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Original Article

Further results on (1,0,0)-F-Face magic mean graphs

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Abstract

A (1,0,0)-F-Face magic mean labeling is an assignment of labels to the vertices of planar graph such that the mean weight of each face including an exterior face is constant. In this paper, the existence of (1,0,0)-F-face magic mean labeling is proven for new graphs obtained by applying graph operations like Cartesian product, subdivision, vertex duplication and edge duplication.

Keywords: labeling, (1,0,0)-F-Face magic mean labeling, (1,0,0)-F-Face magic mean graph

1. Introduction

Throughout this paper, a graph G means a finite, connected, undirected planar graph having neither loops nor multiple edges. A planar graph is a graph which can be drawn in a plane so that no two edges intersect.

Path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n.

Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the cartesian product $G_1 \times G_2$ has the vertex set $\{(u,v): u\epsilon G_1, v\epsilon G_2\}$ and the edges (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$. Vertex duplication of a cycle C_n, denoted by $\widehat{C_n}$, is formed by duplicating all the vertices of $C_n, n \geq 3$ where $n \equiv 0 (mod\ 2)$.

A globe graph is a graph produced by joining two isolated vertices by n paths of length two. It is isomorphic to $K_{2,n}$. A wheel graph consists of a rim and spokes. The rim edges are the edges of a cycle and the spokes are edges connecting the central vertex with each node of the rim.

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Duplication of an vertex v in a graph G by a vertex v' produces a new graph G' where $V(G') = V(G) \cup \{v'\}$ and $E(G') = E(G) \cup \{v'x: x$ is a vertex adjacent to v in $G\}$

Duplication of an edge $e = uv$ in a graph G by a vertex v' produces a new graph G' where $V(G') = V(G) \cup \{v'\}$ and $E(G') = E(G) \cup \{uv', v'v\}$.

A labeling of a graph is a map that carries graph elements to numbers (usually to the positive or non-negative integers). Many kinds of labelings have been studied and an excellent survey of graph labeling can be found in Gallian (2005).

A function ψ is called a mean labeling of graph G if $\psi: V(G) \to \{0,1,2,\ldots,|E(G)|\}$ is injective and the induced edge function $\psi^*: E(G) \to \{1,2,\ldots,|E(G)|\}$ defined as follows is bijective.

For each edge

$$uv, \psi^*(uv) = \begin{cases} \dfrac{\psi(u)+\psi(v)}{2} & if\ \psi(u)+\psi(v)\ is\ even \\ \dfrac{\psi(u)+\psi(v)+1}{2} & if\ \psi(u)+\psi(v)\ is\ odd \end{cases}$$

The graph which admits mean labeling is called a mean graph (Durai Baskar & Arockiaraj, 2016).

A graph G is magic if the edges of G can be labeled by the numbers $1,2,3,\ldots,|E(G)|$, so that the sum of the labels of all the edges incident with any vertex is the same (Hartsfield & Ringel, 1990).

A bijection $\phi: E(G) \to \{1,2,\ldots,|E(G)|\}$ is called a F-Face magic mean labeling of G if the induced face labeling

$$\phi^*(f_i) = \left\lfloor \frac{\text{sum of the labels of the edges in the boundary of } f_i}{deg(f_i)} \right\rfloor$$

$$= \left\lfloor \frac{\sum_{e_j \epsilon f_i} \phi(e_j)}{deg(f_i)} \right\rfloor = k, \text{a constant}$$

for each face f_i, including the exterior face of G, where $deg(f_i)$ is the degree of the face f_i, that is the number of edges that bound the face. The graph which admits F-Face magic mean labeling is called a F-Face magic mean graph (Arockiaraj & Meena Kumari, 2017). Magic meanness property of slanting ladder has been discussed by Meena Kumari and Arockiaraj (2017). Arockiaraj and Meena Kumari (2018) investigated the F-face magic mean labeling for the vertex duplication of cycle C_n. (Durai Baskar *et al*., 2016) discussed the geometric meanness property of the grid graph $P_m \times P_n$.

Motivated by these works, the authors of this paper introduce the (1,0,0)-F-Face magic mean labeling of graphs as follows:

A bijection $\phi: V(G) \to \{1,2,\ldots,|V(G)|\}$ is called a (1,0,0)-F-Face magic mean labeling of G if the induced face labeling

$$\phi^*(f_i) = \left\lfloor \frac{\text{sum of the labels of the vertices in the boundary of } f_i}{deg(f_i)} \right\rfloor$$

$$= \left\lfloor \frac{\sum_{v_j \epsilon f_i} \phi(v_j)}{deg(f_i)} \right\rfloor = k, \text{a constant}$$

for each face f_i, including the exterior face of G,where $deg(f_i)$ is the degree of the face f_i, that is the number of edges that bound the face. The graph which admits (1,0,0)-F-Face magic mean labeling is called a (1,0,0)-F-Face magic mean graph. The weight of the face f of a graph G is defined as the sum of all the vertex labels on the face and the mean weight of any face f is defined as the induced labeling on the face.

In this paper, the existence of (1,0,0)-F-face magic mean labeling is proven for new graphs obtained by applying graph operations like Cartesian product, subdivision, vertex duplication and edge duplication. We investigate the (1,0,0)-magic meanness property of graphs such as Vertex duplication of cycle $\widehat{C_n}$, a graph obtained by subdividing the rim edges of a wheel graph $W_n = C_n + K_1$ by a vertex, globe graph $K_{2,n}$, a graph obtained from cycle C_n by identifying each of its edge with an edge of a copy of C_4, Slanting ladder SL_n and grid graph $P_m \times P_n$.

2. Main Results

Theorem 2.1. The graph $\widehat{C_n}$ is a *(*1,0,0*)*-F-face magic mean graph with face constant n if and only if n is even.

Proof. If n is odd, $\widehat{C_n}$ is a nonplanar graph.
Assume that n is even.

Let $\{v_i: 1 \le i \le n\}$ be the vertices of C_n and u_i be the respective duplicating vertex of v_i, for $1 \le i \le n$.

Then $E(\widehat{C_n}) = \{v_i v_{i+1}: 1 \le i \le n-1\} \cup \{u_i v_{i-1}: 2 \le i \le n\} \cup \{u_i v_{i+1}: 1 \le i \le n-1\} \cup \{u_n v_1, v_n u_1, v_n v_1\}$ and

$$F(\widehat{C_n}) = \{f_i = (v_{i-1}v_i, v_i v_{i+1}, u_i v_{i+1}, u_i v_{i-1}): 2 \le i \le n-1\}$$
$$\cup \{f_1 = (v_1 v_2, v_n v_1, u_1 v_n, u_1 v_2), f_n = (v_{n-1} v_n, v_n v_1, v_1 u_n, v_{n-1} u_n)\}$$
$$\cup \left\{f_I = \left(\bigcup_{i\equiv 0(mod\ 2), i=2}^{n-2} v_{i+1}u_i, \bigcup_{i\equiv 0(mod\ 2), i=2}^{n} v_{i-1}u_i, u_n v_1\right),\right.$$
$$\left. f_0 = \left(\bigcup_{i\equiv 1(mod\ 2), i=1}^{n-1} v_{i+1}u_i, \bigcup_{i\equiv 1(mod\ 2), i=3}^{n-1} v_{i-1}u_i, v_n u_1\right)\right\}.$$

In $\widehat{C_n}$, $|V(\widehat{C_n})| = 2n$ and $|F(\widehat{C_n})| = n+2$.

Define $\phi: V(\widehat{C_n}) \to \{1,2,\ldots,2n\}$ as follows: $For\ 1 \le i \le n, \phi(v_i) = \begin{cases} n+1, & i = 1 \\ i-1, & i \equiv 0(mod2) \\ 2n-i+2, & i \equiv 1(mod2), i \ne 1 \end{cases}$ and

$$\phi(u_i) = \begin{cases} n, & i = 2 \\ i-2, & i \equiv 0(mod2), i \ne 2 \\ 2n-i+1, & i \equiv 1(mod2). \end{cases}$$

Then the induced face labeling ϕ^* in G is obtained as follows

$$\phi^*(f_i) = \left\lfloor \frac{1}{4}\{\phi(v_{i-1}) + \phi(v_i) + \phi(v_{i+1}) + \phi(u_i)\} \right\rfloor$$
$$= n, for\ all\ \ 3 \le i \le n-1, i \equiv 1(mod2)$$
$$\phi^*(f_1) = \left\lfloor \frac{1}{4}\{\phi(v_1) + \phi(v_2) + \phi(u_1) + \phi(v_n)\} \right\rfloor = n,$$
$$\phi^*(f_2) = \left\lfloor \frac{1}{4}\{\phi(v_1) + \phi(v_2) + \phi(v_3) + \phi(u_2)\} \right\rfloor = n,$$
$$\phi^*(f_i) = \left\lfloor \frac{1}{4}\{\phi(v_{i-1}) + \phi(v_i) + \phi(v_{i+1}) + \phi(u_i)\} \right\rfloor$$
$$= n, for\ all\ \ 4 \le i \le n, i \equiv 0(mod2)\ and$$
$$\phi^*(f_0) = \left\lfloor \frac{1}{n}\left\{\sum_{i=1,i\equiv 1(mod2)}^{n-1} \phi(u_i) + \sum_{i=2,i\equiv 0(mod2)}^{n} \phi(v_i)\right\} \right\rfloor = n.$$

Thus $\phi^*(f) = n$, for each face f of G.

So, $G \simeq \widehat{C_n}$ is a (1,0,0)-F-face magic mean graph with face constant n if n is even. A (1,0,0)-F-face magic mean labeling of $\widehat{C_8}$ is shown in Figure 1.

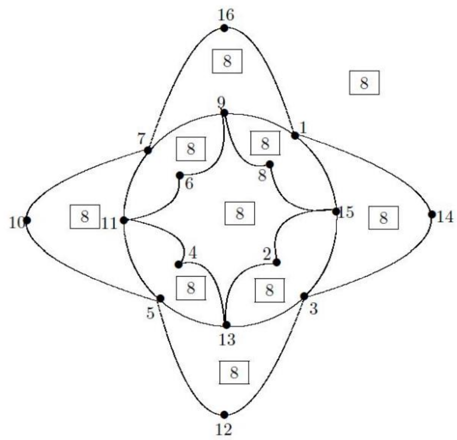
Theorem 2.2. The graph G obtained by subdividing the rim edges of a wheel graph $W_n = C_n + K_1$ by a vertex is a *(*1,0,0*)*-F-Face magic mean graph for $n \ge 3$ with face constant n.

Proof. Let $V(G) = \{u_i: 1 \le i \le n\} \cup \{v_i: 1 \le i \le n\} \cup \{u\}$,

$E(G) = \{uu_i: 1 \le i \le n\} \cup \{u_i v_i: 1 \le i \le n\} \cup \{v_i u_{i+1}: 1 \le i \le n-1\} \cup \{v_n u_1\}$ and

$F(G) = \{f_i = (uu_i, u_i v_i, v_i u_{i+1}, u_{i+1} u): 1 \le i \le n-1\}$

$\cup \{f_n = (uu_n, u_n v_n, v_n u_1, u_1 u)\} \cup \{f_0 = \left(\bigcup_{i=1}^{n} u_i v_i, \bigcup_{i=1}^{n-1} v_i u_{i+1}, v_n u_1\right)\}$



Fi gure 1. A (1,0,0)-F-Face magic mean labeling of $\widehat{C_8}$ with face constant 8.

where f_0 is the outer face of G, $deg(f_i) = 4, 1 \leq i \leq n$ and $deg(f_0) = 2n$, $|V(G)| = 2n + 1$.

Case (i) n is odd.

Define $\phi: V(G) \to \{1,2,\ldots,2n+1\}$ as follows:

$$\phi(u) = (3n-3)/2,$$

$$\phi(u_i) = \begin{cases} \frac{i+1}{2}, & i \equiv 1(mod2) \\ \frac{n+i+1}{2}, & i \equiv 0(mod2) \text{ and} \end{cases}$$

$$\phi(v_i) = \begin{cases} 2n - i + 1, & 1 \leq i \leq \frac{n+3}{2} \\ 2n - i, & \frac{n+5}{2} \leq i \leq n-1 \\ 2n+1, & i = n. \end{cases}$$

Then the induced face labeling ϕ^* on G is obtained as $\phi^*(f) = n$ for each face f of G. Thus ϕ is a (1,0,0)-F-Face magic mean labeling of G.

Case (ii) n is even

Define $\phi: V(G) \to \{1,2,\ldots,2n+1\}$ as follows:

$$\phi(u) = 3n/2,$$

$$\phi(u_i) = \begin{cases} \frac{i+1}{2}, & i \equiv 1(mod2) \\ \frac{n+i}{2}, & i \equiv 0(mod2) \text{ and} \end{cases}$$

$$\phi(v_i) = \begin{cases} 2n - i + 2, & 1 \leq i \leq \frac{n}{2} - 1 \\ 2n - \frac{n}{2} + 1, & i = \frac{n}{2} \\ 2n - i, & \frac{n}{2} + 1 \leq i \leq n-1 \\ 2n - \frac{n}{2} + 2, & i = n. \end{cases}$$

Then the induced face labeling ϕ^* on G is obtained as $\phi^*(f) = n$ for each face f of G. Thus ϕ is a (1,0,0)-F-Face magic mean labeling of G. A (1,0,0)-F-Face magic mean labeling of $C_8 + K_1$ and $C_7 + K_1$ are shown in Figure 2.

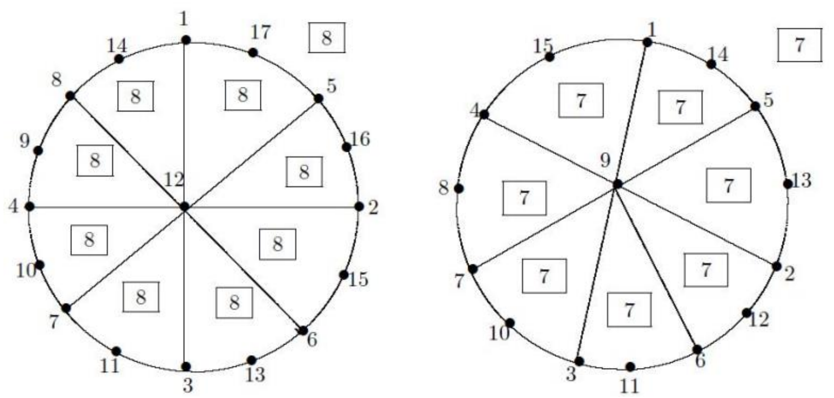


Figure 2. A (1,0,0)-F-Face magic mean labeling of $C_8 + K_1$ and $C_7 + K_1$ having face mean constant 8 and 7 respectively.

Theorem 2.3. The globe graph $K_{2,n}, n \geq 1$ is a *(*1,0,0*)*-F-face magic mean graph.

Proof. Let $V(G) = \{u_i: 1 \leq i \leq 2\} \cup \{v_i: 1 \leq i \leq n\}$,

$E(G) = \{u_1 v_i: 1 \leq i \leq n\} \cup \{u_2 v_i: 1 \leq i \leq n\}$ and

$F(G) = \{f_i = (u_1 v_i, u_i v_{i+1}, u_2 v_i, u_2 v_{i+1}): 1 \leq i \leq n-1\}$

$\cup \{f_0 = (u_1 v_1, u_1 v_n, u_2 v_1, u_2 v_n)\}$

where f_0 is the outer face of G, $deg(f_i) = 4, 0 \leq i \leq n, |V(G)| = 2n + 1$.

Case (i) n is even.

Define $\phi: V(G) \to \{1,2,3,\ldots,2n+1\}$ as follows:

$$For\ i \equiv 1(mod2), \phi(v_i) = \begin{cases} \frac{i+1}{2}, & 1 \leq i \leq 5 \\ \frac{i+3}{2}, & 1 \leq i \leq 5 \\ \frac{n}{2}, & i = 11 \\ \frac{n}{2} - \frac{i-11}{2}, & 13 \leq i \leq n-3 \\ 4 & i = n-1. \end{cases}$$

$$For\ i \equiv 0(mod2), \phi(v_i) = \begin{cases} n - \frac{i}{2} + 3, & 2 \leq i \leq 4 \\ n - \frac{i}{2} + 2, & 6 \leq i \leq 8 \\ n - \frac{i}{2} + 1, & 10 \leq i \leq n-4 \\ n-3, & i = n-2 \\ n, & i = n. \end{cases}$$

$\phi(u_1) = \frac{n}{2} + 1$ and

$\phi(u_2) = \frac{n}{2} + 2.$

Then the induced face labeling ϕ^* on G is obtained as $\phi^*(f) = \frac{n}{2} + 1$ for each face f of G. Thus ϕ is a (1,0,0)-F-Face magic mean labeling of G.

Case (ii) $n \equiv 1(mod4)$

Assume that $n \geq 5$. Define $\phi: V(G) \to \{1,2,3,\ldots,2n+1\}$ as follows:

$$For\ 1 \leq i \leq n, i \neq n-2, \phi(v_i) = \begin{cases} i+1, & i \equiv 1(mod2) \\ n-i, & i \equiv 0(mod2), \end{cases}$$

$\phi(v_{n-2}) = n$, $\phi(u_1) = n - 1$ and

$\phi(u_2) = n + 2.$

Then the induced face labeling ϕ^* on G is obtained as $\phi^*(f) = \frac{3n+3}{4}$ for each face f of G. Thus ϕ is a (1,0,0)-F-Face magic mean labeling of G.

Case (iii) $n \equiv 3(mod4)$

Assume that $n \geq 3$. Define $\phi: V(G) \rightarrow \{1,2,3,\ldots,2n+1\}$ as follows:

$$For\ 1 \leq i \leq n \text{ and } i \neq n-2, \phi(v_i) = \begin{cases} i+1, & i \equiv 1(mod2) \\ n-i, & i \equiv 0(mod2), \end{cases}$$

$$\phi(v_{n-2}) = n,$$
$$\phi(u_1) = n-1 \text{ and}$$
$$\phi(u_2) = n+2.$$

Then the induced face labeling ϕ^* on G is obtained as $\phi^*(f) = \frac{3n+1}{4}$ for each face f of G. Thus ϕ is a (1,0,0)-F-Face magic mean labeling of G. When $n = 1$, the graph $K_{2,1}$ is a tree and so it admits a (1,0,0)-F-Face magic mean labeling. A (1,0,0)-F-Face magic mean labeling of $K_{2,10}, K_{2,7}$ and $K_{2,9}$ are shown in Figures 3, 4 and 5 respectively.

Theorem 2.4. Let G be a graph obtained from cycle C_n by identifying each of its edge with an edge of a copy of C_4. Then G is a (1,0,0)-F-face magic mean graph.

Proof. Let the vertex set of C_n be $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and the vertex set of i^{th} copy of C_4 be $V(C_4) = \{u_{i1}, u_{i2}, u_{i3}, u_{i4}\}$. Let the edge $v_i v_{i+1}$ of cylce C_n be identified with edge $u_{i1}u_{i2}$ of i^{th} copy of C_4 and the edge $v_n v_1$ be identified with the edge

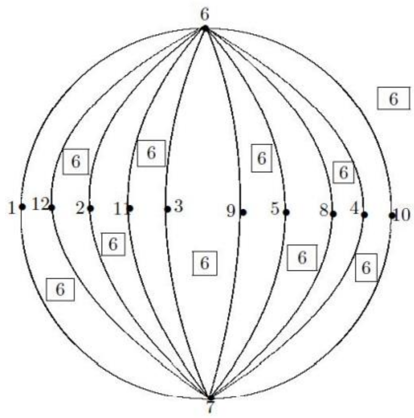


Figure 3. A (1,0,0)-F-Face magic mean labeling of $K_{2,10}$ with face mean constant 7.

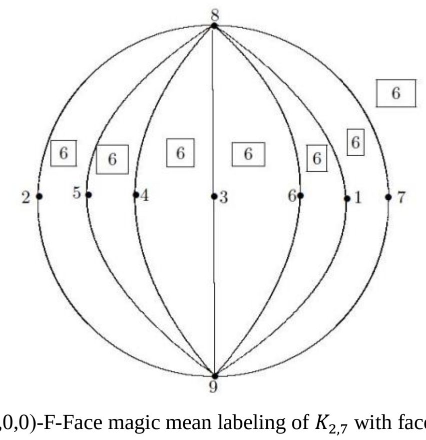


Figure 4. A (1,0,0)-F-Face magic mean labeling of $K_{2,7}$ with face mean constant 6.

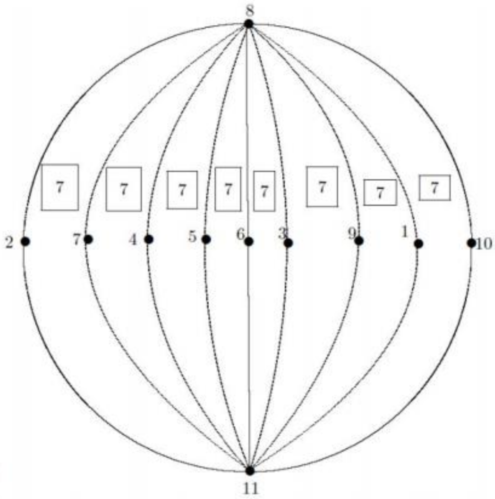


Figure 5. A (1,0,0)-F-Face magic mean labeling of $K_{2,9}$ with face mean constant 7.

$u_{n1}u_{n2}$ of n^{th}copy of C_4. Let $F(G)\{f_i = (v_iv_{i+1}, v_iu_{i3}, u_{i3}u_{i4}, u_{i4}v_{i+1}): 1 \le i \le n-1\} \cup \{f_n = (v_nv_1, v_1u_{n4}, u_{n3}u_{n4}, u_{n3}v_n)\}$ $\cup \left\{f_0 = (\cup_{i=1}^{n} v_iu_{i3}, \cup_{i=1}^{n-1} v_{i+1}u_{i4}, \cup_{i=1}^{n} u_{i3}u_{i4})\right\} \cup \left\{f_I = (\cup_{i=1}^{n-1} v_iv_{i+1}, v_nv_1)\right\}$ where $deg(f_i) = 4, 1 \le i \le n, deg(f_0) = 3n$ and $deg(f_I) = n$

Define $\phi: V(G) \to \{1,2,3,\dots,3n\}$ as follows:

For $1 \le i \le n, \phi(v_i) = n + i,$

$$\phi(u_{i3}) = n - i + 1 \text{ and}$$

$$\phi(u_{i4}) = \begin{cases} 3n - i, & 1 \le i \le n-1 \\ 3n, & i = n \end{cases}$$

Then the induced face labeling ϕ^* on G is obtained as

$$\phi^*(f_i) = \left\lfloor \frac{3n+1}{2} \right\rfloor = \begin{cases} \frac{3n}{2}, & n \equiv 0(mod2) \\ \frac{3n+1}{2}, & n \equiv 1(mod2), \end{cases}$$

Thus ϕ is a (1,0,0)-F-Face magic mean labeling of G. The (1,0,0)-F-face magic mean labeling of C_5 obtained by identifying each edge with an edge of a copy of C_4 is shown in Figure 6.

Theorem 2.5. Slanting ladder SL_n, $n \ge 2$ is a *(*1,0,0*)*-F-Face magic mean graph with face mean constant n.

Proof. Let $G \simeq SL_n$, where $V(G) = \{u_i: 1 \le i \le n\} \cup \{v_i: 1 \le i \le n\}$,
$E(G) = \{u_iu_{i+1}: 1 \le i \le n-1\} \cup \{v_iv_{i+1}: 1 \le i \le n-1\} \cup \{u_iv_{i-1}: 2 \le i \le n\}$ and
$F(G) = \{f_i = (u_{i+1}u_{i+2}, v_iv_{i+1}, u_{i+1}v_i, u_{i+2}v_{i+1}): 1 \le i \le n-2\} \cup \left\{f_0 = \left(\bigcup_{i=1}^{n-1} u_iu_{i+1}, \bigcup_{i=1}^{n-1} v_iv_{i+1}, u_2v_1, u_nv_{n-1}\right)\right\}$
in which f_0 is the outer face of G.

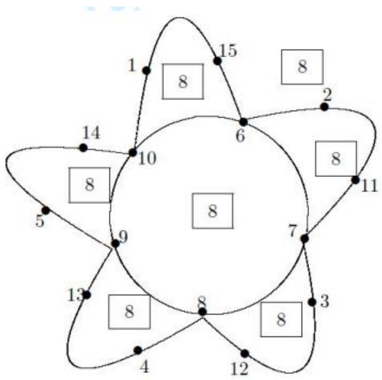


Figure 6. A (1,0,0)-F-face magic mean labeling of C_5 obtained by identifying each edge with an edge of a copy of C_4 having face constant.

In $G, |V(G)| = 2n, deg(f_i) = 4, 1 \le i \le n-2$ and $deg(f_0) = 2n$.

Case (i) n is odd.

Define $\phi: V(G) \to \{1,2,\dots,2n\}$ as follows:

$$\phi(u_i) = \begin{cases} \frac{3n+i}{2}, & i \equiv 1(mod2) \\ \frac{n-i+1}{2}, & i \equiv 0(mod2) \text{ and} \end{cases}$$

$$\phi(v_i) = \begin{cases} \frac{2n+i-1}{2}, & i \equiv 1(mod2) \\ n - \frac{i}{2}, & i \equiv 0(mod2). \end{cases}$$

Then the induced face labeling ϕ^* on G is obtained as $\phi^*(f_i) = n$, for all $1 \le i \le n-2$ and $\phi^*(f_0) = \left\lfloor \frac{2n+1}{2} \right\rfloor = n$

Case (ii) n is even.

When $n = 2, SL_n$ is a tree. So it is a (1,0,0)-F-face magic mean graph.
For $n \ge 2$, Define $\phi: V(G) \to \{1,2,\dots,2n\}$ as follows:

$$\phi(u_i) = \begin{cases} \frac{n-i+1}{2}, & i \equiv 1(mod2) \\ \frac{3n+i}{2}, & i \equiv 0(mod2) \text{ and} \end{cases}$$

$$\phi(v_i) = \begin{cases} \frac{2n-i+1}{2}, & i \equiv 1(mod2) \\ n + \frac{i}{2}, & i \equiv 0(mod2). \end{cases}$$

Then the induced face labeling ϕ^* on G is obtained as $\phi^*(f_i) = \left\lfloor \frac{4n+2}{4} \right\rfloor = n$, for all $1 \le i \le n-2$ and $\phi^*(f_0) = \left\lfloor \frac{2n+1}{2} \right\rfloor = n$.

Thus ϕ is a (1,0,0)-F-Face magic mean labeling of $SL_n, n \ge 2$ with face mean constant n. A (1,0,0)-F-Face magic mean labeling of SL_7 and SL_8 are shown in Figure 7.

Theorem 2.6. The grid graph $P_m \times P_n, m, n \ge 1$ is a *(*1,0,0*)*-F-face magic mean graph with face constant $\frac{mn}{2}$ if either m or n is even or both are even and $\frac{mn+1}{2}$ if both m and n are odd.

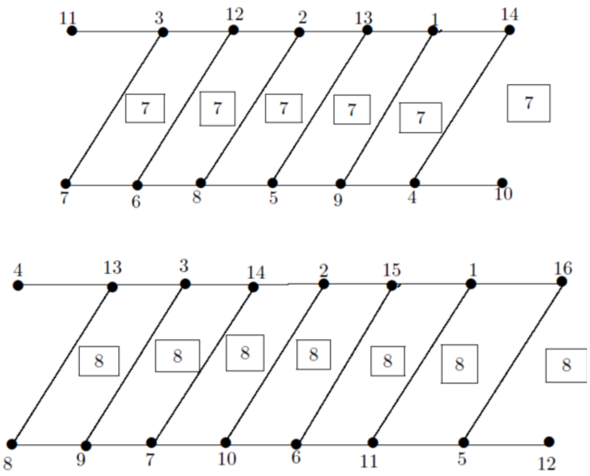


Figure 7. A (1,0,0)-F-Face magic mean labeling of SL_7 and SL_8 having face constant 7 and 8 respectively.

Proof. Let $G = P_m \times P_n$. Then $V(G) = \{u_{ij}: 1 \le i \le m, 1 \le j \le n\}$,

$$E(G) = \{u_{ij}u_{(i+1)j}: 1 \le i \le m-1, 1 \le j \le n\} \cup \{u_{ij}u_{i(j+1)}: 1 \le i \le m, 1 \le j \le n-1\}$$

$$F(G) = \{f_{ij} = (u_{ij}u_{i(j+1)}, u_{i(j+1)}u_{(i+1)(j+1)}, u_{(i+1)(j+1)}u_{(i+1)j}, u_{(i+1)j}u_{ij}): 1 \le i \le m-1, 1 \le j \le n-1\}$$

$$\cup \left\{ f_0 = \bigcup_{j=1}^{n-1} u_{1j}u_{1(j+1)}, \bigcup_{j=1}^{n-1} u_{mj}u_{m(j+1)}, \bigcup_{i=1}^{m-1} u_{i1}u_{(i+1)1}, \bigcup_{i=1}^{m-1} u_{in}u_{(i+1)n} \right\}$$

where f_0 is the outer face of G, $deg(f_{ij}) = 4, 1 \le i \le m-1,\ 1 \le j \le n-1,\ |V(G)| = 2n+1$.

There are $(m-1)(n-1)$ faces of degree 4 and outer face is of degree $2m + 2n - 4$.

$\phi(u_{11}) = 1$,
$\phi(u_{12}) = mn$,

For $3 \le i \le n$, $$\phi(u_{1j}) = \begin{cases} \phi(u_{1(j-2)}) + j - 2, & j \equiv 1(mod2) \\ \phi(u_{1(j-2)}) - j + 2, & j \equiv 0(mod2), \end{cases}$$

$\phi(u_{2n}) = \phi(v_{1n-1}) - n + 1$ and

For $2 \le i \le m, 1 \le j \le n-1$,

$$\phi(u_{ij}) = \begin{cases} \phi(u_{(i-1)(j+1)}) - 1, & i + j \equiv 1(mod2) \\ \phi(u_{(i-1)(j+1)}) + 1, & i + j \equiv 0(mod2). \end{cases}$$

Then the induced face labeling ϕ^* on G is obtained as $\phi^*(f) = mn/2$, for all faces f of G. Thus ϕ is a (1,0,0)-F-Face magic mean labeling of $P_m \times P_n$ with face mean constant $mn/2$. A (1,0,0)-F-Face magic mean labeling of $P_8 \times P_7$ is shown in figure 8.

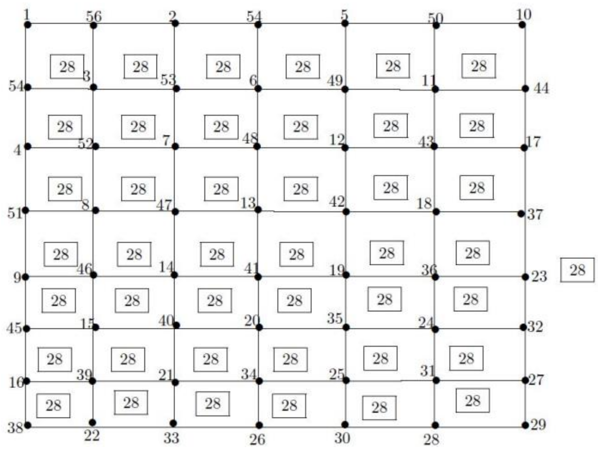


Figure 8. (1,0,0)-F-Face magic mean labeling of $P_8 \times P_7$ having face mean constant 28.

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