Using Polya's Problem Solving Model to Enhance Thelearning Achievement of Grade 10 Bhutanese Students in Trigonometry
การใช้รูปแบบกระบวนการแก้ปัญหาตามแนวคิดของโพลยา (Polya) เพื่อเพิ่มผลสัมฤทธิ์ทางการเรียนของนักเรียนภูฏานเกรด 10 ในวิชาตรีโกณมิติ

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บทคัดย่อ

การวิจัยครั้งนี้ได้ใช้ระเบียบวิธีวิจัยเชิงปริมาณและเชิงคุณภาพในการประเมินการใช้รูปแบบกระบวนการแก้ปัญหาตามแนวคิดของโพลยาเพื่อเพิ่มผลสัมฤทธิ์ทางการเรียนของนักเรียนภูฏานเกรด 10 พร้อมทั้งระบุปัญหาที่เกี่ยวกับการใช้ขั้นตอนการแก้ปัญหาตามแนวคิดของโพลยา การศึกษานี้ได้ใช้การวิจัยเชิงกึ่งทดลอง (Quasi-experimental) ซึ่งแบ่งเป็นก่อนและหลังการทดลอง ทั้งนี้ได้ใช้วิธีการสุ่มตัวอย่างแบบกลุ่ม (Cluster random sampling) เพื่อคัดเลือกนักเรียนเกรด 10 โรงเรียนมัธยมศึกษาตอนปลาย Bajothang ในมณฑลวังดีโปดรัง (Wangduephodrang District) ประเทศภูฏานจำนวน 2 กลุ่มจากทั้งหมด 6 กลุ่ม ได้แก่ กลุ่มทดลองและกลุ่มควบคุม กลุ่มแรกใช้รูปแบบกระบวนการแก้ปัญหาตามแนวคิดของโพลยา ขณะที่กลุ่มควบคุมใช้วิธีการสอนแบบดั้งเดิม ก่อนและหลังเรียนจะใช้แบบทดสอบในวิชาตรีโกณมิติประเมินผลการเรียนรู้ของนักเรียนระดับเกรด 10 ทั้งสองกลุ่ม หลังจากนั้นจะวิเคราะห์ข้อมูลเชิงปริมาณที่เก็บจากการทดสอบผลสัมฤทธิ์การเรียนรู้เป็นเวลา 4 สัปดาห์โดยใช้สถิติเชิงอนุมาน (Inferential Statistics) ได้แก่ สถิติที (t-test) โดยกำหนดระดับนัยสำคัญทางสถิติไว้ที่ $p<0.05$ ซึ่งประกอบด้วยค่าเฉลี่ย (Mean) และค่าเบี่ยงเบนมาตรฐาน (Standard deviation) นอกจากนั้นยังวิเคราะห์ข้อมูลเชิงคุณภาพโดยใช้การเปิดรหัส (open coding) การหาแก่นของรหัส (axial coding) และการเลือกรหัส (selective coding) ที่เก็บจากเครื่องมือวัด เช่น วารสารวิชาการของนักศึกษาและของอาจารย์ ตลอดจนแบบฝึกหัดของนักศึกษา ผลการวิจัยพบว่ากลุ่มนักศึกษาที่ใช้รูปแบบกระบวนการแก้ปัญหาตามแนวคิดของโพลยามีผลการเรียนรู้ดีกว่ากลุ่มที่สอนโดยใช้วิธีการเรียนการสอนแบบเดิม ปัญหาที่พบคือ ความไม่เข้าใจปัญหา ความยุ่งยากในขั้นตอนการวางแผน ความไม่สามารถในการแก้ปัญหาตามแผนที่ได้วางไว้และการไม่ปฏิบัติตามขั้นตอน

***คำสำคัญ**: รูปแบบการแก้ปัญหาตามแนวคิดของโพลยา ผลสัมฤทธิ์ทางการเรียน วิชาตรีโกณมิติ*

Abstract

Both quantitative and qualitative research methodology was used to examine the use of Polya's Problem Solving Model in enhancing the learning achievement of grade 10 Bhutanese students and to identify the problems associated in applying Polya's Problem Solving steps. A quasi-experimental of non-equivalent pre-test and post-test design was used for the study. A cluster random sampling was adopted to select two

sections of grade 10 out of the 6 sections of grade 10 of Bajothang Higher Secondary School of Wangduephodrang District, Bhutan. The experimental group was treated with Polya's Problem Solving Model while the control group was taught using the traditional method. Both groups were pre-tested and post-tested using the learning achievement test prepared in the unit 'Trigonometry' of grade 10. The quantitative data collected from the learning achievement test after 4 weeks were interpreted using inferential statistics t-test with $p<0.05$ level of significance, mean, and standard deviation. The qualitative data was analyzed using the coding system (open, axial, and selective) which were collected from instruments such as student journal, teacher journal, and exercises of the students.The findings showed that the students treated with Polya's Problem Solving Model performed better than those taught using the traditional method. The problems identified were: the failure to understand the problem, the difficulty in the planning phase, the inability to solve the devised plan, and the omission of steps.

***Keywords**: Polya's Problem Solving Model, Learning Achievement, Trigonometry*

1. Introduction

Mthethwa (2011) views mathematics as a branch of study that deals with logic, decision-making, deductions, assumptions, precision, clarity of thoughts and the ability to solve problems in a calculative manner by following a series of steps. Mathematics is an important subject not only from the point of view of getting an academic qualification at school or college, but also is a subject that prepares the students for the future as well, irrespective of which walk of life children choose to be part of (Davis & Hersh, 2012).

Despite the immense importance of the subject, the study carried out by TIMSS in the year 2011 for mathematics showed that only 3 out of 63 countries improved at all four benchmarks since 1995 (Mullis, Martin, Foy & Arora, 2012). Only 13% of students on average across the Organization for Economic Co-operation and Development (OECD) countries were top performers in mathematics during the study conducted by PISA in the year 2012 (OECD, 2014).The results stated by TIMSS and PISA raises a worldwide concern about disparities in students' mathematics achievement and what can be done to address the learning needs of students more effectively.The performance of the Bhutanese students in mathematics is of no different from the other countries in the world. Bhutan Council for School Examinations and Assessment (BCSEA) informed the Nation about the grade 10 students under-performing consistently in mathematics and science subjects in the last three years (Palden, 2015).

According to Papanastasiou (2008), teaching methodology has direct effect on achievements in mathematics and also on the students' attitudes toward mathematics, on class climates, and on students' mathematics self-perception. In addition, students whom are benefited more from high quality instruction are self-regulated, have strong mathematics backgrounds and had low levels of frustration (Jones & Byrnes, 2006).

According to Dannawi (2013) the field of problem solving in education has gained much of an interest to investigate recently due to its compatibility with the requirements of the information age. She adds that the graduates of the information age requires high complex thinking skills that can work under pressure and vague situations. Problem solving is important element in teaching mathematics because it enhances the logical thinking, contributes in increasing curiosity, organization, and analysis to interpret communication of information. Further, research on mathematical problem solving explains the importance of problem solving in improving the potential of the students in mathematics, it improves reasoning over intuition, and in addition, it increases the students' motivation and enthusiasm in solving math problems.

Therefore, this study was designed to assess the use of Polya's Problem Solving Model in enhancing the learning achievement of grade 10 Bhutanese students in 'Trigonometry'.Polya's Problem Solving Model consists of four steps: 1) understand the problem (analysis): students should be able to restate the problem in their own words , 2) devise a plan (planning): students should seek for reasonable ways to solve problem after understanding the problem, 3) carry out the plan (Implementation): students should execute the plan that they have devised in the step 2, and 4) look back (Reflection): it involves self-reflection on what they have done, what worked and what didn't.

2. Research Objectives

2.1 To assess the learning achievement of grade 10 Bhutanese students in trigonometry after using Polya's Problem Solving Model.

2.2 To identify the problems encountered by the Bhutanese students in applying Polya's Problem solving steps.

3. Research Methodology

Theresearch design was a non-equivalent quasi-experimental pre-test post-test design and both quantitative and qualitative approaches were used.

3.1 Population and sample

The population of this study was 210 grade 10 Bhutanese students of Bajothang Higher Secondary School (BHSS) in Wangduephodrang District, Bhutan. The school has grade level starting from grade 7 till grade 12 with the student population of 1135 and 55 teachers. Two sections out of six sections of grade 10 were selected as the subject of the study using the cluster random sampling. Each section consisted of 25 numbers of students. One section of grade 10 was used as experimental group and the other as the control group. Every section consists of the mixed ability students.

3.2 Research Instruments

The Quantitative data in this study was collected by conducting tests to the participants. The pre-test and post-test of 30 multiple choice questions from the unit 'Trigonometry' was administered to assess and compare the achievement level in the experimental and control group before and after the treatment was

given. The pre-test was administered in the beginning of the study and later the post-test was administered at the end of the experiment after the treatment. The same set of questions were used in both the pre-test and post-test. The qualitative data was collected from the students and teacher journals. The journal writing was focused on identifying the problems encountered by the Bhutanese students in applying Polya's Problem Solving steps.

3.3 Research Procedures

A pre-test was administered comprising of 30 multiple choice questions prepared from the unit 'Trigonometry'. Eight detailed lesson plans were employed in teaching the unit "Trigonometry" to the grade 10 students of BHSS. Each lesson was planned for 100 minutes incorporating Polya's Problem Solving model in the experimental group. Two lessons in a week were taught for four weeks. Students wrote the journal in the 1st, 3rd, and 4th week of the study focusing on the problems that they encountered in applying Polya's Problem Solving steps. The teacher maintained journal collectively with a teacher colleague (who frequently observed the lesson) for every lesson for the same purpose. The teacher's journal writing was based on classroom observation and students' daily exercises.Finally, the post-test was administered at the end of the treatment period. The same questions as that of pre-test was used.

4. Data Analysis

The data analysis was done based on the learning achievement test scores, studentjournal, and teacher journal. The details are as presented below.

4.1 Analysis of the test scores

The inferential statistics t-test with $p<0.05$ level of significance, mean, and standard deviation was used to infer the results of the quantitative data.

Table 1 Comparison of pre-test and post-test within the group

Group		Mean	Mean Difference	Standard Deviation	Sig(2-tailed)
Control	Pre-test	7.76	10.4	1.300	0.000
	Post-test	18.16		4.819	
Experimental	Pre-test	8.04	15.16	1.241	0.000
	Post-test	23.2		2.972	

Significance level (p): < 0.05-Significant

The mean of the pre-test and post-test of the control group were 7.76 and 18.16 respectively as shown in the table 1. The mean of the pre-test and the post-test of experimental group were 8.04 and 23.2 respectively as shown in table 1. The difference mean of the control group was 10.4 and experimental group was 15.16 resulting to the significance value (p) to be 0.00 which indicated that, there was statistically significant increase in the scores of the students in the post-test as compared to that of the pre-test in both the groups.

Table 2 Comparison of pre-tests and post-tests between the groups

Group		Mean	Mean Difference	Standard Deviation	Sig(2-tailed)
Control and Experimental	Pre-test	7.76	0.28	1.300	0.356
	Post-test	8.04		1.241	
Control and Experimental	Pre-test	18.16	5.04	4.819	0.000
	Post-test	23.2		2.972	

The difference mean of the pre-tests of the experimental and the control group was 0.28 as shown in table 2. The significance value (p) of pre-tests was 0.36 which indicated that the scores in the pre-tests of both the groups were not significant statistically. The result was as desired as the learning ability of the students in the experimental and the control group should be of same ability in the beginning of the experiment.

The mean difference of the post-tests of the experimental and the control group was 5.04 as shown in the table 4.2. The significance value (p) of post-tests was 0.00 which indicated that the scores in the post-tests of both the groups were statistically significant. It meant that the scores of the post-test of the experimental group were much higher than the scores of the post-test of the control group.

The standard deviation of the means of the pre-tests were 1.30 and 1.24 in the control and experimental group respectively as shown in the table 2. It indicated that the most of the scores in the pre-tests were clustered around the mean in both the control and experimental group. It meant that the students were of same learning ability at the beginning of the study.

The standard deviation of the means of the post-tests were 4.82 and 2.97 in the control and experimental group respectively as shown in the table 2. The standard deviation of the post-test of the control group showed that the scores were scattered around the mean which meant that the control group consisted of students with varied learning ability. While the standard deviation of the post-test of the experimental group indicated that the scores were clustered around the mean. It meant that the students in the experimental group consisted of same learning ability group as compared to the control group.

Hence, the findings from the mean, standard deviation, and the significance value < 0.05 showed that the use of Polya's Problem Solving Model enhanced the learning achievement of the grade 10 Bhutanese students in Trigonometry.

4.2 Analysis of the student and teacher journal

Coding system (open, axial, and selective) of the Grounded theory (Strauss & Corbin, 1998) was used to analyze the student and the teacher journals. Students failed to understand the problem when the questions were presented in twisted or indirect and complex form especially the word problems. Unfamiliar words, lengthy questions, and unfamiliar terms were other reasons stated by the students as well as by the teacher in their journal in failing to understand the problem.

Planning phase was another obstacle for the students as described in the student journal and teacher journal. The reasons stated were being poor in integrating the known and unknown information from

the presented problem and inability in recollecting the prior knowledge. The other reason was that when students were given with word problems, they failed to represent it in mathematical symbols.

It was found that many could fare well with the solving phase of Polya's Problem Solving model as they already faced the challenges in the first two steps. However, few made mistakes in solving the devised plan due to their carelessness or lack of operational skills and arithmetic skills as stated in student and teacher journal.

Despite all the problems, students shared a positive view on Polya's Problem Solving Model. They reflected the model to be helpful, interesting, fun, time saving, and described as a systematic approach to problem solving. Students stated that it was a new experience for them to check and reflect on their own work as it was always the teacher who checked their work in the usual practice.

Hence, due to their positive attitude towards Polya's Problem Solving Model, it lead to the higher academic achievement of the Bhutanese students in the unit 'Trigonometry' of grade 10.

5. Discussion

It discusses the findings of the learning achievement test scores and the findings of the student journal and the teacher journal.

5.1 Test Score Findings

The finding that Polya's Problem Solving Model improved the academic achievement was parallel to the studies carried out by: Ali (2010); Baki and Karatas (2013); Dema (2013); Lee and Chen (2015); Perveen (2010); Olatide, Olaniyan, Esther and Nwankwo (2015) Their finding showed that there was significant gain between pre-test and post-test especially of experimental group who undergoes treatment, i.e. teaching students with Polya's Problem Solving Model. Such finding showed that the students performed better when taught using Polya's Problem Solving Model than they did when taught using the conventional or traditional method.

Dooren and Verschaffel (2013); Gok (2014); Hensberry and Jacobbe (2012) and Splide (2013) also conducted similar research and concluded that Polya's Problem Solving Model resulted in higher academic achievement level. In addition, they stated that the students turned out to be active information receiver from passive listeners, became free self-learner, increased students' analyzing ability and became confident effective problem solver.

The related learning theories such as behaviorist, cognitivist, and constructivist contributed in the gain of academic achievement. In this study, students were made to work in pairs or groups most of the time during classroom activities. Allowing students to work in groups or pairs gave students room to discuss, question each other, and act as a support system to each other in acquiring the solution to the presented solution. Constructivist learning theory states that students learn more when they interact and engage actively in learning. The guided questions which were within the framework of Polya's Problem Solving Model helped the students to link the information from unknown to known whereby gaining new set of knowledge.

Using a real world problem is one among the four facets of constructivist learning theory. So in this study the problems presented were all related to their practical life. Suppose the question: *"A garage is building a ramp so that workers can stand below a car to work on it. If the platform for the car must be 1.7 m above the ground and the angle at which the ramp meets the ground is 20°, how long should the ramp be?"* In this problem, all the terms and objects used are ones which can be found in their everyday life.

Mayer and Wittrock (2006) distinguished among four major cognitive processes in problem solving: Representing, in which the problem solver constructs a cognitive representation of the problem; Planning, in which the problem solver devises a plan for solving the problem; Executing, in which the problem solver carries out the plan; and Self-regulating, in which the problem solver evaluates the effectiveness of cognitive processing during problem solving and adjusts accordingly. The same cognitive process occurs in the Polya's Problem Solving Model where students firstly identifies the known information to represent the problem, followed by devising the plan to solve the problem, next student carries out the plan, lastly they look back to check their work.

Further behaviorist theory believes that students learn better through trial and error. In lieu to this, students in this study were encouraged to try out different strategies in second step in solving the problem when their plan or the first strategy failed. In addition, Yuan (2013) stated that Polya's Problem Solving Model was effective when combined with enough repetition and practice in her study.

5.2 Students and teacher journal findings

The findings related to the difficulties encountered by the students in applying Polya's Problem Solving step were similar to the findings of the studies carried out by Kaur (1995); Lee (2001); and McGinn & Boote (2003); in the similar field.

It was found from the student and teacher journals that the students had difficulties in understanding the problem phase of Polya's Problem Solving Model. According to Polya (2009) and Krulick& Rudnick (1996) understanding the question is a crucial aspect in the problem-solving as first of all question needs to be understood, before problem could be solved. Students failed to understand the problem when the questions were twisted or indirect and complex. Unfamiliar words, lengthy questions, and unfamiliar terms were other reasons stated by the students in failing to understand the problem from both the student and teacher journal. Tambychik & Meerah (2010) stated that language skills and visual spatial skills are vital in the process of understanding problems in mathematics. Lack of skills such as giving meaning to the problem and understanding the objective to be accomplished causes difficulties in bringing meaning to the information and objectives stated in the problem. Consequently, meaningful connection between information in the problems could not be noticed and decision on how to do about would be chaos.Abdullah, Zakaria, and Halim (2012) found in their study that students who could not bring meaning to the problems, did not know how to plan and perform the problem solving strategies.

It was found that the planning phase was another area students failed to cope with, from the student and teacher journal. The reasons stated were due to being poor in integrating the given information and

inability in recollecting the prior knowledge. Students need to organize the strategy to solve the problem and get the answer in this step. Tambychik and Meerah (2010) pointed out that students need to have information skill to organize the problem-solving strategy. They added that students must be aware of how to organize the given information, what concepts to be used, and what are the number facts concerned. According to Garderen (2006), the causes of the failure to successfully apply the planning phase is due to the deficiency in visual-spatial skill which lead to difficulty in differentiating, relating and organizing information. Ibrahim (2010) stated that students found difficult to transform the problem into mathematical sentences which leads to the failure in devising the plan to solve the problem. On the other hand, Stendall (2009) had stated that, the abilities to make meaningful perceptions and to use memory effectively are important factors in solving problem.

Students found the first two steps quite challenging. So it was not difficult to tackle the 3rd step (carry out the plan) for most of them. However, very few students had problem with the 3rd step. The reasons stated were that they were poor with complex numbers and operations and due to their carelessness. Tambychik and Meerah (2010) pointed out that students must be aware of which operational to be carried out, and the order of the operational procedure for successful problem solving. They also stated that sometimes it was due to their carelessness that they failed to do the 3rd step. The study by Sharpe, Krawee, and Fults (2014) also found students' carelessness as one of the weaknesses in problem solving.

Students expressed their thrill and enthusiasm in applying the final phase of Problem Solving Model i.e. look back. Students stated that it was a new experience for them to check and reflect on their own work. They mentioned that it was always the teacher who checked their work. Dema (2013) stated that with the inclusion of Polya's Problem Solving Model, students were encouraged and motivated to apply their own thinking ability, look for any left out information and to come up with their own creative and innovative ways to check their answer. Polya (2009) stated that much can be gained by taking time to reflect and look back at what one has done, what worked and what didn't. This will help predict what strategy to use to solve future problems.

Despite all the problems, students shared a positive view on Polya's Problem Solving Model. They reflected the model to be helpful, interesting, fun, time saving, and described as a systematic approach to problem solving. Hence, due to their positive attitude towards Polya's Problem Solving Model, it lead to the higher academic achievement of the students in the unit 'Trigonometry' of grade 10.

6. Conclusion

The study found out that Polya's Problem Solving Model enhanced the learning achievement in trigonometry of grade 10 and that students did encounter problems in applying the Polya's Problem Solving steps. The use of Polya's Problem Solving Model should be encouraged during teaching and learning of mathematics in schools since it enhanced the performance of students irrespective of their scoring level. The problems identified in this study would help teachers to come up with enriching teaching techniques,

material and activities in the classroom to cater the learning needs of the students.

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