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Two New Iterative Methods for Solving Nonlinear Equations without Derivative

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Abstract

In this paper, we propose two new iterative methods for solving nonlinear equations with one variable without derivative. In convergence theory, the two new iterative methods have second and third order convergence. Some numerical experiments show that the two new derivative free iterative methods outperform the several other existing methods.

Keywords: Non-linear Equations, Order of Convergence, Derivative Free Method

1. Introduction

Solving a nonlinear equation $f(x)=0$ is the most important problems in Numerical analysis. Since it is not always possible to find the exactly solution by the direct method, the numerical iterative methods are useful to obtain an approximate solution of equations. There are many different iterative methods for solving nonlinear equations. One of the classical standard numerical iterative methods is a Newton's method, it given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n \in \mathbb{N} \tag{1}$$

where x_0 is an initial value that we might guess the initial value which near a solution of equations. This method has second order of convergence (Argyros, 2008). However, the Newton's method has a disadvantage, one has to calculate the derivative of $f(x)$. Sometimes, the first derivative function of $f(x)$ is more difficult to calculate or $f(x)$ is not differentiable at a certain point, the Newton's method is not applicable to solve nonlinear equation. Therefore, the Newton's method was modified by many researchers who considered the first derivative function by approximation of derivatives. (Jain, 2007; Hafiz, 2014; Singh, 2017)

Steffensen's method is one of the iterative methods which is based on the Newton's method (Conte & Boor, 1981). The method was modified by substitute the derivative function of function in the Newton's method by using a forward difference approximation, it given by

$$f'(x_n) \approx \frac{f(x_n + f(x_n)) - f(x_n)}{f(x_n)}. \tag{2}$$

Therefore, the Steffensen's method becomes

$$x_{n+1} = x_n - \frac{f(x_n)^2}{f(x_n + f(x_n)) - f(x_n)}, \quad n \in \mathbb{N}. \tag{3}$$

The method still has second order convergence. In 2005, the new iterative method formed by the composition of the Newton's method and the Steffensen's method, namely Newton-Steffensen's method was introduced by Sharma (Sharma, 2005). It given by

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \tag{4}$$

$$x_{n+1} = x_n - \frac{f(x_n)^2}{f'(x_n)(f(x_n) - f(y_n))}, \quad n \in \mathbb{N} \tag{5}$$

which has third order of convergence.

In 2010, Dehghan and Hajarian (Dehghan & Hajarian, 2010) introduced two new third order of convergence methods, called Dehghan I 's method and Dehghan II 's method. The Dehghan I 's method is given by

$$y_n = x_n - \frac{2f(x_n)^2}{f(x_n + f(x_n)) - f(x_n - f(x_n))}, \tag{6}$$

$$x_{n+1} = x_n - \frac{2f(x_n)\left(f(x_n) + f(y_n)\right)}{f(x_n + f(x_n)) - f(x_n - f(x_n))}, \quad n \in \mathbb{N}, \tag{7}$$

and the Dehghan II 's method is given by

$$y_n = x_n + \frac{2f(x_n)^2}{f(x_n + f(x_n)) - f(x_n - f(x_n))}, \tag{8}$$

$$x_{n+1} = x_n - \frac{2f(x_n)\left(f(x_n) + f(y_n)\right)}{f(x_n + f(x_n)) - f(x_n - f(x_n))}, \quad n \in \mathbb{N}. \tag{9}$$

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In this paper, we introduce two new derivative free iterative methods for solving nonlinear equations which replaced the derivative function $f'(x)$ by using the half forward difference and an average between forward and central difference approximation.

The remainder of this paper is organized as follow. In section 2, the two new derivative free iterative methods are provided and order of convergence for iterative methods are established.
In section 3, numerical examples show that both methods are better performance than of each of the existing methods described. Finally, conclusion is given in section 4.

2. Description of the methods and convergence analysis

In this section, we shall describe derivative free iterative methods which by using different approximation of derivatives. Moreover, we show that convergence analysis of our methods. Letting $x^* \in I$ be a simple zero of a sufficiently differentiable function $f: I \subseteq \mathbb{R} \to \mathbb{R}$ in an open interval I.

2.1 A two order of convergence derivative free iterative method (Method I)

To construct the second order convergence derivative free iterative method, we consider the approximation of derivative in the Steffensen's method as form (3) and modify this method by using the half forward difference approximation as figure 1.

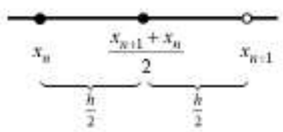

Figure 1. The half forward difference approximation.

By approximation of the derivative, we have

$$f'(x_n) \approx \frac{f\left(\frac{x_{n+1}+x_n}{2}\right) - f(x_n)}{\frac{h}{2}} \approx \frac{2\left(f\left(\frac{2x_n+h}{2}\right) - f(x_n)\right)}{h}, \tag{10}$$

where h is a very small value. As n is a large number and x_0 is close enough to the root x^* we will estimate value of h by $f(x_n)$ So, we have

$$f'(x_n) \approx \frac{2\left(f\left(\frac{2x_n+f(x_n)}{2}\right) - f(x_n)\right)}{f(x_n)}. \tag{11}$$

Then we obtain the new derivative free iterative method as form

$$x_{n+1} = x_n - \frac{f(x_n)^2}{2\left(f\left(\frac{2x_n+f(x_n)}{2}\right) - f(x_n)\right)}, \quad n \in \mathbb{N}. \tag{12}$$

2.2 A three order of convergence derivative free iterative method (Method II)

To construct the third order convergence derivative free iterative method, we consider the Newton-Steffensen's method as form (4),(5) and use the average of forward and central different as figure 2 to approximate the derivative of function at the n^{th} iteration.

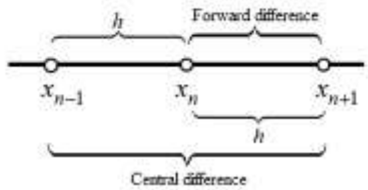

Figure 2. The forward and central difference approximation.

So, the average of forward and central difference approximation is given by

$$\begin{aligned} f'(x_n) &\approx \frac{1}{2}\left(\frac{f(x_n+h)-f(x_n)}{h} + \frac{f(x_n+h)-f(x_n-h)}{2h}\right) \\ &\approx \frac{1}{2}\left(\frac{2f(x_n+h)-2f(x_n)+f(x_n+h)-f(x_n-h)}{2h}\right) \\ &\approx \frac{1}{2}\left(\frac{3f(x_n+h)-2f(x_n)-f(x_n-h)}{2h}\right) \\ &\approx \left(\frac{3f(x_n+h)-2f(x_n)-f(x_n-h)}{4h}\right), \end{aligned} \tag{13}$$

where h is a very small value. Then we substitute the value of h by $f(x_n)$ So, we have

$$f'(x_n) \approx \frac{3f(x_n+f(x_n))-2f(x_n)-f(x_n-f(x_n))}{4f(x_n)}. \tag{14}$$

Therefore, the derivative function in Newton-Steffensen's method as form (4),(5) was replaced by the above approximating the derivative. The new derivative free iterative method becomes

$$y_n = x_n - \frac{4f(x_n)^2}{3f(x_n+f(x_n))-2f(x_n)-f(x_n-f(x_n))} \tag{15}$$

$$x_{n+1} = x_n - \frac{4f(x_n)^2}{(3f(x_n+f(x_n))-2f(x_n)-f(x_n-f(x_n)))(f(x_n)-f(y_n))}, \tag{16}$$

where $n \in \mathbb{N}$.

In the following theorems, we are going to prove that the our iterative methods have orders of convergence 2 and 3, respectively.

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Theorem 2.1 If x_0 is sufficiently close to x^*, then the derivative free iterative method (Method I) has second order of convergence.

Proof. Let x^* be the simple root of $f(x)$, i.e. $f(x^*)=0,\ f'(x^*)\neq 0$, and the error equation is $e_n = x_n - x^*$. Using Taylor series of $f(x_n)$ about x^*, we have

$$f(x_n) = c_1e_n + c_2e_n^2 + c_3e_n^3 + c_4e_n^4 + c_5e_n^5 + c_6e_n^6 + O(e_n^7), \tag{17}$$

where $c_k = \dfrac{f^k(x^*)}{k!}$, $k = 1,2,3,\dots$.

Computing $f(x_n)^2$, it is given as following

$$f(x_n)^2 = c_1^2e_n^2 + 2c_1c_2e_n^3 + (2c_1c_3 + c_2^2)e_n^4 + (2c_1c_4 + 2c_2c_3)e_n^5 + O(e_n^6). \tag{18}$$

Using equation (17) to compute $2\left(f\left(\dfrac{2x_n + f(x_n)}{2}\right) - f(x_n)\right)$. Then this term can be obtained after simplifying follows :

$$\begin{aligned} 2\left(f\left(\frac{2x_n+f(x_n)}{2}\right)-f(x_n)\right) &= c_1^2e_n + \left(3c_1c_2 + \frac{c_1^2c_2}{2}\right)e_n^2 + \left(4c_1c_3 + 2c_2^2 + c_1c_2^2 + \frac{3c_1^2c_3}{2}\right.\\ &\left. + \frac{c_1^3c_3}{4}\right)e_n^3 + \left(5c_1c_4 + 5c_2c_3 + 4c_1c_2c_3 + \frac{c_2^3}{2} + \frac{3c_1^2c_2c_3}{4} + 3c_1^2c_4\right.\\ &\left. + c_1^3c_4 + \frac{c_1^4c_4}{8}\right)e_n^4 + O(e_n^5). \end{aligned} \tag{19}$$

Substituting (18) and (19) in the equation (12), we have

$$e_{n+1} = \left(\frac{c_2}{c_1} + \frac{c_2}{2}\right)e_n^2 + O(e_n^{2+1}). \tag{20}$$

Hence, it follows that the derivative free iterative method which is of the form (12) has second order of convergence.

Theorem 2.2 If x_0 is sufficiently close to x^*, then the derivative free iterative method (Method II) has third order of convergence.

Proof. Let x^* be the simple root of $f(x)$, i.e. $f(x^*)=0,\ f'(x^*)\neq 0$, and the error equation is $e_n = x_n - x^*$. Using Taylor series of $f(x_n)$ about x^*, we have

$$f(x_n) = c_1e_n + c_2e_n^2 + c_3e_n^3 + c_4e_n^4 + c_5e_n^5 + c_6e_n^6 + O(e_n^7), \tag{21}$$

where $c_k = \dfrac{f^k(x^*)}{k!}$, $k = 1,2,3,\dots$.

Computing $f(x_n)^2$, it is given as following

$$f(x_n)^2 = c_1^2e_n^2 + 2c_1c_2e_n^3 + (2c_1c_3 + c_2^2)e_n^4 + (2c_1c_4 + 2c_2c_3)e_n^5 + O(e_n^6). \tag{22}$$

Computing $f(x_n + f(x_n))$ and $f(x_n - f(x_n))$, can be obtained after simplifying as following

$$\begin{aligned} f(x_n + f(x_n)) &= (c_1^2 + c_1)e_n + (c_2c_1^2 + 3c_2c_1 + c_2)e_n^2 + (c_3c_1^3 + 3c_3c_1^2\\ &+ 2c_1c_2^2 + 4c_3c_1 + 2c_2^2 + c_3)e_n^3 + (c_4 + c_2(c_2^2 + 2c_1c_3)\\ &+ 5c_1c_4 + 5c_2c_3 + 6c_1^2c_4 + 4c_1^3c_4 + c_1^4c_4 + 6c_1c_2c_3\\ &+ 3c_1^2c_2c_3)e_n^4 + O(e_n^5) \end{aligned} \tag{23}$$

and

$$\begin{aligned} f(x_n - f(x_n)) &= (-c_1^2 + c_1)e_n + (c_2c_1^2 - 3c_2c_1 + c_2)e_n^2 + (-c_3c_1^3 + 3c_3c_1^2\\ &+ 2c_1c_2^2 - 4c_3c_1 - 2c_2^2 + c_3)e_n^3 + (c_4 + c_2(c_2^2 + 2c_1c_3)\\ &- 5c_1c_4 - 5c_2c_3 + 6c_1^2c_4 - 4c_1^3c_4 + c_1^4c_4 + 6c_1c_2c_3\\ &- 3c_1^2c_2c_3)e_n^4 + O(e_n^5). \end{aligned} \tag{24}$$

Using equation (21),(22),(23) and (24) to compute $\dfrac{4f(x_n)^2}{3f(x_n+f(x_n)) - 2f(x_n) - f(x_n - f(x_n))}$. Then we have

$$\begin{aligned} \frac{4f(x_n)^2}{3f(x_n+f(x_n)) - 2f(x_n) - f(x_n - f(x_n))} &= e_n + \left(-\frac{c_2}{c_1} - \frac{c_2}{2}\right)e_n^2 + \left(-\frac{2c_3}{c_1} + \frac{2c_2^2}{c_1^2} + \frac{c_2^2}{c_1} + \frac{3c_3}{2}\right.\\ &\left. - c_1c_3 + \frac{c_2^2}{4}\right)e_n^3 + \left(-\frac{3c_4}{c_1} + \frac{7c_2c_3}{c_1^2} + \frac{5c_2c_3}{c_1} - \frac{11c_2^3}{2c_1^2}\right.\\ &+ \frac{5c_2c_3}{2} - 3c_4 - 4c_1c_4 - \frac{c_1^2c_4}{2} - \frac{4c_2^3}{c_1^3} - \frac{3c_2^3}{4c_1} + \frac{6c_3}{c_1}\\ &\left. - \frac{c_2^3}{8}\right)e_n^4 + O(e_n^5) \end{aligned} \tag{25}$$

By considering y_n in the equation (15), we have

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$$
\begin{aligned}
y_n = x^* &+ \left(\frac{c_2}{c_1}+\frac{c_2}{2}\right)e_n^2 + \left(\frac{2c_3}{c_1}-\frac{2c_2^2}{c_1^2}-\frac{c_2^2}{c_1}-\frac{3c_3}{2}+c_1c_3\right.\\
&\left.-\frac{c_2^2}{4}\right)e_n^3 + \left(\frac{3c_4}{c_1}-\frac{7c_2c_3}{c_1^2}-\frac{5c_2c_3}{c_1}+\frac{11c_2^3}{2c_1^2}-\frac{5c_2c_3}{2}\right.\\
&\left.+3c_4+4c_1c_4+\frac{c_1^2c_4}{2}+\frac{4c_2^3}{c_1^3}+\frac{3c_2^3}{4c_1}-\frac{6c_3}{c_1}+\frac{c_2^3}{8}\right)e_n^4\\
&+O(e_n^5).
\end{aligned}
\tag{26}
$$

Now, substituting (26) in Taylor series of $f(y_n)$, we have

$$
\begin{aligned}
f(y_n) = &\left(c_2+\frac{c_1c_2}{2}\right)e_n^2 + \left(2c_3-\frac{2c_2^2}{c_1}-c_2^2-\frac{3c_1c_3}{2}+c_1^2c_3\right.\\
&\left.-\frac{c_1c_2^2}{4}\right)e_n^3 + \left(3c_4-\frac{7c_2c_3}{c_1}-5c_2c_3+\frac{11c_2^3}{2c_1}+\frac{5c_1c_2c_3}{2}\right.\\
&\left.+3c_1c_4+4c_1^2c_4+\frac{c_1^3c_4}{2}+\frac{5c_2^3}{c_1^2}+c_2^3-6c_3+\frac{c_1c_2^3}{8}+\frac{c_2^3}{c_1}\right)e_n^4\\
&+O(e_n^5).
\end{aligned}
\tag{27}
$$

Using equation (25) and (27) in the equation (16), we have

$$
e_{n+1} = \left(\frac{c_2^2}{c_1^2}+\frac{c_2^2}{2c_1}+3c_3\right)e_n^3 + O(e_n^{3+1}). \tag{28}
$$

Hence, it follows that the derivative free iterative method which is form of $(15), (16)$ has the third order convergence.

3. Numerical examples

To numerical comparison, we will present some problems of nonlinear equations from the tested function of Jaiswal (Jaiswal, 2013) and Dehghan and Hajarian (Dehghan & Hajarian, 2010) to compare the proposed methods with existing methods which are as follows: the Newton's method is of the form $(2),$ the Steffensen's method is of the form $(3),$ the Newton-Steffensen's method is of the form $(4),(5)$ and the two Dehghan's methods, namely the Dehghan I 's method and the Dehghan II 's method which are of the form $(6),(7)$ and $(8),(9)$, respectively. In these comparisons, we consider the following functions

Table 1. Test functions and their roots.

Functions	Root
$f_1(x) = \sin^2(x) - x^2 + 1$	1.404492
$f_2(x) = \cos(x) - x$	0.739085
$f_3(x) = \cos(x) - xe^x + x^2$	0.639154
$f_4(x) = e^x - 1.5 - \arctan(x)$	0.767653
$f_5(x) = \ln(x^2 + x + 2) - x + 1$	4.152590
$f_6(x) = x^2 - e^x - 3x + 2$	0.257530

In order to compare of the numerical iterative methods, the numerical experiments have been implemented on GNU Octave, version 5.1.0 with different initial value x_0. Then we choose initial value x_0 that will be a suitable test for these functions. We consider the number of iterations n and the absolute value of function $|f(x_n)|$ when $|f(x_n)| \le 10^{-12}$. The algorithm for the our iterative methods are as follows:

<u>*Numerical Algorithm of Method*</u> *I*

Input: The nonlinear function $f(x)$, tolerance TOL and initial value of x_0.

Step 1 Set $n=0$.

Step 2 While $|f(x_0)|>TOL$ do Step 3-4.

Step 3 Calculate

$$
x_{n+1} = x_n - \frac{f(x_n)^2}{2\left(f\left(\frac{2x_n+f(x_n)}{2}\right)-f(x_n)\right)}.
$$

Calculate $|f(x_{n+1})|$.

Set $n=n+1$.

Step 4 If $|f(x_{n+1})| \le TOL$, then we stop the iteration and print the output of the root of function x_{n+1}, the absolute value of function $|f(x_{n+1})|$ and the number of iterations n.

<u>*Numerical Algorithm of Method*</u> *II*

Input: The nonlinear function $f(x)$, tolerance TOL and initial value of x_0.

Step 1 Set $n=0$.

Step 2 While $|f(x_0)|>TOL$ do Step 3-4.

Step 3 Calculate

$$
y_n = x_n - \frac{4f(x_n)^2}{3f(x_n+f(x_n))-2f(x_n)-f(x_n-f(x_n))}
$$

and

$$
x_{n+1} = x_n - \frac{4f(x_n)^2}{(3f(x_n+f(x_n))-2f(x_n)-f(x_n-f(x_n)))(f(x_n)-f(y_n))},
$$

Calculate $|f(x_{n+1})|$.

Set $n=n+1$.

Step 4 If $|f(x_{n+1})| \le TOL$, then we stop the iteration and print the output of the root of function x_{n+1} , the absolute value of function $|f(x_{n+1})|$ and the number of iterations n.

In general, the method with minimum for the number of iteration could be chosen as the best method to find solution of function. Moreover, the method gives the absolute value of function which is closer to zero than the other, is will be the best method.

Table 2. Numerical results of $f_1(x) = \sin^2(x) - x^2 + 1$ with $x_0 = 1$.

Methods	Number of iterations n	$\|f(x_n)\|$
Newton's method	5	7.593925×10^{-13}
Steffensen's method	5	9.992007×10^{-16}
Newton-Steffensen's method	5	1.323386×10^{-13}
Dehghan I 's method	4	4.440892×10^{-16}
Dehghan II 's method	4	6.302736×10^{-13}
Method I	4	3.330669×10^{-16}
Method II	3	3.330669×10^{-16}

Table 3. Numerical results of $f_2(x) = \cos(x) - x$ with $x_0 = 0$.

Methods	Number of iterations n	$\|f(x_n)\|$
Newton's method	5	0
Steffensen's method	4	1.110223×10^{-16}
Newton-Steffensen's method	5	0
Dehghan I 's method	4	0
Dehghan II 's method	3	2.109424×10^{-15}
Method I	4	1.110223×10^{-16}
Method II	3	0

Table 4. Numerical results of $f_3(x) = \cos(x) - xe^x + x^2$ with $x_0 = -2$.

Methods	Number of iterations n	$\|f(x_n)\|$
Newton's method	7	1.110223×10^{-16}
Steffensen's method	7	1.110223×10^{-16}
Newton-Steffensen's method	7	1.110223×10^{-16}
Dehghan I 's method	5	1.110223×10^{-16}
Dehghan II 's method	8	1.110223×10^{-16}
Method I	4	1.110223×10^{-16}
Method II	4	1.221245×10^{-15}

Table 5. Numerical results of $f_4(x) = e^x - 1.5 - \arctan(x)$ with $x_0 = 1$.

Methods	Number of iterations n	$\|f(x_n)\|$
Newton's method	12	1.465494×10^{-13}
Steffensen's method	6	2.220446×10^{-16}
Newton-Steffensen's method	4	2.220446×10^{-16}
Dehghan I 's method	3	1.110223×10^{-16}
Dehghan II 's method	3	5.073719×10^{-14}
Method I	5	4.440892×10^{-16}
Method II	3	0

Table 6. Numerical results of $f_5(x) = \ln(x^2 + x + 2) - x + 1$ with $x_0 = 0$.

Methods	Number of iterations n	$\|f(x_n)\|$
Newton's method	5	4.440892×10^{-16}
Steffensen's method	5	4.440892×10^{-16}
Newton-Steffensen's method	5	0
Dehghan I 's method	4	4.440892×10^{-16}
Dehghan II 's method	4	4.440892×10^{-16}
Method I	5	0
Method II	3	8.171241×10^{-14}

Table 7. Numerical results of $f_6(x) = x^2 - e^x - 3x + 2$ with $x_0 = 2$.

Methods	Number of iterations n	$\|f(x_n)\|$
Newton's method	4	3.728129×10^{-13}
Steffensen's method	6	0
Newton-Steffensen's method	14	0
Dehghan I 's method	17	0
Dehghan II 's method	1330	0
Method I	4	2.392082×10^{-14}
Method II	4	0

From Table 2-7 of numerical results, we can see that the Method II is better than the other iterative methods. However, the Method I is as good as the

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Method II for $f_3(x) = \cos(x) - xe^x + x^2$ with initial $x_0 = -2$ and $f_6(x) = x^2 - e^x - 3x + 2$ with initial $x_0 = 2$. So for the different nonlinear functions, the proposed iterative method (Method II) outperform the other iterative methods.

4. Conclusion

In this paper, the new numerical iterative methods for solving nonlinear equations with one variable without derivative, namely Method I and Method II are presented. Moreover, the convergence analysis of our methods have been established to show that the Method I and Method II have second and third order convergence, respectively. The six nonlinear functions are considered to illustrate that Method II is better than some other methods. But the Method II is more complicated than the Method I. However the result from Method I gave as efficiency as the Method II. For the future work, one may consider to modify the approximating of derivative of function or extended the method to find all solutions of nonlinear equations which contain one or more than one variables.

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