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Original Article

Aggregate production planning using integrated fuzzy multi-objective optimization with α-cut analysis

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Abstract

Aggregate production planning when operating time, costs, customer demand, labor level, and machine capacity are uncertain is studied. This study optimizes the aggregate production plan, yielding the minimum total costs by using an integrated fuzzy multi-objective linear model with α-Cut analysis. The proposed integrated approach simultaneously minimizes the most possible value of the total costs, maximizes the possibility of obtaining the lowest total costs, and minimizes the possibility of obtaining higher total costs as multiple objectives. Then, the α-Cut analysis is introduced to ensure decision makers that the outcome satisfies their preferences based on a specified minimum allowed satisfaction value (α). The outcome of the study can show a possible range of total costs, yielding an optimal and acceptable plan under uncertainty.

Keywords: aggregate production planning, possibilistic linear programming, fuzzy goal programming, α-cut analysis

1. Introduction

Effective Aggregate Production Planning (APP) is used to meet customer requirements and yields the minimum total costs; production cost and costs of changes in labor levels. APP is a strategy by which a decision maker determines the appropriate production, inventory, and workforce level over a planning horizon.

APP can fall into two categories in terms of the number of objective functions. First, a single objective optimization problem only has one objective or all different objectives can be combined into one objective. This problem provides a single solution instead of a set of alternative solutions to decision makers. Karmarkar and Rajaram (2012) developed APP with capacity constraints to maximize the total profit. Hossain, Nahar, Reza, and Shaifullah (2016) studied APP of multiple products under demand uncertainty by considering wastage cost and incentives. Their aim is to minimize the total relevant costs under imprecise demand, production capacity, workforce, inventory control, wastage reduction, and proper incentive for the workforce.

Second, a multi-objective optimization problem has interactions among different objectives, yielding a set of compromised solutions. Baykasoglu and Gocken (2010) proposed ranking methods of fuzzy numbers and tabu search for solving the fuzzy multi-objective APP problem based on triangular fuzzy numbers. Tohidi and Razavyan (2012) studied an L1-norm method for generating all efficient solutions of the multi-objective integer linear programming problem. Multi-objective APP in a green supply chain was proposed by Entezaminia, Heydari, and Rahmani (2016) to minimize the total losses and maximize the total environmental scores of products.

In practice, the main problem that decision makers have to face, which consequently impacts the overall performance of their aggregate production planning, is uncertainty. It is derived from a lack of or misleading information from two sources: (1) Environmental uncertainty due to a supplier's performance and a customer's behavior in terms of supply and demand, and (2) System uncertainty due to the unreliability of operations and processes inside an organization (Cha-ume & Chiadamrong, 2012). To handle these uncertain problems, a theory of fuzzy sets can be introduced to represent the uncertainty. Bellman and Zadeh (1970) are among the

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pioneers to propose a Fuzzy Goal Programming (FGP) model for decision making in an uncertain environment, and later it was developed into multi-objective linear programming by Zimmermann (1978).

Possibilistic Linear Programming (PLP) is another approach that can be used to incorporate fuzzy data based on the triangular or trapezoidal distribution. Wang and Liang (2005) proposed PLP for the APP problem, which yields an efficient APP compromise solution and overall degree of decision maker satisfaction with determined goal values. Özgen, Önüt, Gülsün, Tuzkaya, and Tuzkaya (2008) proposed a two-phase PLP methodology for multi-objective supplier selection and order allocation problems. Kabak and Ülengin (2011) applied the PLP approach to supply chain networking decisions for maximizing the total profit of an organization. Biswal and Barik (2016) also developed the PLP with known means and variances of right hand side parameters of the constraint.

Based on PLP, in this study, the objective function of minimization of the total costs is divided into three sub objective functions: minimizing the most possible total costs, maximizing the possibility of obtaining the lower total costs, and minimizing the risk of obtaining the higher total costs. Even though the PLP model can help to find the optimal solution of each objective function, it lacks the ability to compromise the optimal solution of all objective functions. To handle this issue, integrating PLP with FGP is proposed in this study to help decision makers obtain a better outcome.

FGP is an extension of conventional programming where the aspiration level of each objective is unity. The achievement of the highest degree (unity) of the fuzzy goals of a problem is to solve multi-objective problems with imprecisely defined model parameters in a decision-making environment. It is sometimes called fuzzy mathematical programming with vagueness, where there is flexibility in the given target values of objective functions and/or the elasticity of constraints. There are two methods of FGP: Zimmermann and Weighted Additive (WA) are chosen to optimize fuzzy multi-objective APP. Zimmermann's method is FGP method that treats fuzzy goals and fuzzy constraints equivalently. However, a symmetric model may not be appropriate for multi-objective decision-making problems because the importance of the objectives can be different for each decision maker. Thus, the WA method is introduced to optimize the problem in which different weights can be applied to various objectives based on decision-makers' preferences.

Taghizadeh, Bagherpour, and Mahdavi (2011) also developed an interactive fuzzy goal programming approach for solving multi-period multi-product production planning problem in an imprecise environment. Lachhwani and Poonia (2012) studied a mathematical solution of the multilevel fractional programming problem with a FGP. They solved multilevel fractional programming problems in a large hierarchical decentralized organization using the FGP approach. Biswas and Kurmar (2016) employed a FGP approach for solid waste management under multiple uncertainties. They developed the FGP model to minimize the net system cost of sorting and transporting the wastes and to maximize the revenue generated from different treatment facilities.

Biswas and De (2016) developed an FGP model for municipal solid waste management to minimize the net system cost of sorting and transporting the wastes and to maximize the revenue generated from different treatment facilities. Mokhtari and Hasani (2017) adopted FGP and designed heuristic algorithms to optimize a multi-objective model for a cleaner production-transportation planning problem in manufacturing plants. Saxena *et al.* (2018) applied the FGP to generate supply chain strategic plan of tyre remanufacturing under uncertainty and group decision making environment.

α-Cut analysis can be introduced to guarantee that the satisfactions of fuzzy goals and fuzzy constraints of decision makers is higher than a minimum allowed value (α). It is solved by the fuzzy multi-objective linear programming model to enhance the satisfaction of fuzzy objectives and constraints of the weightless method (Zimmermann's method). Bodjanova (2002) introduced the concept of α-Cut analysis that is important in the relationship between fuzzy sets and crisp sets. Amir and Leila (2011) also used the α-Cut analysis to improve the applicability of the earned value techniques under real-life and uncertain environments.

Yang, Li, and Han (2016) proposed an improved α-cut approach to transform a fuzzy membership function into a basic belief assignment, which provides a bridge between the fuzzy set theory and Dempster–Shafer evidence theory. Purba *et al.* (2017) developed an α-cut method based importance measure to evaluate and rank the importance of basic events for criticality analysis in Fuzzy Probability based Fault Tree Analysis (FPFTA).

The contribution of this study is to extend the concept of PLP to the fuzzy multi-objective linear model. Rather than knowing and optimizing only the most likely total costs of a plan, the proposed approach can optimize and realize the possible lower total costs and the possible higher total costs as a possible range of the total costs. This information is important for decision makers. They can prepare for possible scenarios including both optimistic and pessimistic cases and take a necessary action for future uncertainty. In addition, the proposed hybrid approach also guarantees that the achievement of all objectives is satisfied and acceptable to the decision makers. This is important since failure to satisfy any objective by the decision makers can lead to an unacceptable result. The obtained production plan would be optimal (subject to various uncertainties) and would satisfy all the objectives and constraints (of the decision makers).

The remaining parts of this paper are organized as follows. The problem description, notation, and formulation are described in Section 2. Section 3 proposes the methodology. A case study is demonstrated in Section 4, and the outcomes are presented in Section 5. Lastly, Section 6 is the conclusion of the study.

2. Problem Description

A company wants to establish an APP that consists of *N* types of products over a horizontal time *T* under uncertain customer demand, operating costs, labor level, and machine capacity.

2.1 Nomenclature

To formulate the mathematical model, the symbol $\sim$ refers to ambiguous data that are used in this study.

2.1.1 Indexes

N Types of products (n = 1, …, N)
T Periods (t = 1, …, T)
J Number of fuzzy goals (j = 1, …, J)
I Number of fuzzy constraints (i = 1, …, I)

2.1.2 Parameters

$\widetilde{D_{nt}}$ Forecast demand of product n in period t (units)
$\widetilde{r_{nt}}$ Cost of regular time production per unit of product n in period t ($/unit)
$\widetilde{o_{nt}}$ Cost of overtime production per unit of product n in period t ($/unit)
$\widetilde{s_{nt}}$ Cost of subcontracting per unit of product n in period t ($/unit)
$\widetilde{\iota_{nt}}$ Cost of inventory per unit of product n in period t ($/unit)
$\widetilde{b_{nt}}$ Cost of backordering per unit of product n in period t ($/unit)
$\widetilde{h_t}$ Cost of hiring per worker in period t ($/person-hour)
$\widetilde{f_t}$ Cost of firing per worker in period t ($/person-hour)
$\widetilde{L_{tmax}}$ Maximum labor level available in period t (person-hours)
$\widetilde{M_{tmax}}$ Maximum machine capacity available in period t (machine-hours)
$\widetilde{MH_{nt}}$ Machine's hour usage per unit of product n in period t (machine-hours/unit)
WS_{tmax} Maximum warehouse space available in period t (ft^2/unit)
LH_{nt} Labor's hour usage per unit of product n in period t (person-hours/unit)
ws_{nt} Warehouse space for product n in period t (ft^2/unit)

2.1.3 Decision variables

RQ_{nt} Regular time production quantity of product n in period t (units)
OQ_{nt} Overtime production quantity of product n in period t (units)
SQ_{nt} Subcontracting quantity of product n in period t (units)
IQ_{nt} Inventory quantity of product n in period t (units)
BQ_{nt} Backorder quantity of product n in period t (units)
H_t Number of workers hired in period t (person-hour)
F_t Number of workers fired in period t (person-hour)
d_{nt} Quantity of product n in period t that satisfies the customer demand (units)

2.1.4 Related notations

$\tilde{Z}$ Total costs
λ Overall satisfaction
w_1^p Pessimistic value's weight
w_2^m Most likely value's weight
w_3^o Optimistic value's weight
λ_j Fuzzy goals (j = 1, …, J)
γ_i Fuzzy constraints (i = 1, …, I)
$\mu_j(x)$ Linear membership function of the fuzzy objective
$\mu_i(x)$ Linear membership function of the fuzzy constraint
w_j Coefficient of compensation of the fuzzy objective (j = 1, …, J)
β_i Coefficient of compensation of the fuzzy constraint (i = 1, …, I)

2.2 Problem formulation

2.2.1 Objective function

Minimizing the total costs is a common objective function of the APP problem. However, the coefficients of costs can be imprecise due to incomplete information. The objective function is proposed as:

Minimize total costs = $\sum_{n=1}^{N}\sum_{t=1}^{T}$ (Regular time production cost + Overtime production cost + Subcontracting cost + Inventory cost + Backordering cost) + $\sum_{t=1}^{T}$(Hiring cost + Layoff cost)

$$\text{Min } \tilde{Z} = \sum_{n=1}^{N}\sum_{t=1}^{T}(\tilde{r}_{nt}\, RQ_{nt} + \tilde{o}_{nt}\, OQ_{nt} + \tilde{s}_{nt}\, SQ_{nt} + \tilde{\iota}_{nt}\, IQ_{nt} + \tilde{b}_{nt}\, BQ_{nt}) + \sum_{t=1}^{T}(\tilde{h}_t H_t + \tilde{f}_t F_t) \quad (1)$$

2.2.2 Constraints

1) Carrying inventory

Demand = Previous Ending Inventory – Previous Backordering units + Regular Time Production Units + Overtime Production Units + Subcontracting Units – Current Ending Inventory + Current Backordering units

$$\tilde{D}_{nt} = IQ_{nt-1} - BQ_{nt-1} + RQ_{nt} + OQ_{nt} + SQ_{nt} - IQ_{nt} + BQ_{nt} \quad \forall N, \forall T \quad (2)$$

2) Labor level

Previous Labor Level + Hiring – Firing - Current Labor Level = 0

$$\sum_{n=1}^{N} LH_{nt-1}(RQ_{nt-1} + OQ_{nt-1}) + H_t - F_t - \sum_{n=1}^{N} LH_{nt}(RQ_{nt} + OQ_{nt}) = 0 \quad \forall T \quad (3)$$

Current Labor Level ≤ Maximum Available Labor Level

$$\sum_{n=1}^{N} \widetilde{LH_{nt}}(RQ_{nt} + OQ_{nt}) \le \tilde{L}_{tmax} \quad \forall T \quad (4)$$

3) Machine capacity

Hours of Machine Usage ≤ Maximum Available Machine Capacity

$$\sum_{n=1}^{N} \widetilde{MH}_{nt}(RQ_{nt} + OQ_{nt}) \le \tilde{M}_{tmax} \quad \forall T \quad (5)$$

4) Warehouse capacity

Warehouse Space Usage ≤ Maximum Available Warehouse Space

$$\sum_{n=1}^{N} ws_{nt} IQ_{nt} \le WS_{tmax} \quad \forall T \quad (6)$$

5) Non-negativity

$$RQ_{nt}, OQ_{nt}, SQ_{nt}, IQ_{nt}, BQ_{nt}, H_t, F_t \ge 0 \quad \forall N, \forall T \quad (7)$$

3. Solution Methodology

Figure 1 presents the proposed integrated approach to optimize the APP under uncertainty. Having formulated the objective functions and all constraints, the first step is to defuzzify the fuzzy data to crisp values either by the weighted average method or the fuzzy ranking method. PLP is the second step, to account for fuzzy operating costs. It is used to convert the fuzzy objective function into the constant objective function by dividing the main fuzzy objective function into three cases: most likely, optimistic, and pessimistic. Then, the Multi-Objective Mixed-Integer Linear Programming (MOMILP) model is used to find the boundaries of each objective function for both the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS). Next, Zimmermann's and the WA methods are used to calculate the overall satisfaction level of the APP. The result of Zimmermann's method is set as a benchmark of the optimal solution, in which all objective functions have equal weight (weightless). In contrast, the WA method is an asymmetric model that allows decision makers to assign the weights of each objective based on their experiences. It is used to find the optimal solution, in which different objective functions can have different importances. Finally, α-Cut analysis is introduced to help decision makers increase the satisfaction level of each objective to meet their specified minimum satisfaction value (α).

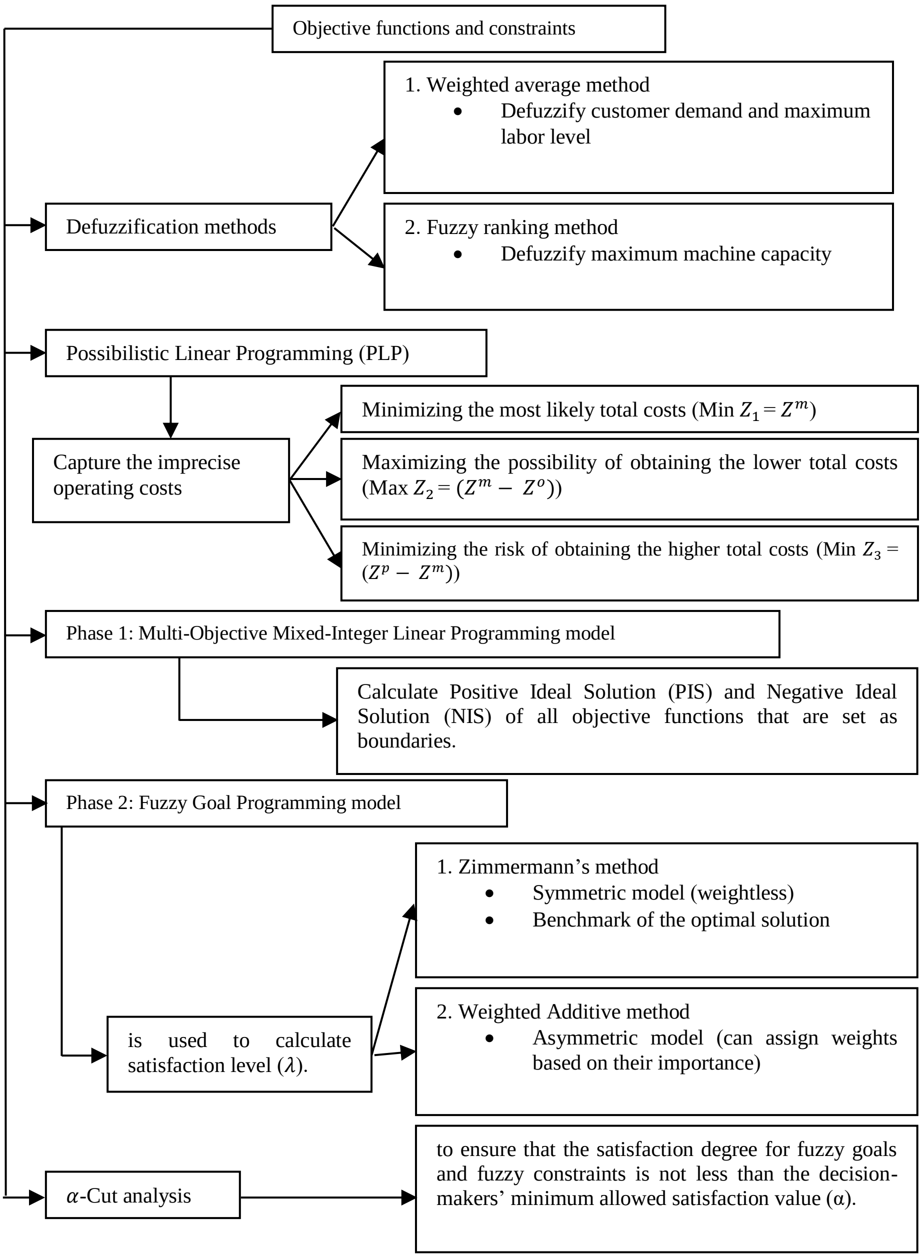


Figure 1. Flow-chart of solution methodology

3.1 Defuzzification methods

A method that can be used to convert imprecise data to crisp data is called a "defuzzification method". The weighted average and fuzzy Ranking methods are two well-known methods for defuzzifing fuzzy numbers. Fuzzy constraints that have fuzzy data on one side of an equation can be defuzzified by the weighted average method (Equation (2)). In contrast, fuzzy constraints where both sides of an equation contain fuzzy data can be defuzzified by the fuzzy ranking method (Equations (4) and (5)).

3.1.1 Weighted average method

Referring to Equation (2), the demand ($\widetilde{D_{nt}}$) has fuzzy values under the triangular distribution and can be defuzzified by applying the weighted average method as follows:

$$IQ_{nt-1} - BQ_{nt-1} + RQ_{nt} + OQ_{nt} + SQ_{nt} - IQ_{nt} + BQ_{nt} = w_1^p D_{nt}^p + w_2^m D_{nt}^m + w_3^o D_{nt}^o \quad (8)$$

where w_1^p, w_2^m, and w_3^o represent the weights of pessimistic, most likely, and optimistic values of the imprecise demand, respectively. The weights w_1^p, w_2^m, and w_3^o can be determined by decision makers based on their experience and $w_1^p + w_2^m + w_3^o = 1$.

3.1.2 Fuzzy ranking method

The FR method can also be used to defuzzify fuzzy data that does not require weight allocation to prioritize the importance of data as in Equation (5). The fuzzy ranking method is introduced as follows:

$$\sum_{n=1}^{N} MH_{nt}^p \left(RQ_{nt} + O_{Qnt}\right) \leq M_{t\,max}^p \quad (9)$$
$$\sum_{n=1}^{N} MH_{nt}^m \left(RQ_{nt} + OQ_{nt}\right) \leq M_{t\,max}^m \quad (10)$$
$$\sum_{n=1}^{N} MH_{nt}^o \left(RQ_{nt} + OQ_{nt}\right) \leq M_{t\,max}^o \quad (11)$$

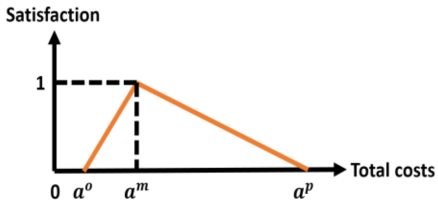
3.2 Possibilistics Linear Programming (PLP)

PLP is introduced into the APP to capture the imprecise operating costs.

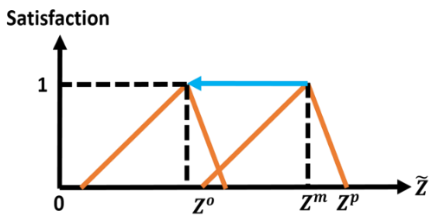
3.2.1 Triangular (possibility) distribution

Triangular fuzzy numbers can be used for representing uncertainty within an interval. The triangular distribution is an optimal transform of the uniform probability distribution. It is the upper envelope of all the possibility distributions, transformed from symmetric probability densities with the same support. It can be used to express the vagueness of information and to represent fuzzy terms in information processing. In principle, membership functions can be of various shapes, but in practice, trapezoidal and triangular membership functions are the most frequently used (Zhang, Ma, & Chen, 2014).

Figure 2(a) shows three prominent points: the most likely value point (a^m), the optimistic value point (a^o), and the pessimistic value point (a^p), which are applied to capture the imprecise operating costs based on the triangular distribution seen in Figure 2(b). Figure 2(b) shows three prominent points: the optimistic cost (z^o), most likely cost (z^m), and the pessimistic cost (z^p) that are used for minimizing the total costs. Because of uncertain costs, the objective function can be divided into 3 objective functions: (1) minimizing the most likely total costs (minimizing z^m), (2) maximizing the lower total costs (maximizing $z^m - z^o$), and (3) minimizing the higher total costs (minimizing $z^p - z^m$) by pushing these three values toward the left. After pushing these three values toward the left, the shape of the triangular distribution can be changed by enlarging the gap between $z^m - z^o$ while reducing the gap between $z^p - z^m$.



(a)



(b)

Figure 2. (a) Triangular distribution, and (b) Minimizing the total costs of logistics

3.2.2 Mathematical model based on PLP

1) Objective functions

- Minimizing the most likely total costs
  Min $Z_1 = Z^m$
  $$= \sum_{n=1}^{N} \sum_{t=1}^{T} [r_{nt}^m RQ_{nt} + o_{nt}^m OQ_{nt} + s_{nt}^m SQ_{nt} + i_{nt}^m IQ_{nt} + b_{nt}^m BQ_{nt}] + \sum_{t=1}^{T} (h_t^m H_t + f_t^m F_t) \quad (12)$$

- Maximizing the possibility of obtaining the lower total costs
  Max $Z_2 = (Z^m - Z^o)$

$= \sum_{n=1}^{N}\sum_{t=1}^{T}[(r_{nt}^{m} - r_{nt}^{o})RQ_{nt} + (o_{nt}^{m} - o_{nt}^{o})OQ_{nt} + (s_{nt}^{m} - s_{nt}^{o})SQ_{nt} + (i_{nt}^{m} - i_{nt}^{o})IQ_{nt} +$
$(b_{nt}^{m} - b_{nt}^{o})BQ_{nt}] + \sum_{t=1}^{T}[(h_{t}^{m} - h_{t}^{o})H_t + (f_t^m - f_t^o)F_t]$ (13)

- Minimizing the risk of obtaining the higher total costs

  Min $Z_3 = (Z^p - Z^m)$

  $= \sum_{n=1}^{N}\sum_{t=1}^{T}[(r_{nt}^{p} - r_{nt}^{m})RQ_{nt} + (o_{nt}^{p} - o_{nt}^{m})OQ_{nt} + (s_{nt}^{p} - s_{nt}^{m})SQ_{nt} + (i_{nt}^{p} - i_{nt}^{m})IQ_{nt}$
  $+ (b_{nt}^{p} - b_{nt}^{m})BQ_{nt}] + \sum_{t=1}^{T}[(h_{t}^{p} - h_{t}^{m})H_t + (f_t^p - f_t^m)F_t]$ (14)

2) Constraints

This model consists of two types of constraints; crisp and fuzzy. Equations (3) and (6) are crisp constraints where there is no uncertainty involved in setting such limitations. The remaining constraints (i.e., Equations (2), (4) and (5)) contain a certain level of uncertainty and must be transformed to crisp constraints by the defuzzification method.

3.3 Integrating the mathematical model of PLP with FGP

To solve the APP, a two-phase approach is applied. The first phase deals with the MOPMILP model, which can be used to convert the fuzzy MOPMILP values to crisp values. FGP is then applied in the second phase to convert the fuzzy MOPMILP values to single-objective possibilistic mixed-integer linear programming values.

3.3.1 Phase 1 (Multi-Objective Mixed-Integer Linear Programming model)

The crisp MOMILP model is stated as follows:

Minimize Z = $[Z_1, -Z_2, Z_3]$

$$Z_1 = Z^m, Z_2 = Z^m - Z^o, Z_3 = Z^p - Z^m \quad (15)$$

The PIS and NIS of all objective functions are set as the boundaries to convert a multiple objective linear programming problem to a single objective linear programming problem. The calculation is below.

Z_1^{PIS} = minimize Z^m , Z_1^{NIS} = maximize Z^m
Z_2^{PIS} = maximize $Z^m - Z^o$, Z_1^{NIS} = minimize $Z^m - Z^o$
Z_3^{PIS} = minimize $Z^p - Z^m$, Z_1^{NIS} = maximize $Z^p - Z^m$ (16)

The linear membership functions for each objective function and constraint are follows:

$$\mu_1(x) = \begin{cases} 1 & if\ Z_1 < Z_1^{PIS} \\ \frac{Z_1^{NIS} - Z_1}{Z_1^{NIS} - Z_1^{PIS}} & if\ Z_1^{PIS} \le Z_1 \le Z_1^{NIS} \\ 0 & if\ Z_1 > Z_1^{NIS} \end{cases} \quad (17)$$

$$\mu_2(x) = \begin{cases} 1 & if\ Z_2 > Z_2^{PIS} \\ \frac{Z_2 - Z_2^{NIS}}{Z_2^{PIS} - Z_2^{NIS}} & if\ Z_2^{NIS} \le Z_2 \le Z_2^{PIS} \\ 0 & if\ Z_2 > Z_2^{NIS} \end{cases} \quad (18)$$

$$\mu_3(x) = \begin{cases} 1 & if\ Z_3 < Z_3^{PIS} \\ \frac{Z_3^{NIS} - Z_3}{Z_3^{NIS} - Z_3^{PIS}} & if\ Z_3^{PIS} \le Z_3 \le Z_3^{NIS} \\ 0 & if\ Z_3 > Z_3^{NIS} \end{cases} \quad (19)$$

$$\mu_g(x) = \begin{cases} \frac{d_{nt} - d_{nt}^{o}}{d_{nt}^{m} - d_{nt}^{o}} & if\ d_{nt}^{o} \le d_{nt} \le d_{nt}^{m} d_{nt} \le d_{nt}^{o} \\ \frac{d_{nt}^{p} - d_{nt}}{d_{nt}^{m}} & if\ d_{nt}^{m} \le d_{nt} \le d_{nt}^{p} \\ 0 & if\ d_{nt} \le d_{nt}^{o}\ and\ d_{nt} \ge d_{nt}^{p} \end{cases} \quad (20)$$

The linear membership functions for minimizing the most likely total costs, maximizing the lower total, minimizing the higher total costs, and fuzzy constraint (customer demand) are described in Appendix.

3.3.2 Phase 2 (FGP model)

A fuzzy decision is a selection of activities, which simultaneously satisfy the objective functions and constraints. It consists of two categories of decision making: symmetric and asymmetric fuzzy. Zimmermann's method belongs to symmetric fuzzy decision-making, which sets equal importance to objectives and constraints (weightless). In contrast, the WA method belongs to asymmetric fuzzy decision making in which a decision maker can assign different weights to objectives and constraints based on their importance, and on the preferences of the decision maker.

1) Zimmermann's method

This method maximizes the lowest satisfaction of objectives, which can guarantee that all satisfaction levels of objectives are higher than the satisfaction level of the lowest objective. It is set as an upper benchmark of the optimal solution, in which all objective functions have equal weight (weightless) and can be expressed as follows:

Maximize $\lambda = \lambda_0$
Subject to:

$$\lambda_0 \geq \mu_f(x), \qquad f = 1,2,3$$
$$x \in F(x) \tag{21}$$

where λ_0 indicates the minimum satisfaction degree of the objective functions, and $F(x)$ denotes the feasible region involving the constraints of the equivalent crisp model.

2) Weighted Additive (WA) method

This method is widely used in vector-objective optimization problems. It attempts to maximize the minimum overall satisfaction of fuzzy objective functions and fuzzy constraints and allows decision makers to assign weights of each objective based on their experiences. It can be expressed as follows:

Maximize $\lambda = \sum_{j=1}^{J} w_j \lambda_j + \sum_{i=1}^{I} \beta_i \gamma_i$
Subject to:

$$\lambda_j \leq \mu_j(x), \qquad j = 1,2,3$$
$$\gamma_i \leq \mu_i(x), \qquad i = 1$$
$$x \in F(x)$$
$$\sum_{j=1}^{J} w_j + \sum_{i=1}^{I} \beta_i = 1 \text{ where } w_j, \beta_i \geq 0 \tag{22}$$

In this study, the main goal is assigned a weight of 30% (equally to each objective), and the constraint is assigned a weight of 10%.

3.4 α-Cut analysis

In most cases, the satisfaction level may not be enough to satisfy the decision makers since a poor performance in one criterion cannot easily be balanced with a good performance in the other criteria. α-Cut analysis can help decision makers to ensure that the satisfaction level for fuzzy goals and fuzzy constraints is not less than the decision-maker's minimum allowed satisfaction value (α). The following constraints can be added to the model.

$$\lambda_j \geq \alpha$$
$$\gamma_i \geq \alpha$$
$$\alpha \in [\alpha^-, \alpha^+] \tag{23}$$

4. Case Study

In the case study, an APP problem with two types of products that are planned to be manufactured in the next 4 periods is used to demonstrate the proposed methodology. The forecast demand, related operating costs, and the plant capacity are uncertain, and are summarized in Table 1.

4.1 Other relevant information

- The initial inventory level of Products 1 and 2 in the first period are 400 and 200 units, respectively. The ending inventory level of Products 1 and 2 in the fourth period are 300 and 200 units, respectively.
- The costs of hiring and firing are imprecise with ($8, $10, $11) and ($2.0, $2.5, $3.2) per worker per hour, respectively.
- The initial labor level is 300 person-hours.
- The labor hours which are used to produce Products 1 and 2 are fixed at 0.05 and 0.07 person-hours per unit in any period, respectively.
- The hours of machine usage per unit are also fuzzy with (0.09, 0.10, 0.11) and (0.07, 0.08, 0.09) machine-hours for Products 1 and 2 in any period, respectively.
- The required warehouse spaces for Products 1 and 2 are 2 ft^2 and 3 ft^2 per unit, respectively.

5. Results

5.1 Multiple-Objective Mixed-Integer Linear Programming (MOMILP)

From Table 2, the PIS and NIS of minimizing the most likely total costs can be calculated by minimizing the most likely total costs. This yields a PIS of $270,075, while maximizing the most likely total costs yields a NIS of $431,260. The PIS and NIS can also be calculated to maximize the lower total costs. This yields the Positive Ideal Solution of $53,718, while minimizing the lower total costs yields the Negative Ideal Solution of $43,667.

5.2 Fuzzy Goal Programming (FGP)

5.2.1 Zimmermann's method

Based on Table 3, the overall satisfaction (λ), which is the maximum value of the minimum satisfaction of the objective functions, is equal to 59.9%. In this case, the satisfaction of each objective is equally set to 59.9% for the relative importance of each objective function (equal weight or

Table 1. Forecast demand and related operating costs of Products 1 and 2, maximum labor level, machine capacity and warehouse space in Periods 1-4

Period	Forecast demand		Cost of regular time production		Cost of overtime production		Cost of subcontracting		Cost of inventory		Cost of backordering		$\widetilde{L_{tmax}}$ (person-hours)	$\widetilde{M_{tmax}}$ (machine-hours)	WS_{tmax} (ft^2)
	$\widetilde{D_{1t}}$ (units)	$\widetilde{D_{2t}}$ (units)	$\widetilde{r_{1t}}$($/unit)	$\widetilde{r_{2t}}$($/unit)	$\widetilde{o_{1t}}$($/unit)	$\widetilde{o_{2t}}$($/unit)	$\widetilde{s_{1t}}$($/unit)	$\widetilde{s_{2t}}$($/unit)	$\widetilde{i_{1t}}$($/unit)	$\widetilde{i_{2t}}$($/unit)	$\widetilde{b_{1t}}$($/unit)	$\widetilde{b_{2t}}$($/unit)			
1	(900,1000,1080)	(900,1000,1080)	(17,20,22)	(8,10,11)	(26,30,33)	(12,15,17)	(22,25,27)	(10,12,13)	(0.27,0.30,0.32)	(0.13,0.15,0.16)	(35,40,44)	(16,20,23)	(175,300,320)	(360,400,430)	10,000
2	(2750,3000,3200)	(450,500,540)	(17,20,22)	(8,10,11)	(26,30,33)	(12,15,17)	(22,25,27)	(10,12,13)	(0.27,0.30,0.32)	(0.13,0.15,0.16)	(35,40,44)	(16,20,23)	(175,300,320)	(450,500,540)	10,000
3	(4600,5000,5300)	(2750,3000,3200)	(17,20,22)	(8,10,11)	(26,30,33)	(12,15,17)	(22,25,27)	(10,12,13)	(0.27,0.30,0.32)	(0.13,0.15,0.16)	(35,40,44)	(16,20,23)	(175,300,320)	(540,600,650)	10,000
4	(1850,2000,2100)	(2300,2500,2650)	(17,20,22)	(8,10,11)	(26,30,33)	(12,15,17)	(22,25,27)	(10,12,13)	(0.27,0.30,0.32)	(0.13,0.15,0.16)	(35,40,44)	(16,20,23)	(175,300,320)	(450,500,540)	10,000

Table 2. Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) of all objective functions

	Positive Ideal Solution (PIS)	Negative Ideal Solution (NIS)
Minimize the most likely total costs ($Z_1 = Z^m$)	$270,075	$431,260
Maximize the lower total costs ($Z_2 = Z^m - Z^o$)	$53,718	$43,667
Minimize the higher total costs ($Z_3 = Z^p - Z^m$)	$26,819	$36,928

Table 3. Optimal solutions from Zimmermann's and Weighted Additive methods

	Zimmermann's method	Weighted Additive method
Overall satisfaction (λ)	59.9%	60.2%
Minimum possible value of the lower total costs ($Z_2 = Z^m - Z^o$)	$261,254	$226,932
Minimum possible value of the most likely total costs ($Z_1 = Z^m$)	$310,940	$270,880
Minimum possible value of the higher total costs ($Z_3 = Z^p - Z^m$)	$341,814	$297,858
Satisfaction from minimizing the most likely total costs (λ_1)	59.9%	99.5%
Satisfaction from maximizing the lower total costs (λ_2)	59.9%	2.7%
Satisfaction from minimizing the higher total costs (λ_3)	59.9%	98.4%
Satisfaction of demand constraint (γ_1)	-	100%

weightless). At 59.9% satisfaction, the minimum value of the most likely total costs (Z_1) is $310,940, the maximum value of the lower total costs (Z_2) is $49,686, and the minimum value of the higher total costs (Z_3) is $30,874.

5.2.2 Weighted Additive (WA) method

The WA method is an asymmetric model in which decision-makers can set different weights for each objective function based on its importance. The WA method attempts to maximize each membership function of fuzzy goals and constraints with their corresponding weights. In this study, $w_1 = 0.30, w_2 = 0.30, and\ w_3 = 0.30$ are assigned to the fuzzy goals and $\beta_1 = 0.10$ is assigned to the fuzzy constraint. This leads to 90% weight for the fuzzy objectives and 10% for the fuzzy constraint.

Based on the results obtained from the WA method (also shown in Table 4), the overall satisfaction (λ) is 60.2%, which is higher than the overall satisfaction from Zimmermann's method (59.9%). Referring to the aforementioned percentages of the satisfaction of each objective function and constraint, the minimum value of the most likely total costs

Table 4. Results comparison

	Weighted Additive method	Fuzzy multi-objective PLP with the α-cut analysis (Scenario 5)
Overall satisfaction (λ)	60.2%	67%
Minimum possible value of the lower total costs ($Z_2 = Z^m - Z^o$)	\$226,932	\$235,919
Minimum possible value of the most likely total costs ($Z_1 = Z^m$)	\$270,880	\$282,150
Minimum possible value of the higher total costs ($Z_3 = Z^p - Z^m$)	\$297,858	\$310,679
Satisfaction from minimizing the most likely total costs (λ_1)	99.5%	92.5%
Satisfaction from maximizing the lower total costs (λ_2)	2.7%	25.5%
Satisfaction from minimizing the higher total costs (λ_3)	98.4%	83.0%
Satisfaction of demand constraint (γ_1)	100%	100%

(Z_1) is \$270,880, the maximum value of the lower total costs (Z_2) is \$43,948, and the minimum value of the higher total costs (Z_3) is \$26,978.

Normally, the obtained values of the satisfaction should follow the weights, which are assigned based on the decision-makers' preferences. In this study, equal weights of the main goals ($w_1 = 0.30, w_2 = 0.30,\ and\ w_3 = 0.30$) are assigned to each objective function but the obtained values of the satisfaction are different ($\lambda_1 = 99.5\%, \lambda_2 = 2.7\%,\ and\ \lambda_3 = 98.4\%$). Typically, it is difficult to obtain lower total costs while minimizing both the most likely total costs and the risk to obtain higher total costs and still satisfy the same amount of the customer's demand.

As a result, the satisfaction values of obtaining the minimization of the most likely total costs ($\lambda_1 = 99.5\%$), reduce the risk of obtaining the higher total costs ($\lambda_3 = 98.4\%$), and achieve the customer's demand ($\lambda_4 = 100\%$) while the satisfaction value of obtaining the lower total costs is only 2.7%. This low satisfaction value will not satisfy the decision makers. So, the α-Cut analysis needs to be introduced to achieve at least the minimum satisfaction level for the decision makers.

5.2.3 α-Cut analysis

This approach can help decision makers increase the satisfaction level of fuzzy objectives and constraints to be not less than their specified minimum allowed satisfaction value (α). In this case, $\alpha^+$is set to 0.599, which is derived from the optimal satisfaction of Zimmermann's method in which all objective functions are equally important (fully symmetric). $\alpha^-$is set to 0.027, which is derived from the lowest satisfaction among fuzzy objective functions and constraints of the WA method in which the fuzzy objective functions and constraints have unequal importance (asymmetric). Thus, α can vary from 0.027 to a maximum level of 0.599 by a step size of 0.057 so that the solution can be changed from asymmetric to fully symmetric decision making.

Based on the WA method, the satisfaction value from maximizing the lower total costs of logistics (λ_2) is equal to 2.7%. This is assumed to be lower than the decision-makers' preferences, which are set to be at least 25%. The process of α-Cut analysis tries to increase the satisfaction level by maximizing the lower total costs (λ_2) to be more than or at least equal to 25%.

Based on Table 5, when the value of α is varied from 0.027 to a maximum level of 0.599 by a step size of 0.057, the results are as follows:

- The satisfaction values from minimizing the most likely total costs (λ_1) and minimizing the higher total costs (λ_3) keep decreasing while the satisfaction values from maximizing the lower total costs (λ_2) keep increasing.
- The satisfaction values of the fuzzy demand constraint (γ_1) for all scenarios achieve 100% or are at their most likely value as a lower or higher amount of demand would reduce the satisfaction values of the total costs.
- A break-even point is shown in Scenario 5 (S5) where the overall satisfaction of the model (λ) is the highest possible at 67% while the satisfaction values of all fuzzy objectives and demand constraint can satisfy the decision-makers preferences (higher than 25% as imposed by the decision makers).

Table 5. Solutions of α-Cut analysis

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
α-Cut	0.027	0.084	0.141	0.198	0.255	0.312	0.369	0.426	0.483	0.540	0.599
Overall satisfaction	70%	69.3%	68.5%	67.8%	67%	66.4%	65.6%	64.9%	64.1%	63.4%	54.5%
Z_1 (\$)	270,490	273,400	276,300	279,220	282,150	284,810	287,680	290,570	293,680	296,320	332,090
Z_2 (\$)	43,938	44,512	45,085	45,674	46,231	46,803	47,376	47,949	48,572	49,097	49,688
Z_3 (\$)	26,970	27,359	27,748	28,151	28,529	28,917	29,307	29,697	30,124	30,478	30,873
λ_1	99.7%	97.9%	96.1%	94.3%	92.5%	90.8%	89.1%	87.2%	85.3%	83.7%	61.5%
λ_2	2.7%	8.4%	14.1%	19.9%	25.5%	31.2%	36.9%	42.6%	48.7%	54%	59.9%
λ_3	98.5%	94.6%	90.8%	86.8%	83.0%	79.25	75.4%	71.5%	67.3%	63.8%	59.9%
γ_1	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Figure 3 shows the plots of the satisfactions of each objective function and demand constraint in each scenario to see the pattern of the satisfaction values of all objectives while the value of α is changed. It also shows the break-even point at Scenario 5 (S5) where the satisfaction values of all fuzzy objectives and demand constraint can satisfy the decision-makers preferences (higher than 25%).

Based on Table 4, the satisfaction values of each objective function and constraint achieve the minimum satisfaction level (25%). After applying the α-cut analysis to the traditional fuzzy multi-objective PLP model (Weighted Additive method), the satisfaction of maximizing the lower total costs (λ_2) is increased by trade-off with satisfaction values of minimizing the most likely total costs (λ_1) and minimizing the higher total costs (λ_3). This can guarantee that applying α-cut analysis can help decision makers to increase the satisfaction level of fuzzy objectives and constraints to be not less than their specified minimum allowed satisfaction value (α).

For the implementation plan, Table 6 shows that there is no overtime production and no backordering in Scenario 5. The manufacturer has to produce 9,832 units of Product 1 and 6,520 units of Product 2 during the regular time in Periods 1-4. Fifty-seven is the total number of workers that are hired, and one hundred and six is the total number of workers that are laid off during Periods 1-4. The optimal total cost of the plan would range from $310,679 for the pessimistic case to $235,919 for the optimistic case with the most likely total costs of $282,150.

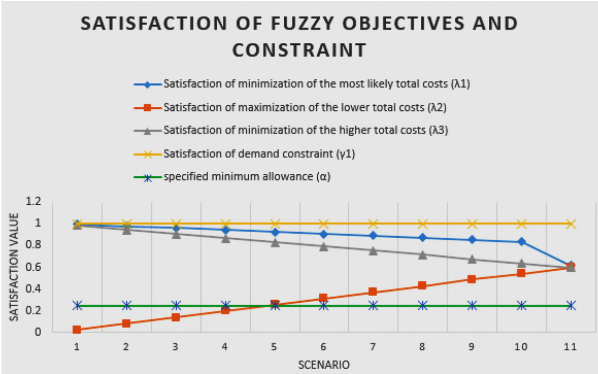


Figure 3. Satisfactions of each objective function and demand constraint

Table 6. Production outcome and changes in labor level for Scenario 5

Product1					
Period	Regular time production (units)	Overtime production (units)	Subcontracting (units)	Backordering level (units)	Ending inventory (units)
1	1,475	0	0	0	878
2	3,188	0	0	0	1,074
3	3,880	0	30	0	0
4	1,289	0	3	0	300
Total	9,832	0	33	0	2,252
Product 2					
Period	Regular time production (units)	Overtime production (units)	Subcontracting (units)	Backordering level (units)	Ending inventory (units)
1	2,975	0	0	0	2,178
2	880	0	0	0	2,559
3	0	0	433	0	0
4	2,665	0	27	0	200
Total	6,520	0	460	0	4,937

Product 1 & Product 2		
Period	Hiring (workers)	Firing (workers)
1	0	18
2	0	61
3	0	27
4	57	0
Total	57	106

6. Conclusions

APP is an intermediate term process of production planning to satisfy customer requirements and achieve a competitive advantage. This study proposed using fuzzy multi-objective optimization with α-Cut analysis to solve the APP problem with imprecise data. It can be used to ensure that all objectives are considered simultaneously, and the satisfactions of fuzzy objectives and constraints are not less than the decision maker's minimum allowed satisfaction value (α).

Normally, the multiple objectives are not treated as equally important. A fuzzy multi-objective linear programming with the WA method was introduced to APP, to assign different weights to various criteria. In addition, α-Cut analysis was also introduced to guarantee that the obtained result is acceptable matching the decision maker's preferences.

Our solutions showed that the proposed fuzzy multi-objective optimization with α-Cut analysis can be used to improve the satisfaction level of fuzzy objectives and constraints. An optimal point (Scenario 5) where the satisfaction values of fuzzy objectives and the demand constraint can satisfy the required minimum satisfaction value from decision makers is selected.

The main limitation of our proposed approach is the assumption of imprecise data. Thus, decision makers should generate and obtain an appropriate distribution, such as triangular or trapezoid distribution, based on subjective judgment and historical data. Further research could explore different weights for individual goals and constraints, to better suit their practical applications.

References

Amir, S., & Leila, M.N. (2011). Evaluating fuzzy earned value indices and estimates by applying alpha cuts. *Expert Systems with Applications*, *38*(1), 8193-8198.

Barik, S. K., & Biswal, M. P. (2016). Possibilistic linear programming problems involving normal random variables. *International Journal of Fuzzy System Applications, 5*(3), 1–13.

Baykasoglu, A., & Gocken, T. (2010). Multi-objective aggregate production planning with fuzzy parameters. *Advances in Engineering Software, 41*(9), 1124–1131.

Bellman, R.G., & Zadeh, L.A. (1970). Decision making a fuzzy environment. *Management Science, 17*(1), 141-164.

Bodjanova, S. (2002). A generalized α-cut. *Fuzzy Sets and Systems, 126*(1), 157-176.

Biswas, A., & De, A. K. (2016). A fuzzy goal programming approach for solid waste management under multiple uncertainties. *Procedia Environmental Sciences, 35* (1), 245–256.

Cha-ume K, & Chiadamrong, N. (2012). Simulation of retail supply chain behaviour and financial impact in an uncertain environment. *International Journal of Logistics Systems and Management, 13*(2), 162.

Entezaminia, A., Heydari, M., & Rahmani, D. (2016). A multi-objective model for multi-product multi-site aggregate production planning in a green supply chain: Considering collection and recycling centers. *Journal of Manufacturing Systems, 40*(1), 63-75.

Hossain, M. M, Nahar, K., Reza, S., & Shaifullah, K. M. (2016) Aggregate production planning under demand uncertainty by considering wastage cost and incentives. *World Review of Business Research*, *6*(2), 170-185.

Kabak, Ö., & Ülengin, F. (2011). Possibilistic linear-programming approach for supply chain networking decisions. *European Journal of Operational Research, 209*(3), 253–264.

Karmarkar, U., & Rajaram, K. (2012). Aggregate production planning for process industries under oligopolistic competition. *European Journal of Operational Research, 223*(3), 680-689.

Lachhwani, K., & Poonia, M. P. (2012). Mathematical solution of multilevel fractional programming problem with fuzzy goal programming approach. *Journal of Industrial Engineering International, 8*(1), 1–8.

Mokhtari, H., & Hasani, A. (2017). A multi-objective model for cleaner production-transportation planning in manufacturing plants via fuzzy goal programming. *Journal of Manufacturing Systems, 44*(1), 230–242.

Özgen, D., Önüt, S., Gülsün, B., Tuzkaya, U. R., & Tuzkaya, G. (2008). A two-phase possibilistic linear programming methodology for multi-objective supplier evaluation and order allocation problems. *Information Sciences, 178*(2), 485–500.

Purba, J. H., Tjahyani, D. S., Widodo, S., & Tjahjono, H. (2017). α-cut method based importance measure for criticality analysis in fuzzy probability – based fault tree analysis. *Annals of Nuclear Energy, 110*(1), 234–243.

Saxena, L. K., Jain, P. K., & Sharma, A. K. (2018). A fuzzy goal programing with carbon tax policy for Brownfield Tyre remanufacturing strategic supply chain planning. *Journal of Cleaner Production, 198*(1), 737–753.

Taghizadeh, K., Bagherpour, M., & Mahdavi, I. (2011). An interactive fuzzy goal programming approach for multi-period multi-product production planning problem. *Fuzzy Information and Engineering*, *3*(4), 393–410.

Tohidi, G., & Razavyan, S. (2012). An L1-norm method for generating all of efficient solutions of multi-objective integer linear programming problem. *Journal of Industrial Engineering International, 8*(1), 1-11.

Wang, R., & Liang, T. (2005). Applying possibilistic linear programming to aggregate production planning. *International Journal of Production Economics, 98*(3), 328-341.

Yang, Y., Li, X. R., & Han, D. (2016). An improved α-cut approach to transforming fuzzy membership function into basic belief assignment. *Chinese Journal of Aeronautics*, *29*(4), 1042–1051.

Zimmermann, H. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, *1*(1), 45-55.

Zhang, X., Ma, W., & Chen, L. (2014). New similarity of triangular fuzzy number and its application. *The Scientific World Journal, 1*(1), 1-7.

Appendix

1. Linear membership functions for the minimization goals (minimize the most likely total costs (Z_1) and minimize the higher total costs (Z_3)) are given as follows:

$$\mu_1(x) = \begin{Bmatrix} 1 & if\ Z_1 < 270{,}075 \\ \frac{Z_1^{NIS} - Z_1}{Z_1^{NIS} - Z_1^{PIS}} & if\ 270{,}075 \leq Z_1 \leq 431{,}260 \\ 0 & if\ Z_1 > 431{,}260 \end{Bmatrix}$$

$$\mu_3(x) = \begin{Bmatrix} 1 & if\ Z_3 < 26{,}819 \\ \frac{Z_3^{NIS} - Z_3}{Z_3^{NIS} - Z_3^{PIS}} & if\ 26{,}819 \leq Z_3 \leq 36{,}928 \\ 0 & if\ Z_3 > 36{,}928 \end{Bmatrix}$$

2. Linear membership functions for the maximization goals (maximize the lower total costs (Z_2) are given as follows:

$$\mu_2(x) = \begin{Bmatrix} 1 & if\ Z_2 > 53{,}718 \\ \frac{Z_2 - Z_2^{NIS}}{Z_2^{PIS} - Z_2^{NIS}} & if\ 43{,}667 \leq Z_2 \leq 53{,}718 \\ 0 & if\ Z_2 > 43{,}667 \end{Bmatrix}$$

3. Linear membership function for a fuzzy constraint (fuzzy demand constraint) is given as follows:

$$\mu_g(x) = \begin{Bmatrix} \frac{d_{nt} - d_{nt}^{o}}{d_{nt}^{m} - d_{nt}^{o}} & if\ 900 \leq d_{nt} \leq 1{,}000 \\ \frac{d_{nt}^{p} - d_{nt}}{d_{nt}^{m}} & if\ 1{,}000 \leq d_{nt} \leq 1{,}080 \\ 0 & if\ d_{nt} \leq 900\ and\ d_{nt} \geq 1{,}080 \end{Bmatrix}$$

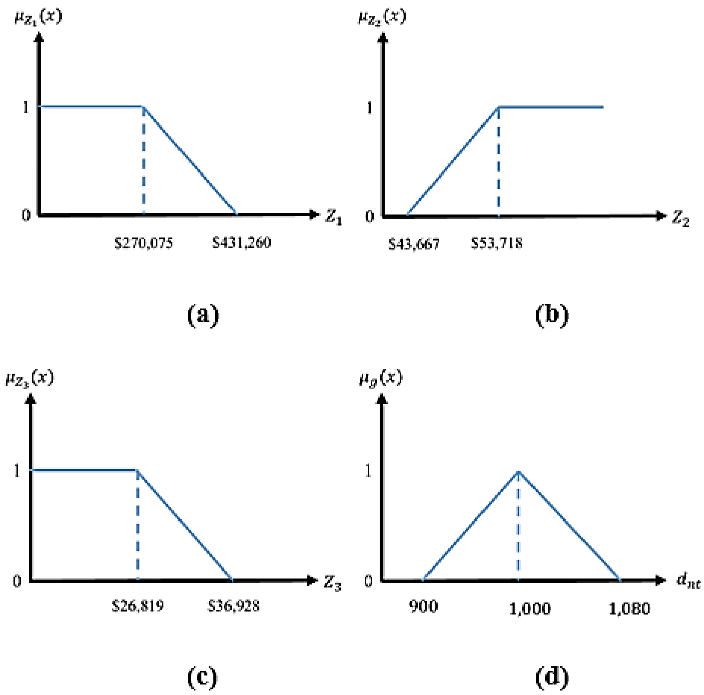


Figure 4. Linear membership functions: (a) minimization of the most likely total costs, (b) maximization of the lower total costs, (c) minimization of the higher total costs, and (d) fuzzy constraint