

CHAPTER 2

THEORIES

Principle and Theory applied in this study are collected for reference and guidance of questionnaire development, planning of operational research as well as a guideline for reporting. Therefore, the study's outcome pattern is framed and directed within technical matter which can be separated into 3 groups as follows.

1. Principle and theory that are applied for community cooperation network prototype establishment are principle and theory of adoption, participation, awareness and a communication.

2. Principle and theory that are employed for waste disposal are principle for municipal waste management, principle for bio-waste usage, principle for biogas production from anaerobic digestion system.

3. Principle of Logistic Regression Analysis which applied for investigated of waste management behavior.

2.1 Principle and Theory for Community Cooperation Network Prototype Establishment

2.1.1 Principle and Theory of Acceptance

2.1.1.1 Definition

Foster [10] provided definition of new technology or new concepts that people can learn through study processes. Learning by oneself will acquire acceptance knowledge. Such good learning comes from practice till that people is sure of benefit of new technology and can be well used.

Rogers [11] provided definition of innovation acceptance that is process of individual passing from innovation technology perception, building attitude and coming to acceptance or denial and final decision of the innovation technology.

The summary is that an acceptance is decision of individual for being willing of newly practice and seen of benefit. So the decision is timely and consequently changes.

2.1.1.2 Acceptance Process

Rogers [12] provided an acceptance process that means individual's mindful psychological process. The stage starts from the first perception of change to acceptance with

practice. The acceptance of new technology for problem solving of Rogers' idea has pass through the acceptance process consisting of 5 stages following.

1. Awareness stage is step of people's alert of newly chanced thing through observation, listening, and viewing without details of those.

2. Interest Stage is step of such perception and will find more details with comparing the existing knowledge and experience.

3. Evaluation Stage is step of decision making concerning with advantage and disadvantage of such new concept as well as trying to practice.

4. Trial Stage is step of new concept testing for make sure that new concept can work or not. After concept trial, Man will makes up his mind for acceptance.

5. Acceptance Stage is step of practice of new concept for maximum benefit as his perception. Acceptance period (short or long) is depended on surrounding factors.

In this connection, Rogers and Shoemaker [12] state that people's innovation acceptance can be separated in three major portions; antecedent, process, and consequences. Details are following;

1. Antecedent is personal's characteristic e.g. personality, attitude on social change, social pattern, information perception etc. All are influenced on decision making of people.

2. Acceptance process that can be divided into 4 steps:

The first step: Knowledge, perception and understanding of innovation technology

The second step: Persuasion, from others media or changing leader or credible information making relation with innovation technology

The third step: Decision, better understanding with innovation technology e.g. testing by himself or result from others' testing and making decision yes or no

The fourth step: confirmation, acceptance or quit of innovation use due to rewarded or returned benefit and information of continuing use or quit

3. Outcomes of innovation use are positive or negative impacts that innovation technology will serve user's need. Such outcomes are clearly and quickly seen or slowly occur and concealment that would depend on other intervening factors.

2.1.2 Theory and Principle of Participation

2.1.2.1 Definition

Turner and John [13] provided a meaning of people's participation that is de-authorization of decision making or resources allocation that cannot be made without common understanding and conscious as well as cooperation contribution of major population through group's representation. This process is for determining and finding guideline of common problem solving. Therefore, it makes individual or group made up their mind for activities which are virtually affected they living especially benefit obtaining of participants. The key participation's components are consisted of following.

1. Cooperation is people's intention of contribution to group's objectives achievement.
2. Coordination is common feeling, relationship, and faithfulness of working together.
3. Responsibility is feeling of concerning with activities and acceptance of practice for group's objectives achievement.

So the summarized equation is below

$$\text{Participation} = \text{Cooperation} + \text{Coordination} + \text{Responsibility}$$

2.1.2.2 Participation Condition

Any development project has to start from people to encourage make self-reliable and self-help capability. A possibility of people's participation building depends on condition at least 3 items below (Turner and John).

1. People have to be freely for participation.
2. People have to have adequate capability for participation.
3. People have to have willingness for participation.

Those conditions are based on daily live style of people e.g. none effect on normal duty, neither waste time for participation nor lose personal benefit etc.

2.1.2.3 Participation Pattern

Participation pattern can be categorized in to 3 types. Defining characteristics of participation as following (Turner and John) [13].

1. Direct participation by group establishment
2. Indirect participation by using group leader or agencies' representative

3. Provided opportunity for people participation by non-representative people organization

Participants are classified into 3 types as following;

1. Actor
2. Affected people
3. Others people who are not direct stakeholder but key man in the process.

2.1.3 Theory and Principle of Awareness Enhancement

2.1.3.1 Definition

Benjamin B. Wolman [14] provided meaning of awareness that is perception or recognition of people on something occur, experience, or object.

Benjamin S. Bloom [15] provided a meaning of awareness in terms that awareness is the lowest level of emotion sector. Awareness is quite similar to emotion and feeling (Affective Domain). Awareness is almost like to knowledge in un-emphasize on stimulator's pattern. But they are different that awareness has not to be occurrence or something. Awareness will be occurred if there is stimulated by stimulator.

Carter V. Good, 1973 [16] stated that awareness means action of recognize, perception or having knowledge, or recognition on the other hand awareness is a feeling of personal perception or showing of people's responsibility for any occurred problem.

2.1.3.2 The Component of Awareness Enhancement

Breckler [17] state that awareness is action came from attitude on stimulators i.e. persons, circumstantial event, social group, and others. Such attitude inclines to be positive or negative respond from learning and experience. There are 3 key components which are detailed below.

1. Cognitive Component is leveled from simple to more development.
2. Affective Component is a feeling of attitude, popularity, prefer or not, good or not that are components for stimulator assessment.
3. Behavioral Component is action of wording and manner that affected by stimulator or tendency of action.

2.1.3.3 Awareness Measurement

Awareness is behavior concerning relative concept for something that comprises perception and cognitive reaction to a condition or event. Therefore, its measurement and assessment have to have specific principle and methodology for making measurement tool with accuracy and reliability. There are many measurement tools such as;

1. Interview Tools may be structural interviewing form that is developed with question and answer with well sequence or non-structural interviewing form that has a key question with freely answer.
2. Questionnaire Tools may be opened or closed end questionnaire or combination between opened end and closed end.
3. Checklist Tools is developed with answer only agree/disagree or having/none of list. It may be marked yes or not on checklist.
4. Qualitative Scale Tools It should be appropriate tool for emotion and recognition measurement that needs to be measured on strength level.

2.1.4 Persuasion Theory

2.1.4.1 Definition

Miller and Burgoon [18] explained convince process that will be applied on anyone if someone intends to be a mind controller over a specific person.

A convincing characteristic could be summarized as following;

- A convincing agent intends to dominate convinced people.
- In general, alternatives of convinced people are more than one choice and convincing agent tries to convince for acceptance of convinced people.
- The convincing agent need is change or creation or maintaining of opinion, attitude, popularity, and belief of convinced people. Moreover, such a need is affected on other factors i.e. emotion, behavior etc.

2.1.4.2. Convincing Message Creation

Pilanowat [19] stated that in a convincing process, a transmitter tries to claim supporting reason. It may support both sides of a single issue or presenting one side. Therefore, it should consider key factors as follows.

1. Both sides of message presentation are appropriate for moderately high education person.



2. Both sides of message presentation are appropriate for person who is not support standing point of transmitter at beginning.

3. Both sides of message presentation are appropriate for person who possibly receives opposite opinion message.

4. One side of message presentation is appropriate for person who agrees with transmitter at beginning and other condition that high possibility such person will not obtain an opposite message in later.

5. An existing attitude and agreement may cause non-different effect on opinion even they obtain either sides or one side message.

2.1.4.3 Message Appeals

In a convincing process, a transmitter has to know key point of convincing message. It may be bio-aspect (Body needs) or Learned Motives. A better result of convincing process will be occurred if convinced person obtains a key point of message that links to conceptual frame of him. The general key points of convincing message are follows:

1. Fear Appeals
2. Emotional Appeals
3. Anger Appeals
4. Humorous Appeals
5. Reward as Appeals
6. Motivation Appeals

The key point of convincing message is one of strategic convincing process. The transmitter may use Fear Appeals, Reward as Appeals, other Emotional Appeals. A different of information message and convincing message is the latter emphasizes on such key points (Pilanowat) [19].



2.2 Principle and Theory for Waste Disposal

2.2.1 Principle of Municipal Solid Waste Management

2.2.1.1 Definition of Municipal Solid Waste

Solid wastes may be defined as useless, unused, unwanted or discarded material available in solid form. Semisolid food waste and municipal sludge may also be included in municipal solid waste (Qaism and Chaing) [20].

Municipal solid waste (MSW) includes wastes such as durable goods, nondurable goods, containers and packaging, food wastes, yard wastes and miscellaneous inorganic waste from residential, commercial, institution and industrial sources. MSW does not include waste from sources such as municipal sludge and combustion ash, and industrial non-hazardous process wastes that might also be disposed of in municipal waste landfill or incinerators (Qaism and Chaing) [20]

2.2.1.2 Sources, Types, Composition and Properties of Municipal Solid Waste [21]

(1) Sources of Municipal Solid Waste

Sources of solid wastes in a community are, in general, related to land use and zoning. Although any number of source classifications can be developed, the following categories are useful: (1) residential, (2) commercial, (3) institutional, (4) construction and demolition, (5) municipal services, (6) treatment plant sites, (7) industrial, and (8) agricultural (Tchobanoglous) [21]. Typical waste generation facilities, activities, or locations associated with each of these sources are reported in Table 2-1, where municipal solid waste (MSW) is normally assumed to include all community wastes with the exception of industrial process wastes and agricultural wastes.

(2) Types of Municipal Solid Waste

As a basic for subsequent discussions, it will be helpful to define the various types of solid wastes that are generated (see Table 2.1). It is important to be aware that the definitions of solid waste terms and the classifications vary greatly in the literature and in the profession. Consequently, the use of published data requires considerable care, judgment, and common sense. The following definitions are intended to serve as a guide and are not meant to be precise in a scientific sense.

Table 2.1: Sources of solid wastes within a community [21]

Source	Typical facilities, activities, or locations where wastes are generated	Types of solid wastes
Residential	Single family and multifamily detached dwellings, low-, medium-, and high-rise apartments, etc.	Food wastes, paper, cardboard, plastics, textiles, leather, yard wastes, wood, glass, tin cans, aluminum, other metals, ashes, street leaves, special wastes (including bulky items, consumer electronics, white goods, yard wastes collected separately, batteries, oil, and tires), household hazardous wastes
Commercial	Stores, restaurants, markets, office buildings, hotels, motels, print shops, service stations, auto repair shops, etc.	Paper, cardboard, plastics, wood, food waste, glass, metals, special wastes (see above), hazardous wastes, etc.
Institutional	Schools, hospitals, prisons, governmental centers	As above in commercial
Construction and demolition	New construction sites, road repair/renovation sites, razing of buildings, broken pavement	Wood, steel, concret, dirt, etc.
Municipal services (excluding treatment facilities)	Street cleaning, landscaping, catch basin cleaning, parks and beaches, other recreational areas	Special wastes, rubbish, street sweepings, landscape and tree trimmings, catch basin debris, general wastes from parks, beaches, and recreational areas
Treatment plant sites	Water, wastewater, and industrial treatment processes	Treatment plant wastes, principally composed of residual sludges
Municipal solid waste	All of the above	All of the above
Industrial	Construction, fabrication, light and heavy manufacturing, refineries, chemical plants, power plants, demolition, etc.	Industrial process wastes, scrap materials, etc. Non-industrial wastes including food wastes, waste,ashes, demolition and construction wastes, special wastes, hazardous wastes
Agricultural	Field and row crops, orchards, vineyards, dairies, feedlots, farms, etc	Spoiled food wastes, agricultural wastes, waste, hazardous wastes

(3) Composition of Municipal Solid Waste

Composition is the term used to describe the individual components that make up a solid waste stream and their relative distribution, usually based on percent by weight. Information on the composition of solid wastes is important in evaluating equipment needs, systems, and management programs and plans. For example, if the solid wastes generated at a commercial facility consist of only paper products, the use of special processing equipment, such as shredders and balers, may be appropriate. Separate collection may also be considered if the city or collection agency is involved in a paper-product recycling program. The distribution of components in residential MSW provided details in Table 2.2.

Municipal solid waste is normally comprised of food wastes, rubbish, demolition and construction wastes, street sweepings, garden wastes, abandoned vehicles and appliances, and treatment plant residues. Quantity and composition of MSW vary greatly for different municipalities and times of the year. Factors influencing the characteristics of MSW are climate, social customs, per capita income, and degree of urbanization and industrialization.

Table 2.2: Typical distribution of components in residential MSW for low, middle, and upper-income countries excluding recycled materials [21]

Component	Low-income Countries	Middle-income countries	Upper-income countries
Organic			
Food wastes	40-85 %	20-65 %	6-30 %
Paper	1-10 %	8-30 %	20-45 %
Cardboard			5-15 %
Plastics	1-5 %	2-6 %	2-8 %
Textiles	1-5 %	2-10 %	2-6 %
Rubber	1-5 %	1-4 %	0-2 %
Leather			0-2 %
Yard wastes	1-5 %	1-10 %	10-20 %
Wood			1-4 %
Inorganic			
Glass	1-10 %	1-10 %	4-12 %
Tin cans			2-8 %
Aluminum	1-5 %	1-5 %	0-1 %
Other metal			1-4 %
Dirt, ash, etc.	1-40 %	1-30 %	0-10 %

2.2.1.3 Integrated Solid Waste Management (ISWM)

Integrated solid waste management (ISWM) can be defined as the selection and application of suitable techniques, technologies, and management programs to achieve specific waste management objectives and goals. Since numerous state and federal laws have been adopted, ISWM is also evolving in response to the regulations developed to implement the various laws. A hierarchy of waste management activities has also been established by recent regulations. A hierarchy (arrangement in order of rank) in waste management can be used to rank actions to implement programs within the community. The ISWM hierarchy adopted by the U.S. Environmental Protection Agency (EPA) is composed of the following elements:

source reduction, recycling, waste combustion, and land filling (Tchobanoglous) [21]. The ISWM hierarchy is source reduction, recycling, waste transformation, and landfilling.

- **Source Reduction:** The highest rank of the ISWM hierarchy, source reduction, involves reducing the amount and/or toxicity of the wastes that are now generated. Source reduction is first in the hierarchy because it is the most effective way to reduce the quantity of waste, the cost associated with its handling, and its environmental impacts. Waste reduction may occur through the design, manufacture, and packaging of products with minimum toxic content, minimum volume of material, or a longer useful life. Waste reduction may also occur at the household, commercial, or industrial facility through selective buying patterns and the reuse of products and materials [9].

- **Recycling:** The second highest rank in the hierarchy is *recycling*, which involves (1) the separation and collection of waste materials; (2) the preparation of these materials for reuse, reprocessing, and remanufacture; and (3) the reuse, reprocessing, and remanufacture of these materials. Recycling is an important factor in helping to reduce the demand on resources and the amount of waste requiring disposal by landfilling.

- **Waste Transformation:** The third rank in the ISWM hierarchy, *waste transformation*, involves the physical, chemical, or biological alteration of wastes. Typically, the physical, chemical, and biological transformations that can be applied to MSW are used (1) to improve the efficiency of solid waste management operations and systems, (2) to recover reusable and recyclable materials, and (3) to recover conversion products (e.g., compost) and energy in the form of heat and combustible biogas. The transformation of waste materials usually results in the decreases use of landfill capacity. The reduction in waste volume through combustion is a well-known example.

- **Landfilling:** Ultimately, something must be done with (1) the solid wastes that cannot be recycled and are of no further use; (2) the residual matter remaining after solid wastes have been separated at a materials recovery facility; and (3) the residual matter remaining after the recovery of conversion products or energy. There are only two alternatives available for the long-term handling of solid wastes and residual matter: disposal on or in the earth's mantle, and disposal at the bottom of the ocean. Landfilling, the fourth rank of the ISWM hierarchy, involves the controlled disposal of wastes on or in the earth's mantle, and it

is by far the most common method of ultimate disposal for waste residuals. Landfilling is the lowest rank in the ISWM hierarchy because it represents the least desirable means of dealing with society's wastes.

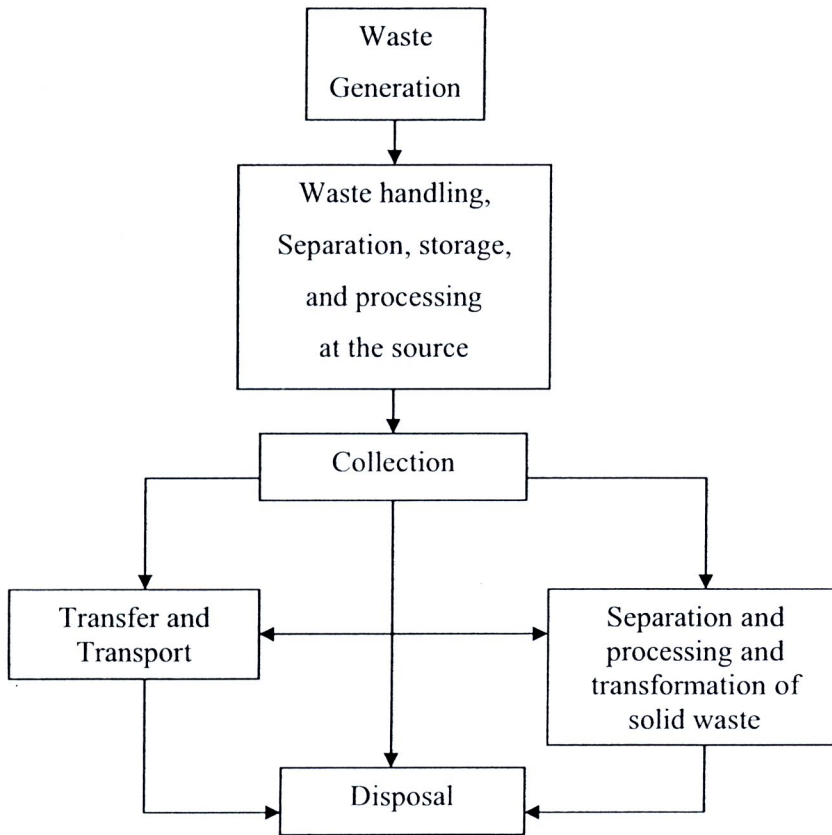


Figure 2.1 Simplified diagram showing the interrelationships between the functional elements in a solid waste management system. [21]

2.2.2 Utilization of Organic Fraction Municipal Solid Waste (OFMSW)

The components that constitute the organic fraction of MSW are food wastes, paper, cardboard, plastics, textiles, rubber, leather, yard wastes, and wood. All of these materials can be recycled, either separately or as commingled waste. The components can be recovered separately by source separation or at a MRF; they can also be recovered from MSW in commingled form by removal of inorganic. The choice of recovery method is dictated by the use of the material or end product. Source-separated materials generally have the least contamination and exhibit physical and recycling properties different from the commingled

components. The reuse and recycling opportunities and the specifications for the commingled materials that compose the organic fraction of MSW are considered in the following discussion.

2.2.2.1 Reused and Recycling Opportunity

The principal reuse and recycling opportunities for the materials that make up the organic fraction of MSW are the production of (1) compost, (2) methane, (3) organic compounds, and (4) refuse-derived fuel (Tchobanoglous,) [21].

(1) Production of Compost. Municipal solid waste typically contains 70 to 80 percent organic material and composting is gaining popularity as a waste management option . Almost all MSW composting systems start with separation of recyclables, metals, and hazardous materials, followed by size reduction and additional separation. The end uses for MSW compost are usually limited to agricultural uses or land reclamation. Few operators sell the finished product, although some give it to public agencies, farmers, and nurseries. Owing to the poor separation of incoming materials, complaints regarding the presence of residual plastics and glass shards have been frequent. In some cases, the compost produced from MSW has been used for intermediate landfill cover.

(2) Production of Methane. The production of methane from the organic materials contained in commingled MSW is accomplished biologically under anaerobic conditions. Typically, methane is produced from the organic fraction of MSW under controlled conditions in either a low-solids (6 to 10 percent solids) or a high-solids (20 to 35 percent solids) anaerobic reactor. Methane can be used for the production of energy and heat, or for the conversion to methanol and/or other products. The production of methanol is of interest because methanol is a clean-burning, storable fuel. The digested solids from the low- and high-solids processes can be composted to produce a usable product or placed in a landfill.

(3) Production of Organic Compounds. The organic materials contained in commingled MSW can also be used for the production of a variety of organic compounds including sugars, alcohols, solvents, organic acids, hydrocarbon gases, and aromatic compounds. For example, source-separated paper is comprised of approximately 61 percent cellulose; 16 percent hemicellulose; 21 percent lignin; and 2 percent protein, ash, and so on. With this composition, waste paper is ideally suited as a feedstock for the production of

ethanol. Similarly, other organic materials in MSW can be used for the production of other organic compounds.

(4) Production of Refuse-Derived Fuel. Refuse-derived fuel (RDF) refers to solid waste that is processed to serve as a fuel for boilers used to produce steam or electricity. RDF is frequently burned in utility boilers and in specially designed combustion systems. RDF has also been mixed and burned with coal. Although *as-discarded* waste is sometimes considered a refuse-derived fuel, the term usually refers to waste that has been sorted, reduced in size, and refined by removal of noncombustible such as metals and glass.

2.2.2.2 Recovery of Conversion Product and Energy

The organic fraction municipal solid waste can be converted to usable products and ultimately to energy in number of ways, including

- 1) Combustion to produce steam and electricity.
- 2) Pyrolysis to produce a synthesis gas, liquid or solid fuel, solid.
- 3) Gasification to produce a synthetic fuel.
- 4) Biological conversion to produce compost.
- 5) Bio-digestion to generate methane and produce organic humus.

2.2.3 Anaerobic Digestion Process [22]

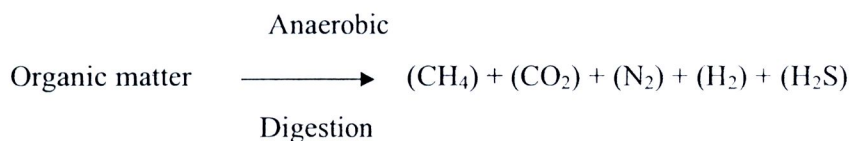
The common raw materials used for biogas generation are of the defined as 'waste materials', e.g. human excreta, animal manure, sewage sludge, and vegetable crop residues, all of which are rich in nutrients suitable for the growth of anaerobic bacteria. Although some of these materials can be used directly as fuels and fertilizers, they could be used for biogas production, to gain some additional heat value (from the biogas) while the other benefits are still retained. Depending on factors such as the composition of the other material, organic loading applied to the digesters, and the time and temperature of anaerobic decomposition, some variations in the composition of biogas can be noticed, but it approximately conforms to following:

Methane	(CH ₄)	55-65	percent
Carbon dioxide	(CO ₂)	35-45	percent
Nitrogen	(N ₂)	0-3	percent
Hydrogen	(H ₂)	0-1	percent
Hydrogen sulphide	(H ₂ S)	0-1	percent

Of the different gases produced, CH_4 is the most desirable gas, because it has a high calorific value ($\approx 9,000 \text{ kcal} / \text{m}^3$). The assumption of biogas yield which from anaerobic digestion is about $70\text{-}140 \text{ m}^3/\text{ton}$ of waste [23]. Base on the heat value of the biogas ($4,500\text{-}6,300 \text{ kcal/m}^3$) that on complete combustion 1 m^3 of biogas is sufficient to:

- run a 1 horsepower engine for 2 h,
- provide 1.25 kW-h of electricity,
- provide 6 h of light equivalent to a 60-W bulb,
- Equivalent to 0.4 kg of diesel, 0.6 kg of petrol and 0.8 kg of coal

The anaerobic digestion of organic material is biochemically a very complicated process, involving hundreds of possible intermediate compounds and reaction, each of which is catalyzed by specific enzymes or catalysts. However, the overall chemical reaction is of the simplified to:



In general, anaerobic digestion is considered to occur in the following stages:

Stage 1: Liquefaction

Many organic wastes consist of complex organic polymers such as proteins, fats, carbohydrates, cellulose, lignin, etc., some of which are in the form of insoluble solids. In this stage these organic polymers are broken down by extra cellular enzymes produced by hydrolytic bacteria, and dissolved in water. The simple soluble, organic components (or monomers) which are formed are easily available to any acid-producing bacteria. It is difficult to distinguish this stage from what is known as stage 2 (acid-formation stage), because some molecules will be absorbed without further breakdown and can be degraded internally.

Stage 2: Acid formation

The monomeric components released by the hydrolytic breakdown due to bacterial action in stage 1 are further converted to acetic acid (acetates), H_2 and CO_2 by the acetogenic bacteria. Volatile fatty acids are produced as the end - products of bacterial metabolism of protein, fat, and carbohydrate; in which acetic, propionic, and lactic acids are the major products. Carbon dioxide and hydrogen gas are also liberated during carbohydrate

catabolism, with methanol, and other simple alcohols being other possible by-products of carbohydrate breakdown. The proportion of these different substrates produced depends on the flora present, as well as on the environmental conditions.

Stage 3: Methane formation

The products of stage 2 are finally converted to CH_4 and other end – products by a group of bacteria called methanogens. Methanogenic bacteria are obligate anaerobes whose growth rate is generally slower than the bacteria in Stages 1 and 2

The methanogenic bacteria use acetic acid, methanol, or carbon dioxide and hydrogen gas to produce methane. Acetic acid or acetate is the single most important substrate for methane formation, with approximately 70 percent of the methane produce from acetic acid. The remaining methane comes from carbon dioxide and hydrogen. A few other substrates can also be used, such as formic acid, but these are not important, as the are not usually present in anaerobic fermentation. The methanogenic bacteria are also dependent on the stage 1 and stage 2 bacteria to provide nutrients in a usable form. For example, organic N compounds must be reduced to ammonia to ensure efficient utilization by the methanogenic bacteria.

The methane formation reaction in Stage 3 is very important in anaerobic digestion. Besides producing CH_4 gas, the methanogens also regulate and neutralize the pH of the digester slurry by converting the volatile fatty acids into CH_4 and other gases. The conversion of H_2 into CH_4 by the methanogens helps reduce the partial pressure of H_2 in the digester slurry, which is beneficial to the activity of the acetogenic bacteria. If the methanogenic bacteria fail to function effectively there will be little or no CH_4 production from the digester, ad so waste stabilization is not achieved because the organic compounds will be converted only to volatile fatty acids, which can cause further pollution if discharged into a water course or on land. As the methanogenic bacteria are obligate anaerobes their growth is inhibited even by small amounts of oxygen, and it is essential that a highly reducing environment be maintained to promote their growth, and methane bacteria are also sensitive to other environmental factors.

2.3 Principle of Logistic Regression Analysis

2.3.1 Principle and Definition

In statistics, logistic regression is used for prediction of the probability of occurrence of an event by fitting data to a logic function or logistic curve. It is a generalized linear model used for binomial regression. Like many forms of regression analysis, it makes use of several predictor variables that may be either numerical or categorical.

Logistic regression is used extensively in the medical and social sciences fields, as well as marketing applications such as prediction of a customer's propensity to purchase a product or cease a subscription.

What distinguishes a logistic regression model from the linear regression model is that the outcome variable, which in logistic regression is binary or dichotomous and linear regression, is assumed to be continuous (Hosmer and Lemeshow) [24].

In the primary model, Y_i is the binary response of an individual or an experimental unit that can take on one of two possible values, denoted by $Y = 1$ if the event happens and $Y = 0$ if the event does not happen. Suppose x is a vector of explanatory variables of the decision and β is the vector of slope parameters, measuring the impact of changes in x on the probability of the decision to happen or not happen. These may write Y_i as a linear function of x and some error term ϵ_i .

$$Y_i = \beta' x_i + \epsilon_i \quad (2.1)$$

Where $\beta' = [\beta_0, \beta_1, \beta_2 \dots \beta_k]$

$x_i = [x_{i1}, x_{i2}, x_{i3}, \dots x_{ik}]$

ϵ_i = the error term.

where β_0 is called the "intercept" and $\beta_1, \beta_2, \beta_3$, and so on, are called the "regression coefficients" of x_1, x_2, x_3 respectively. The intercept is the value of z when the value of all independent variables is zeros (e.g. the value of z in someone with no risk factors). Each of the regression coefficients describes the size of the contribution of that risk factor. A positive regression coefficient means that the explanatory variable increases the probability of the outcome, while a negative regression coefficient means that the variable decreases the probability of that outcome; a large regression coefficient means that the risk factor strongly influences the probability of that outcome; while a near-zero regression coefficient means that

risk factor has little influence on the probability of that outcome. Logistic regression is a useful way of describing the relationship between one or more independent variables (e.g., age, sex, etc.) and a binary response variable, expressed as a probability, that has only two possible values, such as death ("dead" or "not dead"). Extensions of the model cope with multi-category dependent variables and ordinal dependent variables, such as polytomous regression. Multi-class classification by logistic regression is known as multinomial logit modeling. An extension of the logistic model to sets of interdependent variables is the conditional random field.

2.3.2 Formal mathematical specification

In order to simplify notation, we use $\pi(x) = E(Y/x)$ to represent the conditional mean of Y given certain values of x . The probability of the binary response is defined as follows:

$$\text{If } Y_i = 1; P(Y_i = 1) = \pi(x) \quad (2.2)$$

$$Y_i = 0; P(Y_i = 0) = 1 - \pi(x) \quad (2.3)$$

If $E(\epsilon_i) = 0$, the expected value of the response variable is

$$\begin{aligned} E(Y_i) &= 1[\pi(x)] + 0[1 - \pi(x)] \\ &= \pi(x) \end{aligned} \quad (2.4a)$$

This implies that

$$E(Y_i) = \beta' x_i = \pi(x) \quad (2.4b)$$

Hence, the expected response given by the response function $E(Y/x) = \beta' x_i$ is just the probability that the response variable takes on the value 1 (Montgomery, Peck and Vining, 2001) [25].

In the linear regression model $Y = E(Y/x) + \epsilon$, the error term expresses an observation's deviation from the conditional mean. Generally we assume that the deviation is caused by the many other influencing factors of only marginal importance, i.e. ϵ is normal with zero mean and constant variance. Given this assumption, the conditional distribution of the outcome variable for given values of x will also be normal with mean $E(Y/x)$, and constant variance.

However, the linear probability model produces problems, for the dependent variable is dichotomous, and the corresponding distribution describes the distribution of the errors

expressed in terms of the dichotomous outcome variable. Hence the error term $\varepsilon_i = Y_i - \beta'x_i$ must take one of the following two possible values, depending on the value of Y_i :

$$\text{If } Y_i = 1; \varepsilon_i = 1 - \beta'x_i = 1 - \pi(x) \quad (2.5)$$

$$Y_i = 0; \varepsilon_i = -\beta'x_i = -\pi(x) \quad (2.6)$$

Consequently, the error variance is

$$\sigma_{Yi2} = E \{ [Y_i - E(Y_i)]^2 \} \quad (2.7a)$$

$$= [1 - \pi(x)]^2 \pi(x) + [0 - \pi(x)]^2 [1 - \pi(x)] \quad (2.7b)$$

$$= \pi(x)[1 - \pi(x)] \quad (2.7c)$$



The derivation shows that ε_i has a distribution with mean zero and variance equal to $\pi(x)[1 - \pi(x)]$. Hence, ε_i cannot be even approximately normally distributed. In fact, the conditional distribution of the binary variable Y follows a binomial (Bernoulli) distribution with probability given by the condition mean, $\pi(x)$ [24]. The specific functional form of the logistic regression model is as follows:

$$\pi(x) = \frac{1}{[1 + \exp(-\beta_0 - \beta_i x_i)]} \quad (2.8a)$$

By expanding the fraction in (2.8a) by $[\exp(\beta_0 + \beta_i x_i)]$, the following equivalent representation of the logistic model is straightforward:

$$\pi(x) = \frac{[\exp(\beta_0 + \beta_i x_i)]}{[1 + \exp(\beta_0 + \beta_i x_i)]} \quad (2.8b)$$

This equation is easily linearised by taking logarithms on both sides (2.8b). This procedure is called logit-transformation and the result gives the “logit model” or - equivalently - “logistic probability unit”:

$$\text{logit}(\pi(x)) = \log[\pi(x) / 1 - \pi(x)] = \beta_0 + \beta_i x_i \quad (2.8c)$$

In the case of one regressor only, this “logistic distribution functions” (2.4a) is an S-shaped cumulative distribution as shown in Figure 2.2. Its main characteristic is that the function restricts the estimated probabilities $E(Y_i)$ for any given value of x_i to lie between 0 and 1. This reveals that in principle any proper continuous probability function will suffice to give this result, and in many analyses the normal distribution has been used, leading to the so-called “probit model”. However, it is very difficult to justify the choice of one distribution or another on theoretical grounds, and in most applications it seems not to make much difference which one was selected. (Greene) [26]

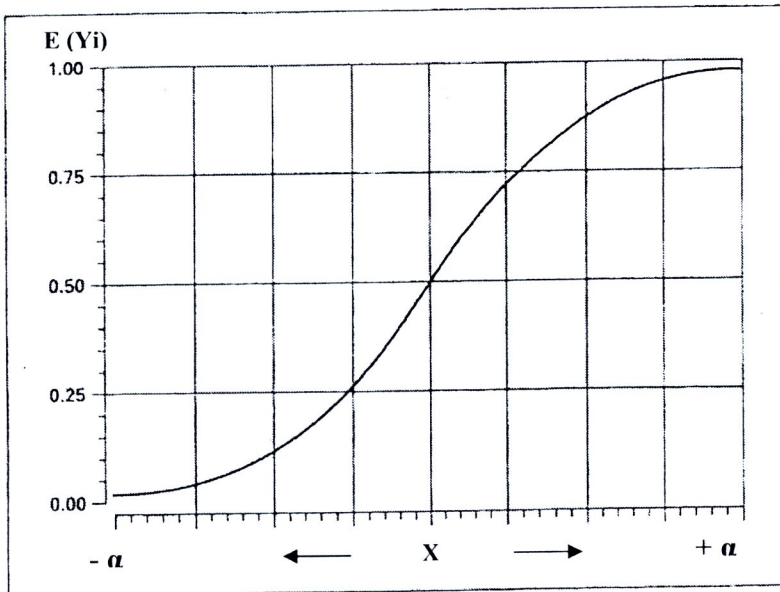


Figure 2.2: Typical function graph for logistic regression (one regressor) [26]