ภาคผนวก

International Association of Computer Science and Information Technology (IACSIT)

Certificate of Oral Presentation

Awarded to

Grienggrai Rajchakit

For successfully presenting his/her paper in MIMT 2013(Bali Island, Indonesia, March 16-17, 2013)

Paper ID: MT 0026

Paper Title: Delay-dependent Asymptotical Stabilization Criterion of Recurrent Neural Networks

Organization: Division of Mathematics, Faculty of Science, Maejo University, Thailand

Conference Chair

Conference Executive

INTERNATIONAL ASSOCIATION OF COMPUTER SCIENCE & INFORMATION TECHNOLOGY ★ IACSIT Regn.No. 53122865K

2013 4th International Conference on International Conference on Mechanical, Industrial, and Manufacturing Technologies

IACSIT WWW.IACSIT.ORG ttp trans tech publications inc. Publishers in Science and Engineering ISI Proceedings Engineering Village Young

INTERNATIONAL CONFERENCE ON APPLIED ANALYSIS AND MATHEMATICAL MODELING (ICAAMM2013)

2-5 June 2013, Istanbul, Turkey

Certificate of Participation

This is to certify that

Grienggrai Rajchakit

has given the following talk

Guaranteed cost control for Hopfield neural networks with interval nondifferentiable time-varying delay

as an oral presenter

during The International Conference on Applied Analysis and Mathematical Modeling held on 2-5 June 2013 at the Yildiz Technical University, Istanbul-Turkey

Prof. Dr. Mustafa Bayram
Chairman

Applied Mechanics and Materials Vol. 330 (2013) pp 1045-1048
Online available since 2013/Jun/27 at www.scientific.net

doi:10.4028/www.scientific.net/AMM.330.1045

DELAY-DEPENDENT ASYMPTOTICAL STABILIZATION CRITERION OF RECURRENT NEURAL NETWORKS

Grienggrai Rajchakit

Division of Mathematics, Faculty of Science, Maejo University, Chiang Mai 50290, Thailand

griengkrai@yahoo.com

Keywords: Neural networks; Time-varying Delay; Stability; Quadratic Lyapunov functional approach.

Abstract. This paper deals with the problem of delay-dependent stability criterion of discrete-time recurrent neural networks with time-varying delays. Based on quadratic Lyapunov functional approach and free-weighting matrix approach, some linear matrix inequality criteria are found to guarantee delay-dependent asymptotical stability of these systems. And one example illustrates the exactness of the proposed criteria.

Introduction

A recurrent neural network (RNNs) is a very important tool for many application areas such as associative memory, pattern recognition, signal processing, model identication and combinatorial optimization. With the development of research on RNNs in theory and application, the model is more and more complex. Parameter uncertainties and nonautonomous phenomena often exist in real systems due to modeling inaccuracies [1, 2]. Particularly when we consider a longterm dynamical behavior of the system and consider seasonality of the changing environment, the parameters of the system usually will change with time [3, 4]. Simultaneously, in implementations of artificial neural networks, time delay may occur due to finite switching speeds of the amplifiers and communication time [5, 6]. In order to model those systems with neural networks, the neural networks with time-varying delay appear in many papers [7, 8]. So in this paper we consider the stability of the following discrete-time recurrent neural networks:

In this paper, we consider control discrete-time system of neural networks of the form

$$v(k+1) = Cv(k) + AS(v(k)) + BS(v(k-h(k))) + Du(k) + f\,, \tag{1}$$

where $v(k) \in \Omega \subseteq \mathbb{R}^n$ is the neuron state vector, $0 < h_2 \le h(k) \le h_2, \forall k = 0, 1, 2, \ldots,$ $C = diag\{c_1, \ldots, c_n\}$, $c_i \ge 0$, $i = 1, 2, \ldots, n$ is the $n \times n$ constant relaxation matrix, A, B are the $n \times n$ constant weight matrix, D is $n \times m$ constant matrix, $u(k) \in \mathbb{R}^m$ is the control vector, $f = (f_1, \ldots, f_n) \in \mathbb{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1), \ldots, s_n(z_n)]^T$ with $s_i \in C^1[\mathbb{R}, (-1, 1)]$ where s_i is the neuron activations and monotonically increasing for each $i = 1, 2, \ldots, n$.

The asymptotic stability of the zero solution of the delay-differential system of Hopfield neural networks has been developed during the past several years. Much less is known regarding the asymptotic stability of the zero solution of the control discrete-time system of neural networks. Therefore, the purpose of this paper is to establish sufficient condition for the asymptotic stability of the zero solution of (1) in terms of certain matrix inequalities.

Preliminaries

The following notations will be used throughout the paper. $\mathbb{R}^+$ denotes the set of all non-negative real numbers; $\mathbb{Z}^+$ denotes the set of all non-negative integers; $\mathbb{R}^n$ denotes the n-finite-dimensional Euclidean space with the Euclidean norm $\|.\|$ and the scalar product between x and y is defined by $x^T y$; $\mathbb{R}^{n\times m}$ denotes the set of all $(n \times m)$-matrices; and A^T denotes the transpose of the matrix A;

Matrix $Q \in \mathbb{R}^{n\times n}$ is positive semidefinite ($Q \geq 0$) if $x^T Qx \geq 0$, for all $x \in \mathbb{R}^n$. If $x^T Qx > 0$($x^T Qx < 0$, resp.) for any $x \neq 0$, then Q is positive (negative, resp.) definite and denoted by $Q > 0$,($Q < 0$, resp.). It is easy to verify that $Q > 0$, ($Q < 0$, resp.) iff $\exists \beta > 0$: $x^T Qx \geq \beta \|x\|^2, \forall x \in \mathbb{R}^n$, ($\exists \beta > 0$: $x^T Qx \leq -\beta \|x\|^2, \forall x \in \mathbb{R}^n$, resp.).

Lemma 1. [1] The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(x): \mathbb{R}^n \to \mathbb{R}^+$ such that

$$\exists \beta > 0 : \Delta V(x(k)) = V(x(k+1)) - V(x(k)) \leq -\beta \|x(k)\|^2,$$

along the solution of the system. In case the above condition holds for all $x(k) \in V_\delta$, we say that the zero solution is asymptotically stable.

Main Results

In this section, we consider the sufficient condition for asymptotic stability of the zero solution v^* of (1) in terms of certain matrix inequalities. Without loss of generality, we can assume that $v^* = 0, S(0) = 0$ and f =0 (for otherwise, we let $x = v - v^*$ and define $S(x) = S(x + v^*) - S(v^*)$). The new form of (1) is now given by

$$x(k+1) = Cx(k) + AS(x(k)) + BS(x(k-h(k))) + Du(k). \tag{2}$$

This is a basic requirement for controller design. Now, we are interested designing a feedback controller for the system (2) as $u(k) = Kx(k)$, where K is $n \times m$ constant control gain matrix.

The new form of (2) is now given by

$$x(k+1) = Cx(k) + AS(x(k)) + BS(x(k-h(k))) + DKx(k). \tag{3}$$

Throughout this paper we assume the neuron activations $s_i(x_i)$, $i = 1,2,\ldots,n$ is bounded and monotonically nondecreasing on $\mathbb{R}$, and $s_i(x_i)$ is Lipschitz continuous, that is, there exist constant $l_i > 0, i = 1,2,\ldots,n$ such that

$$|s_i(r_1) - s_i(r_2)| \leq l_i |r_1 - r_2|, \forall r_1, r_2 \in \mathbb{R}. \tag{4}$$

By condition (4), $s_i(x_i)$ satisfy

$$|s_i(x_i)| \leq l_i |x_i|, \; i = 1,2,\ldots,n. \tag{5}$$

Theorem 1. The zero solution of the control discrete-time system of neural networks (3) is asymptotically stable if there exist symmetric positive definite matrices P, G, W, R satisfying the following matrix inequalities of the form

$$\psi = \begin{pmatrix} (1,1) & 0 \\ 0 & (2,2) \end{pmatrix} < 0, \tag{6}$$

where

$(1,1)= CPC + CPDK + K^T D^T PC + K^T D^T PDK - P + CPAL + L^T A^T PC$

$+K^T D^T PAL + L^T A^T PDK + L^T A^T PAL + \hat{h} W,$

$(1,2)$=$CPBL + K^T D^T PBL + L^T A^T PBL$, $(2,1)$=$L^T B^T PC + L^T B^T PDKx(k) + L^T B^T PAL,$

$(2,2)$=$L^T B^T PBL - G - \hat{h} R$, and $\hat{h} = h_2 - h_1 + 1$.

Proof. Consider the Lyapunov function candidate, where

$V_1(x(k)) = x^T(k) Px(k)$,

$$V_2(x(k)) = \sum_{i=k-h(k)}^{k-1} x^T(i) Gx(i),$$

$$V_3(x(k)) = \sum_{j=k-h_2+1}^{k-h_1} \sum_{i=j}^{k-1} x^T(i)Wx(i),$$

$$V_4(x(k)) = \sum_{i=k-h(k)}^{k-1} (h(k)-k+i)x^T(i)Rx(i).$$

The Lyapunov difference of the system along trajectory of solution of (3) is given by

$$\begin{aligned}\Delta V_1(x(k)) &= V_1(x(k+1)) - V_1(x(k)) \\ &= [Cx(k) + AS(x(k)) + BS(x(k-h(k))) + Du(k)]^T \\ &\quad \times P[Cx(k) + AS(x(k)) + BS(x(k-h(k))) + Du(k)] \\ &\quad - x^T(k)Px(k) \\ &= x^T(k)[CPC + CPDK + K^T D^T PC + K^T D^T PDK - P]x(k) \\ &\quad + x^T(k)CPAS(x(k)) + S^T(x(k))A^T PCx(k) \\ &\quad + x^T(k)CPBS(x(k-h(k))) + S^T(x(k-h(k)))B^T PCx(k).\end{aligned}$$

As a result, we obtain

$$\begin{aligned}\Delta V &\le x^T(k)[CPC + CPDK + K^T D^T PC + K^T D^T PDK - P \\ &\quad + CPAL + L^T A^T PC + K^T D^T PAL + L^T A^T PDK + L^T A^T PAL + \hat{h}W]x(k) \\ &\quad + x^T(k)[CPBL + K^T D^T PBL + L^T A^T PBL]x(k-h(k)) \\ &\quad + x^T(k-h(k))[L^T B^T PC + L^T B^T PDKx(k) + L^T B^T PAL]x(k) \\ &\quad + x^T(k-h(k))[L^T B^T PBL - G - \hat{h}R]x(k-h(k)), \\ &= \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix}^T \begin{pmatrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{pmatrix} \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix} \\ &= y^T(k)\psi y(k),\end{aligned}$$

Where,

$$\begin{aligned}(1,1) &= CPC + CPDK + K^T D^T PC + K^T D^T PDK - P \\ &\quad + CPAL + L^T A^T PC + K^T D^T PAL + L^T A^T PDK + L^T A^T PAL + \hat{h}W, \\ (1,2) &= CPBL + K^T D^T PBL + L^T A^T PBL, \\ (2,1) &= L^T B^T PC + L^T B^T PDKx(k) + L^T B^T PAL, \\ (2,2) &= L^T B^T PBL - G - \hat{h}R, \\ y(k) &= \begin{pmatrix} x(k) \\ x(k-h(k)) \end{pmatrix}.\end{aligned}$$

By the condition (6), $\Delta V(y(k))$ is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \le -\beta\|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from **Lemma 1**. This completes the proof.

Remark 1. Theorem 1 gives a sufficient condition for the asymptotic stability of control discrete-time system of neural networks (3) via matrix inequalities. These conditions are described in terms of certain diagonal matrix inequalities, which can be realized by using the linear matrix inequality algorithm proposed. But [9, 10] these conditions are described in terms of certain symmetric matrix inequalities, which can be realized by using the Schur complement lemma and linear matrix inequality algorithm proposed.

Conclusions

This paper was dedicated to the delay-dependent stability of discrete-time recurrent neural networks with time-varying delay. A less conservative LMI-based globally stability criterion is obtained with quadratic Lyapunov functional approach and free-weighting matrix approach for periodic discrete-time recurrent neural networks with a time-varying delay. One example illustrates the exactness of the proposed criterion.

Ackhowledgment

This work was supported by The Office of Agricultural Research and Extension Maejo University Chiangmai Thailand.

Reference

[1] R.A. Chayjan and Y. Moazez, Estimation of paddy equilibrium moisture sorption using ANNs. J. Applied Sci., Vol. 8 (2008), pp. 346-351.

[2] B. Chen and J.Wang, Global exponential periodicity of a class of recurrent neural networks with oscillating parameters and time-varying delays ,IEEE Trans. on Neural Networks, Vol. 16 (2005), pp. 1440-1449.

[3] E.M. Elabbasy and M.M. El-Dessoky, Adaptive coupled synchronization of coupled chaotic dynamical systems. Trends Applied Sci. Res., Vol. 2 (2007), pp. 88-102.

[3] H. Huang, G. Feng and J. D. Cao, Robust state estimation for uncertain neural networks with time-varying delay, IEEE Trans. on Neural networks, Vol. 19 (2008), pp.1329-1339.

[4] J. Li, Y. F. Diao, Y. Zhang, J. L. Mao and X. Yin, Stability analysis of discrete Hopfield neural networks with weight function matrix, Lecture Notes on Computer Science, Vol. 5370 (2008), pp. 760-768.

[5] J. Li,Y. F. Diao, M. D. Li and X. Yin, Stability analysis of Discrete Hopfield Neural Networks with the nonnegative definite monotone increasing weight function matrix, Discrete Dynamics in Nature and Society, Vol. 2009 (2009), Article ID 673548, 10 pages.

[6] J. Li, W. G. Wu, J. M. Yuan, Q. R. Tan and X. Yin, Delay-dependent stability criterion of arbitrary switched linear systems with time-varying delay, Discrete Dynamics in Nature and Society, Vol. 2010 (2010), Article ID 347129, 16 pages.

[7] J. L. Liang, Z. D. Wang, Y.R. Liu and X. H. Liu, Robust Synchronization of an Array of Coupled Stochastic Discrete-Time Delayed Neural Networks, IEEE Trans. on Neural networks, Vol. 19 (2008), pp.1910-1921.

[8] Y. R. Liu, Z. D. Wang and X. H. Liu, Asymptotic stability for neural networks with mixed time-delays: the discrete-time case, Neural Networks, Vol. 22 (2009), pp. 67-74.

[9] X. Y. Lou and B. T. Cui, Delay-Dependent Criteria for Global Robust Periodicity of Uncertain Switched Recurrent Neural Networks With Time-Varying Delay, IEEE Trans. on Neural networks, Vol. 19 (2008), pp. 549-557.

[10] C. Y. Lu, H. H. Tsai, T. J. Su, J. Tsai and C. W. Liao, A delay-dependent approach to passivity analysis for uncertain neural networks with time-varying delay, Neural Process Lett, Vol. 27 (2008), pp. 237-246.

Materials Engineering and Automatic Control II

10.4028/www.scientific.net/AMM.330

Delay-Dependent Asymptotical Stabilization Criterion of Recurrent Neural Networks

10.4028/www.scientific.net/AMM.330.1045

Niamsup et al. *Journal of Inequalities and Applications* 2013, 2013:292
http://www.journalofinequalitiesandapplications.com/content/2013/1/292

Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

Guaranteed cost control for switched recurrent neural networks with interval time-varying delay

Piyapong Niamsup[1], Manlika Rajchakit[2] and Grienggrai Rajchakit[2*]

*Correspondence:
griengkrai@yahoo.com
[2]Division of Mathematics and Statistics, Faculty of Science, Maejo University, Chiangmai, 50290, Thailand
Full list of author information is available at the end of the article

Abstract

This paper studies the problem of guaranteed cost control for a class of switched recurrent neural networks with interval time-varying delay. The time delay is a continuous function belonging to a given interval, but not necessary differentiable. A cost function is considered as a nonlinear performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. By constructing a set of augmented Lyapunov-Krasovskii functionals, a guaranteed cost controller is designed via memoryless state feedback control, a switching rule for the exponential stabilization for the system is designed via linear matrix inequalities and new sufficient conditions for the existence of the guaranteed cost state-feedback for the system are given in terms of linear matrix inequalities (LMIs). A numerical example is given to illustrate the effectiveness of the obtained result.

Keywords: neural networks; guaranteed cost control; switching design; stabilization; interval time-varying delays; Lyapunov function; linear matrix inequalities

1 Introduction

Stability and control of recurrent neural networks with time delay have attracted considerable attention in recent years [1–8]. In many practical systems, it is desirable to design neural networks which are not only asymptotically or exponentially stable but can also guarantee an adequate level of system performance. In the area of control, signal processing, pattern recognition and image processing, delayed neural networks have many useful applications. Some of these applications require that the equilibrium points of the designed network be stable. In both biological and artificial neural systems, time delays due to integration and communication are ubiquitous and often become a source of instability. The time delays in electronic neural networks are usually time-varying, and sometimes vary violently with respect to time due to the finite switching speed of amplifiers and faults in the electrical circuitry. Guaranteed cost control problem [9–12] has the advantage of providing an upper bound on a given system performance index and thus the system performance degradation incurred by the uncertainties or time delays is guaranteed to be less than this bound. The Lyapunov-Krasovskii functional technique has been among the popular and effective tools in the design of guaranteed cost controls for neural networks with time delay. Nevertheless, despite such a diversity of results available, the most existing works either assumed that the time delays are constant or differentiable [13–16].

Although, in some cases, delay-dependent guaranteed cost control for systems with time-varying delays were considered in [12, 13, 15], the approach used there cannot be applied to systems with interval, non-differentiable time-varying delays. To the best of our knowledge, the guaranteed cost control and state feedback stabilization for switched recurrent neural networks with interval time-varying delay, non-differentiable time-varying delays have not been fully studied yet (see, *e.g.*, [9–12, 15–25] and the references therein). Which are important in both theories and applications. This motivates our research.

In this paper, we investigate the guaranteed cost control for switched recurrent neural networks problem. The novel features here are that the delayed neural network under consideration is with various globally Lipschitz continuous activation functions, and the time-varying delay function is interval, non-differentiable. Specifically, our goal is to develop a constructive way to design a switching rule to exponentially stabilize the system. A nonlinear cost function is considered as a performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. Based on constructing a set of augmented Lyapunov-Krasovskii functionals combined with the Newton-Leibniz formula, new delay-dependent criteria for guaranteed cost control via memoryless feedback control are established in terms of LMIs, which allow simultaneous computation of two bounds that characterize the exponential stability rate of the solution and can be easily determined by utilizing MATLABs LMI control toolbox.

The outline of the paper is as follows. Section 2 presents definitions and some well-known technical propositions needed for the proof of the main result. LMI delay-dependent criteria for guaranteed cost control and a numerical example showing the effectiveness of the result are presented in Section 3. The paper ends with conclusions and cited references.

2 Preliminaries

The following notation will be used in this paper. $\mathbb{R}^+$ denotes the set of all real non-negative numbers; $\mathbb{R}^n$ denotes the n-dimensional space with the scalar product $\langle x,y\rangle$ or x^Ty of two vectors x, y, and the vector norm $\|\cdot\|$; $M^{n\times r}$ denotes the space of all matrices of $(n\times r)$ dimensions. A^T denotes the transpose of matrix A; A is symmetric if $A=A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A)=\max\{\operatorname{Re}\lambda;\lambda\in\lambda(A)\}$. $x_t:=\{x(t+s):s\in[-h,0]\}$, $\|x_t\|=\sup_{s\in[-h,0]}\|x(t+s)\|$; $C^1([0,t],\mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n$-valued continuously differentiable functions on $[0,t]$; $L_2([0,t],\mathbb{R}^m)$ denotes the set of all the $\mathbb{R}^m$-valued square integrable functions on $[0,t]$.

Matrix A is called semi-positive definite ($A\geq 0$) if $\langle Ax,x\rangle\geq 0$ for all $x\in\mathbb{R}^n$; A is positive definite ($A>0$) if $\langle Ax,x\rangle>0$ for all $x\neq 0$; $A>B$ means $A-B>0$. The notation $\operatorname{diag}\{\cdots\}$ stands for a block-diagonal matrix. The symmetric term in a matrix is denoted by $*$.

Consider the following switched recurrent neural networks with interval time-varying delay:

$$\begin{aligned}
\dot{x}(t) &= -A_{\gamma(x(t))}x(t) + W_{0\gamma(x(t))}f\big(x(t)\big) \\
&\quad + W_{1\gamma(x(t))}g\big(x\big(t-h(t)\big)\big) + B_{\gamma(x(t))}u(t), \quad t\geq 0, \qquad (2.1)\\
x(t) &= \phi(t), \quad t\in[-h_1,0],
\end{aligned}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the state of the neural, $u(\cdot) \in L_2([0,t], \mathbb{R}^m)$ is the control; n is the number of neurons, and

$$f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T,$$
$$g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T,$$

are the activation functions; $\gamma(\cdot): R^n \to \mathcal{N} := \{1, 2, \ldots, N\}$ is the switching rule, which is a function depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\gamma(x(t)) = j$ implies that the system realization is chosen as the jth system, $j = 1, 2, \ldots, N$. It is seen that system (2.1) can be viewed as an autonomous switched system in which the effective subsystem changes when the state $x(t)$ hits predefined boundaries.

$A_j = \operatorname{diag}(\overline{a}_{1j}, \overline{a}_{2j}, \ldots, \overline{a}_{nj})$, $\overline{a}_{ij} > 0$, represents the self-feedback term; $B_j \in R^{n\times m}$ are control input matrices; W_{0j}, W_{1j} denote the connection weights and the delayed connection weights, respectively. The time-varying delay function $h(t)$ satisfies the condition

$$0 \le h_0 \le h(t) \le h_1.$$

The initial functions $\phi(t) \in C^1([-h_1, 0], R^n)$, with the norm

$$\|\phi\| = \sup_{t\in[-h_1,0]} \sqrt{\|\phi(t)\|^2 + \|\dot{\phi}(t)\|^2}.$$

In this paper we consider various activation functions and assume that the activation functions $f(\cdot)$, $g(\cdot)$ are Lipschitzian with the Lipschitz constants $f_i, e_i > 0$:

$$\begin{aligned} |f_i(\xi_1) - f_i(\xi_2)| &\le f_i|\xi_1 - \xi_2|, \quad i = 1, 2, \ldots, n, \forall \xi_1, \xi_2 \in \mathbb{R}, \\ |g_i(\xi_1) - g_i(\xi_2)| &\le e_i|\xi_1 - \xi_2|, \quad i = 1, 2, \ldots, n, \forall \xi_1, \xi_2 \in \mathbb{R}. \end{aligned} \tag{2.2}$$

The performance index associated with system (2.1) is the following function:

$$J = \int_0^\infty f^0(t, x(t), x(t-h(t)), u(t))\, dt, \tag{2.3}$$

where $f^0(t, x(t), x(t-h(t)), u(t)): R^+ \times R^n \times R^n \times R^m \to R^+$, is a nonlinear cost function satisfying

$$\exists Q_1, Q_2, \quad R: f^0(t, x, y, u) \le \langle Q_1 x, x\rangle + \langle Q_2 y, y\rangle + \langle Ru, u\rangle \tag{2.4}$$

for all $(t, x, y, u) \in R^+ \times R^n \times R^n \times R^m$ and $Q_1, Q_2 \in R^{n\times n}$, $R \in R^{m\times m}$, are given symmetric positive definite matrices. The objective of this paper is to design a memoryless state feedback controller $u(t) = Kx(t)$ for system (2.1) and the cost function (2.3) such that the resulting closed-loop system

$$\dot{x}(t) = -(A_j - B_j K)x(t) + W_{0j} f(x(t)) + W_{1j} g(x(t-h(t))) \tag{2.5}$$

is exponentially stable and the closed-loop value of the cost function (2.3) is minimized.

Remark 2.1 It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the lower bound of delay is not restricted to zero; therefore, the stability criteria proposed in [4–7, 9–13, 15–18, 21–24] are not applicable to this system.

Remark 2.2 It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the delay function $h(t)$ is non-differentiable; therefore, the stability criteria proposed in [5, 6, 8, 10–12, 14–19, 22–25] are not applicable to this system.

Definition 2.1 Given $\alpha > 0$. The zero solution of closed-loop system (2.5) is α-exponentially stabilizable if there exists a positive number $N > 0$ such that every solution $x(t,\phi)$ satisfies the following condition:

$$\|x(t,\phi)\| \leq Ne^{-\alpha t}\|\phi\|, \quad \forall t \geq 0.$$

Definition 2.2 Consider control system (2.1). If there exist a memoryless state feedback control law $u(t) = Kx(t)$ and a positive number J^* such that the zero solution of closed-loop system (2.5) is exponentially stable and the cost function (2.3) satisfies $J \leq J^*$, then the value J^* is a guaranteed constant and $u(t)$ is a guaranteed cost control law of the system and its corresponding cost function.

We introduce the following technical well-known propositions, which will be used in the proof of our results.

Proposition 2.1 (Schur complement lemma [26]) *Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T$, $Y = Y^T > 0$. Then $X + Z^TY^{-1}Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0.$$

Proposition 2.2 (Integral matrix inequality [27]) *For any symmetric positive definite matrix $M > 0$, scalar $\sigma > 0$ and vector function $\omega : [0,\sigma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds:*

$$\left(\int_0^\sigma \omega(s)\,ds\right)^T M\left(\int_0^\sigma \omega(s)\,ds\right) \leq \sigma\left(\int_0^\sigma \omega^T(s)M\omega(s)\,ds\right).$$

3 Design of guaranteed cost controller

In this section, we give a design of memoryless guaranteed feedback cost control for neural networks (2.1). Let us set

$$w_{11} = -[P + \alpha I]A_j - A_j^T[P + \alpha I] - 2B_jB_j^T + 0.25B_jRB_j^T + \sum_{i=0}^{1} G_i,$$

$$w_{12} = P + A_jP + 0.5B_jB_j^T,$$

$$w_{13} = e^{-2\alpha h_0}H_0 + 0.5B_jB_j^T + A_jP,$$

$$w_{14} = 2e^{-2\alpha h_1}H_1 + 0.5B_jB_j^T + A_jP,$$

$$
\begin{aligned}
&w_{15} = P0.5B_jB_j^T + A_jP,\\
&w_{22} = \sum_{i=0}^{1} W_{ij}D_iW_{ij}^T + \sum_{i=0}^{1} h_i^2H_i + (h_1 - h_0)U - 2P - B_jB_j^T,\\
&w_{23} = P, \qquad w_{24} = P, \qquad w_{25} = P,\\
&w_{33} = -e^{-2\alpha h_0}G_0 - e^{-2\alpha h_0}H_0 - e^{-2\alpha h_1}U + \sum_{i=0}^{1} W_{ij}D_iW_{ij}^T,\\
&w_{34} = 0, \qquad w_{35} = -2\alpha h_1 U,\\
&w_{44} = \sum_{i=0}^{1} W_{ij}D_iW_{ij}^T - e^{-2\alpha h_1}U - e^{-2\alpha h_1}G_1 - e^{-2\alpha h_1}H_1, \qquad w_{45} = e^{-2\alpha h_1}U,\\
&w_{55} = -e^{-2\alpha h_1}U + W_{0j}D_0W_{0j}^T,\\
&E = \operatorname{diag}\{e_i, i = 1,\dots,n\}, \qquad F = \operatorname{diag}\{f_i, i = 1,\dots,n\},\\
&\lambda_1 = \lambda_{\min}\left(P^{-1}\right),\\
&\lambda_2 = \lambda_{\max}\left(P^{-1}\right) + h_0\lambda_{\max}\left[P^{-1}\left(\sum_{i=0}^{1} G_i\right)P^{-1}\right]\\
&\qquad + h_1^2\lambda_{\max}\left[P^{-1}\left(\sum_{i=0}^{1} H_i\right)P^{-1}\right] + (h_1 - h_0)\lambda_{\max}\left(P^{-1}UP^{-1}\right).
\end{aligned}
$$

Theorem 3.1 *Consider control system* (2.1) *and the cost function* (2.3). *If there exist symmetric positive definite matrices* P, U, G_0, G_1, H_0, H_1 *and diagonal positive definite matrices* D_i, $i = 0,1$, *satisfying the following LMIs*:

$$
\mathcal{E}_j = \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} & w_{15}\\ * & w_{22} & w_{23} & w_{24} & w_{25}\\ * & * & w_{33} & w_{34} & w_{35}\\ * & * & * & w_{44} & w_{45}\\ * & * & * & * & w_{55} \end{bmatrix} < 0, \quad j = 1,2,\dots,N, \tag{3.1}
$$

$$
\mathcal{S}_{1j} = \begin{bmatrix} -PA_j - A_j^TP - \sum_{i=0}^{1} e^{-2\alpha h_i}H_i & 2PF & PQ_1\\ * & -D_0 & 0\\ * & * & -Q_1^{-1} \end{bmatrix} < 0, \quad j = 1,2,\dots,N, \tag{3.2}
$$

$$
\mathcal{S}_{2j} = \begin{bmatrix} W_{1j}D_1W_{1j}^T - e^{-2\alpha h_1}U & 2PE & PQ_2\\ * & -D_1 & 0\\ * & * & -Q_2^{-1} \end{bmatrix} < 0, \quad j = 1,2,\dots,N, \tag{3.3}
$$

then

$$
u_j(t) = -\frac{1}{2}B_j^TP^{-1}x(t), \quad t \geq 0, j = 1,2,\dots,N, \tag{3.4}
$$

is a guaranteed cost control and the guaranteed cost value is given by

$$
J^* = \lambda_2\|\phi\|^2.
$$

The switching rule is chosen as $\gamma(x(t)) = j$. Moreover, the solution $x(t,\phi)$ of the system satisfies

$$\|x(t,\phi)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t}\|\phi\|, \quad \forall t \geq 0.$$

Proof Let $Y = P^{-1}$, $y(t) = Yx(t)$. Using the feedback control (2.5), we consider the following Lyapunov-Krasovskii functional:

$$V(t,x_t) = \sum_{i=1}^{6} V_i(t,x_t),$$
$$V_1 = x^T(t)Yx(t),$$
$$V_2 = \int_{t-h_0}^{t} e^{2\alpha(s-t)}x^T(s)YG_0Yx(s)\,ds,$$
$$V_3 = \int_{t-h_1}^{t} e^{2\alpha(s-t)}x^T(s)YG_1Yx(s)\,ds,$$
$$V_4 = h_0\int_{-h_0}^{0}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)YH_0Y\dot{x}(\tau)\,d\tau\,ds,$$
$$V_5 = h_1\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)YH_1Y\dot{x}(\tau)\,d\tau\,ds,$$
$$V_6 = (h_1-h_0)\int_{t-h_1}^{t-h_0}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)YUY\dot{x}(\tau)\,d\tau\,ds.$$

It is easy to check that

$$\lambda_1\|x(t)\|^2 \leq V(t,x_t) \leq \lambda_2\|x_t\|^2, \quad \forall t \geq 0. \tag{3.5}$$

Taking the derivative of V_1, we have

$$\begin{aligned}
\dot{V}_1 &= 2x^T(t)Y\dot{x}(t)\\
&= y^T(t)\left[-PA_j^T - A_jP\right]y(t) - y^T(t)B_jB_j^Ty(t)\\
&\quad + 2y^T(t)W_{0j}f(\cdot)y(t) + 2y^T(t)W_{1j}g(\cdot)y(t)\\
\dot{V}_2 &= y^T(t)G_0y(t) - e^{-2\alpha h_0}y^T(t-h_0)G_0y(t-h_0) - 2\alpha V_2;\\
\dot{V}_3 &= y^T(t)G_1y(t) - e^{-2\alpha h_1}y^T(t-h_1)G_1y(t-h_1) - 2\alpha V_3;\\
\dot{V}_4 &= h_0^2\dot{y}^T(t)H_0\dot{y}(t) - h_1e^{-2\alpha h_0}\int_{t-h_0}^{t}\dot{x}^T(s)H_0\dot{x}(s)\,ds - 2\alpha V_4;\\
\dot{V}_5 &= h_1^2\dot{y}^T(t)H_1\dot{y}(t) - h_1e^{-2\alpha h_1}\int_{t-h_1}^{t}\dot{y}^T(s)H_1\dot{y}(s)\,ds - 2\alpha V_4;\\
\dot{V}_6 &= (h_1-h_0)^2\dot{y}^T(t)U\dot{y}(t) - (h_1-h_0)e^{-2\alpha h_1}\int_{t-h_1}^{t-h_0}\dot{y}^T(s)U\dot{y}(s)\,ds - 2\alpha V_6.
\end{aligned}$$

Applying Proposition 2.2 and the Leibniz-Newton formula

$$\int_s^t \dot{y}(\tau)\,d\tau = y(t) - y(s),$$

we have, for $j = 1, 2$, $i = 0, 1$,

$$\begin{aligned}
-h_i \int_{t-h_i}^{t} \dot{y}^T(s) H_j \dot{y}(s)\,ds &\le -\left[\int_{t-h_i}^{t} \dot{y}(s)\,ds\right]^T H_j \left[\int_{t-h_i}^{t} \dot{y}(s)\,ds\right] \\
&\le -\big[y(t) - y\big(t-h(t)\big)\big]^T H_j \big[y(t) - y\big(t-h(t)\big)\big] \\
&= -y^T(t) H_i y(t) + 2x^T(t) H_j y\big(t-h(t)\big) \\
&\quad - y^T(t-h_i) H_j y(t-h_i).
\end{aligned} \tag{3.6}$$

Note that

$$\int_{t-h_1}^{t-h_0} \dot{y}^T(s) U \dot{y}(s)\,ds = \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s)\,ds + \int_{t-h(t)}^{t-h_0} \dot{y}^T(s) U \dot{y}(s)\,ds.$$

Applying Proposition 2.2 gives

$$\begin{aligned}
\big[h_1 - h(t)\big] \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s)\,ds &\ge \left[\int_{t-h_1}^{t-h(t)} \dot{y}(s)\,ds\right]^T U \left[\int_{t-h_1}^{t-h(t)} \dot{y}(s)\,ds\right] \\
&\ge \big[y\big(t-h(t)\big) - y(t-h_1)\big]^T U \big[y\big(t-h(t)\big) - y(t-h_1)\big].
\end{aligned}$$

Since $h_1 - h(t) \le h_1 - h_0$, we have

$$[h_1 - h_0] \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s)\,ds \ge \big[y\big(t-h(t)\big) - y(t-h_1)\big]^T U \big[y\big(t-h(t)\big) - y(t-h_1)\big],$$

then

$$-[h_1 - h_0] \int_{t-h_1}^{t-h(t)} \dot{y}^T(s) U \dot{y}(s)\,ds \le -\big[y\big(t-h(t)\big) - y(t-h_1)\big]^T U \big[y\big(t-h(t)\big) - y(t-h_1)\big].$$

Similarly, we have

$$-(h_1 - h_0) \int_{t-h(t)}^{t-h_0} \dot{y}^T(s) U \dot{y}(s)\,ds \le -\big[y(t-h_0) - y\big(t-h(t)\big)\big]^T U \big[y(t-h_0) - y\big(t-h(t)\big)\big].$$

Then we have

$$\begin{aligned}
\dot{V}(\cdot) + 2\alpha V(\cdot) \le{}& y^T(t)\big[-PA_j^T - A_j P\big] y(t) - y^T(t) B_j B_j^T y(t) + 2y^T(t) W_{0j} f(\cdot) \\
&+ 2y^T(t) W_{1j} g(\cdot) + y^T(t)\left(\sum_{i=0}^{1} G_i\right) y(t) + 2\alpha\langle Py(t), y(t)\rangle \\
&+ \dot{y}^T(t)\left(\sum_{i=0}^{1} h_i^2 H_i\right)\dot{y}(t) + (h_1 - h_0)\dot{y}^T(t) U \dot{y}(t)
\end{aligned}$$

$$
\begin{aligned}
&- \sum_{i=0}^{1} e^{-2\alpha h_i} y^T(t-h_i) G_i y(t-h_i) \\
&- e^{-2\alpha h_0} \big[y(t) - y(t-h_0)\big]^T H_0 \big[y(t) - y(t-h_0)\big] \\
&- e^{-2\alpha h_1} \big[y(t) - y(t-h_1)\big]^T H_1 \big[y(t) - y(t-h_1)\big] \\
&- e^{-2\alpha h_1} \big[y\big(t-h(t)\big) - y(t-h_1)\big]^T U \big[y\big(t-h(t)\big) - y(t-h_1)\big] \\
&- e^{-2\alpha h_1} \big[y(t-h_0) - y\big(t-h(t)\big)\big]^T U \big[y(t-h_0) - y\big(t-h(t)\big)\big].
\end{aligned}
\tag{3.7}
$$

Using equation (2.5)

$$
P\dot{y}(t) + A_j P y(t) - W_{0j} f(\cdot) - W_{1j} g(\cdot) + 0.5 B_j B_j^T y(t) = 0,
$$

and multiplying both sides by $[2y(t), -2\dot{y}(t), 2y(t-h_0), 2y(t-h_1), 2y(t-h(t))]^T$, we have

$$
\begin{aligned}
&2y^T(t)P\dot{y}(t) + 2y^T(t)AjPy(t) - 2y^T(t)W_{0j}f(\cdot) - 2y^T(t)W_{1j}g(\cdot) \\
&\quad + y^T(t)B_jB_j^T y(t) = 0, \\
&-2\dot{y}^T(t)P\dot{y}(t) - 2\dot{y}^T(t)AjPy(t) + 2\dot{y}^T(t)W_{0j}f(\cdot) \\
&\quad + 2\dot{y}^T(t)W_{1j}g(\cdot) - \dot{y}^T(t)B_jB_j^T y(t) = 0, \\
&2y^T(t-h_0)P\dot{y}(t) + 2y^T(t-h_0)AjPy(t) - 2y^T(t-h_0)W_{0j}f(\cdot) \\
&\quad - 2y^T(t-h_0)W_{1j}g(\cdot) + y^T(t-h_0)B_jB_j^T y(t) = 0, \\
&2y^T(t-h_1)P\dot{y}(t) + 2y^T(t-h_1)AjPy(t) - 2y^T(t-h_1)W_{0j}f(\cdot) \\
&\quad - 2y^T(t-h_1)W_{1j}g(\cdot) + y^T(t-h_1)B_jB_j^T y(t) = 0, \\
&2y^T\big(t-h(t)\big)P\dot{y}(t) + 2y^T\big(t-h(t)\big)AjPy(t) - 2y^T\big(t-h(t)\big)W_{0j}f(\cdot) \\
&\quad - 2y^T\big(t-h(t)\big)W_{1j}g(\cdot) + 2y^T\big(t-h(t)\big)B_jB_j^T y(t) = 0.
\end{aligned}
\tag{3.8}
$$

Adding all the zero items of (3.8) and $f^0(t,x(t),x(t-h(t)),u(t)) - f^0(t,x(t),x(t-h(t)),u(t)) = 0$, respectively, into (3.7) and using the condition (2.4) for the following estimations:

$$
\begin{aligned}
f^0\big(t,x(t),x\big(t-h(t)\big),u(t)\big) &\le \big\langle Q_1x(t),x(t)\big\rangle + \big\langle Q_2x\big(t-h(t)\big),x\big(t-h(t)\big)\big\rangle \\
&\quad + \big\langle Ru(t),u(t)\big\rangle \\
&= \big\langle PQ_1Py(t),y(t)\big\rangle + \big\langle PQ_2Py\big(t-h(t)\big),y\big(t-h(t)\big)\big\rangle \\
&\quad + 0.25\big\langle B_jRB_j^T y(t),y(t)\big\rangle, \\
2\big\langle W_{0j}f(x),y\big\rangle &\le \big\langle W_{0j}D_0W_{0j}^T y,y\big\rangle + \big\langle D_0^{-1}f(x),f(x)\big\rangle, \\
2\big\langle W_{1j}g(z),y\big\rangle &\le \big\langle W_{1j}D_1W_{1j}^T y,y\big\rangle + \big\langle D_1^{-1}g(z),g(z)\big\rangle, \\
2\big\langle D_0^{-1}f(x),f(x)\big\rangle &\le \big\langle FD_0^{-1}Fx,x\big\rangle, \\
2\big\langle D_1^{-1}g(z),g(z)\big\rangle &\le \big\langle ED_1^{-1}Ez,z\big\rangle,
\end{aligned}
$$

we obtain

$$\dot{V}(\cdot) + 2\alpha V(\cdot) \leq \zeta^T(t)\mathcal{E}_j\zeta(t) + y^T(t)S_{1j}y(t) + y^T\big(t-h(t)\big)S_{2j}y\big(t-h(t)\big) \\ - f^0\big(t, x(t), x\big(t-h(t)\big), u(t)\big), \tag{3.9}$$

where $\zeta(t) = [y(t), \dot{y}(t), y(t-h_0), y(t-h_1), y(t-h(t))]$, and

$$\mathcal{E}_j = \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\ * & w_{22} & w_{23} & w_{24} & w_{25} \\ * & * & w_{33} & w_{34} & w_{35} \\ * & * & * & w_{44} & w_{45} \\ * & * & * & * & w_{55} \end{bmatrix},$$

$$S_{1j} = -PA_j - A_j^T P - \sum_{i=0}^{1} e^{-2\alpha h_i} H_i + 4PFD_0^{-1}FP + PQ_1P,$$

$$S_{2j} = W_{1j}D_1W_{1j}^T - e^{-2\alpha h_2}U + 4PED_1^{-1}EP + PQ_2P.$$

Note that by the Schur complement lemma, Proposition 2.1, the conditions $S_{1j} < 0$ and $S_{2j} < 0$ are equivalent to the conditions (3.2) and (3.3), respectively. Therefore, by conditions (3.1), (3.2), (3.3), we obtain from (3.9) that

$$\dot{V}(t, x_t) \leq -2\alpha V(t, x_t), \quad \forall t \geq 0. \tag{3.10}$$

Integrating both sides of (3.10) from 0 to t, we obtain

$$V(t, x_t) \leq V(\phi)e^{-2\alpha t}, \quad \forall t \geq 0.$$

Furthermore, taking condition (3.5) into account, we have

$$\lambda_1 \|x(t,\phi)\|^2 \leq V(x_t) \leq V(\phi)e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t}\|\phi\|^2,$$

then

$$\|x(t,\phi)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t}\|\phi\|, \quad t \geq 0,$$

which concludes the exponential stability of closed-loop system (2.5). To prove the optimal level of the cost function (2.3), we derive from (3.9) and (3.1)-(3.3) that

$$\dot{V}(t, z_t) \leq -f^0\big(t, x(t), x\big(t-h(t)\big), u(t)\big), \quad t \geq 0. \tag{3.11}$$

Integrating both sides of (3.11) from 0 to t leads to

$$\int_0^t f^0\big(t, x(t), x\big(t-h(t)\big), u(t)\big)\, dt \leq V(0, z_0) - V(t, z_t) \leq V(0, z_0),$$

due to $V(t,z_t) \geq 0$. Hence, letting $t \to +\infty$, we have

$$J = \int_0^\infty f^0\big(t, x(t), x\big(t-h(t)\big), u(t)\big)\,dt \leq V(0,z_0) \leq \lambda_2 \|\phi\|^2 = J^*.$$

This completes the proof of the theorem. □

Example 3.1 Consider the switched recurrent neural networks with interval time-varying delays (2.1), where

$$A_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, \qquad W_{01} = \begin{bmatrix} -0.1 & 0.3 \\ 0.2 & -0.8 \end{bmatrix},$$

$$W_{02} = \begin{bmatrix} -0.7 & 0.3 \\ 0.4 & -0.9 \end{bmatrix}, \qquad W_{11} = \begin{bmatrix} -0.4 & 0.2 \\ 0.3 & -0.3 \end{bmatrix}, \qquad W_{12} = \begin{bmatrix} -0.2 & 0.3 \\ 0.1 & -0.4 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \qquad E = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, \qquad F = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.7 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.6 \end{bmatrix}, \qquad R = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.9 \end{bmatrix},$$

$$\begin{cases} h(t) = 0.1 + 1.2652\sin^2 t & \text{if } t \in \mathcal{I} = \bigcup_{k\geq 0}[2k\pi, (2k+1)\pi], \\ h(t) = 0 & \text{if } t \in R^+ \setminus \mathcal{I}. \end{cases}$$

Note that $h(t)$ is non-differentiable, therefore, the stability criteria proposed in [4–7, 9–13, 15–18, 21–24] are not applicable to this system. Given $\alpha = 0.3$, $h_0 = 0.1$, $h_1 = 1.3652$, by using the Matlab LMI toolbox, we can solve for P, U, G_0, G_1, H_0, H_1, D_0, and D_1 which satisfy the conditions (3.1)-(3.3) in Theorem 3.1.

A set of solutions is as follows:

$$P = \begin{bmatrix} 1.5219 & -0.3659 \\ -0.3659 & 2.2398 \end{bmatrix}, \qquad U = \begin{bmatrix} 3.1239 & -0.2365 \\ -0.2365 & 3.0123 \end{bmatrix},$$

$$G_0 = \begin{bmatrix} 1.3225 & 0.0258 \\ 0.0258 & 1.2698 \end{bmatrix}, \qquad G_1 = \begin{bmatrix} 2.2368 & 0.0148 \\ 0.0148 & 3.1121 \end{bmatrix},$$

$$H_0 = \begin{bmatrix} 2.2189 & 0.1238 \\ 0.1238 & 1.2368 \end{bmatrix}, \qquad H_1 = \begin{bmatrix} 2.3225 & 0.0369 \\ 0.0369 & 2.1897 \end{bmatrix},$$

$$D_0 = \begin{bmatrix} 2.9870 & 0 \\ 0 & 3.2589 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 3.2698 & 0 \\ 0 & 4.3258 \end{bmatrix}.$$

Then

$$u_1(t) = 0.2579x_1(t) + 0.2589x_2(t), \quad t \geq 0,$$

$$u_2(t) = 0.1397x_1(t) + 0.2176x_2(t), \quad t \geq 0,$$

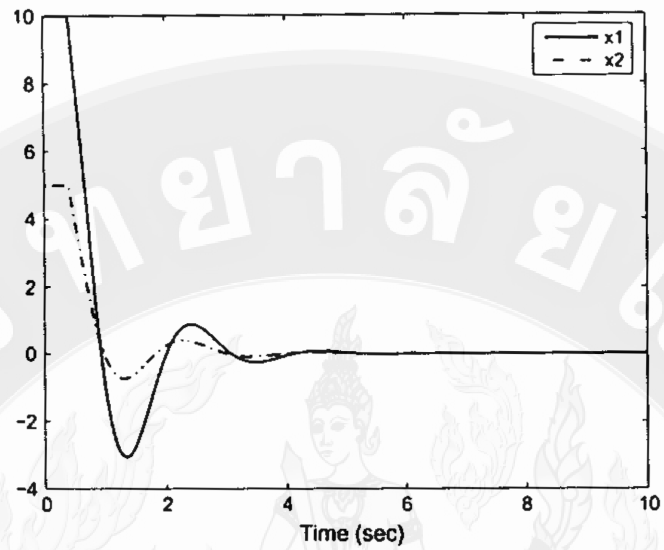

Figure 1 The simulation of the solutions $x_1(t)$ and $x_2(t)$ with the initial condition $\phi(t) = [10\ 5]^T$, $t \in [0, 10]$.

are a guaranteed cost control law and the cost given by

$$J^* = 1.1268\|\phi\|^2.$$

Moreover, the solution $x(t,\phi)$ of the system satisfies

$$\|x(t,\phi)\| \le 2.3257e^{-0.3t}\|\phi\|, \quad \forall t \ge 0.$$

The trajectories of solution of switched recurrent neural networks is shown in Figure 1, respectively.

4 Conclusions

In this paper, the problem of guaranteed cost control for Hopfield neural networks with interval non-differentiable time-varying delay has been studied. A nonlinear quadratic cost function is considered as a performance measure for the closed-loop system. The stabilizing controllers to be designed must satisfy some exponential stability constraints on the closed-loop poles. By constructing a set of time-varying Lyapunov-Krasovskii functionals, a switching rule for the exponential stabilization for the system is designed via linear matrix inequalities. A memoryless state feedback guaranteed cost controller design has been presented and sufficient conditions for the existence of the guaranteed cost state-feedback for the system have been derived in terms of LMIs.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The authors contributed equally and significantly in writing this paper. The authors read and approved the final manuscript.

Author details
[1]Deparment of Mathematics, Faculty of Science, Chiang Mai University, Chiangmai, 50000, Thailand. [2]Division of Mathematics and Statistics, Faculty of Science, Maejo University, Chiangmai, 50290, Thailand.

Acknowledgements
This work was supported by the Office of Agricultural Research and Extension Maejo University, the Thailand Research Fund Grant, the Higher Education Commission and Faculty of Science, Maejo University, Thailand. The first author is supported by the Center of Excellence in Mathematics, Thailand, and Commission for Higher Education, Thailand. The authors thank anonymous reviewers for valuable comments and suggestions, which allowed us to improve the paper.

Received: 14 July 2012 Accepted: 11 May 2013 Published: 10 June 2013

References

1. Hopfield, JJ: Neural networks and physical systems with emergent collective computational abilities. Proc. Natl. Acad. Sci. USA **79**, 2554-2558 (1982)
2. Kevin, G: An Introduction to Neural Networks. CRC Press, Boca Raton (1997)
3. Wu, M, He, Y, She, JH: Stability Analysis and Robust Control of Time-Delay Systems. Springer, Berlin (2010)
4. Arik, S: An improved global stability result for delayed cellular neural networks. IEEE Trans. Circuits Syst. **499**, 1211-1218 (2002)
5. He, Y, Wang, QG, Wu, M: LMI-based stability criteria for neural networks with multiple time-varying delays. Physica D **112**, 126-131 (2005)
6. Kwon, OM, Park, JH: Exponential stability analysis for uncertain neural networks with interval time-varying delays. Appl. Math. Comput. **212**, 530-541 (2009)
7. Phat, VN, Trinh, H: Exponential stabilization of neural networks with various activation functions and mixed time-varying delays. IEEE Trans. Neural Netw. **21**, 1180-1185 (2010)
8. Rajchakit, M, Rajchakit, G: LMI approach to robust stability and stabilization of nonlinear uncertain discrete-time systems with convex polytopic uncertainties. Adv. Differ. Equ. **2012**, 106 (2012)
9. Botmart, T, Niamsup, P: Robust exponential stability and stabilizability of linear parameter dependent systems with delays. Appl. Math. Comput. **217**, 2551-2566 (2010)
10. Chen, WH, Guan, ZH, Lu, X: Delay-dependent output feedback guaranteed cost control for uncertain time-delay systems. Automatica **40**, 1263-1268 (2004)
11. Rajchakit, M, Niamsup, P, Rajchakit, G: A switching rule for exponential stability of switched recurrent neural networks with interval time-varying delay. Adv. Differ. Equ. **2013**, 44 (2013). doi:10.1186/1687-1847-2013-44
12. Ratchagit, K: A switching rule for the asymptotic stability of discrete-time systems with convex polytopic uncertainties. Asian-Eur. J. Math. **5**, 1250025 (2012)
13. Rajchakit, M, Rajchakit, G: Mean square exponential stability of stochastic switched system with interval time-varying delays. Abstr. Appl. Anal. **2012**, Article ID 623014 (2012). doi:10.1155/2012/623014
14. Palarkci, MN: Robust delay-dependent guaranteed cost controller design for uncertain neutral systems. Appl. Math. Comput. **215**, 2939-2946 (2009)
15. Park, JH, Kwon, OM: On guaranteed cost control of neutral systems by retarded integral state feedback. Appl. Math. Comput. **165**, 393-404 (2005)
16. Rajchakit, G, Rojsiraphisal, T, Rajchakit, M: Robust stability and stabilization of uncertain switched discrete-time systems. Adv. Differ. Equ. **2012**, 134 (2012). doi:10.1186/1687-1847-2012-134
17. Park, JH, Choi, K: Guaranteed cost control of nonlinear neutral systems via memory state feedback. Chaos Solitons Fractals **24**, 183-190 (2005)
18. Rajchakit, M, Rajchakit, G: Mean square robust stability of stochastic switched discrete-time systems with convex polytopic uncertainties. J. Inequal. Appl. **2012**, 135 (2012). doi:10.1186/1029-242X-2012-135
19. Fridman, E, Orlov, Y: Exponential stability of linear distributed parameter systems with time-varying delays. Automatica **45**, 194-201 (2009)
20. Xu, S, Lam, J: A survey of linear matrix inequality techniques in stability analysis of delay systems. Int. J. Syst. Sci. **39**(12), 1095-1113 (2008)
21. Xie, JS, Fan, BQ, Young, SL, Yang, J: Guaranteed cost controller design of networked control systems with state delay. Acta Autom. Sin. **33**, 170-174 (2007)
22. Yu, L, Gao, F: Optimal guaranteed cost control of discrete-time uncertain systems with both state and input delays. J. Franklin Inst. **338**, 101-110 (2001)
23. Phat, VN, Ratchagit, K: Stability and stabilization of switched linear discrete-time systems with interval time-varying delay. Nonlinear Anal. Hybrid Syst. **5**, 605-612 (2011)
24. Ratchagit, K, Phat, VN: Stability criterion for discrete-time systems. J. Inequal. Appl. **2010**, Article ID 201459 (2010)
25. Phat, VN, Kongtham, Y, Ratchagit, K: LMI approach to exponential stability of linear systems with interval time-varying delays. Linear Algebra Appl. **436**, 243-251 (2012)
26. Boyd, S, El Ghaoui, L, Feron, E, Balakrishnan, V: Linear Matrix Inequalities in System and Control Theory. SIAM, Philadelphia (1994)
27. Gu, K, Kharitonov, V, Chen, J: Stability of Time-Delay Systems. Birkhäuser, Berlin (2003)

doi:10.1186/1029-242X-2013-292
Cite this article as: Niamsup et al.: **Guaranteed cost control for switched recurrent neural networks with interval time-varying delay.** *Journal of Inequalities and Applications* 2013 **2013**:292.

3rd World Congress on
Information and Communication Technologies
Hanoi, Vietnam 15-18, December 2013

Proceedings of the

2013 Third World Congress on Information and Communication Technologies (WICT 2013)

Hanoi, Vietnam
15-18 December, 2013

Edited by

Long Thanh Ngo, Le Quy Don University, Vietnam
Ajith Abraham, Machine Intelligence Research Labs (MIR Labs), USA
Lam Thu Bui, Le Quy Don University, Vietnam
Emilio Corchado, University of Salamanca, Spain
Choo Yun-Hoy, Universiti Teknikal Malaysia Melaka, Malaysia
Kun Ma, University of Jinan, China

Technically Co-Sponsored by

IEEE Systems Man and Cybernetics Society, Spain Chapter

Technically Supported by

IEEE Systems Man and Cybernetics Society, Technical Committee on Soft Computing

IEEE Catalog Number: CFP1395H-ART
ISBN: 978-1-4799-3230-6

New delay-dependent sufficient conditions for the exponential stability of linear hybrid systems with interval time-varying delays

Manlika Rajchakit and Grienggrai Rajchakit

Major of Mathematics and Statistics
Maejo University
Chiangmai, Thailand
Email: griengkrai@yahoo.com

***Abstract*—This paper is concerned with exponential stability of switched linear systems with interval time-varying delays. The time delay is any continuous function belonging to a given interval, in which the lower bound of delay is not restricted to zero. By constructing a suitable augmented Lyapunov-Krasovskii functional combined with Leibniz-Newton's formula, a switching rule for the exponential stability of switched linear systems with interval time-varying delays and new delay-dependent sufficient conditions for the exponential stability of the systems are first established in terms of LMIs.**

I. Introduction

Switched time-delay systems have been attracting considerable attention during the recent years [1-10], due to the significance both in theory development and practical applications. However, it is worth noting that only the state time delay is considered, and the time delay in the state derivatives is largely ignored in the existing literature. If each subsystem of a switched system has time delay in the state derivatives, then the switched system is called switched neutral system [10–14]. Switched neutral systems exist widely in engineering and social systems, many physical plants can be modelled as switched neutral systems, such as distributed networks and heat exchanges. For example, in [11–16], a switched neutral type delay equation with nonlinear perturbations was exploited to model the drilling system. Unlike other systems, the neutral has time-delay in both the state and derivative. However, it is well-known that time-delay in the system may be a source of instability or bad system performance. Thus many researchers try to study them to find stability criteria for such system with time-delay to be stable.Most of the known results on this problem are derived assuming only that the time-varying delay $h(t)$ is a continuously differentiable function, satisfying some boundedness condition on its derivative: $\dot{h}(t) \leq \delta < 1$. This paper gives the improved results for the exponential stability of switched linear systems with interval time-varying delay. The time delay is assumed to be a time-varying continuous function belonging to a given interval, but not necessary to be differentiable. Specifically, our goal is to develop a constructive way to design switching rule to the exponential stability of switched linear systems with interval time-varying delay. By constructing argument Lyapunov functional combined with LMI technique, we propose new criteria for the exponential stability of the switched linear system. The delay-dependent stability conditions are formulated in terms of LMIs.

The paper is organized as follows: Section II presents definitions and some well-known technical propositions needed for the proof of the main results. Delay-dependent exponential stability conditions of the switched linear system are presented in Section III.

II. Preliminaries

The following notations will be used in this paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the $n-$dimensional space with the scalar product $\langle .,. \rangle$ and the vector norm $\| \ . \ \|$; $M^{n\times r}$ denotes the space of all matrices of $(n \times r)-$dimensions; A^T denotes the transpose of matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\text{min/max}}(A) = \text{min/max}\{\text{Re}\lambda; \lambda \in \lambda(A)\}$; $x_t := \{x(t+s) : s \in [-h, 0]\}, \|x_t\| = \sup_{s\in[-h,0]} \| x(t+s) \|$; $C([0,t], R^n)$ denotes the set of all R^n-valued continuous functions on $[0,t]$; Matrix A is called semi-positive definite ($A \geq 0$) if $\langle Ax, x\rangle \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $\langle Ax, x\rangle > 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$. $*$ denotes the symmetric term in a matrix.

Consider a linear system with interval time-varying delay of the form

$$\begin{aligned} &\dot{x}(t) = A_\gamma x(t) + D_\gamma x(t - h(t)), \quad t \in R^+, \\ &x(t) = \phi(t), t \in [-h_2, 0], \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state; $\gamma(.) : R^n \to \mathcal{N} := \{1, 2, \ldots, N\}$ is the switching rule, which is a function depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\gamma(x(t)) = i$ implies that the system realization is chosen as the i^{th} system, $i = 1, 2, ..., N$. It is seen that the system (1) can be viewed as

978-1-4799-3230-6/13/\$31.00 ©2013

an autonomous switched system in which the effective subsystem changes when the state $x(t)$ hits predefined boundaries. $A_i, D_i \in M^{n\times n}, i = 1, 2, ..., N$ are given constant matrices, and $\phi(t) \in C([-h_2, 0], R^n)$ is the initial function with the norm
$\| \phi \| = \sup_{s\in[-h_2,0]} \| \phi(s) \|$; The time-varying delay function $h(t)$ satisfies

$$0 \le h_1 \le h(t) \le h_2, \quad t \in R^+.$$

The stability problem for switched system (1) is to construct a switching rule that makes the system exponentially stable.

Remark 2.1. It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the lower bound of delay is not restricted to zero.

Definition 2.1. Given $\alpha > 0$. The switched linear system (1) is $\alpha-$exponentially stable if there exists a switching rule $\gamma(.)$ such that every solution $x(t, \phi)$ of the system satisfies the following condition

$$\exists N > 0 : \quad \| x(t, \phi) \| \le Ne^{-\alpha t} \| \phi \|, \quad \forall t \in R^+.$$

We end this section with the following technical well-known propositions, which will be used in the proof of the main results.

Definition 2.2. The system of matrices $\{J_i\}, i = 1, 2, \dots, N$, is said to be strictly complete if for every $x \in R^n \backslash \{0\}$ there is $i \in \{1, 2, \dots, N\}$ such that $x^T J_i x < 0$.

It is easy to see that the system $\{J_i\}$ is strictly complete if and only if

$$\bigcup_{i=1}^{N} \alpha_i = R^n \backslash \{0\},$$

where

$$\alpha_i = \{x \in R^n : \quad x^T J_i x < 0\}, i = 1, 2, ..., N.$$

We end this section with the following technical well-known propositions, which will be used in the proof of the main results.

Proposition 2.1. [17] *The system* $\{J_i\}, i = 1, 2, \dots, N$, *is strictly complete if there exist* $\delta_i \ge 0, i = 1, 2, \dots, N, \sum_{i=1}^{N} \delta_i > 0$ *such that*

$$\sum_{i=1}^{N} \delta_i J_i < 0.$$

If $N = 2$ *then the above condition is also necessary for the strict completeness.*

Proposition 2.2. (Cauchy inequality) *For any symmetric positive definite marix* $N \in M^{n\times n}$ *and* $a, b \in R^n$ *we have*

$$\pm a^T b \le a^T N a + b^T N^{-1} b.$$

Proposition 2.3. [18] *For any symmetric positive definite matrix* $M \in M^{n\times n}$, *scalar* $\gamma > 0$ *and vector function* $\omega : [0, \gamma] \to R^n$ *such that the integrations concerned are well defined, the following inequality holds*

$$\Big(\int_0^\gamma \omega(s)\, ds\Big)^T M \Big(\int_0^\gamma \omega(s)\, ds\Big) \le \gamma \Big(\int_0^\gamma \omega^T(s) M \omega(s)\, ds\Big).$$

Proposition 2.4. [19] *Let* E, H *and* F *be any constant matrices of appropriate dimensions and* $F^T F \le I$. *For any* $\epsilon > 0$, *we have*

$$EFH + H^T F^T E^T \le \epsilon E E^T + \epsilon^{-1} H^T H.$$

Proposition 2.5. (Schur complement lemma [20]). *Given constant matrices* X, Y, Z *with appropriate dimensions satisfying* $X = X^T, Y = Y^T > 0$. *Then* $X + Z^T Y^{-1} Z < 0$ *if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0.$$

III. Main Results

Let us set

$$\mathcal{M}_i = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & 0 & M_{24} & S_2 \\ * & * & M_{33} & M_{34} & S_3 \\ * & * & * & M_{44} & M_{45} \\ * & * & * & * & M_{55} \end{bmatrix},$$

$$J_i = Q - S_1 A_i - A_i^T S_1^T, \tag{2}$$

$$\alpha_i = \{x \in R^n : \quad x^T J_i x < 0\}, \quad i = 1, 2, ..., N,$$

$$\bar{\alpha}_1 = \alpha_1, \quad \bar{\alpha}_i = \alpha_i \setminus \bigcup_{j=1}^{i-1} \bar{\alpha}_j, \quad i = 2, 3, \dots, N,$$

$$\lambda_1 = \lambda_{\min}(P),$$

$$\lambda_2 = \lambda_{\max}(P) + 2h_2 \lambda_{\max}(Q),$$

$$M_{11} = A_i^T P + P A_i + 2\alpha P + Q,$$

$$M_{12} = -S_2 A_i, \quad M_{13} = -S_3 A_i,$$

$$M_{14} = P D_i - S_1 D_i - S_4 A_i, \quad M_{15} = S_1 - S_5 A_i,$$

$$M_{22} = -e^{-2\alpha h_1} Q, \quad M_{24} = -S_2 D_i,$$

$$M_{33} = -e^{-2\alpha h_2} Q, \quad M_{34} = -S_3 D_i,$$

$$M_{44} = -S_4 D_i, \quad M_{45} = S_4 - S_5 D_i,$$

$$M_{55} = S_5 + S_5^T.$$

The main result of this paper is summarized in the following theorem.

Theorem 1. *Given $\alpha > 0$. The zero solution of the switched linear system (1) is $\alpha-$exponentially stable if there exist symmetric positive definite matrices P, Q, and matrices $S_i, i = 1, 2, ..., 5$ such that satisfying the following conditions*

(i) $\exists \delta_i \geq 0, i = 1, 2, \ldots, N, \quad \sum_{i=1}^{N} \delta_i > 0 : \sum_{i=1}^{N} \delta_i J_i < 0.$

(ii) $\mathcal{M}_i < 0, \quad i = 1, 2, ..., N.$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\| x(t, \phi) \| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \| \phi \|, \quad \forall t \in R^+.$$

Proof. We consider the following Lyapunov-Krasovskii functional for the system (1)

$$V(t, x_t) = \sum_{i=1}^{3} V_i,$$

where

$$V_1 = x^T(t) P x(t),$$
$$V_2 = \int_{t-h_1}^{t} e^{2\alpha(s-t)} x^T(s) Q x(s)\, ds,$$
$$V_3 = \int_{t-h_2}^{t} e^{2\alpha(s-t)} x^T(s) Q x(s)\, ds.$$

It easy to check that

$$\lambda_1 \| x(t) \|^2 \leq V(t, x_t) \leq \lambda_2 \| x_t \|^2, \quad \forall t \geq 0, \qquad (3)$$

Taking the derivative of V_1 along the solution of system (1) we have

$$\begin{aligned}\dot{V}_1 =& 2x^T(t) P \dot{x}(t)\\ =& 2x^T(t)[A_i^T P + A_i P] x(t) + 2x^T(t) P D_i x(t - h(t));\\ \dot{V}_2 =& x^T(t) Q x(t) - e^{-2\alpha h_1} x^T(t - h_1) Q x(t - h_1) - 2\alpha V_2;\\ \dot{V}_3 =& x^T(t) Q x(t) - e^{-2\alpha h_2} x^T(t - h_2) Q x(t - h_2) - 2\alpha V_3.\end{aligned}$$

Therefore, we have

$$\begin{aligned}\dot{V}(.) + 2\alpha V(.) \leq& 2x^T(t)[A_i^T P + A_i P + 2\alpha P + 2Q)] x(t)\\ &+ 2x^T(t) P D_i x(t - h(t))\\ &- e^{-2\alpha h_1} x^T(t - h_1) Q x(t - h_1)\\ &- e^{-2\alpha h_2} x^T(t - h_2) Q x(t - h_2).\end{aligned} \qquad (4)$$

By using the following identity relation

$$\dot{x}(t) - A_i x(t) - D_i x(t - h(t)) = 0,$$

we have

$$\begin{aligned}&2x^T(t) S_1 \dot{x}(t) - 2x^T(t) S_1 A_i x(t)\\ &- 2x^T(t) S_1 D_i x(t - h(t)) = 0\\ &2x^T(t - h_1) S_2 \dot{x}(t) - 2x^T(t - h_1) S_2 A_i x(t)\\ &- 2x^T(t - h_1) S_2 D_i x(t - h(t)) = 0\\ &2x^T(t - h_2) S_3 \dot{x}(t) - 2x^T(t - h_2) S_3 A_i x(t)\\ &- 2x^T(t - h_2) S_3 D_i x(t - h(t)) = 0\\ &2x^T(t - h(t)) S_4 \dot{x}(t) - 2x^T(t - h(t)) S_4 A_i x(t)\\ &- 2x^T(t - h(t)) S_4 D_i x(t - h(t)) = 0\\ &2\dot{x}^T(t) S_5 \dot{x}(t) - 2\dot{x}^T(t) S_5 A_i x(t)\\ &- 2\dot{x}^T(t) S_5 D_i x(t - h(t)) = 0\end{aligned} \qquad (5)$$

Adding all the zero items of (5) into (4), we obtain

$$\begin{aligned}\dot{V}(.) + 2\alpha V(.) \leq& x^T(t)[A_i^T P + P A_i + 2\alpha P - S_1 A_i\\ &- A_i^T S_1^T + 2Q] x(t)\\ &+ 2x^T(t)[e^{-2\alpha h_1} R - S_2 A_i] x(t - h_1)\\ &+ 2x^T(t)[-S_3 A_i] x(t - h_2) + 2x^T(t)[P D_i\\ &- S_1 D_i - S_4 A_i] x(t - h(t))\\ &+ 2x^T(t)[S_1 - S_5 A_i] \dot{x}(t)\\ &+ x^T(t - h_1)[-e^{-2\alpha h_1} Q] x(t - h_1)\\ &+ 2x^T(t - h_1)[-S_2 D_i] x(t - h(t))\\ &+ 2x^T(t - h_1) S_2 \dot{x}(t)\\ &+ x^T(t - h_2)[-e^{-2\alpha h_2} Q] x(t - h_2)\\ &+ x^T(t - h_2)[-S_3 D_i] x(t - h(t))\\ &+ 2x^T(t - h_2) S_3 \dot{x}(t)\\ &+ x^T(t - h(t))[-S_4 D_i] x(t - h(t))\\ &+ 2x^T(t - h(t))[S_4 - S_5 D_i] \dot{x}(t)\\ &+ \dot{x}^T(t)[S_5 + S_5^T] \dot{x}(t)\\ =& x^T(t) J_i x(t) + \zeta^T(t) \mathcal{M}_i \zeta(t),\end{aligned} \qquad (6)$$

where

$$\zeta(t) = [x(t), x(t - h_1), x(t - h_2), x(t - h(t)), \dot{x}(t)],$$

$$J_i = Q - S_1 A_i - A_i^T S_1^T,$$

$$\mathcal{M}_i = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & 0 & M_{24} & S_2 \\ * & * & M_{33} & M_{34} & S_3 \\ * & * & * & M_{44} & M_{45} \\ * & * & * & * & M_{55} \end{bmatrix},$$

$$M_{11} = A_i^T P + PA_i + 2\alpha P + Q,$$
$$M_{12} = -S_2 A_i, \quad M_{13} = -S_3 A_i,$$
$$M_{14} = PD_i - S_1 D_i - S_4 A_i, \quad M_{15} = S_1 - S_5 A_i,$$
$$M_{22} = -e^{-2\alpha h_1} Q, \quad M_{24} = -S_2 D_i,$$
$$M_{33} = -e^{-2\alpha h_2} Q, \quad M_{34} = -S_3 D_i,$$
$$M_{44} = -S_4 D_i, \quad M_{45} = S_4 - S_5 D_i,$$
$$M_{55} = S_5 + S_5^T.$$

Therefore, we finally obtain from (6) and the condition (ii) that

$$\dot{V}(.) + 2\alpha V(.) < x^T(t) J_i x(t), \quad \forall i = 1, 2, ..., N, \quad t \in R^+.$$

We now apply the condition (i) and Proposition 2.1., the system J_i is strictly complete, and the sets α_i and $\bar{\alpha}_i$ by (2) are well defined such that

$$\bigcup_{i=1}^{N} \alpha_i = R^n \backslash \{0\},$$

$$\bigcup_{i=1}^{N} \bar{\alpha}_i = R^n \backslash \{0\}, \quad \bar{\alpha}_i \cap \bar{\alpha}_j = \emptyset, i \neq j.$$

Therefore, for any $x(t) \in R^n$, $t \in R^+$, there exists $i \in \{1, 2, \ldots, N\}$ such that $x(t) \in \bar{\alpha}_i$. By choosing switching rule as $\gamma(x(t)) = i$ whenever $\gamma(x(t)) \in \bar{\alpha}_i$, from (6) we have

$$\dot{V}(.) + 2\alpha V(.) \leq x^T(t) J_i x(t) < 0, \quad t \in R^+,$$

and hence

$$\dot{V}(t, x_t) \leq -2\alpha V(t, x_t), \quad \forall t \in R^+. \tag{7}$$

Integrating both sides of (7) from 0 to t, we obtain

$$V(t, x_t) \leq V(\phi) e^{-2\alpha t}, \quad \forall t \in R^+.$$

Furthermore, taking condition (3) into account, we have

$$\lambda_1 \parallel x(t, \phi) \parallel^2 \leq V(x_t) \leq V(\phi) e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t} \parallel \phi \parallel^2,$$

then

$$\parallel x(t, \phi) \parallel \leq \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t} \parallel \phi \parallel, \quad t \in R^+,$$

which concludes the proof by the Lyapunov stability theorem [21].

IV. Conclusion

This paper has proposed a switching design for the exponential stability of switched linear systems with interval time-varying delays. Based on the improved Lyapunov-Krasovskii functional, a switching rule for the exponential stability for the system is designed via linear matrix inequalities.

Acknowledgment

This work was supported by the Office of Agricultural Research and Extension Maejo University Chiangmai Thailand, the Thailand Research Fund Grant, the Higher Education Commission and Faculty of Science, Maejo University, Thailand.

References

[1] M. C. de Oliveira, J. C. Geromel, Liu Hsu, LMI characterization of structural and robust stability: the discrete-time case, *Linear Algebra and its Applications*, **296**(1999), 27-38.

[2] V.N. Phat and P.T. Nam (2007), Exponential stability and stabilization of uncertain linear time-varying systems using parameter dependent Lyapunov function. *Int. J. of Control*, 80, 1333-1341.

[3] V.N. Phat and P. Niamsup, Stability analysis for a class of functional differential equations and applications. *Nonlinear Analysis: Theory, Methods & Applications* **71**(2009), 6265-6275.

[4] V.N. Phat, T. Bormat and P. Niamsup, Switching design for exponential stability of a class of nonlinear hybrid time-delay systems, Nonlinear Analysis: Hybrid Systems, **3**(2009), 1-10.

[5] Y.J. Sun, Global stabilizability of uncertain systems with time-varying delays via dynamic observer-based output feedback, *Linear Algebra and its Applications*, **353**(2002), 91-105.

[6] O.M. Kwon and J.H. Park, Delay-range-dependent stabilization of uncertain dynamic systems with interval time-varying delays, *Applied Math. Conputation*, **208**(2009), 58-68.

[7] H. Shao, New delay-dependent stability criteria for systems with interval delay, *Automatica*, **45**(2009), 744-749.

[8] J. Sun, G.P. Liu, J. Chen and D. Rees, Improved delay-range-dependent stability criteria for linear systems with time-varying delays, *Automatica*, **46**(2010), 466-470.

[9] W. Zhang, X. Cai and Z. Han, Robust stability criteria for systems with interval time-varying delay and nonlinear perturbations, *J. Comput. Appl. Math.*, 234 (2010), 174-180.

[10] V.N. Phat, Robust stability and stabilization of uncertain linear hybrid systems with state delays, IEEE Trans. CAS II, **52**(2005), 94-98.

[11] V.N. Phat and P.T. Nam (2007), Exponential stability and stabilization of uncertain linear time-varying systems using parameter dependent Lyapunov function. *Int. J. of Control*, 80, 1333-1341.

[12] V.N. Phat and P. Niamsup, Stability analysis for a class of functional differential equations and applications. *Nonlinear Analysis: Theory, Methods & Applications* **71**(2009), 6265-6275.

[13] V.N. Phat and P.T. Nam (2007), Exponential stability and stabilization of uncertain linear time-varying systems using parameter dependent Lyapunov function. *Int. J. of Control*, 80, 1333-1341.

[14] V.N. Phat and P. Niamsup, Stability analysis for a class of functional differential equations and applications. *Nonlinear Analysis: Theory, Methods & Applications* **71**(2009), 6265-6275.

[15] V.N. Phat, Y. Khongtham, and K. Ratchagit, LMI approach to exponential stability of linear systems with interval time-varying delays. *Linear Algebra and its Applications* **436**(2012), 243-251.

[16] K. Ratchagit and V.N. Phat, Stability criterion for discrete-time systems, J. Ineq. Appl., **2010**(2010), 1-6.

[17] F. Uhlig, A recurring theorem about pairs of quadratic forms and extensions, Linear Algebra Appl., **25**(1979), 219-237.

[18] K. Gu, An integral inequality in the stability problem of time delay systems, in: *IEEE Control Systems Society and Proceedings of IEEE Conference on Decision and Control*, IEEE Publisher, New York, 2000.

[19] Y. Wang, L. Xie and C.E. de SOUZA, Robust control of a class of uncertain nonlinear systems. *Syst. Control Lett.*, **1991992**), 139–149.

[20] S. Boyd, L.El Ghaoui, E. Feron and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.

[21] J.K. Hale and S.M. Verduyn Lunel, *Introduction to Functional Differential Equations*, Springer-Verlag, New York, 1993.

3rd World Congress on
Information and Communication Technologies
Hanoi, Vietnam 15-18, December 2013

- Home
- Welcome Message
- Organizing Committees
- Plenary Speakers
- Table of Contents
- Author Index
- Search
- Help

Proceedings of the

2013 Third World Congress on Information and Communication Technologies (WICT 2013)

Hanoi, Vietnam
15-18 December, 2013

Edited by

Long Thanh Ngo, Le Quy Don University, Vietnam
Ajith Abraham, Machine Intelligence Research Labs (MIR Labs), USA
Lam Thu Bui, Le Quy Don University, Vietnam
Emilio Corchado, University of Salamanca, Spain
Choo Yun-Hoy, Universiti Teknikal Malaysia Melaka, Malaysia
Kun Ma, University of Jinan, China

Technically Co-Sponsored by

IEEE

IEEE Systems Man and Cybernetics Society, Spain Chapter

Technically Supported by

IEEE Systems Man and Cybernetics Society, Technical Committee on Soft Computing

IEEE Catalog Number: CFP1395H-ART
ISBN: 978-1-4799-3230-6

New design switching rule for the robust mean square stability of uncertain stochastic discrete-time hybrid systems

Grienggrai Rajchakit

Major of Mathematics
Maejo University
Chiangmai, Thailand
Email: griengkrai@yahoo.com

***Abstract*—This paper is concerned with robust mean square stability of uncertain stochastic switched discrete time-delay systems. The system to be considered is subject to interval time-varying delays, which allows the delay to be a fast time-varying function and the lower bound is not restricted to zero. Based on the discrete Lyapunov functional, a switching rule for the robust mean square stability for the uncertain stochastic discrete time-delay system is designed via linear matrix inequalities.**

I. Introduction

Switched systems constitute an important class of hybrid systems. Such systems can be described by a family of continuous-time subsystems (or discrete-time subsystems) and a rule that orchestrates the switching between them. It is well known that a wide class of physical systems in power systems, chemical process control systems, navigation systems, automobile speed change system, and so forth may be appropriately described by the switched model [1-7]. In the study of switched systems, most works have been centralized on the problem of stability. In the last two decades, there has been increasing interest in the stability analysis for such switched systems; see, for example, [8, 9] and the references cited therein. Two important methods are used to construct the switching law for the stability analysis of the switched systems. One is the state-driven switching strategy [9]; the other is the time-driven switching strategy [8]. A switched system is a hybrid dynamical system consisting of a finite number of subsystems and a logical rule that manages switching between these subsystems (see, e.g., [1–10] and the references therein).

The main approach for stability analysis relies on the use of Lyapunov-Krasovskii functional and linear matrix inequality (LMI) approach for constructing a common Lyapunov function [11, 12, 13]. Although many important results have been obtained for switched linear continuous-time systems, there are few results concerning the stability of switched linear discrete systems with time-varying delays. In [14, 15], a class of switching signals has been identified for the considered switched discrete-time delay systems to be stable under the average dwell time scheme.

This paper studies robust mean square stability problem for uncertain stochastic switched linear discrete-time delay with interval time-varying delays. Specifically, our goal is to develop a constructive way to design switching rule to robustly mean square stable the uncertain stochastic linear discrete-time delay systems. By using improved Lyapunov-Krasovskii functional combined with LMIs technique, we propose new criteria for the robust mean square stability of the uncertain stochastic linear discrete-time delay system. Compared to the existing results, our result has its own advantages. First, the time delay is assumed to be a time-varying function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available, the delay function is bounded but not restricted to zero. Second, the approach allows us to design the switching rule for robust mean square stability in terms of of LMIs.

The paper is organized as follows: Section II presents definitions and some well-known technical propositions needed for the proof of the main results. Switching rule for the robust mean square stability is presented in Section III.

II. Preliminaries

The following notations will be used throughout this paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the n-dimensional space with the scalar product of two vectors $\langle x, y\rangle$ or x^Ty; $R^{n\times r}$ denotes the space of all matrices of $(n \times r)-$ dimension. N^+ denotes the set of all non-negative integers; A^T denotes the transpose of A; a matrix A is symmetric if $A = A^T$.
Matrix A is semi-positive definite ($A \geq 0$) if $\langle Ax, x\rangle \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $\langle Ax, x\rangle > 0$ for all $x \neq 0$; $A \geq B$ means $A - B \geq 0$. $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\min}(A) = \min\{Re\lambda : \lambda \in \lambda(A)\}$.

Consider a uncertain stochastic discrete systems with interval time-varying delay of the form

$$\begin{aligned} x(k+1) &= (A_\gamma + \Delta A_\gamma(k))x(k) + (B_\gamma + \Delta B_\gamma(k))x(k-d(k)) \\ &\quad + \sigma_\gamma(x(k), x(k-d(k)), k)\omega(k), \\ &\quad k \in N^+, \quad x(k) = v_k, \quad k = -d_2, -d_2+1, ..., 0, \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the state, $\gamma(.) : R^n \to \mathcal{N} := \{1, 2, \ldots, N\}$ is the switching rule, which is a function depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\gamma(x(k)) = i$ implies that the system realization is chosen as the i^{th} system, $i = 1, 2, ..., N$. It is seen that the system (1) can be viewed as an autonomous switched system in which the effective subsystem changes when the state $x(k)$ hits predefined boundaries. $A_i, B_i, i = 1, 2, ..., N$ are given constant matrices and the time-varying uncertain matrices $\Delta A_i(k)$ and $\Delta B_i(k)$ are defined by: $\Delta A_i(k) = E_{ia}F_{ia}(k)H_{ia}, \Delta B_i(k) = E_{ib}F_{ib}(k)H_{ib}$, where $E_{ia}, E_{ib}, H_{ia}, H_{ib}$ are known constant real matrices with appropriate dimensions. $F_{ia}(k), F_{ib}(k)$ are unknown uncertain matrices satisfying

$$F_{ia}^T(k)F_{ia}(k) \leq I, \quad F_{ib}^T(k)F_{ib}(k) \leq I, \quad k = 0, 1, 2, ..., \tag{2}$$

where I is the identity matrix of appropriate dimension, $\omega(k)$ is a scalar Wiener process (Brownian Motion) on $(\Omega, \mathcal{F}, \mathcal{P})$ with

$$E[\omega(k)] = 0, \quad E[\omega^2(k)] = 1, \quad E[\omega(i)\omega(j)] = 0 (i \neq j), \tag{3}$$

and σ_i: $R^n \times R^n \times R \to R^n, i = 1, 2, ..., N$ is the continuous function, and is assumed to satisfy that

$$\begin{aligned} &\sigma_i^T(x(k), x(k-d(k)), k)\sigma_i(x(k), x(k-d(k)), k) \leq \\ &\rho_{i1}x^T(k)x(k) + \rho_{i2}x^T(k-d(k))x(k-d(k), \\ &x(k), x(k-d(k) \in R^n, \end{aligned} \tag{4}$$

where $\rho_{i1} > 0$ and $\rho_{i2} > 0, i = 1, 2, ..., N$ are known constant scalars. The time-varying function $d(k) : N^+ \to N^+$ satisfies the following condition:

$$0 < d_1 \leq d(k) \leq d_2, \quad \forall k \in N^+$$

Remark 2.1. It is worth noting that the time delay is a time-varying function belonging to a given interval, in which the lower bound of delay is not restricted to zero.

Definition 2.1. The uncertain stochastic switched system (1) is robustly stable if there exists a switching function $\gamma(.)$ such that the zero solution of the uncertain stochastic switched system is robustly stable.

Definition 2.2. The system of matrices $\{J_i\}, i = 1, 2, \dots, N$, is said to be strictly complete if for every $x \in R^n \backslash \{0\}$ there is $i \in \{1, 2, \dots, N\}$ such that $x^T J_i x < 0$.

It is easy to see that the system $\{J_i\}$ is strictly complete if and only if

$$\bigcup_{i=1}^N \alpha_i = R^n \backslash \{0\},$$

where

$$\alpha_i = \{x \in R^n : \quad x^T J_i x < 0\}, i = 1, 2, ..., N.$$

Definition 2.3. The discrete-time system (1) is robustly stable in the mean square if there exists a positive definite scalar function $V(k, x(k) : R^n \times R^n \to R$ such that

$$E[\Delta V(k, x(k))] = E[V(k+1, x(k+1)) - V(k, x(k))] < 0,$$

along any trajectory of solution of the system (1).

Proposition 2.1. [16] *The system $\{J_i\}, i = 1, 2, \dots, N$, is strictly complete if there exist $\delta_i \geq 0, i = 1, 2, \dots, N, \sum_{i=1}^N \delta_i > 0$ such that*

$$\sum_{i=1}^N \delta_i J_i < 0.$$

If $N = 2$ then the above condition is also necessary for the strict completeness.

Proposition 2.2. (Cauchy inequality) *For any symmetric positive definite marix $N \in M^{n\times n}$ and $a, b \in R^n$ we have*

$$\pm a^T b \leq a^T N a + b^T N^{-1} b.$$

Proposition 2.3. [16] *Let E, H and F be any constant matrices of appropriate dimensions and $F^T F \leq I$. For any $\epsilon > 0$, we have*

$$EFH + H^T F^T E^T \leq \epsilon E E^T + \epsilon^{-1} H^T H.$$

III. Main Results

Let us set

$$W_i = \begin{bmatrix} W_{i11} & W_{i12} & W_{i13} \\ * & W_{i22} & W_{i23} \\ * & * & W_{i33} \end{bmatrix},$$

where

$$\begin{aligned} W_{i11} &= Q - P, \\ W_{i12} &= S_1 - S_1 A_i, \\ W_{i13} &= -S_1 B_i, \\ W_{i22} &= P + S_1 + S_1^T + H_{ia}^T H_{ia} + S_1 E_{ib} E_{ib}^T S_1^T, \\ W_{i23} &= -S_1 B_i, \\ W_{i33} &= -Q + 2H_{ib}^T H_{ib} + 2\rho_{i2} I, \\ J_i &= (d_2 - d_1)Q - S_1 A_i - A_i^T S_1^T + 2S_1 E_{ia} E_{ia}^T S_1^T \\ &\quad + S_1 E_{ib} E_{ib}^T S_1^T + H_{ia}^T H_{ia} + 2\rho_{i1} I, \\ \alpha_i &= \{x \in R^n : \quad x^T J_i x < 0\}, \ i = 1, 2, ..., N, \\ \bar{\alpha}_1 &= \alpha_1, \quad \bar{\alpha}_i = \alpha_i \setminus \bigcup_{j=1}^{i-1} \bar{\alpha}_j, \quad i = 2, 3, \dots, N. \end{aligned} \tag{5}$$

The main result of this paper is summarized in the following theorem.

Theorem 1. *The uncertain stochastic switched system (1) is robustly stable in the mean square if there exist symmetric positive definite matrices $P > 0, Q > 0$ and matrix S_1 satisfying the following conditions*

(i) $\exists \delta_i \geq 0, i = 1, 2, \ldots, N, \sum_{i=1}^{N} \delta_i > 0 : \sum_{i=1}^{N} \delta_i J_i < 0.$

(ii) $W_i < 0, \quad i = 1, 2, ..., N.$

The switching rule is chosen as $\gamma(x(k)) = i$, *whenever* $x(k) \in \bar{\alpha}_i$.

Proof. Consider the following Lyapunov-Krasovskii functional for any ith system (1)

$$V(k) = V_1(k) + V_2(k) + V_3(k),$$

where

$$V_1(k) = x^T(k)Px(k), \quad V_2(k) = \sum_{i=k-d(k)}^{k-1} x^T(i)Qx(i),$$

$$V_3(k) = \sum_{j=-d_2+2}^{-d_1+1} \sum_{l=k+j+1}^{k-1} x^T(l)Qx(l),$$

We can verify that

$$\lambda_1 \|x(k)\|^2 \leq V(k). \tag{6}$$

Let us set $\xi(k) = [x(k)\, x(k+1)\, x(k-d(k))\, \omega(k)]^T$, and

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad G = \begin{pmatrix} P & 0 & 0 & 0 \\ I & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix}.$$

Then, the difference of $V_1(k)$ along the solution of the system (1) and taking the mathematical expectation, we obtained

$$\begin{aligned} E[\Delta V_1(k)] &= E[x^T(k+1)Px(k+1) - x^T(k)Px(k)] \\ &= E[\xi^T(k)H\xi(k) - 2\xi^T(k)G^T \begin{pmatrix} 0.5x(k) \\ 0 \\ 0 \\ 0 \end{pmatrix}]. \end{aligned} \tag{7}$$

because of

$$\xi^T(k)H\xi(k) = x(k+1)Px(k+1),$$

$$2\xi^T(k)G^T \begin{pmatrix} 0.5x(k) \\ 0 \\ 0 \\ 0 \end{pmatrix} = x^T(k)Px(k).$$

Using the expression of system (1)

$$\begin{aligned} 0 = &-S_1 x(k+1) + S_1(A_i + E_{ia}F_{ia}(k)H_{ia})x(k) \\ &+S_1(B_i + E_{ib}F_{ib}(k)H_{ib})x(k-d(k)) + S_1\sigma_i\omega(k), \end{aligned}$$

$$\begin{aligned} 0 = &-\sigma_i^T x(k+1) + \sigma_i^T(A_i + E_{ia}F_{ia}(k)H_{ia})x(k) \\ &+\sigma_i^T(B_i + E_{ib}F_{ib}(k)H_{ib})x(k-d(k)) + \sigma_i^T\sigma_i\omega(k), \end{aligned}$$

we have

$$E[-2\xi^T(k)G^T \times \begin{pmatrix} 0.5x(k) \\ [-S_1x(k+1) + S_1(A_i + E_{ia}F_{ia}(k)H_{ia})x(k) \\ +S_1(B_i + E_{ib}F_{ib}(k)H_{ib})x(k-d(k)) + S_1\sigma_i\omega(k)] \\ 0 \\ [-\sigma_i^T x(k+1) + \sigma_i^T(A_i + E_{ia}F_{ia}(k)H_{ia})x(k) \\ +\sigma_i^T(B_i + E_{ib}F_{ib}(k)H_{ib})x(k-d(k)) + \sigma_i^T\sigma_i\omega(k)] \end{pmatrix}]$$

Therefore, from (7) it follows that

$$\begin{aligned} E[\Delta V_1(k)] = E[&x^T(k)[-P - S_1A_i - S_1E_{ia}F_{ia}(k)H_{ia} \\ &- A_i^T S_1^T - H_{ia}^T F_{ia}^T(k)E_{ia}S_1^T]x(k) \\ &+ 2x^T(k)[S_1 - S_1A_i - S_1E_{ia}F_{ia}(k)H_{ia}]x(k+1) \\ &+ 2x^T(k)[-S_1B_i - S_1E_{ib}F_{ib}(k)H_{ib}]x(k-d(k)) \\ &+ 2x^T(k)[-S_1\sigma_i - \sigma_i^T A_i - \sigma_i^T E_{ia}F_{ia}(k)H_{ia}]\omega(k) \\ &+ x(k+1)[S_1 + S_1^T]x(k+1) \\ &+ 2x(k+1)[-S_1B_i - S_1(E_{ib}F_{ib}(k)H_{ib})]x(k-d(k)) \\ &+ 2x(k+1)[\sigma_i^T - S_1\sigma_i]\omega(k) \\ &+ x^T(k-d(k))[-\sigma_i^T B_i - \sigma_i^T E_{ib}F_{ib}(k)H_{ib}]\omega(k) \\ &+ \omega^T(k)[-2\sigma_i^T\sigma_i]\omega(k)], \end{aligned}$$

By asumption (3), we have

$$\begin{aligned} E[\Delta V_1(k)] = E[&x^T(k)[-P - S_1A_i - S_1E_{ia}F_{ia}(k)H_{ia} \\ &- A_i^T S_1^T - H_{ia}^T F_{ia}^T(k)E_{ia}S_1^T]x(k) \\ &+ 2x^T(k)[S_1 - S_1A_i - S_1E_{ia}F_{ia}(k)H_{ia}]x(k+1) \\ &+ 2x^T(k)[-S_1B_i - S_1E_{ib}F_{ib}(k)H_{ib}]x(k-d(k)) \\ &+ x(k+1)[S_1 + S_1^T]x(k+1) \\ &+ 2x(k+1)[-S_1B_i - S_1E_{ib}F_{ib}(k)H_{ib}]x(k-d(k)) \\ &- 2\sigma_i^T\sigma_i], \end{aligned}$$

Applying Proposition 2.2, Proposition 2.3, condition (2) and assumption (4), the following estimations hold

$$-S_1E_{ia}F_{ia}(k)H_{ia} - H_{ia}^T F_{ia}^T(k)E_{ia}^T S_1^T \leq S_1E_{ia}E_{ia}^T S_1^T + H_{ia}^T H_{ia},$$

$$-2x^T(k)S_1E_{ia}F_{ia}(k)H_{ia}x(k+1) \leq$$

$$x^T(k)S_1E_{ia}E_{ia}^T S_1^T x(k) + x(k+1)^T H_{ia}^T H_{ia}x(k+1),$$

$$-2x^T(k)S_1E_{ib}F_{ib}(k)H_{ib}x(k-d(k)) \leq$$

$$x^T(k)S_1E_{ib}E_{ib}^T S_1^T x(k) + x(k-d(k))^T H_{ib}^T H_{ib}x(k-d(k)),$$

$$-2x^T(k+1)S_1E_{ib}F_{ib}(k)H_{ib}x(k-d(k)) \leq$$

$$x^T(k+1)S_1E_{ib}E_{ib}^T S_1^T x(k+1) + x(k-d(k))^T H_{ib}^T H_{ib}x(k-d(k)),$$

$$-\sigma_i^T(x(k), x(k-d(k)), k)\sigma_i(x(k), x(k-d(k)), k) \leq$$

$$\rho_{i1}x^T(k)x(k) + \rho_{i2}x^T(k-d(k))x(k-d(k)).$$

Therefore, we have

$$\begin{aligned}E[\Delta V_1(k)] = E[&x^T(k)[-P - S_1A_i - A_i^TS_1^T \\ &+ 2S_1E_{ia}E_{ia}^TS_1^T \\ &+ S_1E_{ib}E_{ib}^TS_1^T + S_2E_{ia}E_{ia}^TS_2^T \\ &+ H_{ia}^TH_{ia} + 2\rho_{i1}I]x(k) \\ &+ 2x^T(k)[S_1 - S_1A_i]x(k+1) \\ &+ 2x^T(k)[-S_1B_i - S_2A_i]x(k-d(k)) \\ &+ x(k+1)[S_1 + S_1^T + H_{ia}^TH_{ia} \\ &+ S_1E_{ib}E_{ib}^TS_1^T]x(k+1) \\ &+ 2x(k+1)[S_2 - S_1B_i]x(k-d(k)) \\ &+ x^T(k-d(k))[2H_{ib}^TH_{ib} \\ &+ 2\rho_{i2}I]x(k-d(k))],\end{aligned} \tag{8}$$

The difference of $V_2(k)$ is given by

$$\begin{aligned}E[\Delta V_2(k)] &= E[\sum_{i=k+1-d(k+1)}^{k} x^T(i)Qx(i) \\ &\quad - \sum_{i=k-d(k)}^{k-1} x^T(i)Qx(i)] \\ &= E[\sum_{i=k+1-d(k+1)}^{k-d_1} x^T(i)Qx(i) \\ &\quad + x^T(k)Qx(k) - x^T(k-d(k))Qx(k-d(k)) \\ &\quad + \sum_{i=k+1-d_1}^{k-1} x^T(i)Qx(i) \\ &\quad - \sum_{i=k+1-d(k)}^{k-1} x^T(i)Qx(i)].\end{aligned} \tag{9}$$

Since $d(k) \geq d_1$ we have

$$\sum_{i=k+1-d_1}^{k-1} x^T(i)Qx(i) - \sum_{i=k+1-d(k)}^{k-1} x^T(i)Qx(i) \leq 0,$$

and hence from (9) we have

$$\begin{aligned}E[\Delta V_2(k)] \leq E[&\sum_{i=k+1-d(k+1)}^{k-d_1} x^T(i)Qx(i) \\ &+ x^T(k)Qx(k) - x^T(k-d(k))Qx(k-d(k))].\end{aligned} \tag{10}$$

The difference of $V_3(k)$ is given by

$$\begin{aligned}E[\Delta V_3(k)] &= E[\sum_{j=-d_2+2}^{-d_1+1} \sum_{l=k+j}^{k} x^T(l)Qx(l) \\ &\quad - \sum_{j=-d_2+2}^{-d_1+1} \sum_{l=k+j+1}^{k-1} x^T(l)Qx(l)] \\ &= E[\sum_{j=-d_2+2}^{-d_1+1} [\sum_{l=k+j}^{k-1} x^T(l)Qx(l) + x^T(k)Q(\xi)x(k) \\ &\quad - \sum_{l=k+j}^{k-1} x^T(l)Qx(l) \\ &\quad - x^T(k+j-1)Qx(k+j-1)]] \\ &= E[\sum_{j=-d_2+2}^{-d_1+1} [x^T(k)Qx(k) \\ &\quad - x^T(k+j-1)Qx(k+j-1)]] \\ &= E[(d_2-d_1)x^T(k)Qx(k) \\ &\quad - \sum_{j=k+1-d_2}^{k-d_1} x^T(j)Qx(j)].\end{aligned} \tag{11}$$

Since $d(k) \leq d_2$, and

$$\sum_{i=k=1-d(k+1)}^{k-d_1} x^T(i)Qx(i) - \sum_{i=k+1-d_2}^{k-d_1} x^T(i)Qx(i) \leq 0,$$

we obtain from (10) and (11) that

$$\begin{aligned}E[\Delta V_2(k) + \Delta V_3(k)] \leq E[&(d_2 - d_1 + 1)x^T(k)Qx(k) \\ &- x^T(k-d(k))Qx(k-d(k))].\end{aligned} \tag{12}$$

Therefore, combining the inequalities (8), (12) gives

$$E[\Delta V(k)] \leq E[x^T(k)J_ix(k) + \psi^T(k)W_i\psi(k)], \tag{13}$$

where

$$\psi(k) = [x(k)\, x(k+1)\, x(k-d(k))]^T,$$

$$W_i = \begin{bmatrix} W_{i11} & W_{i12} & W_{i13} \\ * & W_{i22} & W_{i23} \\ * & * & W_{i33} \end{bmatrix},$$

$$\begin{aligned}W_{i11} &= Q - P, \\ W_{i12} &= S_1 - S_1A_i, \\ W_{i13} &= -S_1B_i, \\ W_{i22} &= P + S_1 + S_1^T + H_{ia}^TH_{ia} + S_1E_{ib}E_{ib}^TS_1^T, \\ W_{i23} &= -S_1B_i, \\ W_{i33} &= -Q + 2H_{ib}^TH_{ib} + 2\rho_{i2}I,\end{aligned}$$

and

$$\begin{aligned}J_i = (d_2-d_1)Q - S_1A_i - A_i^TS_1^T + 2S_1E_{ia}E_{ia}^TS_1^T + S_1E_{ib}E_{ib}^TS_1^T \\ + H_{ia}^TH_{ia} + 2\rho_{i1}I.\end{aligned}$$

Therefore, we finally obtain from (13) and the condition (ii) that

$$E[\Delta V(k)] < E[x^T(k)J_i x(k)], \quad \forall i = 1, 2, ..., N,$$
$$k = 0, 1, 2, ...$$

We now apply the condition (i) and Proposition 2.1., the system J_i is strictly complete, and the sets α_i and $\bar{\alpha}_i$ by (5) are well defined such that

$$\bigcup_{i=1}^{N} \alpha_i = R^n \backslash \{0\},$$

$$\bigcup_{i=1}^{N} \bar{\alpha}_i = R^n \backslash \{0\}, \quad \bar{\alpha}_i \cap \bar{\alpha}_j = \emptyset, i \neq j.$$

Therefore, for any $x(k) \in R^n, k = 1, 2, ...,$ there exists $i \in \{1, 2, \ldots, N\}$ such that $x(k) \in \bar{\alpha}_i$. By choosing switching rule as $\gamma(x(k)) = i$ whenever $x(k) \in \bar{\alpha}_i$, from the condition (13) we have

$$E[\Delta V(k)] \leq E[x^T(k)J_i x(k)] < 0, \quad k = 1, 2, ...,$$

which, combining the condition (6), Definition 2.3 and the Lyapunov stability theorem [16], concludes the proof of the theorem in the mean square.

IV. Conclusion

This paper has proposed a switching design for the robust stability of uncertain stochastic switched discrete time-delay systems with interval time-varying delays. Based on the discrete Lyapunov functional, a switching rule for the robust stability for the uncertain stochastic switched discrete time-delay system is designed via linear matrix inequalities.

Acknowledgement

This work was supported by the Office of Agricultural Research and Extension Maejo University Chiangmai Thailand, the Thailand Research Fund Grant, the Higher Education Commission and Faculty of Science, Maejo University, Thailand.

References

[1] D. Liberzon, A.S. Morse, Basic problems in stability and design of switched systems, IEEE Control Syst. Mag., **19**(1999), 57-70.
[2] A.V. Savkin and R.J. Evans, Hybrid Dynamical Systems: Controller and Sensor Switching Problems, Springer, New York, 2001.
[3] Z. Sun and S.S. Ge, Switched Linear Systems: Control and Design, Springer, London, 2005.
[4] VN. Phat, Y. Kongtham, and K. Ratchagit, LMI approach to exponential stability of linear systems with interval time-varying delays, Linear Algebra Appl., **436**(2012), 243-251.
[5] VN. Phat, K. Ratchagit, Stability and stabilization of switched linear discrete-time systems with interval time-varying delay, Nonlinear Analysis: Hybrid Systems, **5**(2011), 605-612.
[6] K. Ratchagit, VN. Phat, Stability criterion for discrete-time systems, Journal of Inequalities and Applications, **2010**(2010), 1-6.
[7] K. Ratchagit, Asymptotic stability of nonlinear delay-difference system via matrix inequalities and application, International Journal of Computational Methods, **6**(2009), 389-397.
[8] F. Gao, S. Zhong and X. Gao, Delay-dependent stability of a type of linear switching systems with discrete and distributed time delays, Appl. Math. Computation, **196**(2008), 24-39.
[9] C.H. Lien, K.W. Yu, Y.J. Chung, Y.F. Lin, L.Y. Chung and J.D. Chen, Exponential stability analysis for uncertain switched neutral systems with interval-time-varying state delay, Nonlinear Analysis: Hybrid systems, 3(2009),334–342.
[10] G. Xie, L. Wang, Quadratic stability and stabilization of discrete-time switched systems with state delay, In: Proc. of the IEEE Conference on Decision and Control, Atlantics, December 2004, 3235-3240.
[11] S. Boyd, L.E. Ghaoui, E. Feron and V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, SIAM, Philadelphia, 1994.
[12] D.H. Ji, J.H. Park, W.J. Yoo and S.C. Won, Robust memory state feedback model predictive control for discrete-time uncertain state delayed systems, Appl. Math. Computation, **215**(2009), 2035-2044.
[13] G.S. Zhai, B. Hu, K. Yasuda, and A. Michel, Qualitative analysis of discrete- time switched systems. In: Proc. of the American Control Conference, 2002, 1880-1885.
[14] W.A. Zhang, Li Yu, Stability analysis for discrete-time switched time-delay systems, Automatica, **45**(2009), 2265-2271.
[15] F. Uhlig, A recurring theorem about pairs of quadratic forms and extensions, Linear Algebra Appl., **25**(1979), 219-237.
[16] R.P. Agarwal, Difference Equations and Inequalities, Second Edition, Marcel Dekker, New York, 2000.