass.

5.4 Since there are several dominant species of phytoplankton in the Songkhla Lake, each of which has different characteristics, a lot of specific data are needed to describe the complex influence correctly. In this study it is assumed that the pool of phytoplankton community has similar growth and uptake rates.

5.5 The effects of salinity on phytoplankton growth and nutrient uptake are described by an expression in the form of the Monod's equation.

Results and Discussion

1. Formulation of dispersion model

1.1 Governing equation

The mass balance equation, Eq.(55), is the basis governing equation for the dispersion model. The two dimensional vertical average mass balance equation for non-conservative substance with assumed first order decaying rate can be written as

$$\frac{\partial c}{\partial t} + u\left(\frac{\partial c}{\partial x}\right) + v\left(\frac{\partial c}{\partial y}\right) - \frac{1}{h}\left\{\frac{\partial}{\partial x}\left(hD_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(hD_y \frac{\partial c}{\partial y}\right)\right\} - S_c = 0 \tag{55}$$

where

c	is the vertically average concentration of substance,
u	is the vertically average flow velocities in x-directions,
v	is the vertically average flow velocities in y-directions,
h	is the water depth,
D_x	is the dispersion coefficients in x-directions,
D_y	is the dispersion coefficients in x-directions,
S_c	is the regeneration rate of substance per unit volume.

The governing mass balanced equation expresses the advection term, the dispersion term and the source/sink term. The dispersion and the advection term of mass balance equation of selected parameters are obtained from the identical parameter, the source/sink term is relation to the reaction or transformation processes of each parameter. In this section, the factors governing the transformation processes of selected parameters was studied and represented by mathematical expressions. Theses expressions are then included in the source/sink term of mass balance equation as shown in sub-model of selected parameters as follows.

1.2 Phytoplankton sub-model

To simplify for computation process, in this study phytoplankton nitrogen is used as a measure of phytoplankton biomass. In summary the mass balance equation for computing phytoplankton distribution is expressed in Eq.(56).

$$\frac{\partial P_n}{\partial t} + u\left(\frac{\partial P_n}{\partial x}\right) + v\left(\frac{\partial P_n}{\partial y}\right) - \frac{1}{h}\left\{\frac{\partial}{\partial x}\left(hD_x \frac{\partial P_n}{\partial x}\right) + \frac{\partial}{\partial y}\left(hD_y \frac{\partial P_n}{\partial y}\right)\right\} - S_{pn} = 0 \qquad (56)$$

where

$$S_{pn} = \left(G_{pn} - R_{pn}\right)P_n - G_z Z - Q_{pn} \qquad (57)$$

in which

P_{n} is phytoplankton nitrogen concentration,

G_{pn} is growth rate of phytoplankton,

R_{pn} is respiration rate of phytoplankton,

G_z is phytoplankton nitrogen grazing rate per unit weight of zooplankton,

Z is zooplankton concentration,

Q_{pn} is increasing rate of phytoplankton nitrogen from other sources.

The plankton growth rate is affected by light intensity, temperature, concentrations of limited nutrients (i.e. nitrogen and phosphorus) and salinity. In this study, the growth rate; G_{pn} is expressed by:

$$G_{pn} = G_{\max} F(I).\alpha(T).\beta(A_n, N_{ta}, I_p)F(S) \qquad (58)$$

in which

$$F(I)=\frac{2.718}{k_e H}\left[\exp\left\{\frac{I_0}{I_s}e^{-k_e H}\right\}-\exp\left\{-\frac{I_0}{I_s}\right\}\right] \quad (59)$$

$$\alpha(T)=\theta^{T-20} \quad (60)$$

$$\beta = \min\left[\frac{A_n + N_{ta}}{k_{mN} + A_n + N_{ta}}, \frac{I_p}{k_{mP} + I_P}\right] \quad (61)$$

$$F(S)=\left[\frac{k_{Sal}}{k_{Sal} + Sal}\right] \quad (62)$$

1.3 Organic nitrogen sub-model

Organic nitrogen in various forms which excreted from phytoplankton zooplankton and decomposing organic matter will gradually decomposed to ammonia nitrogen. The mass balance equation of organic nitrogen can be written as

$$\frac{\partial O_n}{\partial t}+u\left(\frac{\partial O_n}{\partial x}\right)+v\left(\frac{\partial O_n}{\partial y}\right)-\frac{1}{h}\left\{\frac{\partial}{\partial x}\left(hD_x\frac{\partial O_n}{\partial x}\right)+\frac{\partial}{\partial y}\left(hD_y\frac{\partial O_n}{\partial y}\right)\right\}-S_{On} = 0 \quad (63)$$

where

$$S_{on} = (U_{pn} - G_{pn} + R_{pn})P_n - R_{on}O_n - Q_{on} \quad (64)$$

in which

O_n is organic nitrogen concentration,

U_{pn} is nitrogen uptake rate per unit weight of phytoplankton nitrogen,

R_{on} is mineralization/hydrolysis rate of organic nitrogen to ammonia,

Q_{pn} is increasing rate of organic nitrogen from other sources.

The nutrient uptake rate is also affected by light intensity, temperature and salinity. In this study the value U_{pn} is expressed as:

$$U_{pn} = F(I).\alpha(T).\lambda(A_n, N_{ta}) \quad (65)$$

in which

$$F(I) = \frac{2.718}{k_e H}\left[\exp\left\{\frac{I_0}{I_s}e^{-k_e H}\right\} - \exp\left\{-\frac{I_0}{I_s}\right\}\right] \quad (66)$$

$$\alpha(T) = \theta^{T-20} \quad (67)$$

$$\lambda(A_n, N_{ta}) = U_{A_n,\max}\frac{A_n}{K_{an} + A_n} + U_{N_{ta},\max}\frac{N_{ta}}{K_{nta} + N_{ta}} \quad (68)$$

1.4 Ammonia nitrogen sub-model

Amonia nitrogen is utilized by phytoplankton as nutrient to form complex organic matter under photosynthesis process. Another removal of ammoia nitrogen is its oxidation to nitrite nitrogen under aerobic condition. The mass balance equation of ammonia nitrogen can be written as

$$\frac{\partial A_n}{\partial t} + u\left(\frac{\partial A_n}{\partial x}\right) + v\left(\frac{\partial A_n}{\partial y}\right) - \frac{1}{h}\left\{\frac{\partial}{\partial x}\left(hD_x\frac{\partial A_n}{\partial x}\right) + \frac{\partial}{\partial y}\left(hD_y\frac{\partial A_n}{\partial y}\right)\right\} - S_{an} = 0 \quad (69)$$

where

$$S_{an} = R_{an} O_n - U_{an} P_n - R_{an} A_n + Q_{an} \quad (70)$$

in which

A_n is ammonia nitrogen concentration,

U_{an} is rate of ammonia nitrogen uptake per unit weight of phytoplankton nitrogen,

R_{an} is rate of ammonia nitrogen oxidation to nitrite nitrogen,

Q_{an} is increasing rate of ammonia nitrogen from other sources.

1.5 Nitrite nitrogen sub-model

Nitrite nitrogen is resulted from oxidation of ammonia nitrogen under aerobic condition. The mass balance equation of nitrite nitrogen can be written as

$$\frac{\partial NO2}{\partial t} + u\frac{\partial NO2}{\partial x} + v\frac{\partial NO2}{\partial y} - \frac{1}{h}\left\{\frac{\partial}{\partial x}\left(hD_x\frac{\partial NO2}{\partial x}\right) + \frac{\partial}{\partial y}\left(hD_y\frac{\partial NO2}{\partial y}\right)\right\} - R_{NO2} = 0 \quad (71)$$

where

$$S_{ti} = R_{an} A_n - R_{ti} A_n - R_{ti} N_{ti} + Q_{ti} \quad (72)$$

in which

N_{ti} is nitrite nitrogen concentration,

R_{ti} is rate of nitrite nitrogen oxidation to nitrate nitrogen,

Q_{ti} is increasing rate of nitrite nitrogen from other sources.

1.6 Nitrate nitrogen sub-model

Nitrate nitrogen is utilized by phytoplankton as nutrient. It is the final product of nitrification process. Under anaerobic condition nitrate nitrogen can be reduced to nitrite and ammonia nitrogen. The mass balance equation of nitrate nitrogen can be written as

$$\frac{\partial N_{ta}}{\partial t} + u\left(\frac{\partial N_{ta}}{\partial x}\right) + v\left(\frac{\partial N_{ta}}{\partial y}\right) - \frac{1}{h}\left\{\frac{\partial}{\partial x}\left(hD_x \frac{\partial N_{ta}}{\partial x}\right) + \frac{\partial}{\partial y}\left(hD_y \frac{\partial N_{ta}}{\partial y}\right)\right\} - S_{ta} = 0 \qquad (73)$$

where

$$S_{ta} = R_{ti} N_{ti} - U_{ta} P_n + Q_{ta} \qquad (74)$$

in which

N_{ta} is nitrate nitrogen concentration,

U_{ta} is rate of nitrate nitrogen uptake per unit weight of phytoplankton nitrogen,

Q_{ta} is increasing rate of nitrate nitrogen from other sources.

1.7 Organic phosphorus sub-model

Phosphorus kinetics are basically similar to nitrogen kinetics except there is no process analogous to denitrification. Inorganic phosphorus is utilized by phytoplankton for growth and is incorporated into phytoplankton biomass. The various forms of organic phosphorus undergo settling, hydrolysis and mineralization, and are converted to inorganic phosphorus at temperature dependent rate. In this study dissolved organic phosphorus (OP) and dissolved inorganic phosphorus (PO_4^{-3}) are selected to study.

The mass balance equation of organic phosphorus can be written as

$$\frac{\partial O_p}{\partial t} + u\left(\frac{\partial O_p}{\partial x}\right) + v\left(\frac{\partial O_p}{\partial y}\right) - \frac{1}{h}\left\{\frac{\partial}{\partial x}\left(hD_x \frac{\partial O_p}{\partial x}\right) + \frac{\partial}{\partial y}\left(hD_y \frac{\partial O_p}{\partial y}\right)\right\} - S_{Op} = 0 \quad (75)$$

where

$$S_{op} = (U_{ip} - G_{ip} + R_{op})P_n - R_{op} O_p - Q_{op} \quad (76)$$

in which

O_P is organic phosphorus concentration,
U_{ip} is inorganic phosphorus uptake rate per unit weight of phytoplankton nitrogen,
G_{ip} is growth rate of phytoplankton multiplied with ratio of phosphorus to nitrogen in phytoplankton cell,
R_{op} is respiration rate of phytoplankton multiplied with ratio of phosphorus to nitrogen in phytoplankton cell,
M_{op} is mineralization/hydrolysis rate of organic phosphorus to inorganic phosphorus,
Q_{ip} is increasing rate of organic phosphorus from other sources.

1.8 Inorganic phosphorus sub-model

As same as the inorganic nitrogen, the mass balance equation of inorganic phosphorus is the result of mineralization and uptake by phytoplankton as shown in Eq. (68).

$$\frac{\partial I_p}{\partial t} + u\left(\frac{\partial I_p}{\partial x}\right) + v\left(\frac{\partial I_p}{\partial y}\right) - \frac{1}{h}\left\{\frac{\partial}{\partial x}\left(hD_x \frac{\partial I_p}{\partial x}\right) + \frac{\partial}{\partial y}\left(hD_y \frac{\partial I_p}{\partial y}\right)\right\} - S_{ip} = 0 \quad (77)$$

where

$$S_{ip} = U_{ip} P_n - R_{op} O_p + Q_{ip} \tag{78}$$

in which

I_P is inorganic phosphorus concentration,

U_{ip} is rate of inorganic phosphorus uptake per unit weight of phytoplankton nitrogen,

Q_{ip} is increasing rate of inorganic phosphorus from other sources.

2. Finding approximated solution of the formulated model

2.1 The discretized equation

The finite element method with Galerkin's weighted residual method is used to find approximated solution of the formulated model. In weighted residual finite element method the dependent variables in the mass balance equation are approximated by a set of trial functions which describe the variables in terms of their nodal values and some independent variables. Substitution of these trial functions in the dispersion equation results in a set of error functions known as the residuals.

The exact solution c in Eq. (55) is approximated by trial functions $\hat{c}$ which describe the variables in terms of their nodal values (c_i). This leads to piecewise continuous function of $\hat{c}$ in the form

$$\hat{c} = \sum_{i=1}^{n} N_i C_i = N_1 C_1 + N_2 C_2 + N_3 C_3 + N_4 C_4 \tag{79}$$

where

N_i = interpolation function

Integral of the product of the residual and a weighting function over the study domain is set to zero, resulting in a weighted residual equation as follows

$$\iint_{\Omega} w_c R_L \, dA = 0 \tag{80}$$

In Galerkin's technique, the interpolation functions ϕ_i (i=1,2,.., n) are used as the weighting function. This results in n weighted residual equations which can be written in the matrix form as:

$$\iint_{\Omega} N R_L \, dA = 0 \tag{81}$$

where

N is the column matrix of N_i (i=1,2,..,n).

From the general mass balance equation (Eq.55) which is the basic governing equation of the dispersion model, we obtain:

$$\iint_{\Omega} N \left[\frac{\partial \hat{c}}{\partial t} + u\left(\frac{\partial \hat{c}}{\partial x}\right) + v\left(\frac{\partial \hat{c}}{\partial y}\right) - \frac{1}{h}\left\{ \frac{\partial}{\partial x}\left(hD_x \frac{\partial \hat{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(hD_y \frac{\partial \hat{c}}{\partial y}\right)\right\} - S_c \right] dA = 0 \tag{82}$$

Eq. (82) can be written as

$$\iint_{\Omega} N \left[\frac{\partial \hat{c}}{\partial t} + u\left(\frac{\partial \hat{c}}{\partial x}\right) + v\left(\frac{\partial \hat{c}}{\partial y}\right) - S_c \right] dA - \iint_{\Omega} \frac{N}{h} \left[\frac{\partial}{\partial x}\left(hD_x \frac{\partial \hat{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(hD_y \frac{\partial \hat{c}}{\partial y}\right)\right] dA = 0 \tag{83}$$

which can be expanded to

$$\iint_{\Omega} N\left[\frac{\partial \hat{c}}{\partial t} + u\left(\frac{\partial \hat{c}}{\partial x}\right) + v\left(\frac{\partial \hat{c}}{\partial y}\right) - S_c\right]dA - \iint_{\Omega} \frac{N}{h}\left(D_x \frac{\partial h}{\partial x}\frac{\partial \hat{c}}{\partial x} + D_y \frac{\partial h}{\partial y}\frac{\partial \hat{c}}{\partial y}\right)dA$$

$$- \iint_{\Omega} N\left[\frac{\partial}{\partial x}\left(D_x \frac{\partial \hat{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial \hat{c}}{\partial y}\right)\right]dA = 0 \qquad (84)$$

Applying partial differential to the third term of Eq.(84)

$$\iint_{\Omega} N\left[\frac{\partial}{\partial x}\left(D_x \frac{\partial \hat{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial \hat{c}}{\partial y}\right)\right]dA = \iint_{\Omega} \left[\frac{\partial}{\partial x}\left(ND_x \frac{\partial \hat{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(ND_y \frac{\partial \hat{c}}{\partial y}\right)\right]dA$$

$$- \iint_{\Omega} \left(D_x \frac{\partial N}{\partial x}\frac{\partial \hat{c}}{\partial x} + D_y \frac{\partial N}{\partial y}\frac{\partial \hat{c}}{\partial y}\right)dA \qquad (85)$$

Then Eq.(85) can be written as

$$\iint_{\Omega} N\left[\frac{\partial \hat{c}}{\partial t} + u\left(\frac{\partial \hat{c}}{\partial x}\right) + v\left(\frac{\partial \hat{c}}{\partial y}\right) - S_c\right]dA - \iint_{\Omega} \frac{N}{h}\left(D_x \frac{\partial h}{\partial x}\frac{\partial \hat{c}}{\partial x} + D_y \frac{\partial h}{\partial y}\frac{\partial \hat{c}}{\partial y}\right)dA$$

$$+ \iint_{\Omega} \left(D_x \frac{\partial N}{\partial x}\frac{\partial \hat{c}}{\partial x} + D_y \frac{\partial N}{\partial y}\frac{\partial \hat{c}}{\partial y}\right)dA - \iint_{\Omega} \left[\frac{\partial}{\partial x}\left(ND_x \frac{\partial \hat{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(ND_y \frac{\partial \hat{c}}{\partial y}\right)\right]dA = 0 \qquad (86)$$

From Green's Theorem

$$\iint \left(\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}\right)dxdy = \oint Mdy + Ndx$$

Equation 86 become

$$\iint_{\Omega} N\left[\frac{\partial \hat{c}}{\partial t} + u\left(\frac{\partial \hat{c}}{\partial x}\right) + v\left(\frac{\partial \hat{c}}{\partial y}\right) - S_c\right]dA - \iint_{\Omega} \frac{N}{h}\left(D_x \frac{\partial h}{\partial x}\frac{\partial \hat{c}}{\partial x} + D_y \frac{\partial h}{\partial y}\frac{\partial \hat{c}}{\partial y}\right)dA$$

$$+ \iint_{\Omega} \left(D_x \frac{\partial N}{\partial x}\frac{\partial \hat{c}}{\partial x} + D_y \frac{\partial N}{\partial y}\frac{\partial \hat{c}}{\partial y}\right)dA - \oint\left(ND_x \frac{\partial \hat{c}}{\partial x}dy + ND_y \frac{\partial \hat{c}}{\partial y}dx\right) = 0 \qquad (87)$$

The term $\left(D_x \frac{\partial \hat{c}}{\partial x} dy + D_y \frac{\partial \hat{c}}{\partial y} dx \right)$ represents the rate of dispersive flux into domain per unit depth through the boundary (mass/(time × unit distance)), i.e. $Q_c dS$ Then Eq.87 can be written as

$$\iint_{\Omega} N \left[\frac{\partial \hat{c}}{\partial t} + u\left(\frac{\partial \hat{c}}{\partial x} \right) + v\left(\frac{\partial \hat{c}}{\partial y} \right) - S_c \right] dA - \iint_{\Omega} \frac{N}{h} \left(D_x \frac{\partial h}{\partial x} \frac{\partial \hat{c}}{\partial x} + D_y \frac{\partial h}{\partial y} \frac{\partial \hat{c}}{\partial y} \right) dA$$

$$+ \iint_{\Omega} \left(D_x \frac{\partial N}{\partial x} \frac{\partial \hat{c}}{\partial x} + D_y \frac{\partial N}{\partial y} \frac{\partial \hat{c}}{\partial y} \right) dA - \oint Q_c N dS = 0 \quad (88)$$

The approximated function $(\hat{c})$, current velocity (u, v), water depth (h) can be explained as

$$\hat{c} = \sum_{i=1}^{k} NC_i = \mathrm{N}^T C$$

$$u = \sum_{i=1}^{k} NU_i = N^T U$$

$$v = \sum_{i=1}^{k} NV_i = N^T V$$

$$h = \sum_{i=1}^{k} NH_i = N^T H$$

where C, U, V, H is the matrix of C_i, U_i, V_i, H_i at node i = 1, 2, 3,…,n substitute in Eq.88 as

$$\iint_{\Omega} N\left[\frac{\partial(N^T C)}{\partial t} + N^T U\left(\frac{\partial N^T C}{\partial x}\right) + N^T V\left(\frac{\partial N^T C}{\partial y}\right) - S_c\right]dA$$
$$- \iint_{\Omega} \frac{N}{H}\left(D_x \frac{\partial(N^T H)}{\partial x}\frac{\partial(N^T C)}{\partial x} + D_y \frac{\partial(N^T H)}{\partial y}\frac{\partial(N^T C)}{\partial y}\right)dA \qquad (89)$$
$$+ \iint_{\Omega}\left(D_x \frac{\partial(N^T C)}{\partial x}\frac{\partial N}{\partial x} + D_y \frac{\partial(N^T C)}{\partial y}\frac{\partial N}{\partial y}\right)dA - \oint Q_c N dS = 0$$

In the finite element method, the whole domain is divided into elements in which the dependent variables are replaced by the approximated functions, defined in term of the nodal values of the variables in each element and the interpolation functions. The continuous problem then becomes a discrete problem with unknown variables at nodal points. In each element, the weighted residual integral is obtained. The domain integral is obtained from summation of these elements weighted residual integrals.

$$\iint_{\Omega} f(x)dx = \sum_{e=1}^{m}\iint_{A^e} f^e(x)dA$$

From eq.89 can be written in the form of finite element equation as

$$\sum_{e=1}^{m}\left(\begin{array}{l} \iint_{A^e} N^e\left[\frac{\partial(N^{eT} C^e)}{\partial t} + N^{eT} U^e\left(\frac{\partial N^{eT} C^e}{\partial x}\right) + N^{eT} V^e\left(\frac{\partial N^{eT} C^e}{\partial y}\right) - S_c^e\right]dA \\ - \iint_{A^e} \frac{N^e}{H^e}\left(D_x \frac{\partial(N^{eT} H^e)}{\partial x}\frac{\partial(N^{eT} C^e)}{\partial x} + D_y \frac{\partial(N^{eT} H^e)}{\partial y}\frac{\partial(N^{eT} C^e)}{\partial y}\right)dA \\ + \iint_{A^e}\left(D_x \frac{\partial(N^{eT} C^e)}{\partial x}\frac{\partial N^e}{\partial x} + D_y \frac{\partial(N^{eT} C^e)}{\partial y}\frac{\partial N^e}{\partial y}\right)dA - \oint Q_c N^e dS = 0 \end{array}\right) = 0 \qquad (90)$$

Eq. 90 can be expanded to

$$\sum_{e=1}^{m}\left(\begin{array}{l}\iint_{A^e} N^e N^{eT} dA\left(\frac{\partial C^e}{\partial t}\right) + \iint_{A^e} N^e N^{eT} U^e \frac{\partial N^{eT}}{\partial x} dA(C^e) + \iint_{A^e} N^e N^{eT} V^e \frac{\partial N^{eT}}{\partial y} dA(C^e) \\ - S_c^e \iint_{A^e} N^e dA - \frac{D_x^e}{H} \iint_{A^e} N^e \frac{\partial N^{eT}}{\partial x} H^e \frac{\partial N^{eT}}{\partial x} dA(C^e) \\ - \frac{D_y^e}{H} \iint_{A^e} N^e \frac{\partial N^{eT}}{\partial y} H^e \frac{\partial N^{eT}}{\partial y} dA(C^e) + D_x^e \iint_{A^e} \frac{\partial N^e}{\partial x} \frac{\partial N^{eT}}{\partial x} dA(C^e) \\ + D_y^e \iint_{A^e} \frac{\partial N^e}{\partial y} \frac{\partial N^{eT}}{\partial y} dA(C^e) - Q_c \oint N^e dS \end{array}\right) = 0 \qquad (91)$$

which can be written in compact form as

$$\sum_{e=1}^{m}\left(\begin{array}{l} M^e\left(\frac{\partial C^e}{\partial t}\right) + M_u^e(C^e) + M_v^e(C^e) - M_s^e - M_{D_{hx}}^e(C^e) \\ - M_{D_{hy}}^e(C^e) + M_{D_x}^e(C^e) + M_{D_y}^e(C^e) - M_q^e \end{array}\right) = 0 \qquad (92)$$

Eq. 92 can be rearranged as

$$\sum_{e=1}^{m}\left[M^e \frac{\partial C^e}{\partial t} + (M_u^e + M_v^e + M_k^e - M_{D_{hx}}^e - M_{D_{hy}}^e + M_{D_x}^e + M_{D_y}^e)C^e\right] = M_s^e + M_q^e \qquad (93)$$

2.2 Gauss Integration of Isoparametric Elements

As previous mentioned, differential area (dA) are definded in terms of dξ and dη by

$$dA = |J|\, d\xi\, d\eta$$

By transforming isoparametric element and then substituted in Eq.(93), element matrix can be written as

$$M^{e}\frac{\partial C^{e}}{\partial t}+(M_{u}^{e}+M_{v}^{e}+M_{k}^{e}-M_{D_{hx}}^{e}-M_{D_{hy}}^{e}+M_{D_{x}}^{e}+M_{D_{y}}^{e})C^{e}=M_{s}^{e}-M_{q}^{e} \qquad (94)$$

where

$$M^{e} = \int_{0}^{1}\int_{0}^{1} N^{e} N^{eT} \left|J\right|^{e} d\xi d\eta$$

$$M_{u}^{e} = \int_{0}^{1}\int_{0}^{1} N^{e} N^{eT} U^{e} \frac{\partial N^{eT}}{\partial x} \left|J\right|^{e} d\xi d\eta$$

$$M_{v}^{e} = \int_{0}^{1}\int_{0}^{1} N^{e} N^{eT} V^{e} \frac{\partial N^{eT}}{\partial y} \left|J\right|^{e} d\xi d\eta$$

$$M_{D_{hx}}^{e} = \frac{D_{x}^{e}}{H^{e}}\int_{0}^{1}\int_{0}^{1} N^{e} \frac{\partial N^{eT}}{\partial x}.H^{e}.\frac{\partial N^{eT}}{\partial x} \left|J\right|^{e} d\xi d\eta$$

$$M_{D_{hy}}^{e} = \frac{D_{y}^{e}}{H^{e}}\int_{0}^{1}\int_{0}^{1} N^{e} \frac{\partial N^{eT}}{\partial y}.H^{e}.\frac{\partial N^{eT}}{\partial y} \left|J\right|^{e} d\xi d\eta$$

$$M_{D_{x}}^{e} = D_{x}^{e}\int_{0}^{1}\int_{0}^{1} \frac{\partial N^{e}}{\partial x}.\frac{\partial N^{eT}}{\partial x} \left|J\right|^{e} d\xi d\eta$$

$$M_{D_{y}}^{e} = D_{y}^{e}\int_{0}^{1}\int_{0}^{1} \frac{\partial N^{e}}{\partial y}.\frac{\partial N^{eT}}{\partial y} \left|J\right|^{e} d\xi d\eta$$

$$M_s^e = S^e \int_0^1 \int_0^1 N^e \left|J\right|^e d\xi d\eta$$

$$M_q^e = Q_c^e \oint N^e dS$$

The element matrix can be calculated by using the numerical integration method according to Gauss-Legendre integration as

$$\int_{-1}^{1}\int_{-1}^{1} f(\xi,\eta)\left|\mathrm{J}(\xi,\eta)\right| d\xi d\eta = \sum_{i=1}^{m}\sum_{j=1}^{n} w_i w_j f(\xi_i,\eta_j)\left|\mathrm{J}(\xi,\eta)\right| \quad (95)$$

where

m and n are the number of Gauss points

W_i and W_j are weights of Gauss points at i = 1, 2, 3, 4 and j = 1, 2, 3, 4 respectively

ξ_i and η_j are Gauss points locations

$\left|\mathrm{J}(\xi,\eta)\right|$ is the determinant of Jacobian matrix.

In a case of rectangular element with m = n = 2, weights of Gauss points W_i =1, W_j = 1 and Gauss point locations $\xi_i = \pm \frac{1}{\sqrt{3}}$, and $\eta_j = \pm \frac{1}{\sqrt{3}}$.

2.3 Computation of system equation

The substance dispersion equation which obtained from these element matrices can be expressed in term of system equation as follows.

$$M \frac{\partial C}{\partial t} + (M_u + M_v - M_{D_x/H} - M_{D_y/H} + M_{D_x} + M_{D_y})C = M_s + M_q \quad (96)$$

which can be written in compact form as

$$M\left(\frac{\partial C}{\partial t}\right) + F.C = G \tag{97}$$

Where

$$F = M_u + M_v - M_{D_{hx}} - M_{D_{hy}} + M_{D_x} + M_{D_y}$$

$$G = M_s + M_q$$

In the case of unsteady state condition, the value of substance concentration (C) at the next time step ($t + \Delta t$) can be computed from

$$C^{t+\Delta t} = \left(\frac{M}{\Delta t} + \frac{F}{2}\right)^{-1}\left[\left(\frac{M}{\Delta t} - \frac{F}{2}\right)C^t + G\right] \tag{98}$$

3. Verification of dispersion model

For a uniform channel with constant flow velocity and assuming constant value of the dispersion coefficient, the substance equation can be written as

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial t} - D_x\frac{\partial^2 c}{\partial x^2} + kc = 0 \tag{99}$$

where

c is the substance concentration

u is the flow velocity

D_x is the longitudinal dispersion coefficient, and

k is the decaying rate of substance

Once the initial and boundary conditions are specified, the solution can be obtained. The following cases are considered.

3.1 Channel with specified concentrations at one end

For a uniform channel with specified substance concentration at the upper end. (Figure 6)

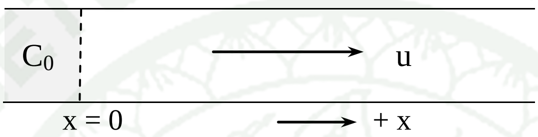


Figure 6 Uniform channel with specified substance concentration at the upper end.

The boundary and initial condition are given as

$c(0,t) = C_0$; $t \geq 0$

$c(\infty,t) = 0$; $t \geq 0$

$c(x,t) = 0$; $x \geq 0$

The following solution is obtained (Odata and Banks, 1961)

$$\frac{c}{C_0} = \frac{1}{2} e^{\frac{xu}{2D_x}} \left[\begin{array}{l} \exp\left\{ \frac{x}{2D_x}\sqrt{u^2 + 4D_x k} \right\} erfc\left\{ \frac{x + \left(\sqrt{u^2 + 4D_x k}\right)t}{\sqrt{4D_x t}} \right\} \\ + \exp\left\{ -\frac{x}{2D_x}\sqrt{u^2 + 4D_x k} \right\} erfc\left\{ \frac{x - \left(\sqrt{u^2 + 4D_x k}\right)t}{\sqrt{4D_x t}} \right\} \end{array} \right] \qquad (100)$$

For steady state we obtain

$$\frac{c}{C_0} = \exp\left\{\frac{x}{2D_x}\left(u + \sqrt{u^2 + 4D_x k}\right)\right\}$$

when the substance concentration at the lower end is specified (figure 7)

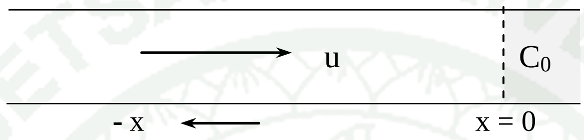


Figure 7 Uniform channel with specified substance concentration at the lower end.

The boundary and initial condition are given by

$C(0,t) = C_0$; $t \geq 0$

$c(-\infty,t) = 0$; $t \geq 0$

$c(x,t) = 0$; $-\infty < x < 0$

The solution can be obtained by substituting x and u in eq.100 by –x and –u, respectively, i.e.

$$\frac{c}{C_0} = \frac{1}{2} e^{\frac{xu}{2D_x}} \left[\begin{array}{l} \exp\left\{-\frac{x}{2D_x}\sqrt{u^2 + 4D_x k}\right\} erfc\left\{\frac{-x + \left(\sqrt{u^2 + 4D_x k}\right)t}{\sqrt{4D_x t}}\right\} \\ + \exp\left\{\frac{x}{2D_x}\sqrt{u^2 + 4D_x k}\right\} erfc\left\{\frac{-x - \left(\sqrt{u^2 + 4D_x k}\right)t}{\sqrt{4D_x t}}\right\} \end{array} \right] \quad (101)$$

For steady state we obtained

$$\frac{c}{C_0} = \exp\left\{\frac{x}{2D_x}\left(u + \sqrt{u^2 + 4D_x k}\right)\right\} \tag{102}$$

The finite element grid used for verifying the developed dispersion model of the above cases is shown in Figure 8 a uniform channel of 20 km. in length, 1 km. in width, and 5 m. in depth is divided into 20 elements with 42 nodes. In the case that the substance concentration is specified at the lower end, the flow velocity of 0.1 m/s, dispersion coefficient of 100 m^2/s, and decaying rate of 1 day^{-1} are specified. The computation is started with zero substance concentration throughout the channel. The concentration distribution at time t=6 hours and steady state condition are shown in figure 9

In the case that the substance concentration at the lower end is specified, the same values of flow velocity and the dispersion coefficient are used but with no decaying rate (k=0). The obtained distribution pattern at steady state condition is shown in figure 10.

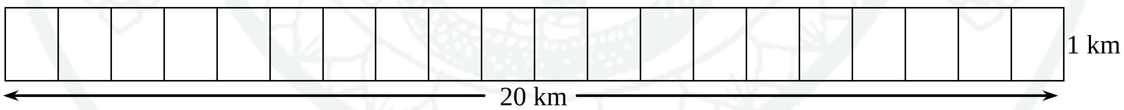


Figure 8 Finite element grid for uniform channel with specified concentration at one end

In this verification, the time increments of 3,600 and 7,200 seconds have been tried. The results obtained with the both time increments are quite close to the analytical solution as shown in Figure 9.

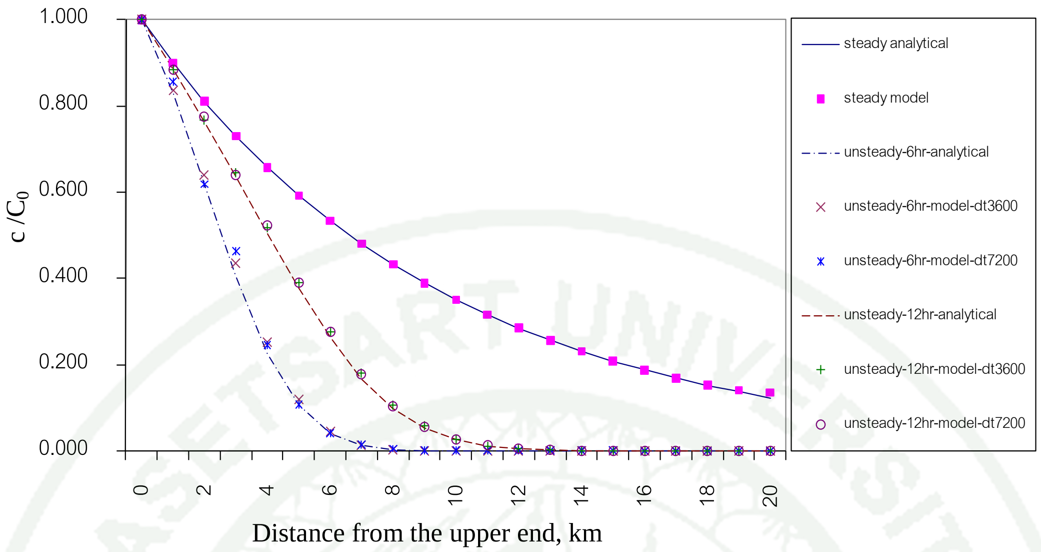


Figure 9 Dispersion on a channel with specified concentration at the upper end (decaying rate = 1.0 day^{-1}, D_x=100 m^2/s)

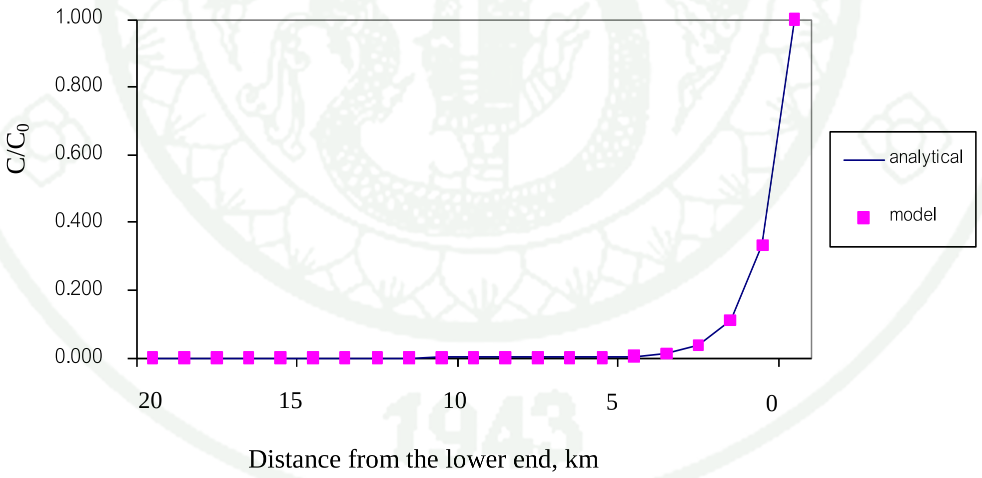


Figure 10 Steady state dispersion in a channel with specified concentration at the lower end (decaying rate = 1.0 day^{-1}, D_x=100 m^2/s)

5. Model Input Data

The amounts of waste loads discharged into the Songkhla Lake was estimated based on existing land use pattern, population density, livestock, aquaculture, and farming activities. The results obtained from field sampling and laboratory analyses were used as initial conditions. The results obtained from the hydrodynamic model using the same element configuration were also fed as input data of these sets of water quality models. The important parameters used for calculation in the models are shown in Table 8.

Table 8 The important parameters for model calculation.

parameter	description	value
G_{pn}	maximum growth rate of phytoplankton	0.06 day^{-1}
G_z	phytoplankton nitrogen grazing rate per unit weight of zooplankton,	5 m^3/g
R_{pn}	respiration rate of phytoplankton	0.004 day^{-1}
R_{on}	Mineralization / hydrolysis rate of organic nitrogen to ammonia	0.15 day^{-1}
R_{an}	Rate of ammonia nitrogen oxidation to nitrite nitrogen	0.1 day^{-1}
R_{ti}	Rate of nitrite nitrogen oxidation to nitrate nitrogen	0.3 day^{-1}
R_{op}	Respiration rate of phytoplankton phosphorus	0.001 day^{-1}
PhyN_Chl	Ratio of nitrogen in phytoplankton cell to chlorophyll-a	10
P_PhyN	Ratio of phosphorus to nitrogen in phytoplankton cell	0.1
k_{mN}	Michaelis-Menton’s constant for Nitrogen limiting growth	0.03 µg-N
k_{mP}	Michaelis-Menton’s constant for Phosphorus limiting growth	0.01 µg-P
k_{Sal}	Michaelis-Menton’s constant for Salinity limiting growth	0.07

Table 8 (Continue)

parameter	description	value
$U_{An,max}$	Ammonia nitrogen uptake rate per unit weight of phytoplankton nitrogen	0.06 hr^{-1}
$U_{ta,\ max}$	Nitrate nitrogen uptake rate per unit weight of phytoplankton nitrogen	0.03 hr^{-1}

Source: Gonanone, 2004

6. Model Calibration

The calibration of dispersion models were undertaken with measured field data. Firstly, the salinity model was applied to the Songkhla Lake. Since the total dissolved salt can be considered as conservative substance, the term representing decaying rate of salinity was set to zero. The salinity at the lake mouth boundary (about 30 ppt.) was specified as boundary value in the salinity model. The dispersion coefficient was varied from 50-100 m^2/sec. From the model simulation it was found that the dispersion coefficient of 50 m^2/sec could provide satisfactory agreement between observed salinity and computed salinity (Figure 11). The error was in range of 0.001 – 3.9 ppt. with average % difference about 14.4%.

The value of dispersion coefficient obtained from the salinity model calibration was then used in the other models. The phytoplankton model calibration with measured field data provided satfactorry result as shown in figure 12. The error was in range of -0.04 – 0.18 mg-N/l with average % difference about 15.19%. Then the calibration of nutrient dispersion models were computed. The parameters in the models were calibrated within the normal ranges suggested by Gonanone (2004). It was found that simulation results could provide satisfactory agreement with measured field data.

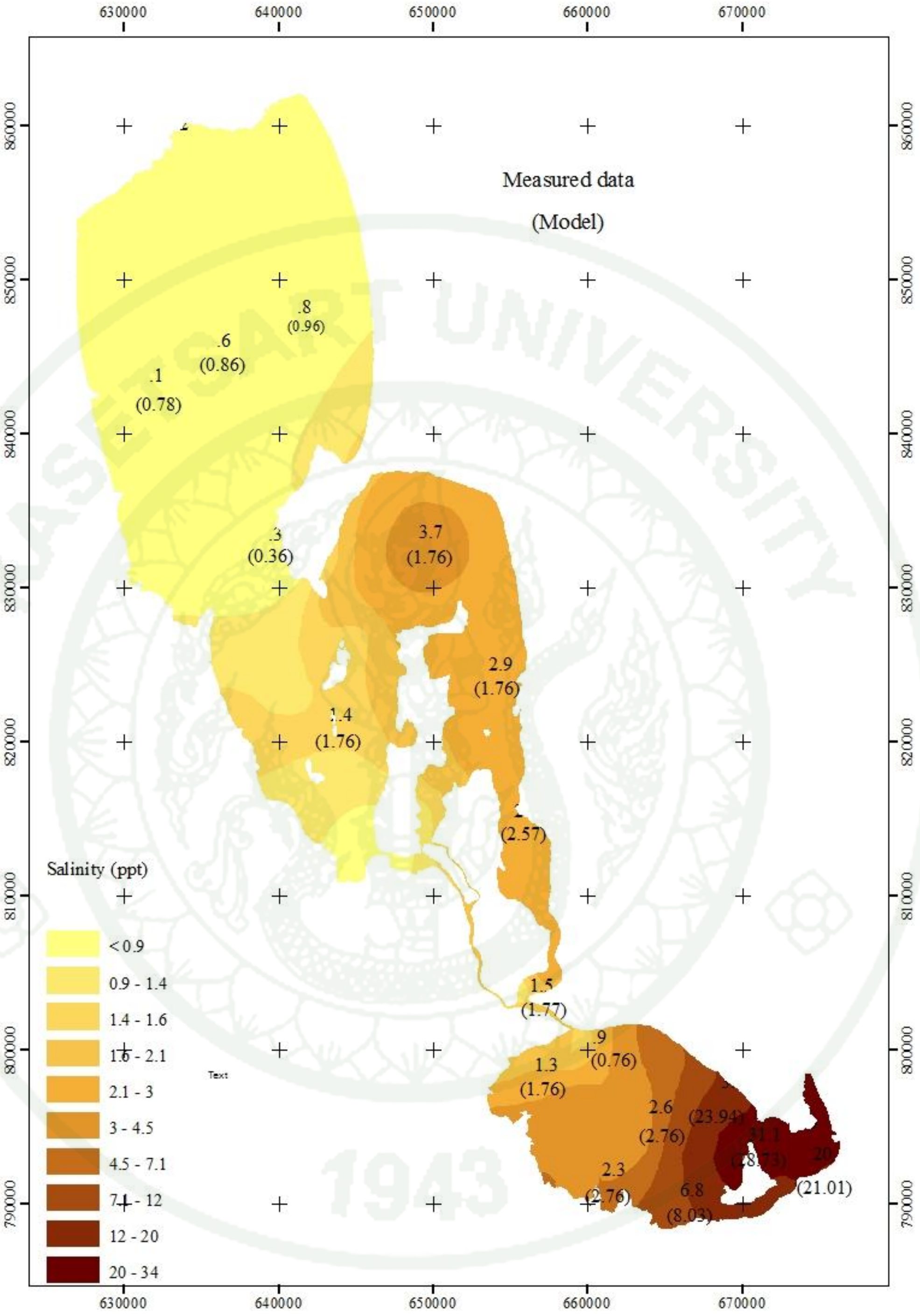


Figure 11 Salinity calibration with measured field data

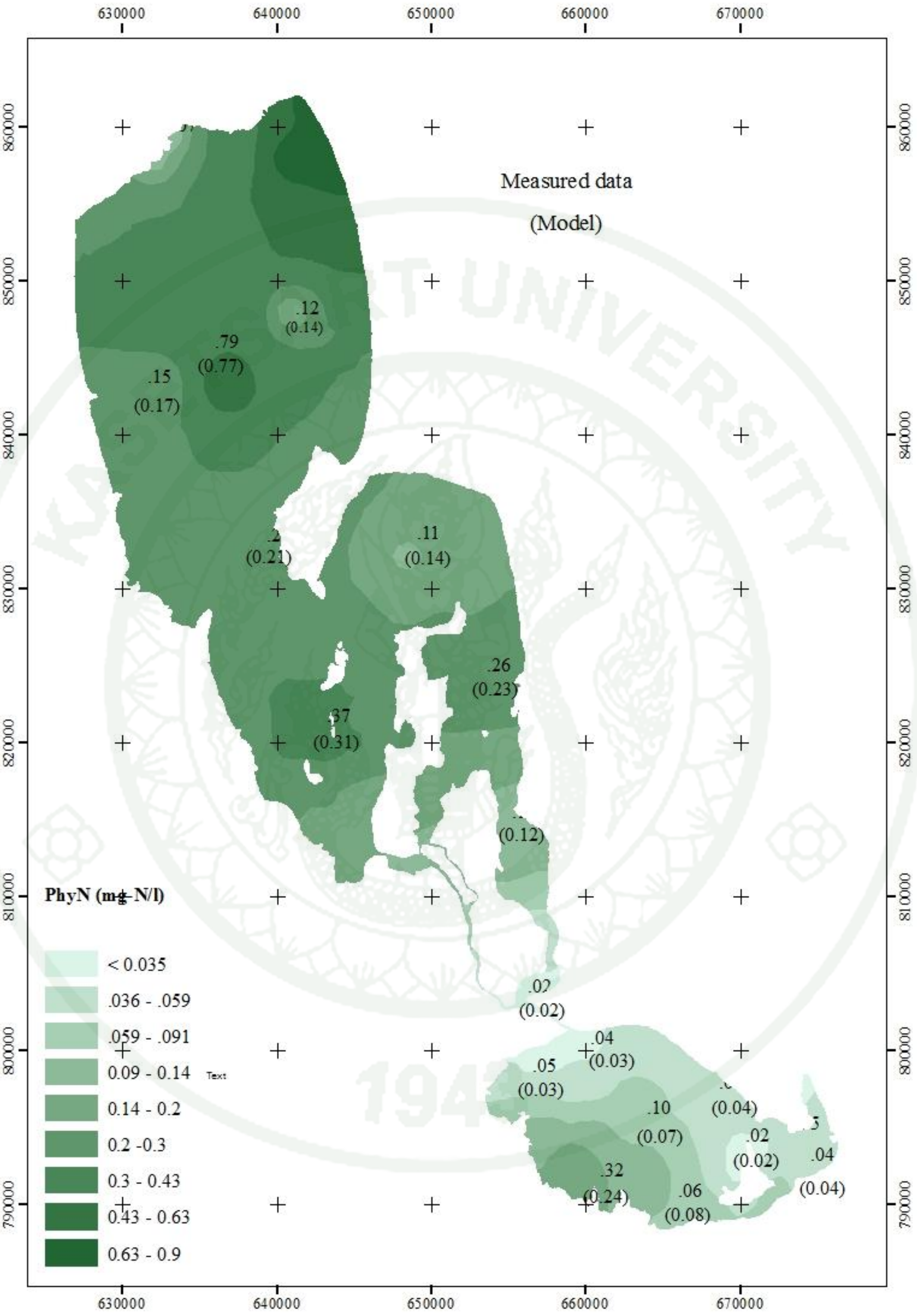


Figure 12 Phytoplankton nitrogen calibration with measured field data

7. Application to Songkhla Lake

The developed phytoplankton model is applied to compute the distribution pattern of phytoplankton population relation to nutrients concentration (i.e.dissolved nitrogen and phosphorus). The area of 1,182 square-kilometer is divided into 138 iso-parametric elements with 220 nodal points. The finite element grid of Songkhla Lake shown in figure 13 is used in application of phytoplankton dispersion models.

The nutrient loading data is obtained from the study of Regional Environment Office 16 (Table 9) and the initial value is the field data which average for each part of Songkhla lake as shown in table 10.

Table 9 Nutrients loading data

Channel	Q^* (m^3/s)	cross section area* (m^2)	Nutrients concentration					
			OrgN mgN/l	NH_4 mgN/l	NO_2 mgN/l	NO_3 mgN/l	OrgP mgP/l	PO_4 mgP/l
Ra Node channel	2.2	56×1.25	0.0263	0.1494	0.2589	0.0235	0.0040	0.1554
Pak Pra channel	67.38	118×2.9	0.1775	0.0701	0.0781	0.0641	0.0273	0.0420
Lum Pum channel	29.19	60.5×1.4	0.0500	0.1405	0.0227	0.0600	0.0333	0.0431
Tha Chead channel	1.99	25×1.4	0.3628	0.0445	0.0149	0.0495	0.0009	0.0887
Pa Bon channel	1.47	28×1.25	0.1688	0.0295	0.0769	0.0236	0.0334	0.0269
Rattaphum channel	9.18	44×1.25	0.1047	0.2083	0.2702	0.7832	0.0008	0.2128
U-Taphao channel	57.45	68×2.1	0.2150	1.500	0.8630	2.6825	0.0391	0.1703

Source: Regional Environment Office 16

Table 10 Initial value condition

	PhyN	OrgN	NH4	NO2	NO3	OP	PO4
Upper lake	0.3768	0.2576	0.0233	0.0069	0.0947	0.0275	0.0425
Middle lake	0.1365	0.1975	0.0741	0.0147	0.1099	0.0298	0.0519
Lower lake	0.0747	0.1640	0.0588	0.0120	0.1507	0.04673	0.0232

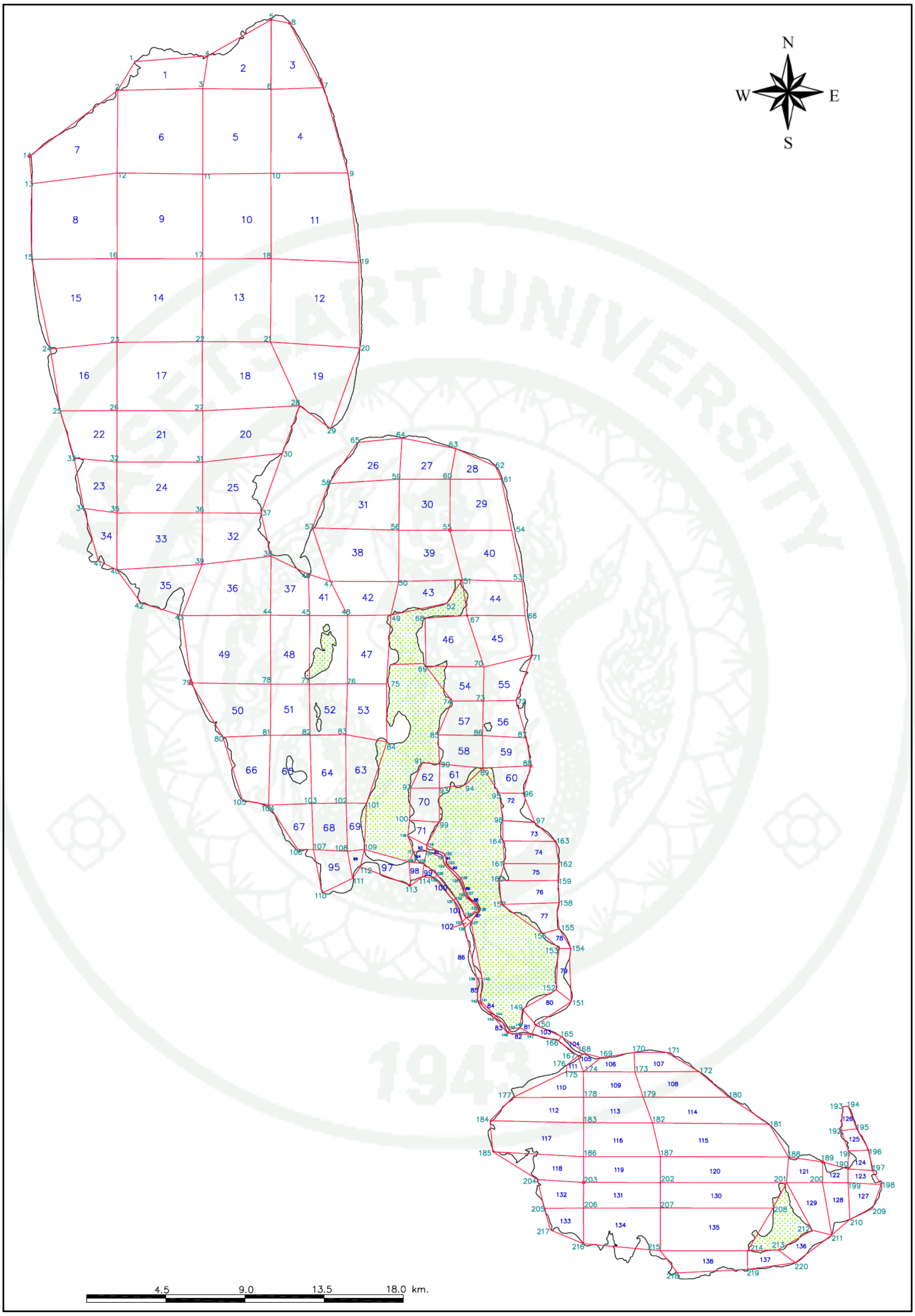
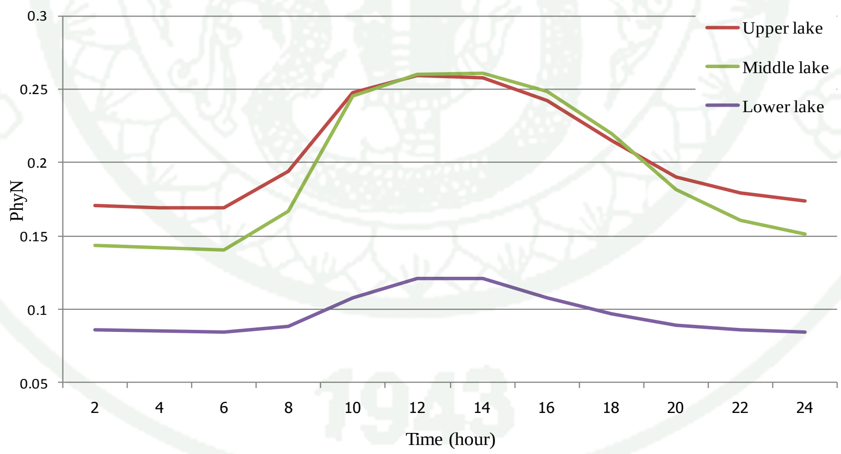


Figure 13 Finite element grid of the Songkhla Lake.

The developed phytoplankton and nutrient dispersion models is applied to Songkhla lake. The computation time is 1 month which has high nutrients loading discharged into Songkhla lake, i.e. November 2009. The temporal variation and spatial distribution of phytoplankton and various form of nutrients computed from the model are described as follow.

7.1 Temporal variation

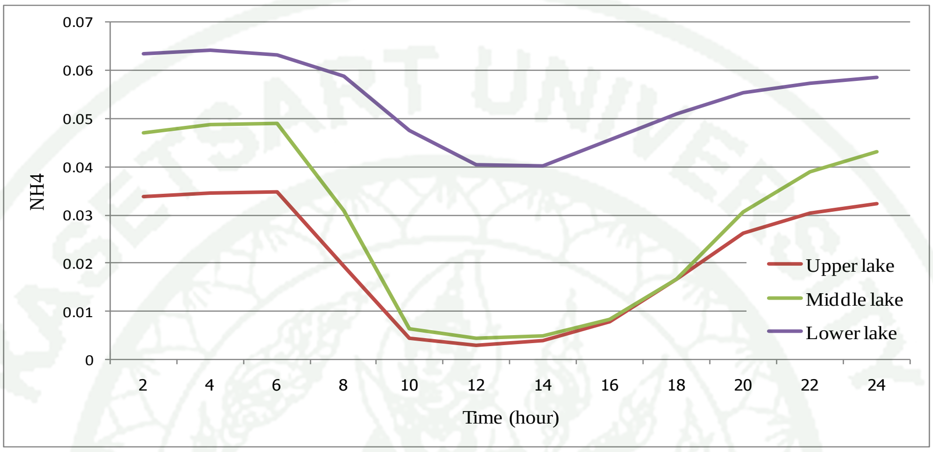
The daily variation of phytoplankton and nutrients concentration was shown in figure 14. During day time, the phytoplankton concentration gradually increases during 6.00 a.m. to 12.00 a.m. (figure 14 (a)), this may due to the higher growth rate than respiration rate. The gradually decreasing concentration is found during 12.00 a.m. to 6.00 p.m. which the light limitation factor is decrease. During nighttime, before 6.00 a.m. and after 6.00 p.m. the phytoplankton concentration is little vary due to the dispersion and advection transport.



(a) Phytoplankton

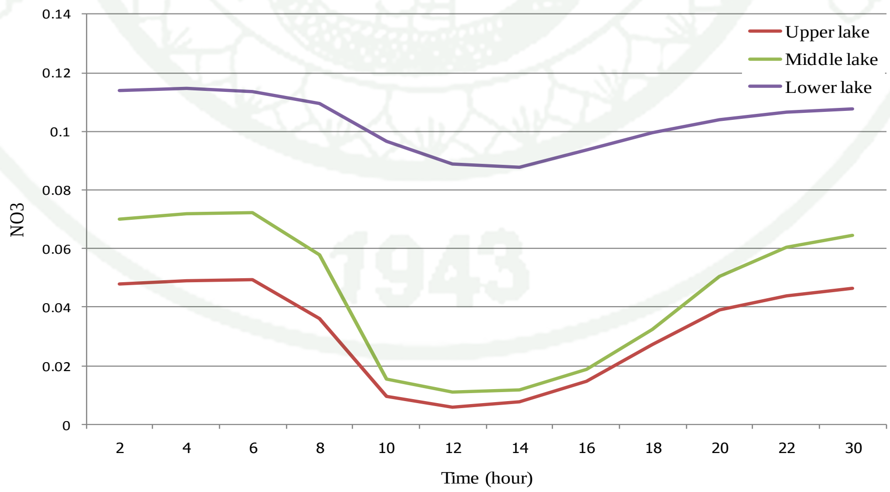
Figure 14 Daily variation of phytoplankton and nutrients concentration

The daily variation concentration of ammonia nitrogen, nitrate nitrogen, inorganic phosphorus are shown in figure 14 (b) – (d), respectively. These nutrient form are available for phytoplankton uptake. Therefore, in high uptake rate of phytoplankton (6.00 a.m. to 12.00 a.m.) the nutrients concentration gradually decrease. While the uptake rate of phytoplankton gradually decrease, the concentration of these form of nutrient gradually increase.



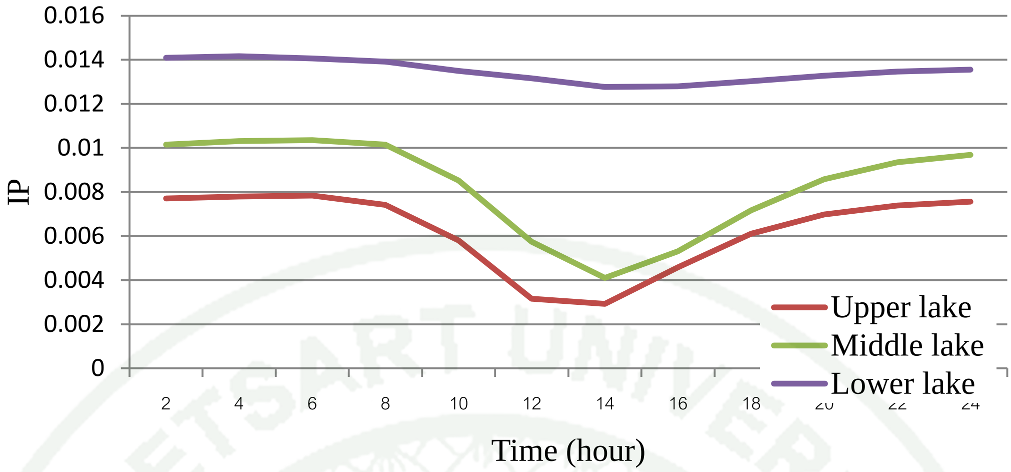
(b) Ammonia nitrogen

Figure 14 (Continued)



(c) Nitrate nitrogen

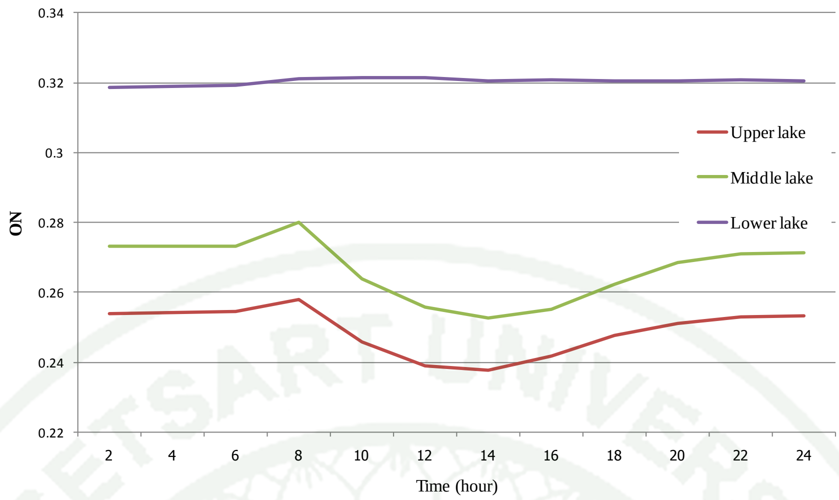
Figure 14 (Continued)



(d) Inorganic phosphorus

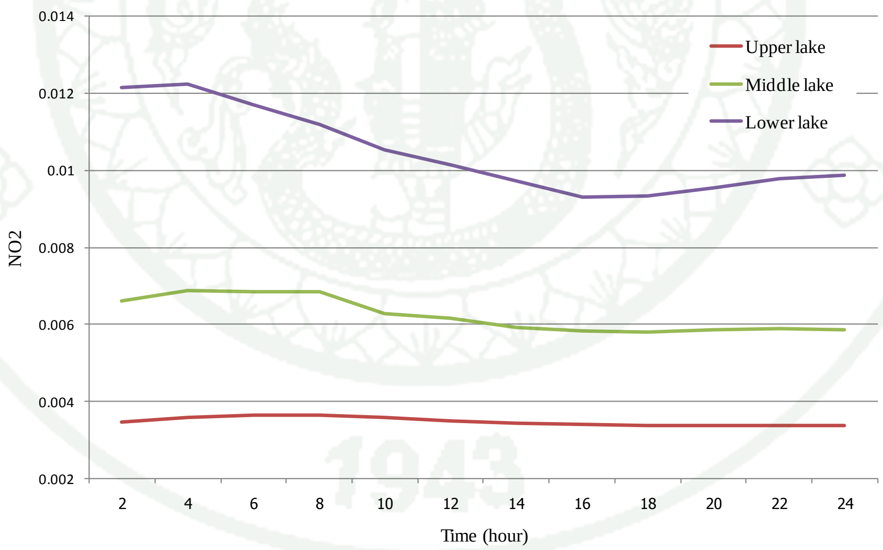
Figure 14 (Continued)

The daily variation of remaining nutrients concentration, i.e. organic nitrogen, nitrite nitrogen, and organic phosphorus, are shown in figure 14 (e) – (g). The pattern of concentration variation is the same as the concentration variation pattern of ammonia nitrogen, nitrate nitrogen and inorganic phosphorus. However the range of variation is less than the variation concentration of available nutrients group.



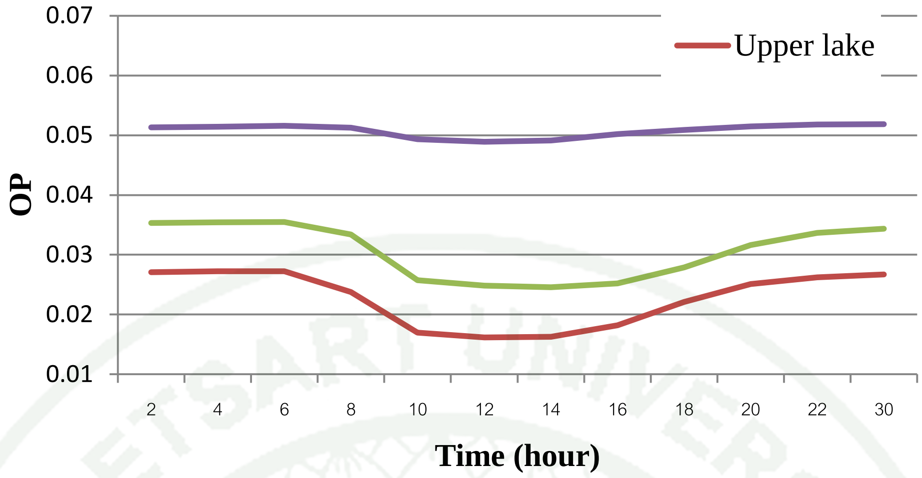
(e) Organic nitrogen

Figure 14 (Continued)



(f) Nitrite nitrogen

Figure 14 (Continued)



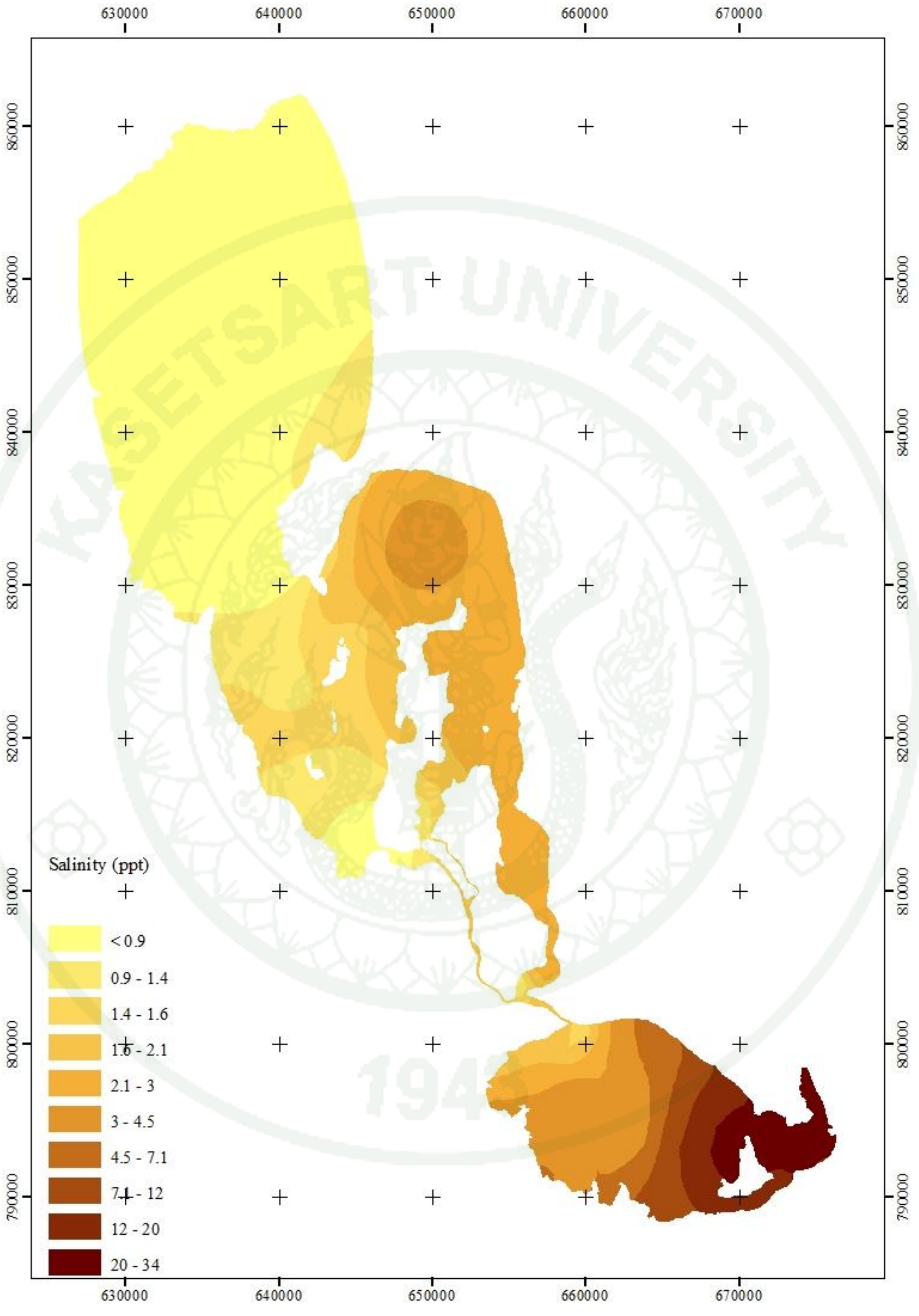
(f) Organic hosphorus

Figure 14 (Continued)

7.2 Spatial distribution

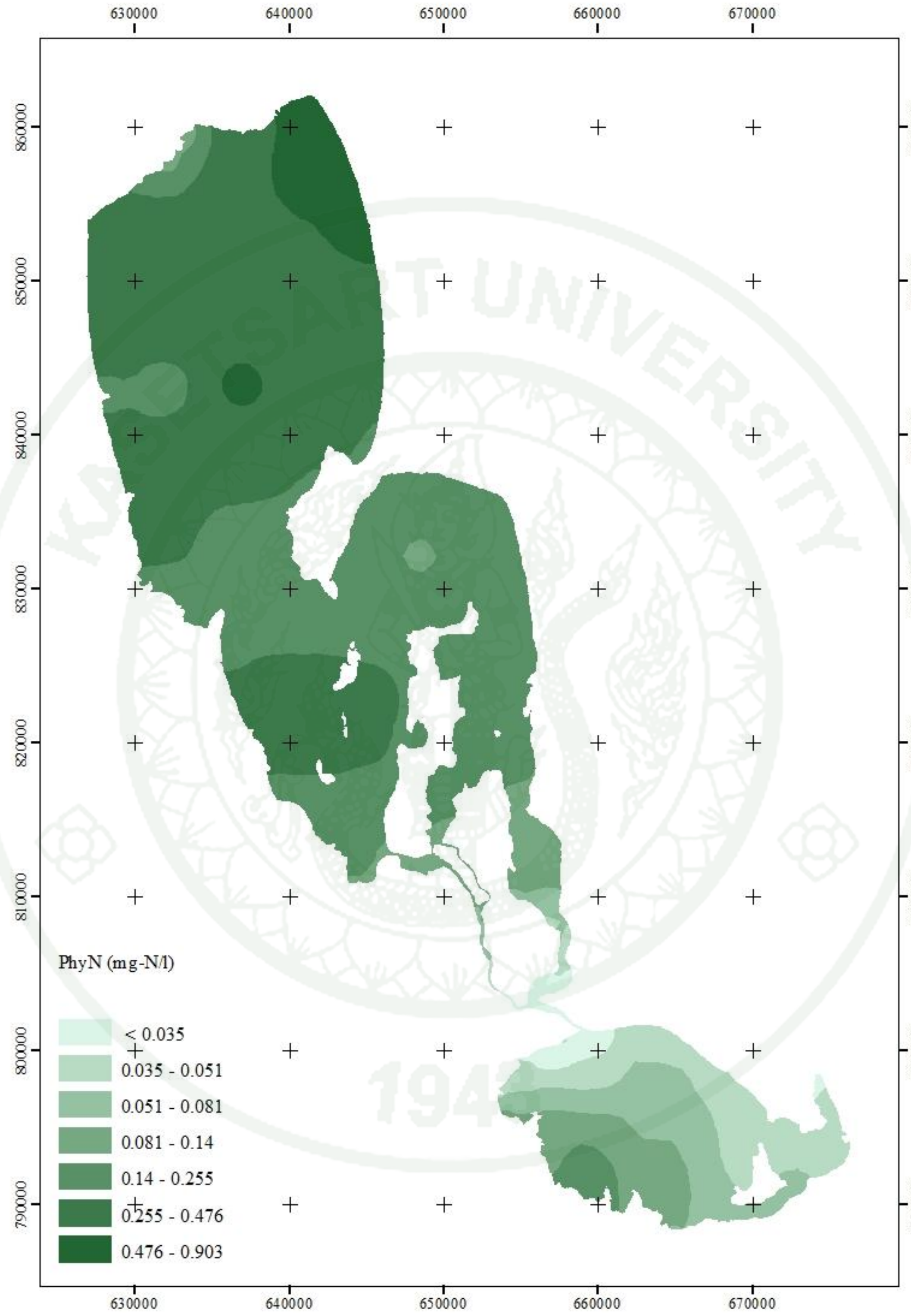
The spatial distribution of phytoplankton and various forms of nutrients are shown in figure 15. The results obtained from model computation correlate to the measured field data. In the Upper Lake and the Middle Lake, the phytoplankton concentration were higher than the phytoplankton concentration in the Lower Lake (figure 15 (b)). It might due to the shallow water depth and lower salinity value in the Upper Lake and the Middle Lake which are suitable for phytoplankton growth.

On the other hand, the concentrations of various nutrients were highest in the Lower Lake (figure 15 (c) – (h)). This might due to the highest nutrient loading from high population communities and factories located along U-Taphao channel and Pak Phawong channel particularly in the rainy season (La-ongsiriwong et al, 2004).



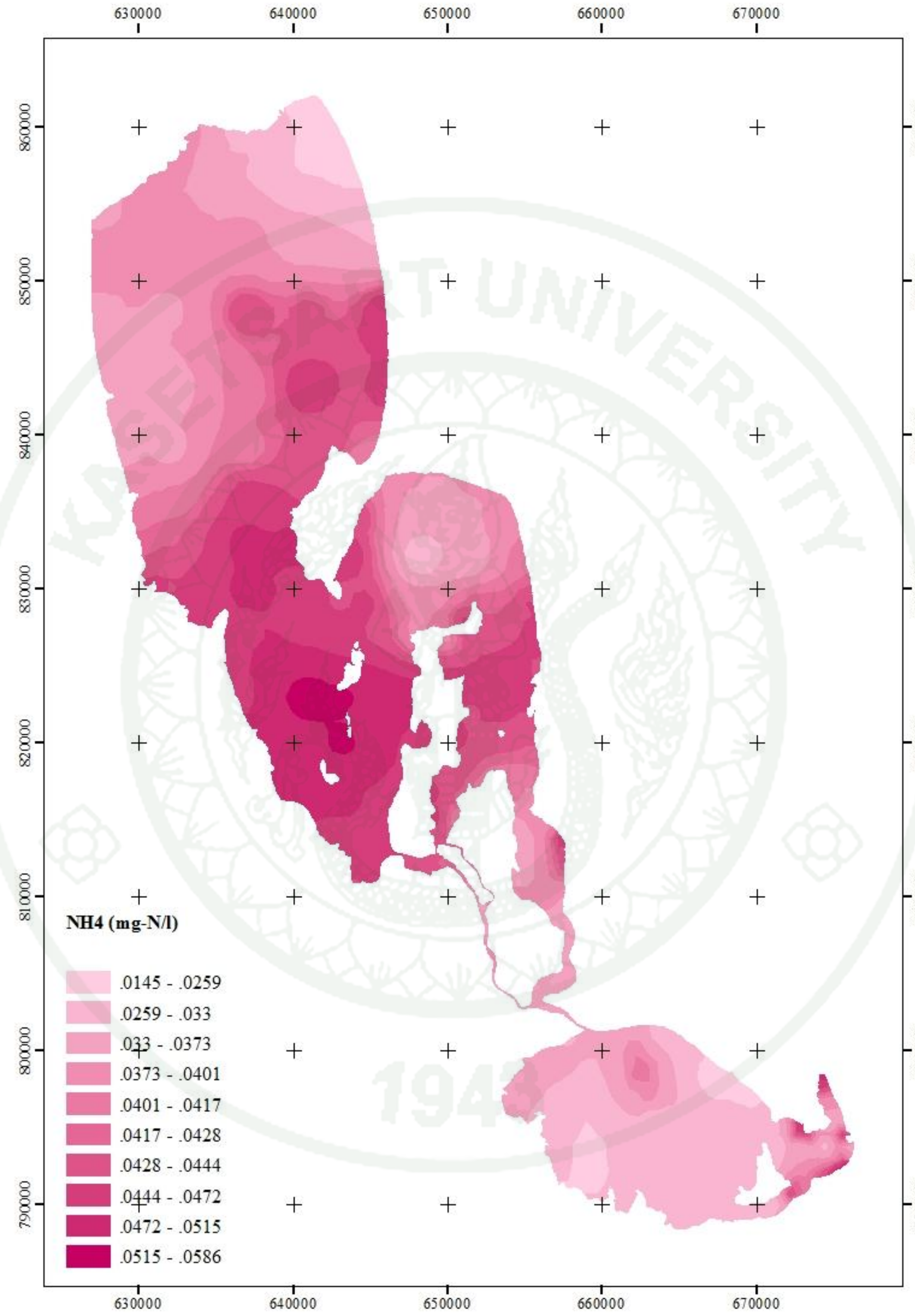
(a) salinity

Figure 15 Spatial distribution of phytoplankton and nutrient in the Songkhla Lake



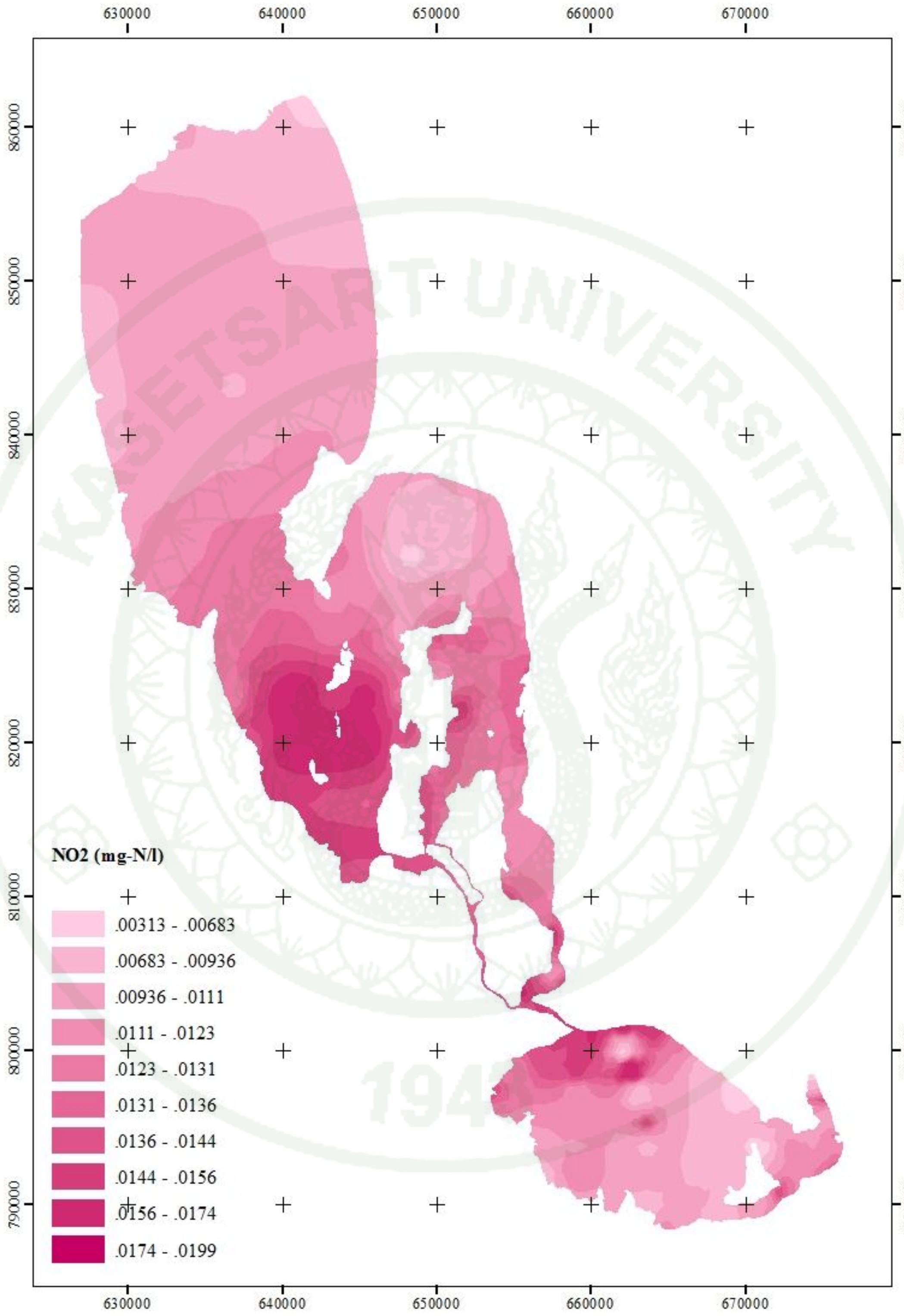
(b) Phtoplankton

Figure 15 (Continued)



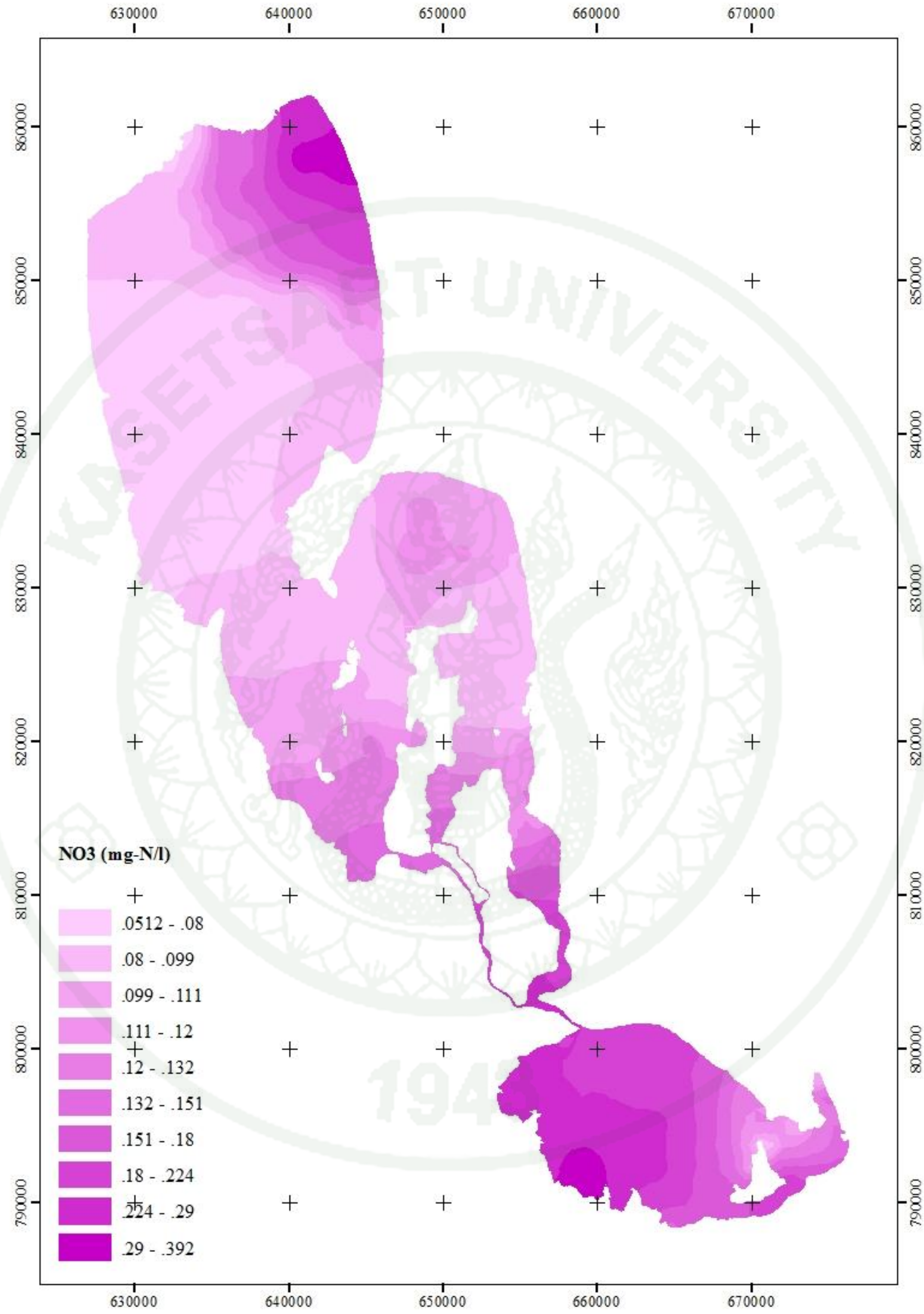
(c) Ammonia nitrogen

Figure 15 (Continued)



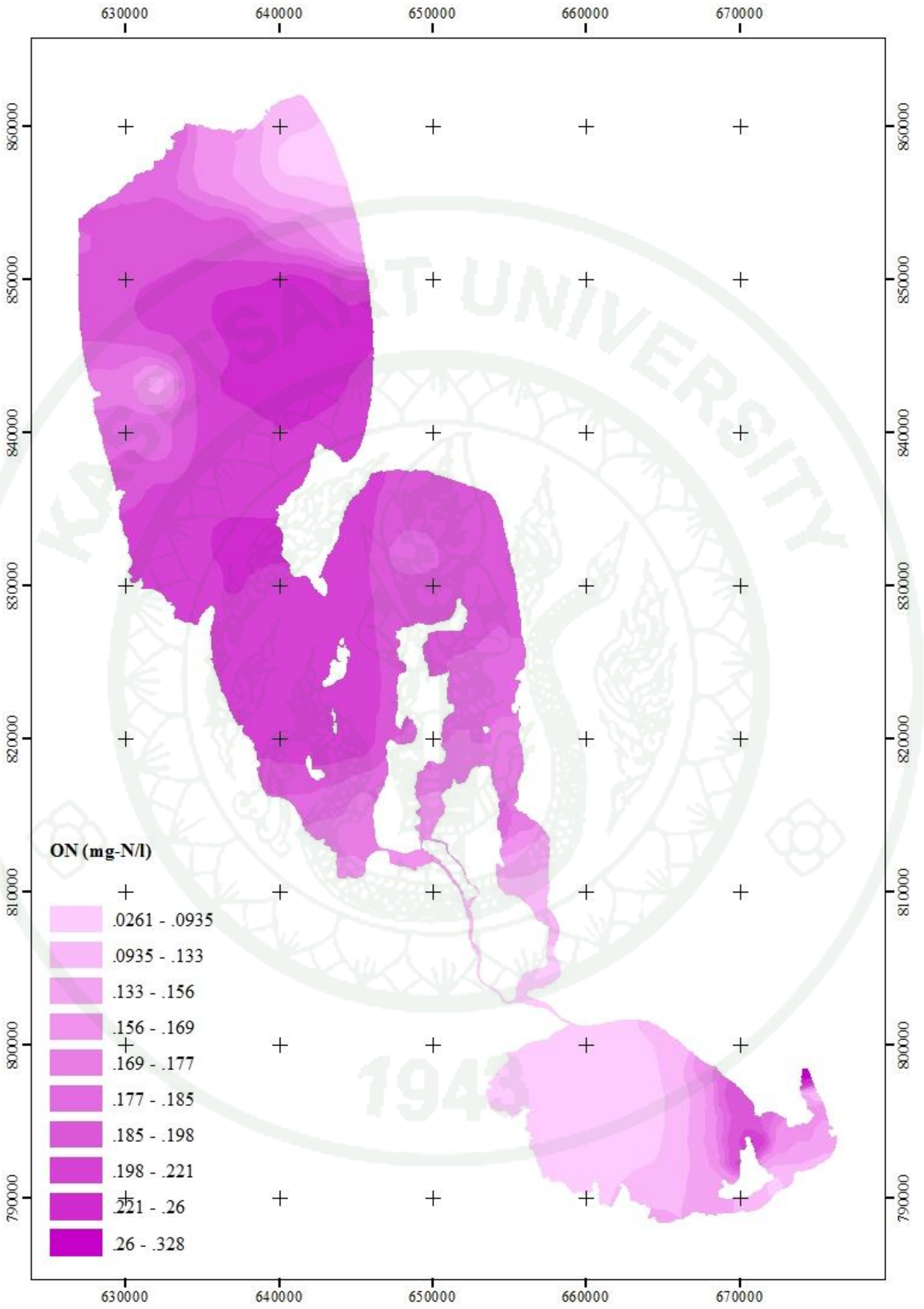
(d) Nitrite nitrogen

Figure 15 (Continued)



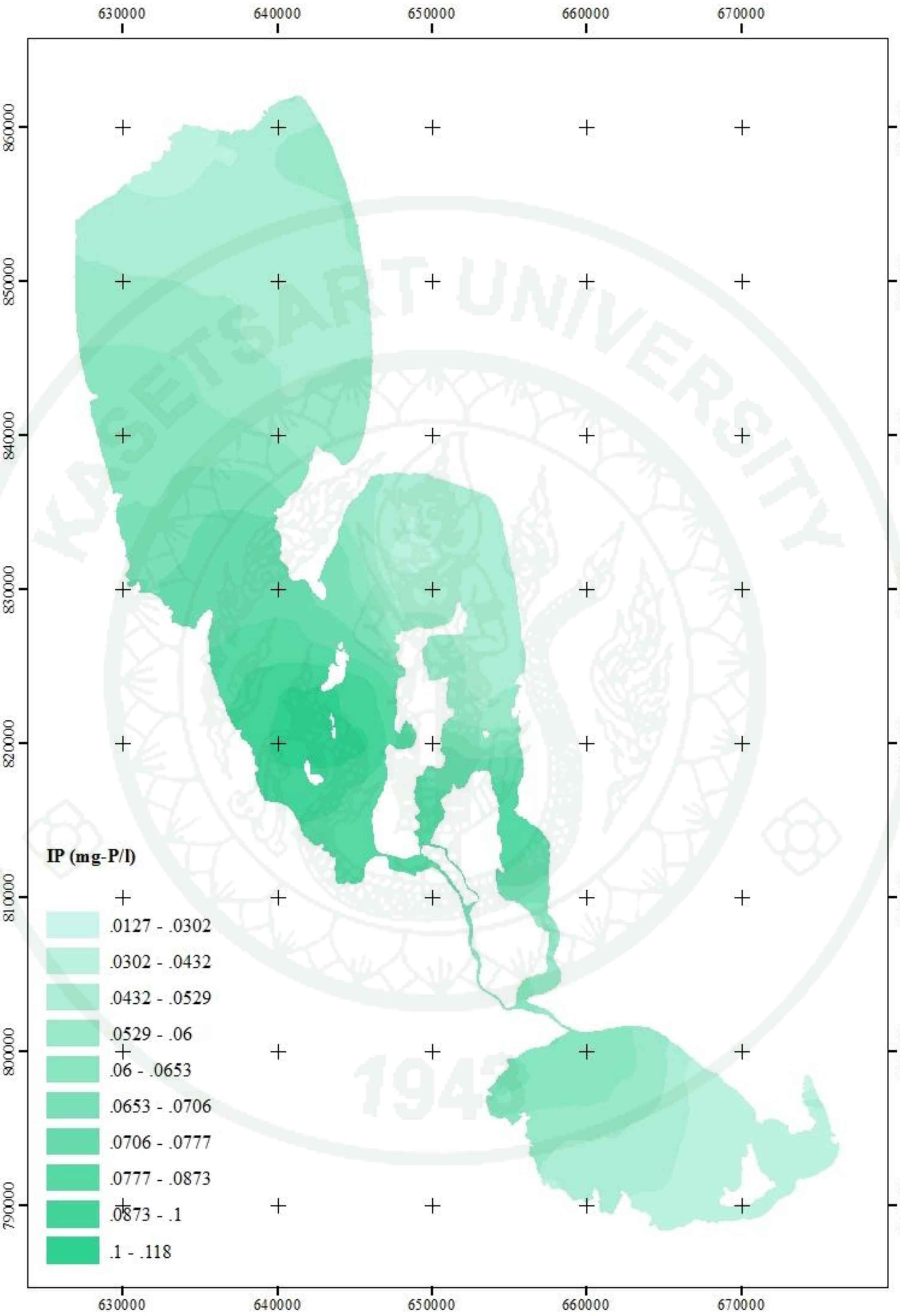
(e) Nitrate nitrogen

Figure 15 (Continued)



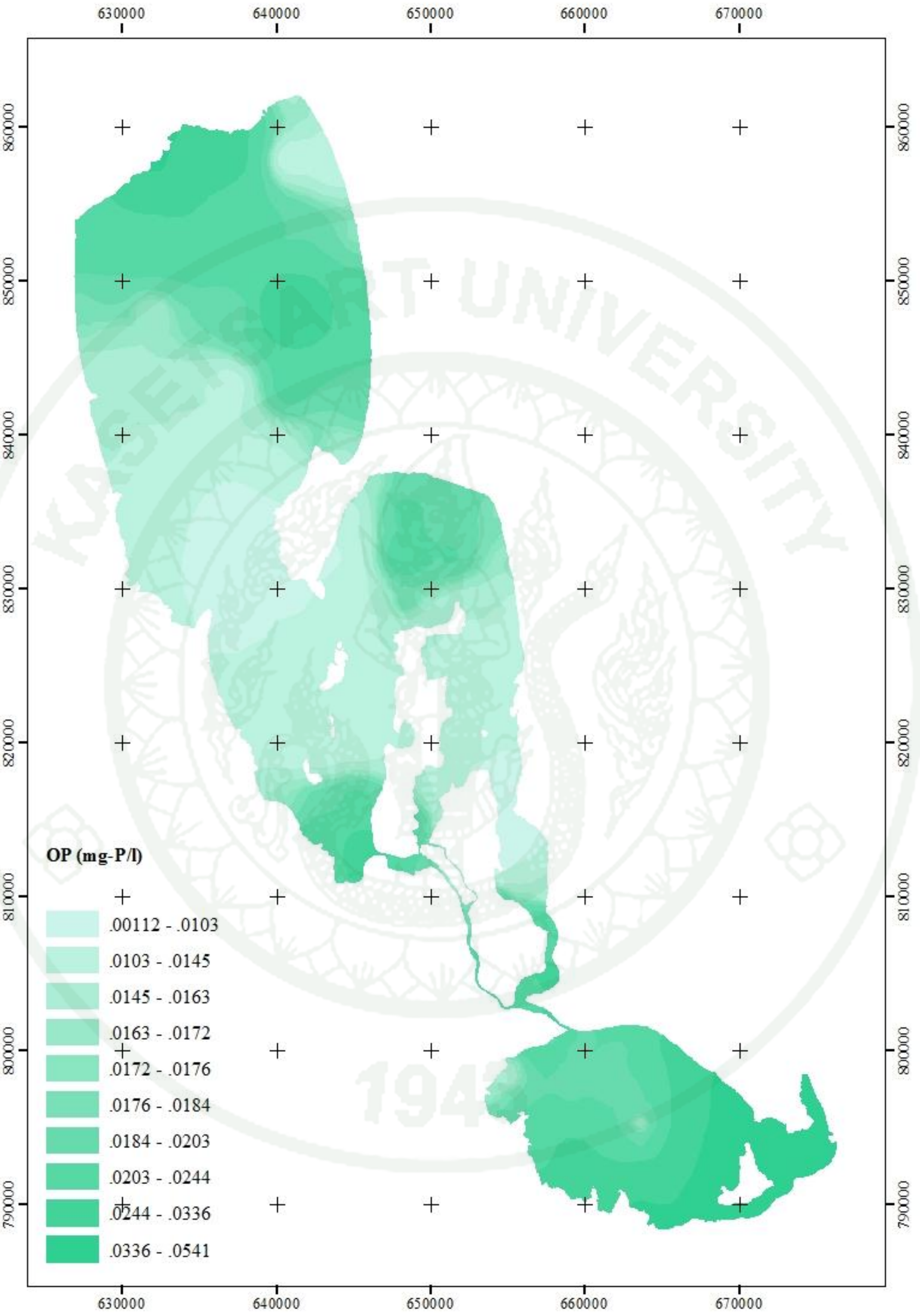
(f) Organic nitrogen

Figure 15 (Continued)



(g) Inorganic phosphorus

Figure 15 (Continued)



(h) Organic phosphorus

Figure 15 (Continued)

CONCLUSION AND RECOMMENDATION

Conclusion

In this study, the mathematical models describing dispersion of phytoplankton and nutrients were developed. The dispersion model was formulated base on the 2 dimensional vertically average mass balance equation. The select parameters in this study included phytoplankton nitrogen, organic nitrogen, ammonia nitrogen, nitrite nitrogen, nitrate nitrogen, organic phosphorus, and inorganic phosphorus. The finite element method with Galerkin's weighted residual technique was used in the formulation of the numerical model. The iso parametric element with linear interpolation function was used. Computer program used in this study was MATLAB.

All the developed water quality models were run to simulate salinity, phytoplankton and nutrient concentrations simultaneously during November – December 2009. The results obtained from the models showed that there were some variations in concentrations of phytoplankton nitrogen as well as various forms of nitrogen and phosphorus between day and night in each day. This was due the effect of sun light intensity on phytoplankton uptake and growth rates. However, these variations were rather small. The daily averaged spatial distributions of phytoplankton and various forms of nutrients was found that in the Upper Lake and the Middle Lake, the phytoplankton concentrations were higher than in the Lower lake. It might be due to the shallower water depth and lower salinity value in the Upper and Middle lakes which are suitable for phytoplankton growth. On the other hand, the concentrations of various nutrients in the Lower Lake were higher due to higher discharge loading.

In General the reliability of all models simulation were depended on the completion and adequacy of required data. This study had introduced all available data and calibrated each model correctly as much as possible. The conclusions in this model works were described as follows;

1) Water depth in the lake is an important factor which affects phytoplankton

growth. High contents of phytoplankton are found in the parts of the lake with shallow water.

2) The relationships between phytoplankton growth and nutrient concentrations are not so obvious in this study.

3) More detailed studies on suitable values of various parameters in the models are still required.

Recommendation

1. The three dimentional model should be considered for the vertical mixing that would enhanced the released of phosphorus from the bottom sediment. The laboratory experiment should be done for examine the correlation of nitrogen and phosphorus between water body and the sediment.
2. Songkhla Lake has diversity ecosystem due to the mixing of freshwater and saline water from the adjoining sea. This salinity gradient has a pronounced effect on the seasonal variability of the freshwater and marine phytoplankton in the lake including the effect on nutrient solubility. Therefore salinity limiting factor could be studied in model development.
3. The remote sensing had conducted as a new approach for calibrating the spatial distribution of chlorophyll-a by comparing with the satellite images. This may contribute to get the great valuable for model development.

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APPENDICES

Appendix A

Source Code of Main Program

Source Code of Main Program

```
%=============================================================
%%                    MAIN PROGRAM
%=============================================================
%%--------------------------------------------------------------------
%          Input node squence, x-y coordinate
%%--------------------------------------------------------------------
%
clear
%
NodeE=xlsread('elem_node');                                    % input element node
Node = [NodeE(:,2) NodeE(:,3) NodeE(:,4) NodeE(:,5)];       % node squence
[e,a] = size (Node);
NumEle = e;                                                    % number of elements
coordinate = xlsread('coorxy');                                % input coordinate x, y
[n,b] = size (coordinate);
NumNode = n;                                                   % number of total nodes
in system
%%-----------------define element coordinate----------------------
x=zeros(NumEle,4);
y=zeros(NumEle,4);
for i = 1:NumEle
   for j = 1 : 4
      x(i,j) = coordinate(Node(i,j),2);            % x coordinate
      y(i,j) = coordinate(Node(i,j),3);            % y coordinate
   end
end
%%-------------element volume calculation-------------------------
Area=zeros(NumEle,1);
for i=1:NumEle
   Lx1=x(i,1);
   Lx2=x(i,2);
   Lx3=x(i,3);
   Lx4=x(i,4);
   Ly1=y(i,1);
   Ly2=y(i,2);
   Ly3=y(i,3);
   Ly4=y(i,4);
   L1=sqrt(((Lx2-Lx1)^2)+((Ly2-Ly1)^2));
   L2=sqrt(((Lx3-Lx2)^2)+((Ly3-Ly2)^2));
   L3=sqrt(((Lx4-Lx3)^2)+((Ly4-Ly3)^2));
   L4=sqrt(((Lx1-Lx4)^2)+((Ly1-Ly4)^2));
   L13=sqrt(((Lx3-Lx1)^2)+((Ly3-Ly1)^2));
   L24=sqrt(((Lx4-Lx2)^2)+((Ly4-Ly2)^2));
   Area(i,1)=0.25*sqrt((4*(L13^2)*(L24^2))-(((L2^2)+(L4^2)-(L1^2)-(L3^2))^2));
end
```

```
%%-----------------input data for initial condition----------------
PhyN_Ct = xlsread('initialV.xls','PhyN');       % initial value of PhyN
PhyP_Ct=0.1*PhyN_Ct;
ON_Ct   = xlsread('initialV.xls','ON');          % initial value of ON
NH4_Ct  = xlsread('initialV.xls','NH4');         % initial value of NH4
NO2_Ct  = xlsread('initialV.xls','NO2');         % initial value of NO3
NO3_Ct  = xlsread('initialV.xls','NO3');         % initial value of NO3
IP_Ct   = xlsread('initialV.xls','IP');          % initial value of IP
OP_Ct   = xlsread('initialV.xls','OP');          % initial value of ON
%%------------input light intensity data ----------------------------
sunNov
%%------------------input Water level(H)--------------------------
load('HNov.mat');
Hdata=HNov;
load('UNov.mat');
Udata=UNov;
load('VNov.mat');
Vdata=VNov;
%%----------------------input Q data------------------------------
QON=xlsread('QEdata.xlsx','ON');
QNH4=xlsread('QEdata.xlsx','NH4');
QNO2=xlsread('QEdata.xlsx','NO2');
QNO3=xlsread('QEdata.xlsx','NO3');
QOP=xlsread('QEdata.xlsx','OP');
QIP=xlsread('QEdata.xlsx','IP');
QPhyN=xlsread('QEdata.xlsx','PhyN');
[qr,qc]=size(QIP);
%%------------------------------------------------------------------------
%%     Set Answer Metrix (concetration at each node vary by time)
%%------------------------------------------------------------------------
ON_Time=ON_Ct;                      % ON Answer at t=o
NH4_Time=NH4_Ct;                    % NH4 Answer at t=o
NO2_Time=NO2_Ct;                    % NO2 Answer at t=o
NO3_Time=NO3_Ct;                    % NO3 Answer at t=o
OP_Time=OP_Ct;                      % OP Answer at t=o
IP_Time=IP_Ct;                      % IP Answer at t=o
PhyN_Time=PhyN_Ct;
%%-----------------------------------------------------------------------
%%     Define delta t and t stop
%%-----------------------------------------------------------------------
t=0;                                % set t start
dt=300;                             % delta t (sec)
PhyNAns=zeros(NumNode,1);           % PhyN Answer at t=o
ONAns=zeros(NumNode,1);             % ON Answer at t=o
NH4Ans=zeros(NumNode,1);            % NH4 Answer at t=o
NO2Ans=zeros(NumNode,1);            % NO2 Answer at t=o
NO3Ans=zeros(NumNode,1);            % NO3 Answer at t=o
```

```
OPAns=zeros(NumNode,1);              % OP Answer at t=o
IPAns=zeros(NumNode,1);              % IP Answer at t=o
while t<2592000                      % set t stop=30 day(t<2592000 sec)
t=t+dt;
MmetrixCal
PhyN_unsteady
ON_unsteady
NH4_unsteady
NO2_unsteady
NO3_unsteady
OP_unsteady
IP_unsteady
end
%%============================================
%%              Save Answer File (Node by time)
%%------------------------------------------------------------------------
PhyN_Time;
save PhyN_Time;
xlswrite('PhyN_ans.xls',PhyN_Time);
ON_Time;
save  ON_Time;
xlswrite('ON_ans.xls',ON_Time);
NH4_Time;
save  NH4_Time;
xlswrite('NH4_ans.xls',NH4_Time);
NO2_Time;
save NO2_Time;
xlswrite('NO2_ans.xls',NO2_Time);
NO3_Time;
save NO3_Time;
xlswrite('NO3_ans.xls',NO3_Time);
OP_Time;
xlswrite('OP_ans.xls',OP_Time);
save OP_Time;
IP_Time;
save IP_Time;
xlswrite('IP_ans.xls',IP_Time);

%%==========================================================
============
%%      M Metrix Calculation : UNSTEADY STAGE MODEL
%%==========================================================
============
%%-----------------------------------------------------------------------
%%     Input U, V ,H data in each dt
%%-----------------------------------------------------------------------
%
```

```
Ux=(Udata(:,(t/300)));                    % velocity in x direction (m/s)
Vy=(Vdata(:,(t/300)));                    % velocity in y direction (m/s)
Dx=50;                    % diffusion coefficient in x direction (m^2/s)
Dy=50;                    % diffusion coefficient in x direction (m^2/s)
H=Hdata(:,(t/300));                        % water level(m)
%%-------------------- calculate H average --------------------------
Haverage=zeros(NumEle,1);
HNode=zeros(NumEle,4);
for i=1:NumEle
 HNode(i,:) = [H(Node(i,1),1) H(Node(i,2),1) H(Node(i,3),1) H(Node(i,4),1)];
 HN        = [HNode(i,1) HNode(i,2) HNode(i,3) HNode(i,4)];
 Haverage(i,1)=mean(HN);
 end
%%----------------------------------------------------------------------
%%    calculate element+system matrix M
%%----------------------------------------------------------------------
%%     initial system matrix M
%%----------------------------------------------------------------------
MSys=zeros(NumNode);
MuSys=zeros(NumNode);
MvSys=zeros(NumNode);
MdxHSys=zeros(NumNode);
MdyHSys=zeros(NumNode);
MdxSys=zeros(NumNode);
MdySys=zeros(NumNode);
%%-----------------------------------------------------------------------
%%   calculate element matrix
%%-----------------------------------------------------------------------
%%
%%-------input zi etha w-----------%%
z = [0.25;  0.75;  0.75; 0.25];
et = [0.25; 0.75; 0.75;  0.25];
wi=0.25;
%%--------------------------------------%%
for i=1:NumEle
%%------initial element matrix M---------%%
MEle=zeros(4);
MuEle=zeros(4);
MvEle=zeros(4);
MdxHEle=zeros(4);
MdyHEle=zeros(4);
MdxEle=zeros(4);
MdyEle=zeros(4);
%%------input element co-ordinate, Ux, Vy---------%%
  nd=Node(i,:);
  XY  = [x(i,1)   y(i,1);
        x(i,2)   y(i,2);
```

```
        x(i,3)   y(i,3);
        x(i,4)   y(i,4)];
 Uxe = [Ux(Node(i,1));
        Ux(Node(i,2));
        Ux(Node(i,3));
        Ux(Node(i,4))];
 Vye = [Vy(Node(i,1));
        Vy(Node(i,2));
        Vy(Node(i,3));
        Vy(Node(i,4))];
 He  = HNode(i,:);
 Havg = Haverage(i,1);
 nd=Node(i,:);
 %%--------calculate phi, B, J, d_phix, d_phiy------%%
for j=1:4
   zi=z(j);
   eta=et(j);
   Phi = [(1-zi)*(1-eta);
           zi*(1-eta);
           zi*eta;
           (1-zi)*eta];
   B   = [ eta-1   1-eta  eta  -eta;
           zi-1   -zi     zi    1-zi];
   J   =  B*XY;
 J_inv = inv(J);
 d_phi = J_inv*B;
 d_phixT= d_phi(1,:);
 d_phiyT= d_phi(2,:);
%%------calculate element matrix M-------------%%
 MEle   = MEle + wi*(Phi*Phi'*(det(J)));
 MuEle  = MuEle + wi*(Phi*(Phi'*Uxe)*d_phixT*abs(det(J)));
 MvEle  = MvEle + wi*(Phi*(Phi'*Vye)*d_phiyT*abs(det(J)));
 MdxHEle  = MdxHEle +
wi*((Dx/(Phi'*He'))*Phi*d_phixT*He'*d_phixT*abs(det(J)));
 MdyHEle  = MdyHEle +
wi*((Dy/(Phi'*He'))*Phi*d_phiyT*He'*d_phiyT*abs(det(J)));
 MdxEle   = MdxEle + wi*(Dx*d_phixT'*d_phixT*abs(det(J)));
 MdyEle   = MdyEle + wi*(Dy*d_phiyT'*d_phiyT*abs(det(J)));
end
%%-------system matrix M------------------------%%
   for ni = 1:4;
       ii = nd(ni);
       for nj = 1:4;
           jj=nd(nj);
           MSys(ii,jj)  = MSys(ii,jj)+MEle(ni,nj);
           MuSys(ii,jj) = MuSys(ii,jj)+MuEle(ni,nj);
           MvSys(ii,jj) = MvSys(ii,jj)+MvEle(ni,nj);
```

```
%           MkSys(ii,jj) = MkSys(ii,jj)+MkEle(ni,nj);
          MdxHSys(ii,jj)= MdxHSys(ii,jj)+MdxHEle(ni,nj);
          MdyHSys(ii,jj)= MdyHSys(ii,jj)+MdyHEle(ni,nj);
          MdxSys(ii,jj)= MdxSys(ii,jj)+MdxEle(ni,nj);
          MdySys(ii,jj)= MdySys(ii,jj)+MdyEle(ni,nj);
      end
   end
end
```

Appendix B

Source Code of Phytoplankton Sub-model

Source Code of Phytoplankton Sub-model

```
%%=============================================================
%%      PHYTOPLANKTON NITROGEN UNSTEADY STAGE MODEL
%%=============================================================
%%
%%---------------------------------------------------------------------
%%     calculate element+system matrix Ms
%%---------------------------------------------------------------------
RRPhyN
MsSys=zeros(NumNode,1);
for i=1:NumEle
  MsEle = zeros(4,1);
     nd = Node(i,:);
  XY  = [x(i,1)   y(i,1);
         x(i,2)   y(i,2);
         x(i,3)   y(i,3);
         x(i,4)   y(i,4)];
  Se  =  RPhyN(i,1);
  AreaE=Area(i,1);
  HEle=Haverage(i,1);
  VolE=AreaE*HEle;
     for j=1:4
        zi =z(j);
        eta=et(j);
    Phi = [(1-zi)*(1-eta);
            zi*(1-eta);
            zi*eta;
            (1-zi)*eta];
    B   = [ eta-1   1-eta  eta  -eta;
            zi-1   -zi     zi    1-zi];
        J   =  B*XY;
       MsEle  = MsEle + wi*Phi*abs(det(J));
      end
      MSEle  = (Se)*MsEle;
for ni = 1:4
        MsSys(Node(i,ni),1)= MsSys(Node(i,ni),1)+MSEle(ni,1);
    end
end
%%
%%----input Q----%%
%QPhyN=xlsread('qedata.xlsx','PhyN');
[qr,qc]=size(QPhyN);
```

```
%%----------------------------------------------------------------------
%%     calculate element+system matrix Mq
%%----------------------------------------------------------------------
MqSys=zeros(NumNode,1);
for i=1:qr;
  MqEle = zeros(4,1);
    qe=QPhyN(i,1);
    Qci=QPhyN(i,4);
    QNode=[QPhyN(i,2), QPhyN(i,3)];
    QNX1=coordinate(QNode(1,1),2);
    QNX2=coordinate(QNode(1,2),2);
    QNY1=coordinate(QNode(1,1),3);
    QNY2=coordinate(QNode(1,2),3);
    Li=sqrt(((QNX2-QNX1)^2)+((QNY2-QNY1)^2));
    nd=Node(qe,:);
    Qi=zeros(4,1);
    for qni=1:4
       Qi(qni,1)=nd(1,qni);
        if Qi(qni,1)==QNode(1,1)
            Qi(qni,1)=0.5;
        elseif Qi(qni,1)==QNode(1,2)
                Qi(qni,1)=0.5;
            else Qi(qni,1)=0;
        end
    end
         MqEle  = MqEle + ((Qci*Li)*Qi);
    for ni = 1:4
            MqSys(Node(qe,ni),1)= MqSys(Node(qe,ni),1)+MqEle(ni,1);
        end
end

%%----------------------------------------------------------------------
%%     element matrix in compacted form
%%----------------------------------------------------------------------

F=MuSys+MvSys-MdxHSys-MdyHSys+MdxSys+MdySys;
G=MsSys+MqSys;
MFP=(MSys/dt)+(F/2);
MFM=(MSys/dt)-(F/2);
PhyN_Right=(MFM*PhyN_Ct)+G;

%%-------input data for boundary condition-------%%
Bound=[193, 0.0747;
              194, 0.0747];
[bb,oo]=size(Bound);
bound=bb;
```

```
%%----------------------------------------------------------------------
%%    apply boundary condition value
%%----------------------------------------------------------------------
 for i=1:bound
    MFP(Bound(i,1),:) = zeros(1,NumNode);  %set all the i node row to zero
    MFP(Bound(i,1),Bound(i,1))=1;        %set the i node diagonal to unity
    PhyN_Right(Bound(i,1))=Bound(i,2);   %put the constrained value in the column
 end
%%----------------------------------------------------------------------
%%    solve the matrix equation
%%----------------------------------------------------------------------
PhyNAns =inv(MFP)*PhyN_Right;
PhyN_Ct=PhyNAns;
it=rem(t,7200);
if it==0
PhyN_Time=[PhyN_Time,PhyNAns];
end

%%=============================================================
%%            RPhyN : PHYTOPLANKTON NITROGEN SUB MODEL
%%=============================================================
%%
%%-------------------- Effect of Light Intensity -----------------------%%
%%

I0=600;
Is=300;
PChl = zeros(NumEle,1);
for i=1:NumEle
   PANode(i,:) = [PA(Node(i,1),1) PA(Node(i,2),1) PA(Node(i,3),1) PA(Node(i,4),1)]
   PAN         = [PANode(i,1) PANode(i,2) PANode(i,3) PANode(i,4)]
   PChl(i,1)   = mean(PAN)
end
Kew = 1;
KeChl = (0.0088*PChl)+(0.054*(PChl.^(2/3)));
Ke=Kew+KeChl;
for i=1:NumEle
   FI(i,1)=(2.718/(Ke(i,1)*Haverage(i,1)))*(exp((I0/Is)*2.718^(-
Ke(i,1)*Haverage(i,1)))-exp(-(I0/Is)));
end

%%----------------- Effect of Water Temperture ---------------------------%%
kT=1.045

%%------------------- Effect of Nutrient Limitation ---------------------%%
NH4 = zeros(NumEle,1);
for i=1:NumEle
```

```
   PNH4Node(i,:) = [PNH4(Node(i,1),1) PNH4(Node(i,2),1) PNH4(Node(i,3),1)
PNH4(Node(i,4),1)]
   PNH4N        = [PNH4Node(i,1) PNH4Node(i,2) PNH4Node(i,3) PNH4Node(i,4)]
   NH4(i,1)   = mean(PNH4N)
end
NO3 = zeros(NumEle,1);
for i=1:NumEle
   PNO3Node(i,:) = [PNO3(Node(i,1),1) PNO3(Node(i,2),1) PNO3(Node(i,3),1)
PNO3(Node(i,4),1)]
   PNO3N        = [PNO3Node(i,1) PNO3Node(i,2) PNO3Node(i,3) PNO3Node(i,4)]
   NO3(i,1)   = mean(PNO3N)
end
IP = zeros(NumEle,1);
for i=1:NumEle
   PIPNode(i,:) = [PIP(Node(i,1),1) PIP(Node(i,2),1) PIP(Node(i,3),1)
PIP(Node(i,4),1)]
   PIPN        = [PIPNode(i,1) PIPNode(i,2) PIPNode(i,3) PIPNode(i,4)]
   IP(i,1)   = mean(PIPN)
end
kmN = 0.025;
kmP = 0.001;
for i=1:NumEle
   FNU (i,1) =
min(((NH4(i,1)+NO3(i,1))/(kmN+NH4(i,1)+NO3(i,1))),(IP(i,1))/(kmP+IP(i,1)))
         end

%%----------------- Effect of Salinity Limitation ----------------------%%
Sal = zeros(NumEle,1);
PSalNode=zeros(NumEle,4);
for i=1:NumEle
   PSalNode(i,:) = [Sal_Ct(Node(i,1),1) Sal_Ct(Node(i,2),1) Sal_Ct(Node(i,3),1)
Sal_Ct(Node(i,4),1)];
   PSalN        = [PSalNode(i,1) PSalNode(i,2) PSalNode(i,3) PSalNode(i,4)];
   Sal(i,1)   = mean(PSalN);
end
kSal = 0.7;
FSal = zeros(NumEle,1);
for i=1:NumEle
   FSal(i,1) = kSal/(kSal+Sal(i,1));
end

%%-------------------- Phytoplankton Nitrogen Growth -------------------%%
GrPhyN = 0.267/86400;
for i=1:NumEle;
     GPhyN(i,1)=GrPhyN*FI(i,1)*kT*FNU(i,1)*FSal(i,1);
end
```

```
%%----------------------- Phytoplankton Nitrogen Respiration ------------------------%%
rPhyN=0.125/86400
kRPhyN=1.045
RrPhyN=rPhyN*(kRPhyN*(25-20))
%%---------------------------- Grazing by Zooplankton ------------------------------%%
ZP=ones(NumEle,1)
GZr=0/86400
GZ=GZr*ZP
%%----------------------- Source/Sink of Phytoplankton Nitrogen ----------------------
%%
PhyN = zeros(NumEle,1);
for i=1:NumEle
   PPhyNNode(i,:) = [PPhyN(Node(i,1),1) PPhyN(Node(i,2),1) PPhyN(Node(i,3),1)
PPhyN(Node(i,4),1)]
   PPhyNN        = [PPhyNNode(i,1) PPhyNNode(i,2) PPhyNNode(i,3)
PPhyNNode(i,4)]
   PhyN(i,1)   = mean(PPhyNN)
end
RPhyN=zeros(NumEle,1)
for i=1:NumEle
    RPhyN(i,1)=((GPhyN(i,1)-RrPhyN-GZ(i,1))*PhyN(i,1))-(SPhyN(i,1)*PhyN(i,1))
end
```

Appendix C

Source Code of Organic Nitrogen Sub-model

Source Code of Organic Nitrogen Sub-model

```
%%=============================================================
%%      ORGANIC NITROGEN UNSTEADY STAGE MODEL
%%=============================================================
%%----------------------------------------------------------------------
%%     calculate element+system matrix Ms
%%----------------------------------------------------------------------
RRON
MsSys=zeros(NumNode,1);
for i=1:NumEle
  MsEle = zeros(4,1);
    nd = Node(i,:);
  XY  = [x(i,1)   y(i,1);
        x(i,2)   y(i,2);
        x(i,3)   y(i,3);
        x(i,4)   y(i,4)];
  Se  =  RON(i,1);
  AreaE=Area(i,1);
  HEle=Haverage(i,1);
  VolE=AreaE*HEle;
    for j=1:4
       zi =z(j);
       eta=et(j);
   Phi = [(1-zi)*(1-eta);
           zi*(1-eta);
           zi*eta;
           (1-zi)*eta];
   B   = [ eta-1   1-eta  eta  -eta;
           zi-1   -zi     zi    1-zi];
       J   =  B*XY;
      MsEle  = MsEle + wi*Phi*abs(det(J));
     end
    MSEle  = Se*MsEle;
 for ni = 1:4
       MsSys(Node(i,ni),1)= MsSys(Node(i,ni),1)+MSEle(ni,1);
 end
end
%%----------------------------------------------------------------------
%%----input Q----%%
 [qr,qc]=size(QON);
```

```
%%
%%     calculate element+system matrix Mq
%%--------------------------------------------------------------------
MqSys=zeros(NumNode,1);
for i=1:qr;
  MqEle = zeros(4,1);
    qe=QON(i,1);
    Qci=QON(i,4)*Haverage(qe,1);
    QNode=[QON(i,2), QON(i,3)];
    QNX1=coordinate(QNode(1,1),2);
    QNX2=coordinate(QNode(1,2),2);
    QNY1=coordinate(QNode(1,1),3);
    QNY2=coordinate(QNode(1,2),3);
    Li=sqrt(((QNX2-QNX1)^2)+((QNY2-QNY1)^2));
    nd=Node(qe,:);
    Qi=zeros(4,1);
    for qni=1:4
       Qi(qni,1)=nd(1,qni);
        if Qi(qni,1)==QNode(1,1)
            Qi(qni,1)=0.5;
        elseif Qi(qni,1)==QNode(1,2)
                Qi(qni,1)=0.5;
            else Qi(qni,1)=0;
        end
    end
         MqEle  = MqEle + ((Qci*Li)*Qi);
    for ni = 1:4
            MqSys(Node(qe,ni),1)= MqSys(Node(qe,ni),1)+MqEle(ni,1);
    end
end

%%--------------------------------------------------------------------
%%     element matrix in compacted form
%%--------------------------------------------------------------------
F=MuSys+MvSys-MdxHSys-MdyHSys+MdxSys+MdySys;
G=MsSys+MqSys;
MFP=(MSys/dt)+(F/2);
MFM=(MSys/dt)-(F/2);
ON_Right=(MFM*ON_Ct)+G;

%%--------------------------------------------------------------------
%%-------input data for boundary condition-------%%
Bound=[193, 0.3276;
              194, 0.3276];
[bb,oo]=size(Bound);
bound=bb;
```

```
%%     apply boundary condition value
%%--------------------------------------------------------------------
 for i=1:bound
     MFP(Bound(i,1),:) = zeros(1,NumNode);  %set all the i node row to zero
     MFP(Bound(i,1),Bound(i,1))=1;        %set the i node diagonal to unity
     ON_Right(Bound(i,1))=Bound(i,2);   %put the constrained value in the column
 end
%%--------------------------------------------------------------------
%%     solve the matrix equation
%%--------------------------------------------------------------------
ONAns =inv(MFP)*ON_Right;
ON_Ct=ONAns;
it=rem(t,7200);
tx=0;
if it==0
tx=[tx,t];
ON_Time=[ON_Time,ONAns];
end


%%=================================================================
%%                  RRON : ORGANIC NITROGEN SUB MODEL                %%
%%=================================================================
%%
%%---------------- Nitrogen Uptake by Phytoplankton -------------------%%
%%-------------------- Effect of Light Intensity -----------------------%%
%%
I0=I0t((t/300),1);
Is=300/86400;
PChl = zeros(NumEle,1);
PANode=zeros(NumEle,4);
for i=1:NumEle
    PANode(i,:) = [PhyN_Ct(Node(i,1),1) PhyN_Ct(Node(i,2),1)
PhyN_Ct(Node(i,3),1) PhyN_Ct(Node(i,4),1)];
    PANN        = [PANode(i,1) PANode(i,2) PANode(i,3) PANode(i,4)];
    PAN         =  10*PANN;
    PChl(i,1)   = mean(PAN);
end
Kew = 0.15;
KeChl = (0.0088*PChl)+(0.054*(PChl.^(2/3)));
Ke=Kew+KeChl;
FI=zeros(NumEle,1);
for i=1:NumEle
    FI(i,1)=(2.718/(Ke(i,1)*Haverage(i,1)))*(exp((I0/Is)*(2.718^(-
Ke(i,1)*Haverage(i,1))))-exp(-(I0/Is)));
end
```

```
%%----------------- Effect of Water Temperture ----------------------------%%
ktT=1.04;
GrPhyN = 0.4/86400;
kT=GrPhyN*(ktT^(25-20));
%%------------------- Effect of Nutrient Limitation ---------------------%%
NH4 = zeros(NumEle,1);
PNH4Node=zeros(NumEle,4);
for i=1:NumEle
   PNH4Node(i,:) = [NH4_Ct(Node(i,1),1) NH4_Ct(Node(i,2),1)
NH4_Ct(Node(i,3),1) NH4_Ct(Node(i,4),1)];
   PNH4N        = [PNH4Node(i,1) PNH4Node(i,2) PNH4Node(i,3) PNH4Node(i,4)];
   NH4(i,1)   = mean(PNH4N);
end
NO3 = zeros(NumEle,1);
PNO3Node=zeros(NumEle,4);
for i=1:NumEle
   PNO3Node(i,:) = [NO3_Ct(Node(i,1),1) NO3_Ct(Node(i,2),1)
NO3_Ct(Node(i,3),1)         NO3_Ct(Node(i,4),1)];
   PNO3N        = [PNO3Node(i,1) PNO3Node(i,2) PNO3Node(i,3) PNO3Node(i,4)];
   NO3(i,1)   = mean(PNO3N);
end
UrNH4  =  0.15/3600;
UrNO3  =  0.01/3600;
kmNH4 = 0.03;
kmNO3 = 0.03;
FNUP=zeros(NumEle,1);
for i=1:NumEle
   FNUP (i,1)=
((UrNH4*(NH4(i,1)/(kmNH4+NH4(i,1))))+(UrNO3*(NO3(i,1)/(kmNO3+NO3(i,1))))
);
end
%%    Nitrogen Uptake
UPhyN=zeros(NumEle,1);
for i = 1:NumEle
UPhyN(i,1)=FI(i,1)*kT*FNUP(i,1);
end

%%-------------------- Phytoplankton Nitrogen Growth -------------------%%
IP = zeros(NumEle,1);
PIPNode=zeros(NumEle,4);
for i=1:NumEle
   PIPNode(i,:) = [IP_Ct(Node(i,1),1) IP_Ct(Node(i,2),1) IP_Ct(Node(i,3),1)
IP_Ct(Node(i,4),1)];
   PIPN        = [PIPNode(i,1) PIPNode(i,2) PIPNode(i,3) PIPNode(i,4)];
   IP(i,1)   = mean(PIPN);
end
kmP  = 0.005;
```

```
kmN  = 0.03;
FNU=zeros(NumEle,1);
for i=1:NumEle
   FNU (i,1) =
min(((NH4(i,1)+NO3(i,1))/(kmN+NH4(i,1)+NO3(i,1))),(IP(i,1))/(kmP+IP(i,1)));
end

%%-------------------- Phytoplankton Nitrogen Growth -------------------%%
GrPhyN = 0.4/86400;
GPhyN=zeros(NumEle,1);
for i=1:NumEle;
   GPhyN(i,1)=GrPhyN*FI(i,1)*kT*FNU(i,1);
end

%%----------------------- Phytoplankton Nitrogen Respiration ------------------------%%
rPhyN=0.001/86400;
kRPhyN=1.045;
RrPhyN=rPhyN*(kRPhyN^(25-20));

%%---------------------- Organic Nitrogen Mineralization ------------------------------%%
MrON=0.04/86400;
kMON=1.02;
MON=MrON*(kMON^(25-20));

%%----------------------- Source/Sink of Phytoplankton Nitrogen ----------------------
%%
PhyN = zeros(NumEle,1);
PPhyNNode=zeros(NumEle,4);
for i=1:NumEle
   PPhyNNode(i,:) = [PhyN_Ct(Node(i,1),1) PhyN_Ct(Node(i,2),1)
PhyN_Ct(Node(i,3),1) PhyN_Ct(Node(i,4),1)];
   PPhyNN        = [PPhyNNode(i,1) PPhyNNode(i,2) PPhyNNode(i,3)
PPhyNNode(i,4)];
   PhyN(i,1)   = mean(PPhyNN);
end
ON = zeros(NumEle,1);
PONNode=zeros(NumEle,4);
for i=1:NumEle
   PONNode(i,:) = [ON_Ct(Node(i,1),1) ON_Ct(Node(i,2),1) ON_Ct(Node(i,3),1)
ON_Ct(Node(i,4),1)];
   PONN        = [PONNode(i,1) PONNode(i,2) PONNode(i,3) PONNode(i,4)];
   ON(i,1)   = mean(PONN);
end
RON=zeros(NumEle,1);
for i=1:NumEle
```

```
%    RON(i,1)=((UPhyN(i,1)-GPhyN(i,1)+RrPhyN)*PhyN(i,1))-(MON*ON(i,1))-
(SON(i,1)*ON(i,1));
    RON(i,1)=((UPhyN(i,1)-GPhyN(i,1)+RrPhyN)*PhyN(i,1))-(MON*ON(i,1));
%    RON(i,1)=(UPhyN(i,1)-GPhyN(i,1)+RrPhyN)*PhyN(i,1);
end
```

ppendix D

Source Code of Ammonia Nitrogen sub-model

Source Code of Ammonia Nitrogen sub-model

```
%%==============================================================
%%      NH4 UNSTEADY STAGE MODEL
%%==============================================================
%%----------------------------------------------------------------------
%%     calculate element+system matrix Ms
%%----------------------------------------------------------------------
RRNH4
MsSys=zeros(NumNode,1);
for i=1:NumEle
  MsEle = zeros(4,1);
    nd = Node(i,:);
  XY  = [x(i,1)   y(i,1);
        x(i,2)   y(i,2);
        x(i,3)   y(i,3);
        x(i,4)   y(i,4)];
  Se  =  RNH4(i,1);
  AreaE=Area(i,1);
  HEle=Haverage(i,1);
  VolE=AreaE*HEle;
    for j=1:4
      zi =z(j);
      eta=et(j);
   Phi = [(1-zi)*(1-eta);
          zi*(1-eta);
          zi*eta;
          (1-zi)*eta];
   B   = [ eta-1   1-eta  eta  -eta;
          zi-1   -zi     zi    1-zi];
      J   =  B*XY;
      MsEle  = MsEle + wi*Phi*abs(det(J));
     end
     MSEle  = (Se)*MsEle;
  for ni = 1:4
      MsSys(Node(i,ni),1)= MsSys(Node(i,ni),1)+MSEle(ni,1);
   end
end
%%----------------------------------------------------------------------
 [qr,qc]=size(QNH4);
```

```
%%----------------------------------------------------------------------
%%     calculate element+system matrix Mq
%%----------------------------------------------------------------------
MqSys=zeros(NumNode,1);
for i=1:qr;
  MqEle = zeros(4,1);
    qe=QNH4(i,1);
    Qci=QNH4(i,4)*Haverage(qe,1);
    QNode=[QNH4(i,2), QNH4(i,3)];
    QNX1=coordinate(QNode(1,1),2);
    QNX2=coordinate(QNode(1,2),2);
    QNY1=coordinate(QNode(1,1),3);
    QNY2=coordinate(QNode(1,2),3);
    Li=sqrt(((QNX2-QNX1)^2)+((QNY2-QNY1)^2));
    nd=Node(qe,:);
    Qi=zeros(4,1);
    for qni=1:4
       Qi(qni,1)=nd(1,qni);
        if Qi(qni,1)==QNode(1,1)
            Qi(qni,1)=0.5;
        elseif Qi(qni,1)==QNode(1,2)
                Qi(qni,1)=0.5;
            else Qi(qni,1)=0;
        end
    end
         MqEle  = MqEle + ((Qci*Li)*Qi);
    for ni = 1:4
            MqSys(Node(qe,ni),1)= MqSys(Node(qe,ni),1)+MqEle(ni,1);
        end
end
%%----------------------------------------------------------------------
%%     element matrix in compacted form
%%----------------------------------------------------------------------
F=MuSys+MvSys-MdxHSys-MdyHSys+MdxSys+MdySys;
G=MsSys+MqSys;
MFP=(MSys/dt)+(F/2);
MFM=(MSys/dt)-(F/2);
NH4_Right=(MFM*NH4_Ct)+G;
%%-------input data for boundary condition-------%%
Bound=[193, 0.0492;
       194, 0.0492];
[bb,oo]=size(Bound);
bound=bb;
```

```
%%-----------------------------------------------------------------------
%%     apply boundary condition value
%%---------------------------------------------------------------------
 for i=1:bound
     MFP(Bound(i,1),:) = zeros(1,NumNode);  %set all the i node row to zero
     MFP(Bound(i,1),Bound(i,1))=1;        %set the i node diagonal to unity
     NH4_Right(Bound(i,1))=Bound(i,2);   %put the constrained value in the column
 end
%%---------------------------------------------------------------------
%%     solve the matrix equation
%%---------------------------------------------------------------------
NH4Ans =inv(MFP)*NH4_Right;
NH4_Ct=NH4Ans;
it=rem(t,7200);
if it==0
NH4_Time=[NH4_Time,NH4Ans];
end



%%============================================================
%%                RRNH4 : AMMONIA NITROGEN SUB MODEL                %%
%%============================================================
%%---------------------- Organic Nitrogen Mineralization ------------------------------%%
MrON=0.15/86400;
kMON=1.04;
MON=MrON*(kMON^(25-20));
%%---------------- Nitrogen Upatake by Phytoplankton -------------------%%
%%  Uptake = FI.FT.FNUP
%%-------------------- Effect of Light Intensity -----------------------%%
I0=I0t((t/300),1);
Is=300/86400;
PChl = zeros(NumEle,1);
PANode=zeros(NumEle,4);
for i=1:NumEle
    PANode(i,:) = [PhyN_Ct(Node(i,1),1) PhyN_Ct(Node(i,2),1)
PhyN_Ct(Node(i,3),1) PhyN_Ct(Node(i,4),1)];
    PANN       = [PANode(i,1) PANode(i,2) PANode(i,3) PANode(i,4)];
    PAN        =  10*PANN;
    PChl(i,1)   = mean(PAN);
end
Kew = 0.15;
KeChl = (0.0088*PChl)+(0.054*(PChl.^(2/3)));
Ke=Kew+KeChl;
FI=zeros(NumEle,1);
for i=1:NumEle
```

```
   FI(i,1)=(2.718/(Ke(i,1)*Haverage(i,1)))*(exp((I0/Is)*(2.718^(-
Ke(i,1)*Haverage(i,1))))-exp(-(I0/Is)));
end

%%----------------- Effect of Water Temperature ----------------------------%%
kT=1.045;

%%-------- Effect of Nutrient Limitation on Phytoplankton Uptake ---------%%
NH4 = zeros(NumEle,1);
PNH4Node=zeros(NumEle,4);
for i=1:NumEle
   PNH4Node(i,:) = [NH4_Ct(Node(i,1),1) NH4_Ct(Node(i,2),1)
NH4_Ct(Node(i,3),1) NH4_Ct(Node(i,4),1)];
   PNH4N        = [PNH4Node(i,1) PNH4Node(i,2) PNH4Node(i,3) PNH4Node(i,4)];
   NH4(i,1)   = mean(PNH4N);
end
UrNH4  = 0.015/3600;
kmNH4 = 0.03;
FNH4UP=zeros(NumEle,1);
for i=1:NumEle
   FNH4UP (i,1)= UrNH4*(NH4(i,1)/(kmNH4+NH4(i,1)));
end
%%    Nitrogen Uptake
UNH4=zeros(NumEle,1);
for i = 1:NumEle
UNH4(i,1)=FI(i,1)*kT*FNH4UP(i,1);
end
%%----------------------- Nitrification ------------------------%%
rNitriNH4=0.1/86400;
kNitriON=1.02;
NitriNH4=rNitriNH4*(kNitriON^(25-20));
%%----------------------- Source/Sink of Phytoplankton Nitrogen ----------------------
%%
PhyN = zeros(NumEle,1);
PPhyNNode=zeros(NumEle,4);
for i=1:NumEle
   PPhyNNode(i,:) = [PhyN_Ct(Node(i,1),1) PhyN_Ct(Node(i,2),1)
PhyN_Ct(Node(i,3),1) PhyN_Ct(Node(i,4),1)];
   PPhyNN        = [PPhyNNode(i,1) PPhyNNode(i,2) PPhyNNode(i,3)
PPhyNNode(i,4)];
   PhyN(i,1)   = mean(PPhyNN);
end
ON = zeros(NumEle,1);
PONNode=zeros(NumEle,4);
for i=1:NumEle
   PONNode(i,:) = [ON_Ct(Node(i,1),1) ON_Ct(Node(i,2),1) ON_Ct(Node(i,3),1)
ON_Ct(Node(i,4),1)];
```

```
   PONN        = [PONNode(i,1) PONNode(i,2) PONNode(i,3) PONNode(i,4)];
   ON(i,1)   = mean(PONN);
end
RNH4=zeros(NumEle,1);
for i=1:NumEle
    RNH4(i,1)=(MON*ON(i,1))-(UNH4(i,1)*PhyN(i,1))-(NitriNH4*NH4(i,1))-(SNH4(i,1)*NH4(i,1));
end
```

Appendix E

Source Code of Nitrite Nitrogen Sub-model

Source Code of Nitrite Nitrogen Sub-model

```
%%================================================================
%%      NO2 UNSTEADY STAGE MODEL
%%================================================================
%%----------------------------------------------------------------
%%    calculate element+system matrix Ms
%%----------------------------------------------------------------

RRNO2
MsSys=zeros(NumNode,1);
for i=1:NumEle
  MsEle = zeros(4,1);
    nd = Node(i,:);
  XY  = [x(i,1)   y(i,1);
         x(i,2)   y(i,2);
         x(i,3)   y(i,3);
         x(i,4)   y(i,4)];
  Se  =  RNO2(i,1);
  AreaE=Area(i,1);
  HEle=Haverage(i,1);
  VolE=AreaE*HEle;
    for j=1:4
      zi =z(j);
      eta=et(j);
   Phi = [(1-zi)*(1-eta);
          zi*(1-eta);
          zi*eta;
          (1-zi)*eta];
   B   = [ eta-1   1-eta  eta  -eta;
          zi-1   -zi    zi    1-zi];
      J   =  B*XY;
      MsEle  = MsEle + wi*Phi*abs(det(J));
    end
    MSEle  = (Se)*MsEle;
for ni = 1:4
      MsSys(Node(i,ni),1)= MsSys(Node(i,ni),1)+MSEle(ni,1);
   end
end
%%----------------------------------------------------------------
[qr,qc]=size(QNO2);
```

```
%%
%%     calculate element+system matrix Mq
%%---------------------------------------------------------------------
MqSys=zeros(NumNode,1);
for i=1:qr;
  MqEle = zeros(4,1);
    qe=QNO2(i,1);
    Qci=QNO2(i,4)*Haverage(qe,1);
    QNode=[QNO2(i,2), QNO2(i,3)];
    QNX1=coordinate(QNode(1,1),2);
    QNX2=coordinate(QNode(1,2),2);
    QNY1=coordinate(QNode(1,1),3);
    QNY2=coordinate(QNode(1,2),3);
    Li=sqrt(((QNX2-QNX1)^2)+((QNY2-QNY1)^2));
    nd=Node(qe,:);
    Qi=zeros(4,1);
    for qni=1:4
       Qi(qni,1)=nd(1,qni);
        if Qi(qni,1)==QNode(1,1)
            Qi(qni,1)=0.5;
        elseif Qi(qni,1)==QNode(1,2)
                Qi(qni,1)=0.5;
            else Qi(qni,1)=0;
        end
    end
        MqEle  = MqEle + ((Qci*Li)*Qi);
    for ni = 1:4
            MqSys(Node(qe,ni),1)= MqSys(Node(qe,ni),1)+MqEle(ni,1);
        end
end


%%---------------------------------------------------------------------
%%     element matrix in compacted form
%%---------------------------------------------------------------------
F=MuSys+MvSys-MdxHSys-MdyHSys+MdxSys+MdySys;
G=MsSys+MqSys;
MFP=(MSys/dt)+(F/2);
MFM=(MSys/dt)-(F/2);
NO2_Right=(MFM*NO2_Ct)+G;

%%-------input data for boundary condition-------%%
Bound=[193, 0.0054;
       194, 0.0054];
[bb,oo]=size(Bound);
bound=bb;
```

```
%%---------------------------------------------------------------------
%%     apply boundary condition value
%%---------------------------------------------------------------------
 for i=1:bound
     MFP(Bound(i,1),:) = zeros(1,NumNode);  %set all the i node row to zero
     MFP(Bound(i,1),Bound(i,1))=1;        %set the i node diagonal to unity
     NO2_Right(Bound(i,1))=Bound(i,2);   %put the constrained value in the column
 end
%%---------------------------------------------------------------------
%%     solve the matrix equation
%%---------------------------------------------------------------------
NO2Ans =inv(MFP)*NO2_Right;
NO2_Ct=NO2Ans;
it=rem(t,7200);
if it==0
NO2_Time=[NO2_Time,NO2Ans];
end


%%=========================================================================
%%
%%                RRNO2 : NITRITE NITROGEN SUB MODEL                %%
%%=========================================================================
%%
%%---------------------- Oxidation of NH4 to NO2 ------------------------------%%
MrNH4=0.1/86400;
kMNH4=1.02;
MNH4=MrNH4*(kMNH4^(25-20));
%%----------------------- Oxidation of NO2 to NO3 ------------------------%%
MrNO2=0.3/86400;
kMNO2=1.06;
%%
MNO2=MrNO2*(kMNO2^(25-20));
%%----------------------- Source/Sink of Phytoplankton Nitrogen ----------------------
%%
NH4 = zeros(NumEle,1);
PNH4Node=zeros(NumEle,4);
for i=1:NumEle
    PNH4Node(i,:) = [NH4_Ct(Node(i,1),1) NH4_Ct(Node(i,2),1)
NH4_Ct(Node(i,3),1) NH4_Ct(Node(i,4),1)];
    PNH4N        = [PNH4Node(i,1) PNH4Node(i,2) PNH4Node(i,3) PNH4Node(i,4)];
    NH4(i,1)   = mean(PNH4N);
end
NO2 = zeros(NumEle,1);
PNO2Node=zeros(NumEle,4);
for i=1:NumEle
```

```
    PNO2Node(i,:) = [NO2_Ct(Node(i,1),1) NO2_Ct(Node(i,2),1)
NO2_Ct(Node(i,3),1) NO2_Ct(Node(i,4),1)];
    PNO2N        = [PNO2Node(i,1) PNO2Node(i,2) PNO2Node(i,3) PNO2Node(i,4)];
    NO2(i,1)   = mean(PNO2N);
end
RNO2=zeros(NumEle,1);
for i=1:NumEle
     RNO2(i,1)=(MNH4*NH4(i,1))-(MNO2*NO2(i,1))-(SNO2(i,1)*NO2(i,1));
end
```

Appendix F

Source Code of Nitrate Nitrogen Sub-model

Source Code of Nitrate Nitrogen Sub-model

```
%%==================================================================
%%      NO3 UNSTEADY STAGE MODEL
%%==================================================================
%%-----------------------------------------------------------------------
%%     calculate element+system matrix Ms
%%-----------------------------------------------------------------------
RRNO3
MsSys=zeros(NumNode,1);
for i=1:NumEle
  MsEle = zeros(4,1);
    nd = Node(i,:);
  XY  = [x(i,1)   y(i,1);
        x(i,2)   y(i,2);
        x(i,3)   y(i,3);
        x(i,4)   y(i,4)];
  Se  =  RNO3(i,1);
  AreaE=Area(i,1);
  HEle=Haverage(i,1);
  VolE=AreaE*HEle;
    for j=1:4
      zi =z(j);
      eta=et(j);
   Phi = [(1-zi)*(1-eta);
          zi*(1-eta);
          zi*eta;
          (1-zi)*eta];
   B   = [ eta-1   1-eta  eta  -eta;
          zi-1   -zi     zi    1-zi];
      J   =  B*XY;
      MsEle  = MsEle + wi*Phi*abs(det(J));
     end
     MSEle  = (Se)*MsEle;
     for ni = 1:4
      MsSys(Node(i,ni),1)= MsSys(Node(i,ni),1)+MSEle(ni,1);
     end
end
```

```
%%-----------------------------------------------------------------------
%%----input Q----%%
 [qr,qc]=size(QNO3);
%%    calculate element+system matrix Mq
%%-----------------------------------------------------------------------
MqSys=zeros(NumNode,1);
for i=1:qr;
  MqEle = zeros(4,1);
    qe=QNO3(i,1);
    Qci=QNO3(i,4)*Haverage(qe,1);
    QNode=[QNO3(i,2), QNO3(i,3)];
    QNX1=coordinate(QNode(1,1),2);
    QNX2=coordinate(QNode(1,2),2);
    QNY1=coordinate(QNode(1,1),3);
    QNY2=coordinate(QNode(1,2),3);
    Li=sqrt(((QNX2-QNX1)^2)+((QNY2-QNY1)^2));
    nd=Node(qe,:);
    Qi=zeros(4,1);
    for qni=1:4
       Qi(qni,1)=nd(1,qni);
        if Qi(qni,1)==QNode(1,1)
            Qi(qni,1)=0.5;
        elseif Qi(qni,1)==QNode(1,2)
                Qi(qni,1)=0.5;
            else Qi(qni,1)=0;
        end
    end
        MqEle  = MqEle + ((Qci*Li)*Qi);
    for ni = 1:4
            MqSys(Node(qe,ni),1)= MqSys(Node(qe,ni),1)+MqEle(ni,1);
    end
end

%%-----------------------------------------------------------------------
%%    element matrix in compacted form
%%-----------------------------------------------------------------------
F=MuSys+MvSys-MdxHSys-MdyHSys+MdxSys+MdySys;
G=MsSys+MqSys;
MFP=(MSys/dt)+(F/2);
MFM=(MSys/dt)-(F/2);
NO3_Right=(MFM*NO3_Ct)+G;
%%-------input data for boundary condition-------%%
Bound=[193, 0.0981;
       194, 0.0981];
[bb,oo]=size(Bound);
bound=bb;
```

```
%%----------------------------------------------------------------------
%%     apply boundary condition value
%%----------------------------------------------------------------------
 for i=1:bound
     MFP(Bound(i,1),:) = zeros(1,NumNode);  %set all the i node row to zero
     MFP(Bound(i,1),Bound(i,1))=1;        %set the i node diagonal to unity
     NO3_Right(Bound(i,1))=Bound(i,2);   %put the constrained value in the column
 end
%%----------------------------------------------------------------------
%%     solve the matrix equation
%%----------------------------------------------------------------------
NO3Ans =inv(MFP)*NO3_Right;
NO3_Ct=NO3Ans;
it=rem(t,7200);
if it==0
NO3_Time=[NO3_Time,NO3Ans];
end



%%=====================================================================
%%                  RRNO3 : NITRATE NITROGEN SUB MODEL                  %%
%%=====================================================================
%%---------------------- Oxidation of NO2 to NO3 ------------------------------%%
MrNO2=0.3/86400;
kMNO2=1.06;
MNO2=MrNO2*(kMNO2^(25-20));
%%---------------- Nitrogen Upatake by Phytoplankton -------------------%%
%%-------------------- Effect of Light Intensity -----------------------%%
%%
I0=I0t((t/300),1);
Is=300/86400;
PChl = zeros(NumEle,1);
PANode=zeros(NumEle,4);
for i=1:NumEle
    PANode(i,:) = [PhyN_Ct(Node(i,1),1) PhyN_Ct(Node(i,2),1)
PhyN_Ct(Node(i,3),1) PhyN_Ct(Node(i,4),1)];
    PANN       = [PANode(i,1) PANode(i,2) PANode(i,3) PANode(i,4)];
    PAN        = 10*PANN;
    PChl(i,1)   = mean(PAN);
end
Kew = 0.15;
KeChl = (0.0088*PChl)+(0.054*(PChl.^(2/3)));
Ke=Kew+KeChl;
FI=zeros(NumEle,1);
for i=1:NumEle
```

```
   FI(i,1)=(2.718/(Ke(i,1)*Haverage(i,1)))*(exp((I0/Is)*(2.718^(-
Ke(i,1)*Haverage(i,1))))-exp(-(I0/Is)));
end

%%----------------- Effect of Water Temperture ----------------------------%%
kT=1.045;

%%-------- Effect of Nutrient Limitation on Phytoplankton Uptake ---------%%
NO3 = zeros(NumEle,1);
PNO3Node=zeros(NumEle,4);
for i=1:NumEle
   PNO3Node(i,:) = [NO3_Ct(Node(i,1),1) NO3_Ct(Node(i,2),1)
NO3_Ct(Node(i,3),1) NO3_Ct(Node(i,4),1)];
   PNO3N        = [PNO3Node(i,1) PNO3Node(i,2) PNO3Node(i,3) PNO3Node(i,4)];
   NO3(i,1)   = mean(PNO3N);
end
UrNO3  =  0.01/3600;
kmNO3 = 0.03;
FNO3UP=zeros(NumEle,1);
for i=1:NumEle
   FNO3UP (i,1)= UrNO3*(NO3(i,1)/(kmNO3+NO3(i,1)));
end
%%    Nitrogen Uptake
UNO3=zeros(NumEle,1);
for i = 1:NumEle
UNO3(i,1)=FI(i,1)*kT*FNO3UP(i,1);
end
%%----------------------- Denitrification ------------------------%%
%%
MrNO3=0.1/86400;
kMNO3=1.02;
%%
MNO3=MrNO3*(kMNO3^(25-20));


%%----------------------- Source/Sink of Phytoplankton Nitrogen ----------------------
%%
PhyN = zeros(NumEle,1);
PPhyNNode=zeros(NumEle,4);
for i=1:NumEle
   PPhyNNode(i,:) = [PhyN_Ct(Node(i,1),1) PhyN_Ct(Node(i,2),1)
PhyN_Ct(Node(i,3),1) PhyN_Ct(Node(i,4),1)];
   PPhyNN        = [PPhyNNode(i,1) PPhyNNode(i,2) PPhyNNode(i,3)
PPhyNNode(i,4)];
   PhyN(i,1)   = mean(PPhyNN);
end
NO2 = zeros(NumEle,1);
```

```
PNO2Node=zeros(NumEle,4);
for i=1:NumEle
   PNO2Node(i,:) = [NO2_Ct(Node(i,1),1) NO2_Ct(Node(i,2),1)
NO2_Ct(Node(i,3),1) NO2_Ct(Node(i,4),1)];
   PNO2N        = [PNO2Node(i,1) PNO2Node(i,2) PNO2Node(i,3) PNO2Node(i,4)];
   NO2(i,1)   = mean(PNO2N);
end
RNO3=zeros(NumEle,1);
for i=1:NumEle
    RNO3(i,1)=(MNO2*NO2(i,1))-(UNO3(i,1)*PhyN(i,1))-(MNO3*NO3(i,1))-
(SNO3(i,1)*NO3(i,1));
end
```

Appendix G

Source Code of Organic Phosphorus Sub-model

Source Code of Organic Phosphorus Sub-model

```
%%=====================================================================
%%      OP UNSTEADY STAGE MODEL
%%=====================================================================
%%-------------------------------------------------------------------
%%     calculate element+system matrix Ms
%%-------------------------------------------------------------------
RROP
MsSys=zeros(NumNode,1);
for i=1:NumEle
  MsEle = zeros(4,1);
    nd = Node(i,:);
  XY  = [x(i,1)   y(i,1);
        x(i,2)   y(i,2);
        x(i,3)   y(i,3);
        x(i,4)   y(i,4)];
  Se  =  ROP(i,1);
  AreaE=Area(i,1);
  HEle=Haverage(i,1);
  VolE=AreaE*HEle;
    for j=1:4
      zi =z(j);
      eta=et(j);
   Phi = [(1-zi)*(1-eta);
          zi*(1-eta);
          zi*eta;
          (1-zi)*eta];
   B   = [ eta-1   1-eta  eta  -eta;
          zi-1   -zi     zi    1-zi];
      J   =  B*XY;
      MsEle  = MsEle + wi*Phi*abs(det(J));
     end
%      MSEle  = (Se/VolE)*MsEle;
     MSEle  = (Se)*MsEle;
     for ni = 1:4
       MsSys(Node(i,ni),1)= MsSys(Node(i,ni),1)+MSEle(ni,1);
     end
end
%%-------------------------------------------------------------------
%%----input Q----%%
 [qr,qc]=size(QOP);
```

```
%%    calculate element+system matrix Mq
%%---------------------------------------------------------------------
MqSys=zeros(NumNode,1);
for i=1:qr;
  MqEle = zeros(4,1);
    qe=QOP(i,1);
    Qci=QOP(i,4)*Haverage(qe,1);
    QNode=[QOP(i,2), QOP(i,3)];
    QNX1=coordinate(QNode(1,1),2);
    QNX2=coordinate(QNode(1,2),2);
    QNY1=coordinate(QNode(1,1),3);
    QNY2=coordinate(QNode(1,2),3);
    Li=sqrt(((QNX2-QNX1)^2)+((QNY2-QNY1)^2));
    nd=Node(qe,:);
    Qi=zeros(4,1);
    for qni=1:4
       Qi(qni,1)=nd(1,qni);
        if Qi(qni,1)==QNode(1,1)
            Qi(qni,1)=0.5;
        elseif Qi(qni,1)==QNode(1,2)
                Qi(qni,1)=0.5;
            else Qi(qni,1)=0;
        end
    end
          MqEle  = MqEle + ((Qci*Li)*Qi);
    for ni = 1:4
            MqSys(Node(qe,ni),1)= MqSys(Node(qe,ni),1)+MqEle(ni,1);
        end
end
%%---------------------------------------------------------------------
%%    element matrix in compacted form
%%---------------------------------------------------------------------
F=MuSys+MvSys-MdxHSys-MdyHSys+MdxSys+MdySys;
G=MsSys+MqSys;
MFP=(MSys/dt)+(F/2);
MFM=(MSys/dt)-(F/2);
OP_Right=(MFM*OP_Ct)+G;
%%---------------------------------------------------------------------
%%-------input data for boundary condition-------%%
Bound=[193, 0.0541;
       194, 0.0541];
[bb,oo]=size(Bound);
bound=bb;
```

```
%%-----------------------------------------------------------------------
%%     apply boundary condition value
%%-----------------------------------------------------------------------
 for i=1:bound
    MFP(Bound(i,1),:) = zeros(1,NumNode);  %set all the i node row to zero
    MFP(Bound(i,1),Bound(i,1))=1;        %set the i node diagonal to unity
    OP_Right(Bound(i,1))=Bound(i,2);   %put the constrained value in the column
 end
%%-----------------------------------------------------------------------
%%     solve the matrix equation
%%-----------------------------------------------------------------------
OPAns =inv(MFP)*OP_Right;
OP_Ct=OPAns;
it=rem(t,7200);
if it==0
OP_Time=[OP_Time,OPAns];
end


%%=====================================================================
%%
%%                  RROP : ORGANIC PHOSPHORUS SUB MODEL                %%
%%=====================================================================
%%
%%
%%---------------- Nitrogen Upatake by Phytoplankton -------------------%%
%%-------------------- Effect of Light Intensity -----------------------%%
%%
I0=I0t((t/300),1);
Is=300/86400;
PChl = zeros(NumEle,1);
PANode=zeros(NumEle,4);
for i=1:NumEle
    PANode(i,:) = [PhyN_Ct(Node(i,1),1) PhyN_Ct(Node(i,2),1)
PhyN_Ct(Node(i,3),1) PhyN_Ct(Node(i,4),1)];
    PANN       = [PANode(i,1) PANode(i,2) PANode(i,3) PANode(i,4)];
    PAN        =  10*PANN;
    PChl(i,1)   = mean(PAN);
end
Kew = 0.15;
KeChl = (0.0088*PChl)+(0.054*(PChl.^(2/3)));
Ke=Kew+KeChl;
FI=zeros(NumEle,1);
for i=1:NumEle
   FI(i,1)=(2.718/(Ke(i,1)*Haverage(i,1)))*(exp((I0/Is)*(2.718^(-
Ke(i,1)*Haverage(i,1))))-exp(-(I0/Is)));
end
```

```
%%----------------- Effect of Water Temperture ----------------------------%%
kT=1.045;
%%------------------- Effect of Nutrient Limitation ---------------------%%
IP = zeros(NumEle,1);
PIPNode=zeros(NumEle,4);
for i=1:NumEle
   PIPNode(i,:) = [IP_Ct(Node(i,1),1) IP_Ct(Node(i,2),1) IP_Ct(Node(i,3),1)
IP_Ct(Node(i,4),1)];
   PIPN        = [PIPNode(i,1) PIPNode(i,2) PIPNode(i,3) PIPNode(i,4)];
   IP(i,1)   = mean(PIPN);
end
UrIP  = 0.001/3600;
kmIP = 0.001;
FPUP=zeros(NumEle,1);
for i=1:NumEle
   FPUP (i,1)= (UrIP*(IP(i,1)/(kmIP+IP(i,1))));
end
%%    Phosphorus Uptake
UPhyP=zeros(NumEle,1);
for i = 1:NumEle
UPhyP(i,1)=FI(i,1)*kT*FPUP(i,1);
end
%%-------------------- Phytoplankton Nitrogen Growth -------------------%%
kmN  = 0.02;
kmP  = 0.005;
NH4 = zeros(NumEle,1);
PNH4Node=zeros(NumEle,4);
for i=1:NumEle
   PNH4Node(i,:) = [NH4_Ct(Node(i,1),1) NH4_Ct(Node(i,2),1)
NH4_Ct(Node(i,3),1) NH4_Ct(Node(i,4),1)];
   PNH4N        = [PNH4Node(i,1) PNH4Node(i,2) PNH4Node(i,3) PNH4Node(i,4)];
   NH4(i,1)   = mean(PNH4N);
end
NO3 = zeros(NumEle,1);
PNO3Node = zeros(NumEle,4);
for i=1:NumEle
   PNO3Node(i,:) = [NO3_Ct(Node(i,1),1) NO3_Ct(Node(i,2),1)
NO3_Ct(Node(i,3),1) NO3_Ct(Node(i,4),1)];
   PNO3N        = [PNO3Node(i,1) PNO3Node(i,2) PNO3Node(i,3) PNO3Node(i,4)];
   NO3(i,1)   = mean(PNO3N);
end
FNU=zeros(NumEle,1);
for i=1:NumEle
   FNU (i,1) =
min(((NH4(i,1)+NO3(i,1))/(kmN+NH4(i,1)+NO3(i,1))),(IP(i,1))/(kmP+IP(i,1)));
end
```

```
%%-------------------- Phytoplankton Phosphorus Growth -------------------%%
GrPhyP = 0.4/86400;
GPhyP=zeros(NumEle,1);
for i=1:NumEle;
   GPhyP(i,1)=GrPhyP*FI(i,1)*kT*FNU(i,1);
end
%%----------------------- Phytoplankton Phosphorus Respiration ------------------------
%%
rPhyP=0.001/86400;
kRPhyP=1.02;
%%
RrPhyP=rPhyP*(kRPhyP^(25-20));
%%
%%---------------------- Organic Phosphorus Mineralization ------------------------------
%%
%%
MrOP=0.06/86400;
kMOP=1.008;
MOP=MrOP*(kMOP^(25-20));
%%
%%----------------------- Source/Sink of Phytoplankton Phosphorus ----------------------
%%
%%
PhyP = zeros(NumEle,1);
PPhyPNode=zeros(NumEle,4);
%PhyP_Ct=0.1*PhyN_Ct;
for i=1:NumEle
   PPhyPNode(i,:) = [PhyP_Ct(Node(i,1),1) PhyP_Ct(Node(i,2),1)
PhyP_Ct(Node(i,3),1) PhyP_Ct(Node(i,4),1)];
   PPhyPN        = [PPhyPNode(i,1) PPhyPNode(i,2) PPhyPNode(i,3)
PPhyPNode(i,4)];
   PhyP(i,1)   = mean(PPhyPN);
end
OP = zeros(NumEle,1);
POPNode=zeros(NumEle,4);
for i=1:NumEle
   POPNode(i,:) = [OP_Ct(Node(i,1),1) OP_Ct(Node(i,2),1) OP_Ct(Node(i,3),1)
OP_Ct(Node(i,4),1)];
   POPN        = [POPNode(i,1) POPNode(i,2) POPNode(i,3) POPNode(i,4)];
   OP(i,1)   = mean(POPN);
end
ROP=zeros(NumEle,1);
for i=1:NumEle
    ROP(i,1)=((UPhyP(i,1)-GPhyP(i,1)+RrPhyP)*PhyP(i,1))-(MOP*OP(i,1))-
(SOP(i,1)*OP(i,1));
end
```

Appendix H

Source Code of Inorganic Phosphorus Sub-model

Source Code of Inorganic Phosphorus Sub-model

```
%%=========================================================
%%      IP UNSTEADY STAGE MODEL
%%=========================================================
%%
%%---------------------------------------------------------------------
%%     calculate element+system matrix Ms
%%---------------------------------------------------------------------
RRIP
MsSys=zeros(NumNode,1);
for i=1:NumEle
  MsEle = zeros(4,1);
     nd = Node(i,:);
  XY  = [x(i,1)   y(i,1);
        x(i,2)   y(i,2);
        x(i,3)   y(i,3);
        x(i,4)   y(i,4)];
  Se  =  RIP(i,1);
  AreaE=Area(i,1);
  HEle=Haverage(i,1);
  VolE=AreaE*HEle;
     for j=1:4
        zi =z(j);
        eta=et(j);
    Phi = [(1-zi)*(1-eta);
            zi*(1-eta);
            zi*eta;
            (1-zi)*eta];
    B   = [ eta-1   1-eta  eta  -eta;
            zi-1   -zi     zi    1-zi];
        J   =  B*XY;
       MsEle  = MsEle + wi*Phi*abs(det(J));
      end
 %     MSEle  = (Se/VolE)*MsEle;
       MSEle  = (Se)*MsEle;
 for ni = 1:4
        MsSys(Node(i,ni),1)= MsSys(Node(i,ni),1)+MSEle(ni,1);
    end
end
%%----input Q----%%
 [qr,qc]=size(QIP);
```

```
%%
%%    calculate element+system matrix Mq
%%-------------------------------------------------------------------
MqSys=zeros(NumNode,1);
for i=1:qr;
  MqEle = zeros(4,1);
    qe=QIP(i,1);
    Qci=QIP(i,4)*Haverage(qe,1);
    QNode=[QIP(i,2), QIP(i,3)];
    QNX1=coordinate(QNode(1,1),2);
    QNX2=coordinate(QNode(1,2),2);
    QNY1=coordinate(QNode(1,1),3);
    QNY2=coordinate(QNode(1,2),3);
    Li=sqrt(((QNX2-QNX1)^2)+((QNY2-QNY1)^2));
    nd=Node(qe,:);
    Qi=zeros(4,1);
    for qni=1:4
       Qi(qni,1)=nd(1,qni);
        if Qi(qni,1)==QNode(1,1)
            Qi(qni,1)=0.5;
        elseif Qi(qni,1)==QNode(1,2)
                Qi(qni,1)=0.5;
            else Qi(qni,1)=0;
        end
    end
          MqEle  = MqEle + ((Qci*Li)*Qi);
    for ni = 1:4
            MqSys(Node(qe,ni),1)= MqSys(Node(qe,ni),1)+MqEle(ni,1);
        end
end

%%-------------------------------------------------------------------
%%    element matrix in compacted form
%%-------------------------------------------------------------------
%F=MuSys+MvSys+MkSys-MdxHSys-MdyHSys+MdxSys+MdySys;
F=MuSys+MvSys-MdxHSys-MdyHSys+MdxSys+MdySys;
G=MsSys+MqSys;
MFP=(MSys/dt)+(F/2);
MFM=(MSys/dt)-(F/2);
IP_Right=(MFM*IP_Ct)+G;
```

```
%%----------------------------------------------------------------------
%%-------input data for boundary condition-------%%
%Bound=[1, IP_Ct(1,1);
%       2, IP_Ct(2,1);
%     193, IP_Ct(193,1);
%     194, IP_Ct(194,1)];                          %Bound(Node Number,  Value)
%Bound=[1, 0.0069;
%       2, 0.0069;
Bound=[193, 0.0127;
       194, 0.0127];
[bb,oo]=size(Bound);
bound=bb;

%%----------------------------------------------------------------------
%%     apply boundary condition value
%%----------------------------------------------------------------------
 for i=1:bound
    MFP(Bound(i,1),:) = zeros(1,NumNode);  %set all the i node row to zero
    MFP(Bound(i,1),Bound(i,1))=1;        %set the i node diagonal to unity
    IP_Right(Bound(i,1))=Bound(i,2);   %put the constrained value in the column
 end
%%----------------------------------------------------------------------
%%     solve the matrix equation
%%----------------------------------------------------------------------
IPAns =inv(MFP)*IP_Right;
IP_Ct=IPAns;
it=rem(t,7200);
if it==0
IP_Time=[IP_Time,IPAns];
end
```

```
%%=============================================================
%%
%%          RIP : DISSOLVED INORGANIC PHOSPHORUS SUB MODEL
%%
%%=============================================================
==
%%------------------Nitrogen Upatake by Phytoplankton -------------------%%
%%-------------------- Effect of Light Intensity -----------------------%%
%%
%I0=I0t((t/300),1);
%Is=300/(24*60*60);
PChl = zeros(NumEle,1);
PANode=zeros(i,4);
for i=1:NumEle
   PANode(i,:) = [PhyN_Ct(Node(i,1),1) PhyN_Ct(Node(i,2),1)
PhyN_Ct(Node(i,3),1)
   PhyN_Ct(Node(i,4),1)];
   PANN       = [PANode(i,1) PANode(i,2) PANode(i,3) PANode(i,4)];
   PChl(i,1)   = mean(PAN);
end
Kew = 0.15;
KeChl = (0.0088*PChl)+(0.054*(PChl.^(2/3)));
Ke=Kew+KeChl;
FI=zeros(NumEle,1);
for i=1:NumEle
   FI(i,1)=(2.718/(Ke(i,1)*Haverage(i,1)))*(exp((I0/Is)*(2.718^(-
Ke(i,1)*Haverage(i,1))))-exp(-(I0/Is)));
end
%%----------------- Effect of Water Temperture ----------------------------%%
kT=1.045;
%%------------------- Effect of Nutrient Limitation ---------------------%%
IP = zeros(NumEle,1);
PIPNode=zeros(NumEle,4);
for i=1:NumEle
   PIPNode(i,:) = [IP_Ct(Node(i,1),1) IP_Ct(Node(i,2),1) IP_Ct(Node(i,3),1)
IP_Ct(Node(i,4),1)];
   PIPN        = [PIPNode(i,1) PIPNode(i,2) PIPNode(i,3) PIPNode(i,4)];
   IP(i,1)   = mean(PIPN);
end
UrIP  =  0.001/3600;
kmIP = 0.001;
FPUP=zeros(NumEle,1);
for i=1:NumEle
   FPUP (i,1)= (UrIP*(IP(i,1)/(kmIP+IP(i,1))));
end
%%    Phosphorus Uptake
UPhyP=zeros(NumEle,1);
```

```
for i = 1:NumEle
UPhyP(i,1)=FI(i,1)*kT*FPUP(i,1);
end
%%---------------------- Organic Phosphorus Mineralization ------------------------------
%%
MrOP=0.06/86400;
kMOP=1.08;
MOP=MrOP*(kMOP^(25-20));
%%----------------------- Source/Sink of Phytoplankton Phosphorus ----------------------
%%
PhyP = zeros(NumEle,1);
PPhyPNode=zeros(NumEle,4);
for i=1:NumEle
   PPhyPNode(i,:) = [PhyP_Ct(Node(i,1),1) PhyP_Ct(Node(i,2),1)
PhyP_Ct(Node(i,3),1) PhyP_Ct(Node(i,4),1)];
   PPhyPN        = [PPhyPNode(i,1) PPhyPNode(i,2) PPhyPNode(i,3)
PPhyPNode(i,4)];
   PhyP(i,1)   = mean(PPhyPN);
end
OP = zeros(NumEle,1);
POPNode=zeros(NumEle,4);
for i=1:NumEle
   POPNode(i,:) = [OP_Ct(Node(i,1),1) OP_Ct(Node(i,2),1) OP_Ct(Node(i,3),1)
OP_Ct(Node(i,4),1)];
   POPN        = [POPNode(i,1) POPNode(i,2) POPNode(i,3) POPNode(i,4)];
   OP(i,1)   = mean(POPN);
end
RIP=zeros(NumEle,1);
for i=1:NumEle
    RIP(i,1)=(MOP*OP(i,1))-(UPhyP(i,1)*PhyP(i,1))-(SOP(i,1)*IP(i,1));
End
```

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