CHAPTER 3 THEORETICAL MODEL

3.1 Overview

This chapter concerns with a model to predict the valve leakage rate. Since a theoretical or physical model can generalize better than empirical models, this chapter considers a theoretical model instead of experimental models to determine the leakage rate. The theoretical relationship and theoretical model for predicting the rate are derived. This chapter is organized as follows. Firstly, Section 3.1 begins by overview of the chapter. Background of theoretical model and assumption of this proposed method are addressed in Sections 3.2 and 3.3, respectively. Sections 3.4 and 3.5 express the development of theoretical relationship and its validation, respectively. Finally, development of theoretical model is presented in Section 3.6.

3.2 Background of Theoretical Model

In experimental model for predicting valve leakage rate, typically, Most of the referred researches derived correlations between AE signal, process and fluid variables from experimental results [32, 33, 34, 35]. Therefore, one of the most common problems is to train a model and to obtain information required to account for valve leakage rates. This affected many process variables, for instance, inlet pressure level, valve size and type etc. Additionally, the fluid variables correlated to AE signal are turbulence jet velocity, sound velocity in the fluid, fluid density, gas constant, temperature etc. Hence, specific tests are performed to provide a great deal of database to model the valve leakage rate empirically.

In the field of noise and vibration control, for a number of years, it has been known that flow control valves can be a significant source of noise in industrial facilities [75, 76, 77, 78, 79, 80]. As such, much work has been done on noise from control valves in air and water systems [77, 78]. In 1995, the International Electrotechnical Commission (IEC) released a standard method [81] for predicting the aerodynamic noise generated by control valves using ideal gases. This standard method was based on the free jet noise studies first published by Lighthill [82, 83] and the confined jet studies of Curle [84]. In the field of AE, however, many researches just use the concept of Lighthill to explain the sources of AE without any theoretical correlation between both fields [33, 34, 35, 47, 48]. Nevertheless, few researches have been investigating relationship between the leakage rate and AE parameter, but they suggested that it was necessary to know the dimension of leak path and fluid velocity through the path in order to estimate the volumetric flow using the AE signal as well as known fluid properties such as density and sound velocity [56]. Certainly, in the real problem nothing is known about size or character of the damage of the valve seat. In addition, the velocity of leaking fluid is also unknown.

Thus, the main contribution presented in this chapter is an attempt to derive a theoretical model to predict leakage rate of valve focused on the known process variable such as valve size and inlet pressure level and the concerned fluid variable by using AE method based on Lighthill's equation.

3.3 Assumptions

For released acoustic emission from valve leakage illustrated in Figure 3.1, the elastic wave from the sound source propagates along wave propagation paths, i.e., valve body and AE measurement system. Since geometrical shapes and materials of valve body are assumed in scopes of this dissertation, it is rather complicated to investigate quantitatively theoretical model of the wave propagation paths and the AE signal. Although effect of wave propagation path is critical in the study of AE applications, influence of redistribution of acoustic field leads to the complicated physical characteristic of AE wave. However, the aim of this dissertation is to investigate a theoretical model to predict the leakage rate. It should be noted that the effect of wave propagation path is excluded from the model in this work.

By neglecting the attenuation, interference, reflection, reflexion, mode conversion of sound wave occurred within the wave propagation path, and the response characteristic of the AE sensor, a hypothesis can be made about the theoretical relationship and model ,i.e., the relation between the sound power (P_s) generated by valve leakage and the AE signal power in form of AE_{RMS}^2.

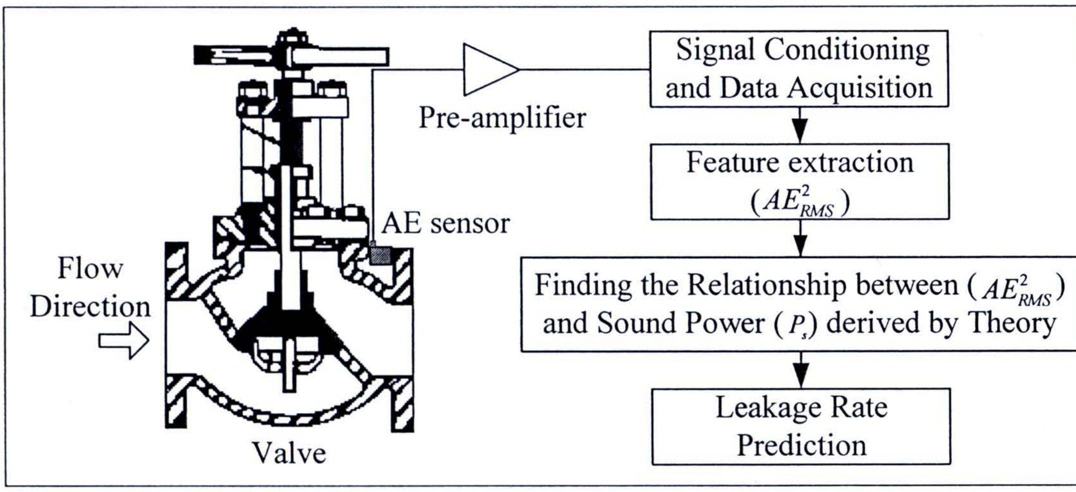

Figure 3.1 Process to predict valve leakage rate by acoustic emission.

3.3.1 RMS Voltage

Sound from previously-mentioned sources propagates through valve body into an AE sensor mounted on the valve body. The AE sensor then converts the sound wave inform of an elastic wave to an electrical signal. The AE signal of valve leakage is of a continuous type. Typically, waveform of the recorded signal is converted to AE_{RMS} to analyze the results. The frequently-used AE_{RMS} quantity for that type is the Root-Mean-Square (RMS) of the AE signal as shown in Figure 3.2 and can be expressed as follows.

For continuous-time system,

$$AE_{RMS} = \sqrt{\frac{1}{T_0}\int_{t_0}^{t_o+T_0} v^2(t)dt} \tag{3.1}$$

where T_0 is the period of time (μs),

t_0 is the initial time (μs), and

$v(t)$ is the voltage signal from an AE sensor in continuous-time system (mV).

For discrete-time system,

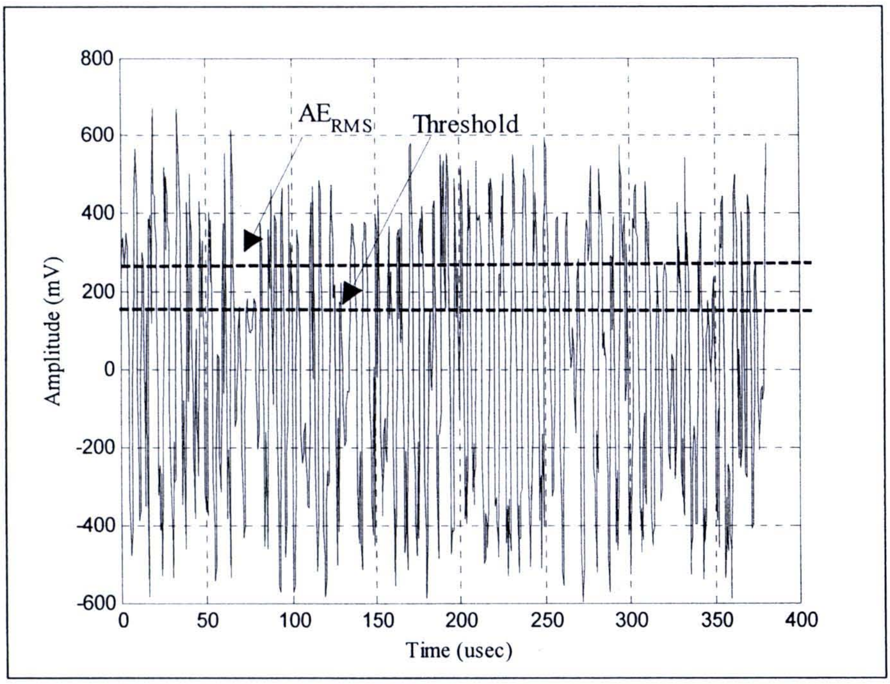

Figure 3.2 AE_{RMS} voltage of AE signal.

$$AE_{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} v^2(i)} \tag{3.2}$$

where N is the number of discrete AE data within the period of time and $v(i)$ is the voltage signal from an AE sensor in the discrete-time system.

3.3.2 AE Signal Power

The energy received from a signal could be calculated from

$$Energy = \int_{t_0}^{t_o + T_0} v^2(t)dt \tag{3.3}$$

where $v(t)$ is the voltage signal (mV).

From signal energy in Equation 3.3, the energy rate contained in the AE signal was found related to AE_{RMS} by [85]

$$\frac{Energy}{T} \propto AE_{RMS}^2 . \tag{3.4}$$

Accordingly, the assumption of theoretical relationship and model to predict valve leakage rate is that the AE signal power, AE_{RMS}^2, is a function of sound power (P_s) of turbulence jet based Lighthill's theory, as expressed by

$$AE_{RMS}^2 = f(P_s) \tag{3.5}$$

where AE_{RMS}^2 is the AE signal power (mV^2) and

P_s is the sound power (Watt).

3.4 Development of Theoretical Relationship

This section aims to investigate theoretical modeling for predicting leakage rate of valve. Although, energy detected by the AE measuring system is a part of total energy released from the leakage. The other parts are heat and vibration etc. However, several researchers found that AE energy and various leakage parameters had a good correlation [12, 33, 86].

To investigate the theoretical relationship, the assumption, which is that the AE signal power in form of AE_{RMS}^2 is a function of the sound power (P_s) generated by valve leakage, is considered as restated by

$$AE_{RMS}^2 = f(P_s) . \tag{3.6}$$

The case of sound generation by unsteady or turbulent jets has been of great interest since the introduction of the first aircraft jet engines about fifty years ago. The modern study of the problem began with the work of Lighthill [35, 39, 82], who developed a theory of sound generation by turbulence flows. The principle is called Lighthill's equation. The aerodynamic sources radiated sound power by turbulence flow is simplified as

$$P_s = \frac{\rho v_2^8 D^2}{\alpha^5} \tag{3.7}$$

where v_2 is the turbulence jet velocity (m/s),

α is the sound velocity in the fluid (m/s),

ρ is the fluid density (kg/m^3), and

D is the valve size (m).

To predict the leakage rate of *in-situ* valve, it should relate AE parameters to some known process variables such as valve size, inlet pressure level, and temperature etc. However, the problem of applying the Lighthill's equation (Equation 3.7) is that the turbulence jet velocity (v_2) is unknown *in-situ*. Thus, it should be replaced by other known fluid or process variables.

The model assumes that the principle to detect the flow through a sharp edge orifice can be used to calculate the flow rate of fluid through valves conveniently and accurately [87]. Typically, the mass flow rate of gas through a sharp edge orifice is shown by

$$W_1 = \frac{c_f P_1 \pi d^2}{4\sqrt{RT_1}} \sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \tag{3.8}$$

where W_1 is the mass flow rate (kg/s),

c_f is the orifice coefficient (dimensionless),

γ is the specific heat ratio (dimensionless),

P_1 is the inlet pressure level (N/m^2),

d is the orifice or leakage diameter (m),

R is the gas constant ($N.m.kg^{-1}.K^{-1}$), and

T_1 is the temperature (K).

Divided by the density, the mass flow rate is converted to a volumetric flow rate. The volumetric flow rate of gas through a sharp edge orifice is given by

$$Q = \frac{c_f P_1 \pi d^2}{4\rho\sqrt{RT_1}} \sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \qquad (3.9)$$

where Q is the volumetric flow rate (m^3/s), and $\kappa = \sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}$ and it can be represented by

$$d = \left(\frac{4Q\rho\sqrt{RT_1}}{\pi c_f \kappa P_1}\right)^{\frac{1}{2}}. \qquad (3.10)$$

The gas velocity at the orifice or leakage hole (v_1) [87] is described by

$$v_1 = \frac{c_f}{\sqrt{1 - \left(\frac{d}{D}\right)^4}} \sqrt{\frac{2(P_1 - P_2)}{\rho}}. \qquad (3.11)$$

The correlation of the velocity at the orifice or leakage hole (v_1) and jet velocity (v_2) can be expressed as

$$v_1 A_1 \approx v_2 A_2 \qquad (3.12)$$

where A_1 is the area of orifice hole and A_2 is the area of downstream pipe. It can be rearranged to

$$v_1 \approx v_2 \left(\frac{D}{d}\right)^2. \qquad (3.13)$$

Let $\beta = \sqrt{1 - \left(\frac{d}{D}\right)^4}$ and substitute the result in Equation 3.11 into Equation 3.13, to obtain

$$v_2 = \frac{c_f}{\beta}\sqrt{\frac{2(P_1 - P_2)}{\rho}}\left(\frac{d}{D}\right)^2. \tag{3.14}$$

Typically, d is about 38% of D or smaller, the increase in velocity (v_2) is only 1% or less. In other words, when d is small compared to D, there is no significant increase in velocity (v_2). In case of valve leakage, $d << D$ and $P_2 << P_1$, so we can omit β and P_2. In order to simplify the sound power of turbulence in form of the process variables, we replace v_2 in Equation 3.7 as

$$P_s = c_0 \frac{P_1^4 d^{16}}{\alpha^5 \rho^3 D^{14}} \tag{3.15}$$

where c_0 is a constant.

To simplify the sound power of turbulence in form of the process variables, Substitute Equation 3.10 into Equation 3.15 to obtain

$$P_s = c_0 \left(\frac{\rho}{\alpha}\right)^5 \left(\frac{RT_1}{P_1}\right)^4 \frac{Q^8}{D^{14}}. \tag{3.16}$$

The density of gas is strongly affected by the inlet pressure level and changed by varying either the gas constant or the temperature. According to Boyle's law, the density of an ideal gas is $\rho = \frac{P_1}{RT_1}$, thus

$$P_s = c_0 \frac{P_1 Q^8}{RT_1 \alpha^5 D^{14}}. \tag{3.17}$$

Substituting Equation 3.17 into Equation 3.6, it obtains

$$AE_{RMS}^{2} = f\left(c_0 \frac{P_1 Q^8}{RT_1 \alpha^5 D^{14}} \right). \qquad (3.18)$$

where f is a function.

3.5 Validation of Theoretical Relationship

3.5.1 Experimental Set-up

As described previously in Equation 3.18, it is essential to set the experiment for validating the theoretical relationship for investigating the characteristics of AE signals. In this section, a preliminary validation of this theoretical relationship was carried out. The experimental system consists of two main parts: (i) pressurizing system and (ii) AE measurement system shown in Figure 3.3. These are described in the following sections. In addition, prediction part illustrated in Figure 3.3 are attempted to express a systematic investigation for solving the theoretical model derived in Section 3.6.

1. Pressurizing System

Pressurizing system is important to investigate the relationship in this dissertation. The primary aim here is to generate the leakage relating with the detected AE signals. There are many components of pressurizing system such as: valve specimen, air compressor, pressure gauge, flow meter, pipeline, tee joint etc. The major components directly dealing with pressurizing system are described as follows.

Valve Test Rig

For valve test rig, manual flange ball valves were selected as the test subjects in the experiment since they are widely used in industries. Different sizes of soft-valve-seat ball valves of 25.4, 50.8, and 76.2 mm inside diameters were made to produce leakage by being loosely closed. The ball valves were made from cast iron material with JIS 10K standard. An example of ball valve used in the experiment was shown in Figure 3.4.

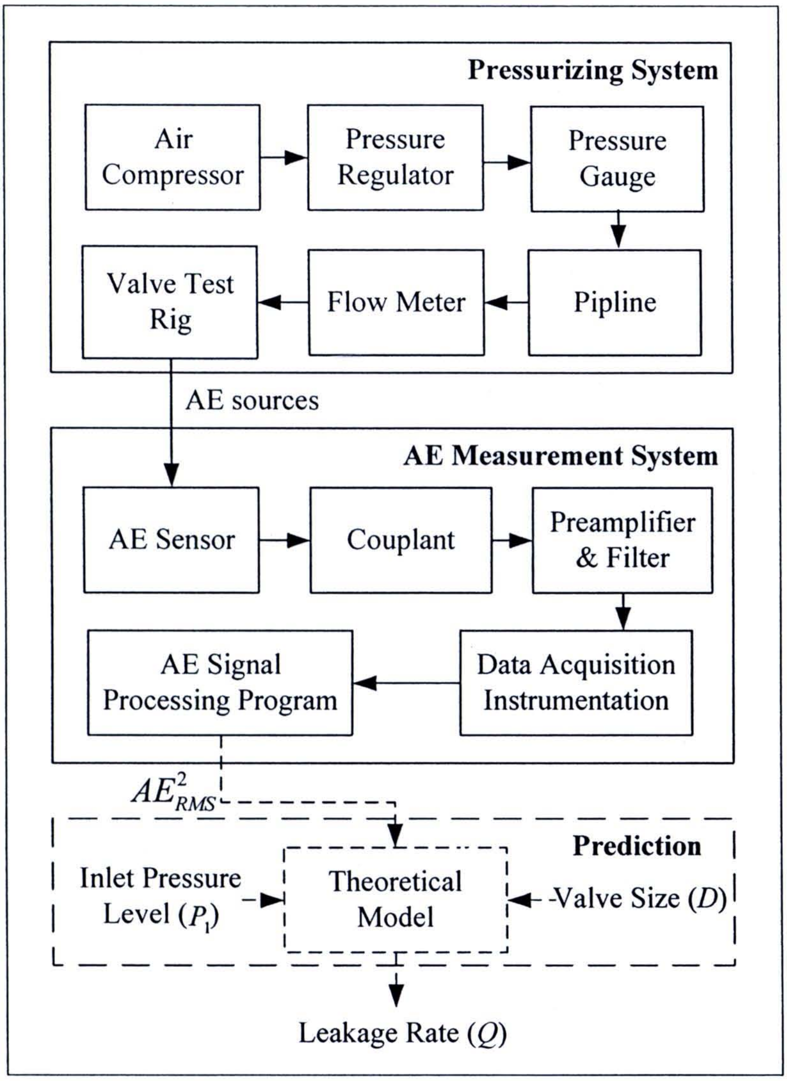

Figure 3.3 Schematic diagram of the AE experimental set-up.

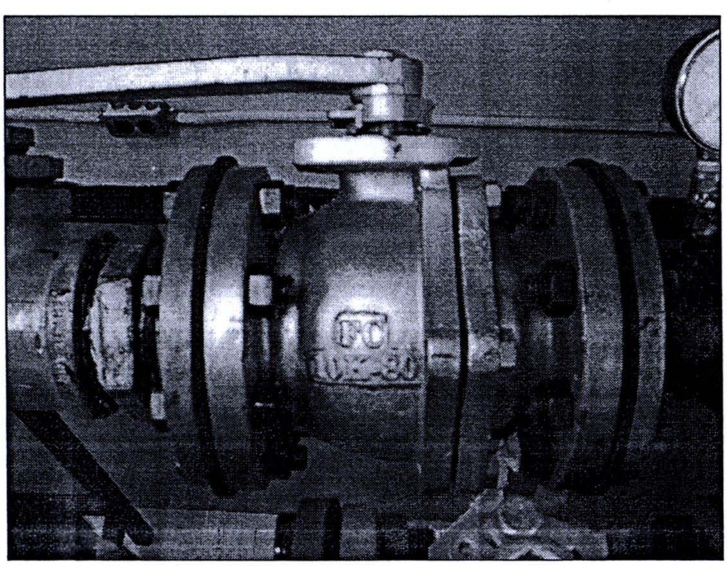

Figure 3.4 Ball valve test rig.

Air Compressor

As previously mentioned in the scope of dissertation, the provided compressible fluid in this work is air because of its accommodation and safety. The piston type air compressor was used to generate air flow with the inlet pressure levels between 100 kPa to 500 kPa, as shown in Figure 3.5.

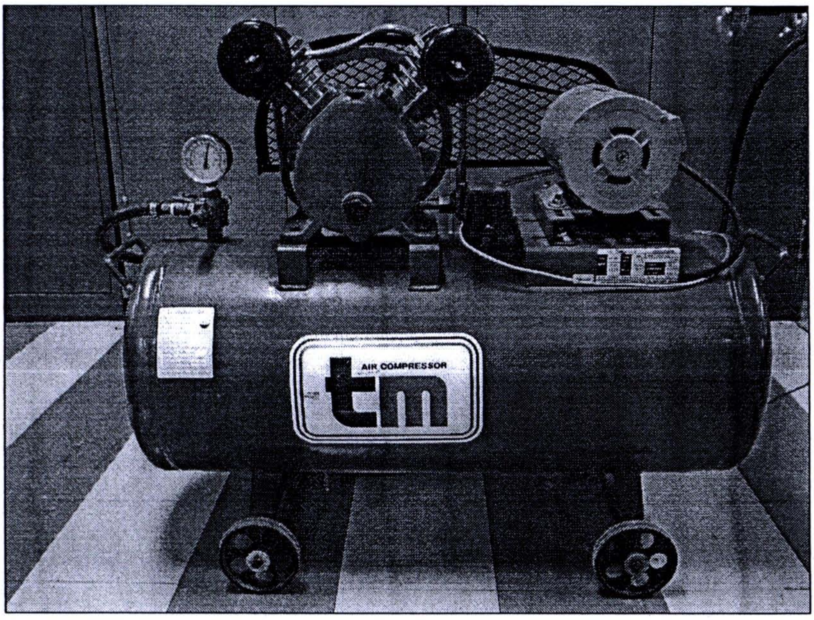

Figure 3.5 A air compressor.

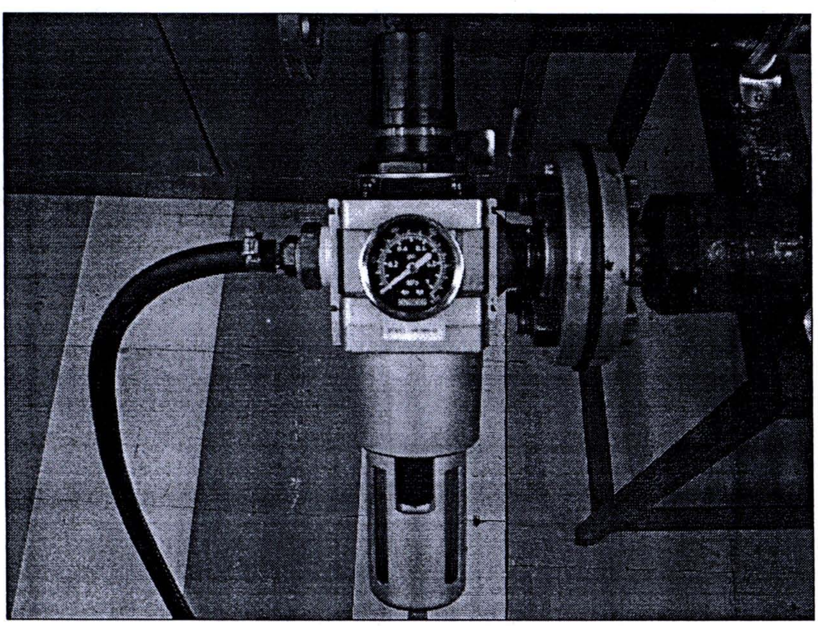

Figure 3.6 Pressure regulator.

Pressure Regulator

A pressure regulator utilized in this research is a device that automatically keeps the flow of an air at a certain pressure range. Regulators are used to allow high-pressure fluid supply lines to be reduced to safe or to be increased to usable pressures. A primary function of the pressure regulator is to match the flow of air through the regulator to the demand for air placed upon the pressurizing system. If the load flow decreases, then the regulator flow must decrease as well. If the load flow increases, then the regulator flow must increase in order to keep the controlled pressure from decreasing due to a shortage of air in the pressurizing system. It can operate in pressure range from 0 to 1000 kPa. An installed pressure regulator was used to keep the inlet pressure levels steady and also to filter out water and dust from the system, as shown Figure 3.6.

Pressure Gauge

Typically, the pressure gauge was used to express the pressures in pressurizing system. In this work, a bourdon tube gauge, of which construction elements are made of brass, was used, as shown in Figure 3.7. The pressure gauges were installed in front and back of the valve to measure the inlet pressure level of the pressurizing system. Each has pressure range from 0 to 1000 kPa (0 to 10 bars) and resolution 10 kPa (0.1 bars).

Flowmeter

In this work, two flowmeters are instruments used to measure volumetric flow rate of an air. Both flowmeters which are rotameter type from *Dwyer* are installed. To measure the leakage rate, it is possible to get a high resolution when a narrow span of scale is chosen. The provided rotameters operate in range of 0 to 5 l/min and resolution is 0.1 l/min and in range of 0 to 10 l/min and resolution is 0.5 l/min, as shown in Figure 3.8 (a) and 3.8 (b), respectively.

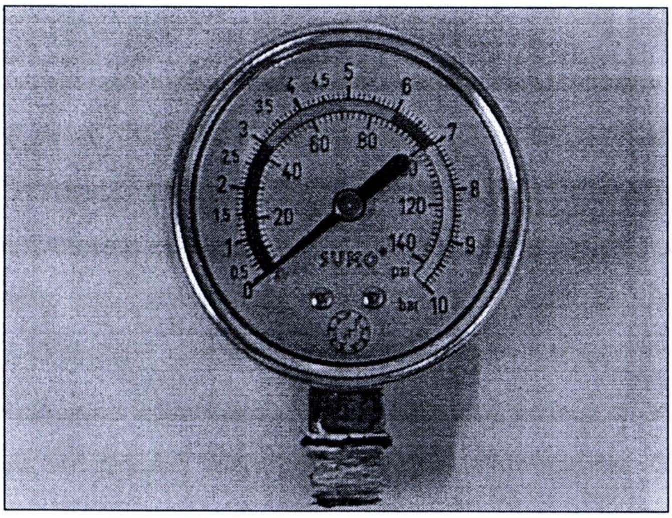

Figure 3.7 Pressure gauge.

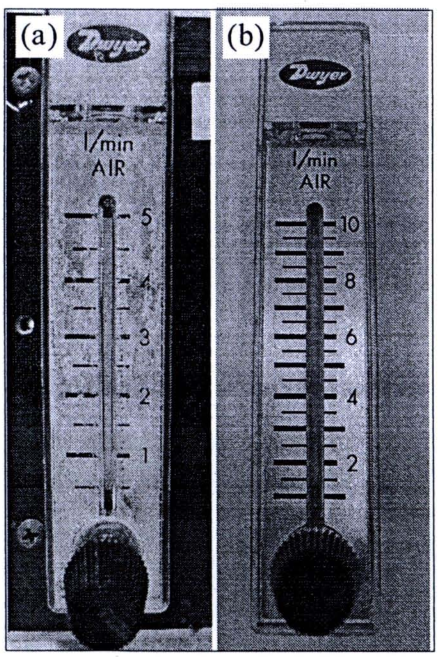

Figure 3.8 Flowmeters (a) resolution of 0.1 l/min and (b) resolution of 0.5 l/min.

2. AE Measurement System

AE measurement system for valve leakage measurement generally contains AE sensors, couplants, preamplifiers, filters, amplifiers, and data acquisition system connected to a valve body. However, R15 sensor having a resonant response center on 150 kHz and operating frequency range of the sensor ranging from 50 kHz up to 200 kHz was utilized to collect the AE information to prove the derivation. This is because the resonance type of AE sensor (R15) has better sensitivity than the wideband type (WD).

However, wideband type (WD) was also used to study the AE characteristics from valve leakage in Section 3.5.3. A preamplifier, PAC model 1220A, was used for a signal conditioner. It was set at the gain of 60 dB and connected to an analogue band-pass filter with a cut-off frequency between 100 kHz and 1200 kHz to eliminate mechanical and background noises that prevails frequency bands outside this range. In order to eliminate background noise, a band-pass filter operated in the range of 100 kHz to 400 kHz is normally used. Finally, the LOCAN320 was proposed to amplify AE signals to useable voltage levels with a gain of 20 dB, and spectrum analyzer model HP 89410A, was also used to record the AE waveforms both the time and the frequency domains. The set-up of the pressurizing system and AE measurement system were shown in Figure 3.9.

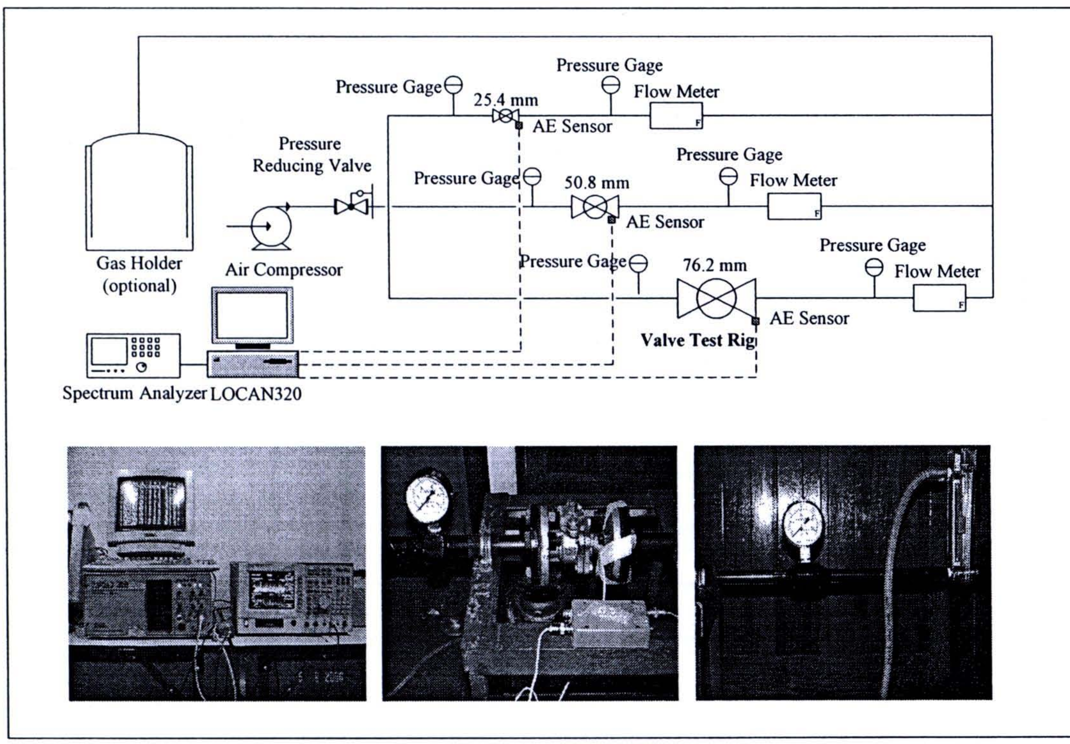

Figure 3.9 Set-up of the pressurizing system and AE measurement system.

3.5.2 Experimental Design

Sound power calculated from theory and the AE signal power (AE_{RMS}^2) obtained by an AE measurement system were investigated at various inlet pressure levels, valve sizes and valve leakage rates to verify the relationship (Equation 3.18) developed in the previous section. The experimental design is shown in Table 3.1. The ball valves were selected as the test subjects. Different sizes of soft-valve-seat ball valves of 25.4, 50.8, and 76.2 mm inside diameters producing leakage by being loosely closed were set. Air flow generated by an air compressor was used (for safety reason) with the setting inlet pressure levels at 100, 300, and 500 kPa. The leakage rates were also set at 1.2, 2.4, 4.2, and 6.0 l/min.

Table 3.1 Experimental design sheet.

Number	**Variables**			
	Valve size (D) (mm)	**Leakage rate (Q) (l/min)**	**Inlet pressure level (P_1) (kPa)**	AE_{RMS} **(mV)**
1	25.4	1.2	300	
2	50.8	4.2	500	
3	50.8	6.0	100	
4	25.4	1.2	100	
5	50.8	2.4	100	
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71	50.8	1.2	100	
72	50.8	2.4	300	

3.5.3 Result of AE Signal Characteristics

This section study the characteristics of AE signal measured from valve leakage in both time and frequency domains. The objectives of this study are to choose method to analyze the leakage signal and to choose an AE sensor to detect the signal.

1. AE Signal Characteristics in Time Domain

The typical AE signals in time domain detected from pressurizing valve leakage are random signals. The result confirmed that a signal received from the AE sensor, model WD is of continuous type, as evidenced in Figure 3.10. Since in the time history of the leakage signal, the signal did not repeat itself and was neither periodic nor transient, it is complicated to be described by an explicit mathematical relationship. Accordingly, the leakage signal should be described in terms of probability statements and statistical averages. Prior to any AE data collection to derive the relationship, the evaluation of AE signal drift during long time measurement should be studied; it was given in Appendix A. Studies were found that the AE signals generated by valve leakage were not drift during long time measurement.

2. AE Signal Characteristics in Magnitude Domain

Probability density functions, providing information about the statistical properties of the signal in the magnitude analysis such as mean and the variance, were also studied in Appendix B. PDF and statistical parameters of the leakage signal were also determined to confirm that valve leakage processes are (at least wide sense) stationary and time-invariant. It was clear that the leakage signals were measured from stationary, random and time-invariant process. When time was varied, mean value are close to zero and variance are not significant changed with time.

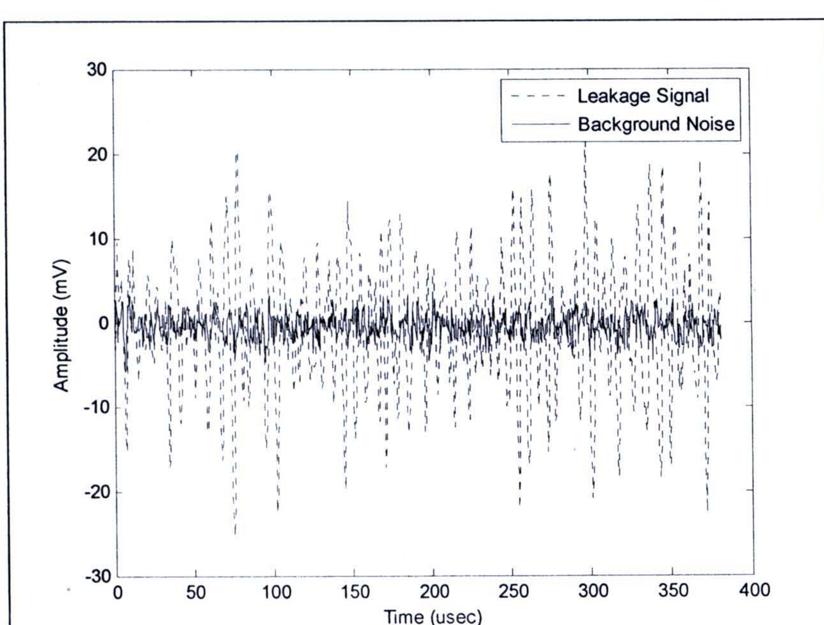

Figure 3.10 AE signal from valve leakage in time domain.

Therefore, magnitude analysis, especially RMS value, providing information about the statistical properties of the AE signal was valuable to utilize in deriving the theoretical relationship.

3. AE Signal Characteristics in Frequency Domain

AE sensors are generally classified into two types: wide bandwidth type that possess a constant sensitivity across its cutoff frequencies, and narrow bandwidth type (resonance type) that are highly sensitive at a specific frequency (resonance frequency).

To study the spectral characteristics of the signal in the frequency domain, AE signal obtained by the AE sensor, WD model, at 0.2 l/min were presented in Figure 3.11. In experiments, leakage rates were prior adjusted at 0 l/min and varied until 0.2 l/min because the AE measurement system can initially detect the leakage signal.

The result affirmed that the ability of this system to detect a small leakage in valves was approximate 0.2 l/min. Moreover, it was found that the characteristic signal from compressed air varied between its frequencies range. The frequency of leakage expressed significant peak amplitude at over 160 kHz to 180 kHz.

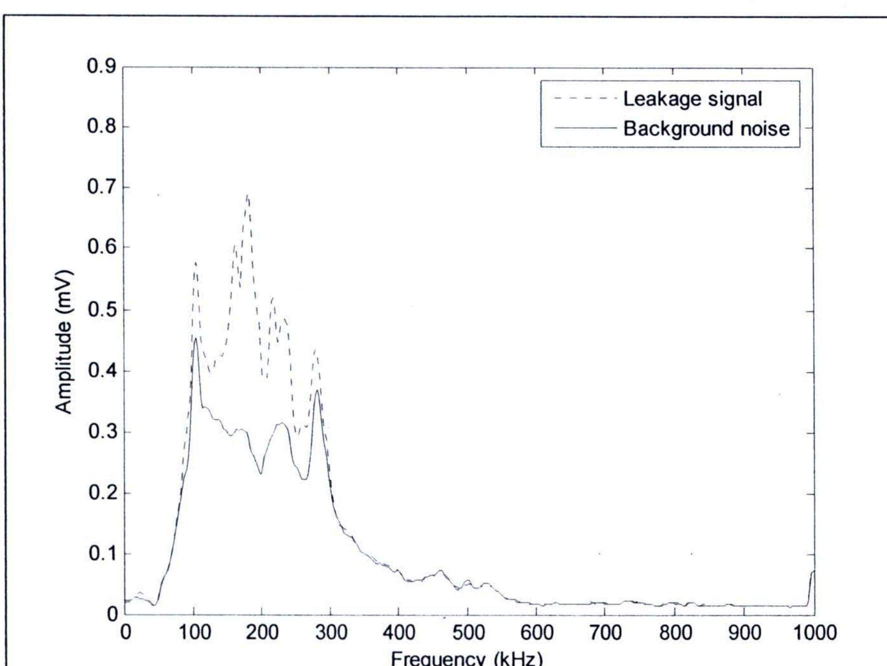

Figure 3.11 AE signal from valve leakage measured by WD sensor in frequency domain.

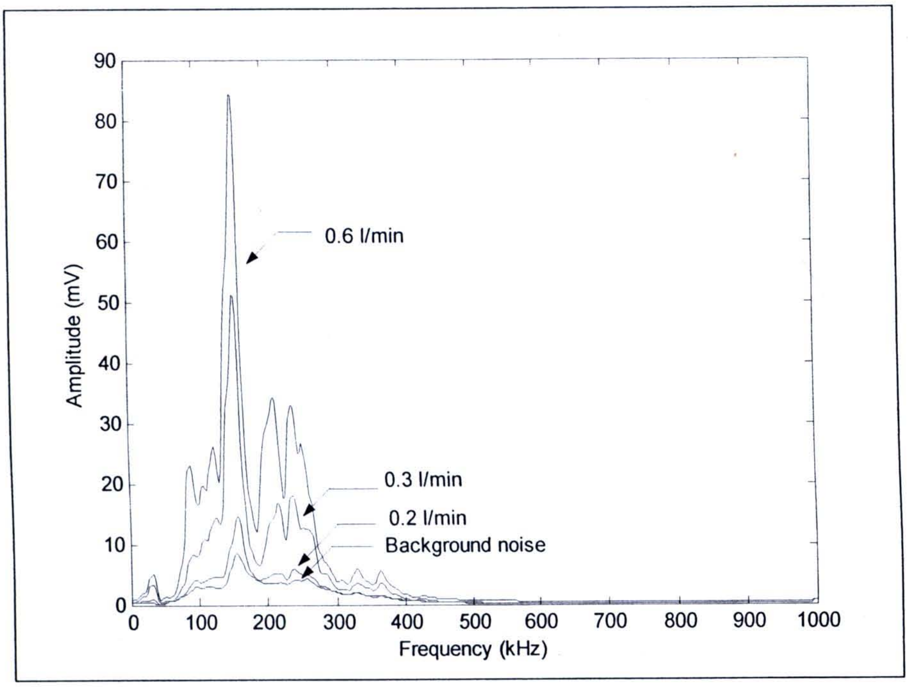

Figure 3.12 AE signal from valve leakage measured by R15 sensor in frequency domain.

Although the choice of sensor model depends on the goal of each application, WD model is typically utilized to search the frequency at the peak amplitude. After the leakage frequencies were detected by WD model, the AE sensor, model R15, having resonance frequency of 150 kHz (from PAC) is mainly chosen to detect leakage signal in this research. The spectral characteristic of the leakage signal obtained by the AE sensor, model R15, was illustrated in Figure 3.12. The result shown that the peak amplitude of AE signal occurred mostly at certainly resonant frequencies under various leakage rates Moreover, it was found that the amplitude of the signal responded increasingly with the leakage rate.

3.5.4 Result of Relation between AE Parameter and Process Variables

From experimental data given in Appendix C, the relationships between AE parameter and fluid variables are shown in Figures 3.13 and 3.14. Figure 3.13 shows the result of AE^2_{RMS} at valve size of 25.4 mm with inlet pressure levels varied from 100 kPa to 500 kPa. It can be seen that the AE^2_{RMS} increases with the increment in inlet pressure level. In addition, the results provided similar trend for the valves of size 50.8 and 76.2 mm at the same inlet pressure condition. This agrees with the result done by Shack et al., 1980 [39]. The may be due to the direct influence of the jet velocity ($v_2(Q,P_1)$) on P_s as described in Equation 3.7 and the AE_{RMS}^2 as implied by Equation 3.6. In other words, the experiments were given that inlet pressures levels (P_1) were correlated and agreed with P_s and AE_{RMS}^2 according to Equations 3.17 and 3.18.

On the other hand, the AE_{RMS}^2 decreased when the valve size increased at various leakage rates as shown in Figure 3.14. In addition, the results provided similar trend for the inlet pressure levels of 100 kPa and 300 kPa at the same valve size. This may be due to the space behind the leakage hole effecting on the jet velocity which varies reversely with valve size. This explained that at the same size of leakage hole *(d)*, smaller valves presented higher jet velocity than bigger valves. This result agrees with the proposed relationships (Equations 3.17 and 3.18).

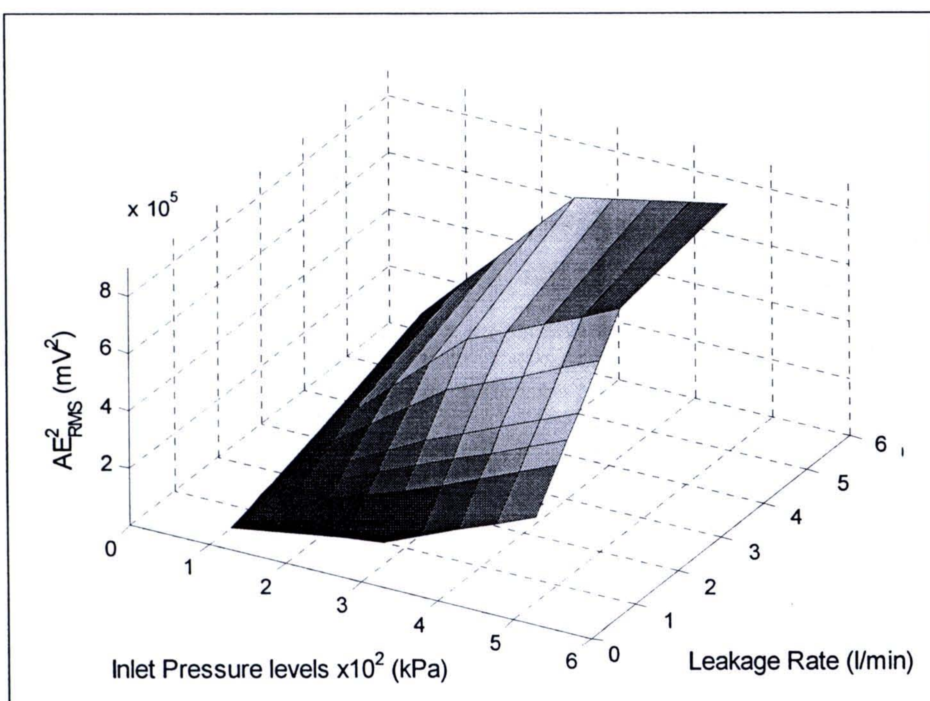

Figure 3.13 AE signal power obtained from experiment at different valve leakage rates and various inlet pressure levels for valve size of 25.4 mm.

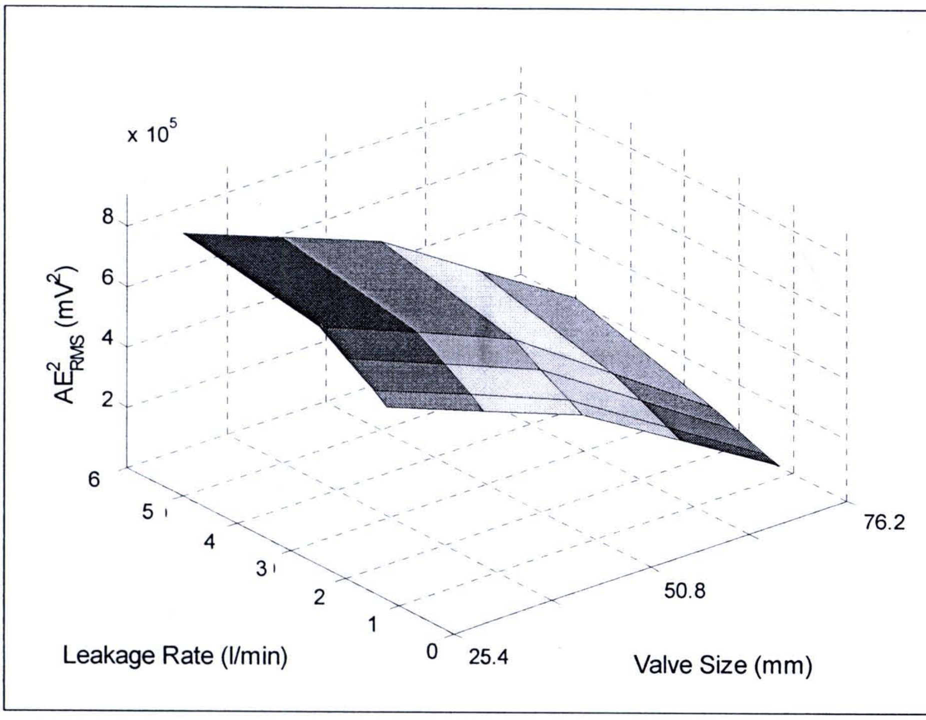

Figure 3.14 AE signal power obtained from experiment at different valve leakage rates and various valve sizes at the inlet pressure level of 500 kPa.

3.5.5 Results of Theoretical Relationship

This section presents the results of theoretical relationship of leakage rate. Experiment set at 25.4 mm and 50.8 mm of ball valve and various inlet pressure levels, 100 kPa to 500 kPa. The value of sound power (P_s), or in the other words, model sound power, expressed on y-axis of Figures 3.15 and 3.16 can be calculated from the term on the right hand side of Equation 3.17 excluding c_0 constant. AE signal power (AE_{RMS}^2) shown in Figures 3.15 and 3.16, expressed on the left hand side of term of Equation 3.6 could be measured from the experimental work. It was confirmed that the solid line was well fit by a function. It implied that sound power has a strong non-linear relationship with the AE signal power. In other words, the AE_{RMS}^2 was directly related to the P_s derived by Lighthill's equation in air valve leakage application. This result corresponded to the derived theoretical relationship shown in Equation 3.18.

Note: Order of power magnitude in y-axis of Figure 3.15 and 3.16 are small. This is ordinary in sound inducted by vibration. Typically, an acoustic efficiency of the jet and a reference decibel of the sound are in order 10^{-5} and 10^{-12} Watt, respectively [58, 62].

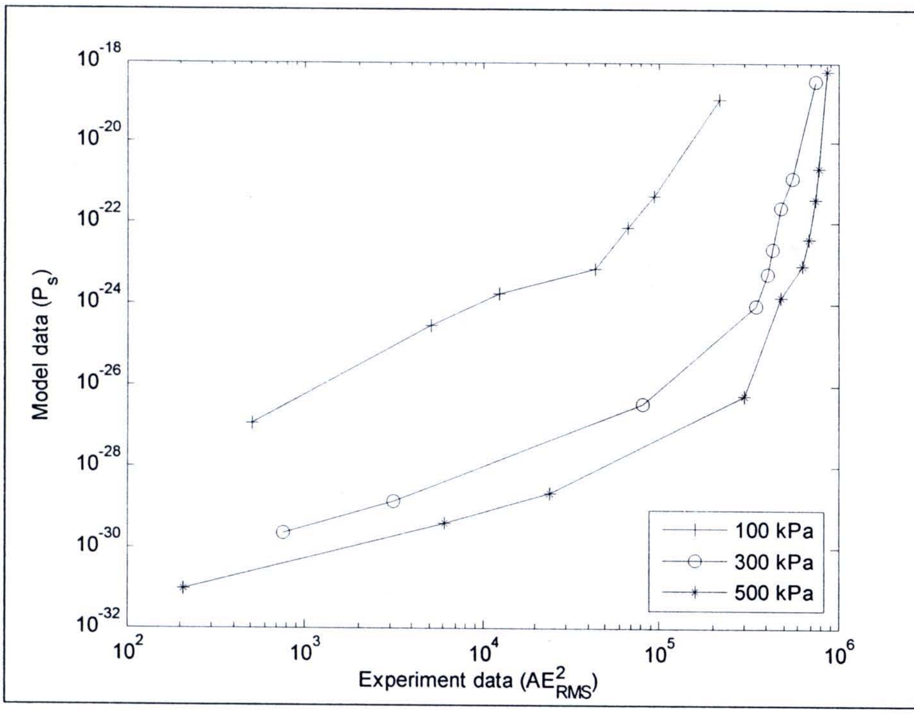

Figure 3.15 Power of predicting model and experimental data of the 25.4 mm of ball valve at a varied 100 kPa to 500 kPa an inlet pressure levels and various leakage rate conditions.

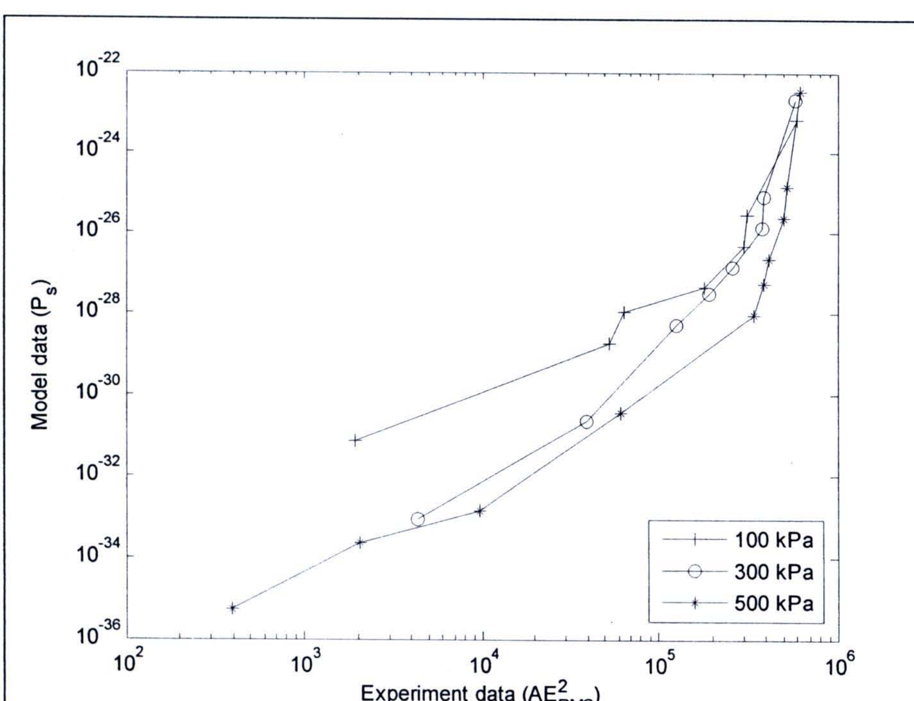

Figure 3.16 Power of predicting model and experimental data of the 50.8 mm of ball valve at a varied 100 kPa to 500 kPa an inlet pressure levels and various leakage rate conditions.

3.6 Development of Theoretical Model

3.6.1 Verification of Significant Variables

Considering Equation 3.18, AE_{RMS}^2, measured from the leakage of air through valves at various tested conditions, is depend on the sound power, P_s. The sound power is related to fluid variables such as inlet pressure level, valve size, gas density, sound velocity, etc. as calculated by Equation 3.17. From the experiment, it can be seen from the ratios (δ) of AE_{RMS}^2 to P_s at various air leakage rate conditions of valve as shown in Figure 3.17.

Figure 3.17 shows a result of δ when inlet pressure level (P_1) is increased. It was found that at any leakage rate of each valve, increasing inlet pressure level (P_1) from 100 kPa to 300 kPa caused insignificant change in δ. However, only the effect of P_1 is shown here because applying different types of gas and temperature also gives similar trend of response as considered by Equation 3.17. In this research, hence, air was only used as a media of fluid. Moreover, all of conditions were tested under room temperature. The reason is to reduce variables and tested conditions. This relationship between each of these variables: inlet pressure level (P_1), type of gas (R) and temperature (T_1) to δ is merely in a lower order as implied by Equation 3.18, and hence the changing of these variables affects δ slightly.

If the leakages of valves of size 25.4 mm and 50.8 mm are considered at the same condition, it was found that the valve size had significant effect on the value of δ as evidenced in Figure 3.17. Changing the size of valve by one step affects on the changing of δ by a magnitude power of order 4 (shown in the y-axis). Therefore, it can be concluded that the rate of response of δ depends significantly on the size of valve. Consequently, it is possible to develop theoretical model to predict leakage rate by considering the significant process variables such as the valve size (D) and the inlet pressure level (P_1).

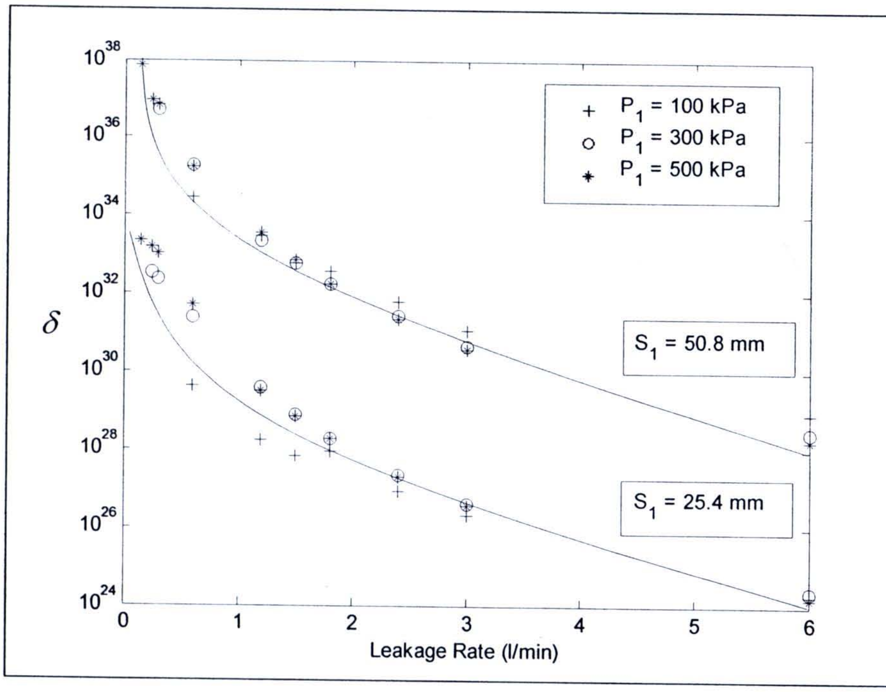

Figure 3.17 Correlation between δ and leakage rate (Q) at various parameters.

3.6.2 Dimensional Analysis

Referring Equation 3.18, the derived theoretical relationship is a non-linear function between AE parameter (AE_{RMS}^2) and various fluid variables which are in term of sound power (P_s) as confirmed in Figures 3.15 and 3.16. Since the aim of this dissertation is to predict leakage rate of valve quantitatively, we must find a mathematical function which has various arguments such as response variable, i.e., the leakage rate, AE_{RMS}^2, and various independent variables described in Equation 3.18. This leads to complication in solving the equation analytically. The mathematical function should be tested by an empirical technique so that its behavior should be observed and reported as experimental data.

Dimensional analysis is a method used to solve this problem to which all engineering and physical sciences are utilized. The method is used to categorize types of physical variables (or quantities) and units (or dimensions) of various arguments depending upon their relations. To find a proper mathematical function, all variables of the theoretical relationships should have dimensionless quantities. Typically, basic (or fundamental) dimensions in fluid mechanics represent the fundamental quantities i.e. mass (M), time

(T), length (L), and temperature (Θ) (or called $MTL\Theta$ system) in the International System of Unites, SI unit. However, the variables can be converted to dimensionless quantities by ratios of fundamental or relevantly derived quantities which are independent of each other, based on $MTL\Theta$ system. Finally, the mathematical function of developed theory or the relationship can be fitted by experiment.

In quantitative analysis of valve leakage, the theoretical relationship as derived in Equation 3.18 can be rewritten to a function consisted of a set of relevant variables as follows.

$$AE_{RMS}^2 = g(Q, \alpha, D, R, T_1, P_1) \qquad (3.19)$$

where g is any non-linear function.

In practice, Equation 3.19 indicates that the dependent (or response) variable, AE_{RMS}^2, depends on six different independent variables. This is a complete set of variables, as derived in Equation 3.18, but this is not an independent set. From the Boyle's law, i.e., the density of an ideal gas $\rho = \frac{P_1}{RT_1}$ [93], this research therefore replaces the inlet pressure level (P_1), the gas constant (R), and the temperature (T_1) by the fluid density (ρ). Accordingly, the following relationship expresses the AE_{RMS}^2 in a complete set of the independent variables as follows:

$$AE_{RMS}^2 = g(Q, \alpha, D, \rho) \qquad (3.20)$$

To achieve the aim of research, thus, we must find the dimensionless quantities and dimensionless function $g(Q, \alpha, D, \rho)$.

3.6.3 Buckingham's Pi Theorem

In 1914, Buckingham proposed a general theory of dimensionless analysis that is now called the *Buckingham's pi theory*. The name *pi* comes from the mathematical notation,

Π, meaning the product of variables. A number of the relevant variables, as shown in Equation 3.20, can be grouped in term of a small number of dimensionless group (pi) by power product of the variables, denoted by $\Pi_1, \Pi_2, ... \Pi_n$.

Typically, the steps involved to find the dimensionless groups are as follow: Firstly, list and count n relevant variables obtained in the problem. List the dimensions of each variable according to $MTL\Theta$ system. The dimensions of physical quantities are given in Table 3.2. Find j as repeating variables and combine it into one term or form of a power product. Find k number of desired pi group when k equals $n-j$. Algebraically find the exponents which make the product dimensionless. Finally, write the final dimensionless relation.

Considering Equation 3.20, there are five relevant variables ($n=5$) that are the AE signal power (AE_{RMS}^2), the leakage rate (Q), the sound velocity (α), the valve size (D), and the fluid density (ρ). The dimension of each variable is shown in Table 3.2. Three variables: α, D, and ρ, are chosen as repeating variables ($j=3$). Therefore, this has two pi groups that are dimensionless ($k=2$) as follows:

$$\Pi_1 = h(\Pi_2) \tag{3.21}$$

Table 3.2 Dimensions or units of physical quantities.

Quantity	**Symbol**	**Dimensions**
Valve size	D	L
Inlet pressure level	P_1	$ML^{-1}T^{-2}$
Leakage rate	Q	L^3T^{-1}
Fluid density	ρ	ML^{-3}
Temperature	T	Θ
Gas Constant	R	$L^2T^{-2}\Theta^{-1}$
Sound power	AE_{RMS}^2	ML^2T^{-3}
Sound velocity	α	LT^{-1}

Note: The AE signal power, AE_{RMS}^2, is a electrical property that can be equivalent to mechanical property in Watt or J/s.

where h is any non-linear function.

To find the first dimensionless group, Π_1, Combine α, D, and ρ with one additional variable, and add AE_{RMS}^2 together. Then, place any exponents $(a_i, b_i, c_i, \ldots)$ on this additional term, and place these in the numerator or denominator to any variables. Set the power product term to $M^0 L^0 T^0$, and solve for the exponents:

$$\Pi_1 = AE_{RMS}^2 \alpha^{a_1} D^{b_1} \rho^{c_1} = (ML^2T^{-3})(LT^{-1})^{a_1}(L)^{b_1}(ML^{-3})^{c_1} = M^0 L^0 T^0 \tag{3.22}$$

Equate exponents:

Length (L): $a_1 + b_1 - 3c_1 + 2 = 0$

Mass (M): $c_1 + 1 = 0$

Time (T): $-a_1 - 3 = 0$

Solving these equations simultaneously, lead to $a_1 = -3$, $b_1 = -2$, and $c_1 = -1$, in other words,

$$\Pi_1 = AE_{RMS}^2 \alpha^{-3} D^{-2} \rho^{-1} = \frac{AE_{RMS}^2}{\alpha^3 D^2 \rho}. \tag{3.23}$$

To find the second dimensionless group, Π_2, repeat the same process as Π_1, however, replace AE_{RMS}^2 by Q. The last *pi* group is expressed as follows:

$$\Pi_2 = Q\alpha^{a_2} D^{b_2} \rho^{c_2} = (L^3T^{-1})(LT^{-1})^{a_2}(L)^{b_2}(ML^{-3})^{c_2} = M^0 L^0 T^0, \tag{3.24}$$

Equate exponents:

Length: $a_2 + b_2 - 3c_2 + 3 = 0$

Mass: $c_2 = 0$

Time: $-a_2 - 1 = 0$

Therefore, we have $a_2 = -1$, $b_2 = -2$, and $c_2 = 0$, and

$$\Pi_2 = Q\alpha^{-1}D^{-2}\rho^0 = \frac{Q}{\alpha D^2}. \tag{3.25}$$

The dimensionless relation for this problem is therefore

$$\frac{AE_{RMS}^2}{\alpha^3 D^2 \rho} = h\left(\frac{Q}{\alpha D^2}\right). \tag{3.26}$$

Since the main contribution of this research is an attempt to predict leakage rate of valve focused on the known process variable such as valve size (D) and inlet pressure level (P_1) based on the AE method. From Boyle's law, the density of an ideal gas is the $\rho = \frac{P_1}{RT_1}$, the dimensionless relation for valve leakage problem can be rewritten as

$$\frac{AE_{RMS}^2}{\alpha^3 D^2 P_1 / RT_1} = h\left(\frac{Q}{\alpha D^2}\right). \tag{3.27}$$

In real application, the leakage rate (Q) should be a response variable. When the arguments of this function are dimensionless, therefore, the arguments and values of the function remain invariant when the function is transverse. Thus, this function should be rearranged as follows:

$$\frac{Q}{\alpha D^2} = i\left(\frac{AE_{RMS}^2}{\alpha^3 D^2 P_1 / RT_1}\right) \tag{3.28}$$

where i is a non-linear function. This equation is a complex two-variable function which is dimensionless.

To imply this analysis, relationships between physical quantities may be represented by mathematical relationships between their values. Furthermore, a mathematical equation that correctly describes a physical relationship between quantities should follow *the principle of dimensionally homogeneous* (PDH) or consistent set of unit, for example, if the function is represented by

$$Q_1 = j(Q_2, Q_3, ..., Q_n) \quad (3.29)$$

where j is a non-linear function.

The numerical value of a leakage rate quantity (Q_1) is determined by the numerical values of a set $Q_2, Q_3, ..., Q_n$, and these must have the same dimension. In implementation, accordingly, Equation 3.28 should be multiplied by group of power monomial derived by dimensional analysis, that is αD^2 value. The dimensional relation of leakage rate related to the relevant variables is

$$Q = \left[i\left(\frac{AE^2_{RMS}}{\alpha^3 D^2 P_1 / RT_1} \right) \right] \times \left(\alpha D^2 \right) \quad (3.30)$$

Equation 3.30 suggests that units in left and right hand sides are the same (in volumetric flow rate).

3.6.4 The Proposed Model

The various relevant variables were obviously significant with leakage rate as shown in Equation 3.30 and the evidence resulting in Section 3.6.1. This equation can be rewritten as a function related to those variables, that is

$$Q = k(AE^2_{RMS}, \alpha, D, R, T_1, P_1). \quad (3.31)$$

where k is a function.

Denote that $D[\ ]$ refer to dimensions of arguments. Referring to Table 3.2, $D[L^3T^{-1}]$, $D[ML^2T^{-3}]^{a_3}$, $D[LT^{-1}]^{b_3}$, $D[L]^{c_3}$, $D[L^2T^{-2}\Theta^{-1}]^{d_3}$, $D[\Theta]^{e_3}$, and $D[ML^{-1}T^{-2}]^{f_3}$ are the dimensions of the leakage rate (Q), the AE signal power (AE^2_{RMS}), the sound velocity (α), the valve size (D), the type of gas (R), temperature (T_1), and the inlet pressure level (P_1), respectively, where any exponents a_3, b_3, c_3, d_3, e_3, and f_3 are letters that represent numbers in algebraic equations.

As mentioned previously, if an equation expresses a proper relationship between variables in a physical process, it will be *dimensionally homogeneous*, i.e., each of its additive terms will have the same dimension. To archive this problem, it is possible to balance or solve for the values by expressing each side of Equation 3.31 as the same power of basic dimensions.

Equate exponents:

Length (L): $2a_3 + b_3 + c_3 + 2d_3 - f_3 = 3$

Mass (M): $a_3 + f_3 = 0$

Time (T): $-3a_3 - b_3 - 2d_3 - 2f_3 = -1$

Temperature (Θ): $-d_3 + e_3 = 0$

One of the methods used to solve a linear equation is the Gaussian elimination method. Solving these equations simultaneously, lead to $a_3 = \frac{1}{8}$, $b_3 = \frac{5}{8}$, $c_3 = \frac{14}{8}$, $d_3 = \frac{1}{8}$, $e_3 = \frac{1}{8}$, and $f_3 = -\frac{1}{8}$.

By replacing the exponent in each argument, the expression between the leakage rates, the AE signal power, and the fluid variables can be written as

$$Q = k(\left[\frac{AE_{RMS}^2 \alpha^5 D^{14} RT_1}{P_1}\right]^{\frac{1}{8}}). \tag{3.32}$$

Equation 3.32 describes how the fluid mechanic variables and AE signal powers affect the leakage rate. To determine the theoretical model for predicting the leakage rate, this work uses a regression model to estimate the function k in Equation 3.32. The regression between variables, i.e., leakage rate (Q) and $\left[\frac{AE_{RMS}^{2}\alpha^{5}D^{14}RT_{1}}{P_{1}}\right]^{\frac{1}{8}}$ was established.

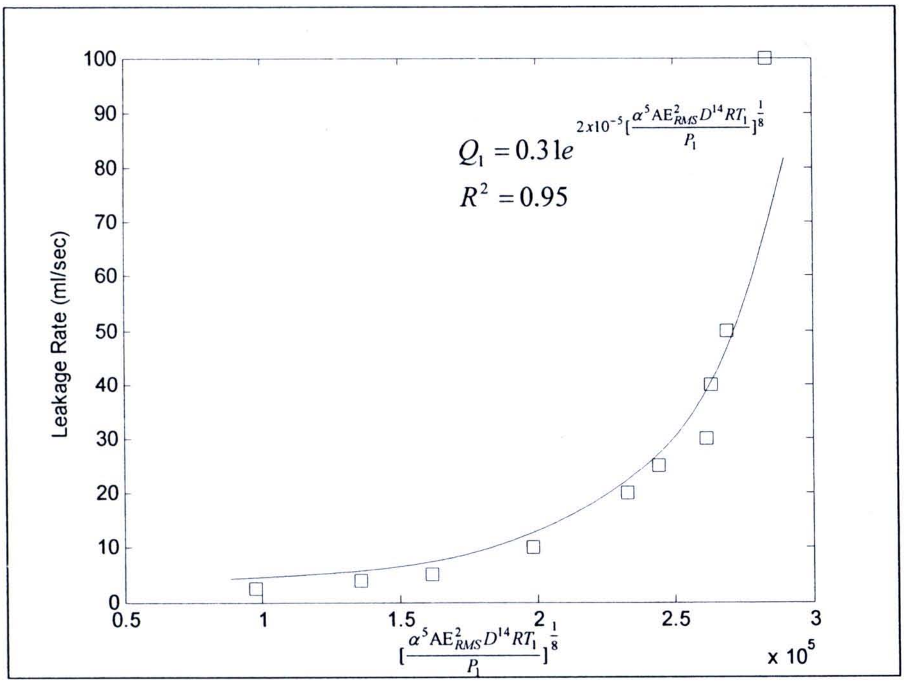

Figure 3.18 Curve fitting of leakage rate (Q_1) and any variables.

We generated air flowing through a leaking valve so that the sound velocity in the fluid (α) was 345.8 m/s, the gas constant (R) was 286.9 N.m.kg^{-1}. K^{-1}, the temperature (T_1) was 298.2 K, and the specific heat ratio (γ) was 1.4. It can be seen that the leakage rate is close to an exponential function of the variable as shown in Figure 3.18.

In this case, a mathematical modeling for the predicting leakage rate of valve is found to be

$$Q_1 = 0.31e^{2x10^{-5}[\frac{\alpha^5 AE_{RMS}^2 D^{14} RT_1}{P_1}]^{\frac{1}{8}}} \quad . \tag{3.33}$$

where Q_1 is the predicted leakage rate (ml/s): 1 m^3/s equals 10^6 ml/sand 1 ml/s equals 0.06 l/min. This theoretical model, which has proven to be successful at predicting the aerodynamic noise downstream of valves throttling air, is however, limited to single phase of gases, isentropic valves (valves with no frictional losses or heat transfer between the inlet and the point of maximum velocity) and, ball valve made of cast iron. This chapter is concerned with a physical relationship and a theoretical model to predict the valve leakage rate. An assumption is presented to develop the relationship and the model based on AE method. The model, based on Lighthill's theory concerning with sound power of turbulent jet (P_s), is related to AE signal power (AE_{RMS}^2). Additionally, the experiment was also set to study the characteristic of AE signal generated by valve leakage and to verify the proposed derivation.