รายงานวิจัยฉบับสมบูรณ์

โครงการ

เสถียรภาพและการทำให้มีเสถียรภาพสำหรับระบบควบคุม
ไม่เชิงเส้นที่ตัวหน่วงแปรผันตามเวลาเป็นช่วง
ซึ่งไม่มีการหาอนุพันธ์และการประยุกต์

Stability and stabilization for nonlinear control systems
with interval non-differentiable time-varying delay
and applications

(ทุนพัฒนาศักยภาพในการทำงานวิจัยของอาจารย์รุ่นใหม่)

โดย

ผศ.ดร.คณิต มุกดาใส และคณะ

มิถุนายน 2559

สัญญาเลขที่ TRG5780069

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ผศ.ดร.คณิต มุกดาใส ภาควิชาคณิตศาสตร์ คณะวิทยาศาสตร์
(หัวหน้าโครงการวิจัยผู้รับทุน) มหาวิทยาลัยขอนแก่น

รศ.ดร.ปิยะพงศ์ เนียมทรัพย์ ภาควิชาคณิตศาสตร์ คณะวิทยาศาสตร์
(นักวิจัยที่ปรึกษา) มหาวิทยาลัยเชียงใหม่

สนับสนุนโดยสำนักงานกองทุนสนับสนุนการวิจัยและมหาวิทยาลัยขอนแก่น

บทคัดย่อ

รหัสโครงการ : TRG5780069

ชื่อโครงการ : เสถียรภาพและการทำให้มีเสถียรภาพสำหรับระบบควบคุมไม่เชิงเส้นที่ตัวหน่วงแปรผันตามเวลาเป็นช่วงซึ่งไม่มีการหาอนุพันธ์และการประยุกต์

ชื่อนักวิจัย : ผศ.ดร. คณิต มุกดาใส
ภาควิชาคณิตศาสตร์ คณะวิทยาศาสตร์ มหาวิทยาลัยขอนแก่น

นักวิจัยที่ปรึกษา: รศ.ดร. ปิยะพงศ์ เนียมทรัพย์
ภาควิชาคณิตศาสตร์ คณะวิทยาศาสตร์ มหาวิทยาลัยเชียงใหม่

อีเมล์: kanit@kku.ac.th (คณิต มุกดาใส)
piyapong.n@cmu.ac.th (ปิยะพงศ์ เนียมทรัพย์)

ระยะเวลาโครงการ : วันที่ 2 มิถุนายน 2557 ถึงวันที่ 1 มิถุนายน 2559

ในงานวิจัยนี้เสนอการวิเคราะห์เสถียรภาพ และการทำให้มีเสถียรภาพของระบบไม่เชิงเส้นตัวหน่วงที่ขึ้นกับเวลาเป็นช่วง ฟังก์ชันตัวหน่วงที่ถูกให้มาเป็นฟังก์ชันต่อเนื่องซึ่งมีขอบเขตบนและขอบเขตล่าง แต่ฟังก์ชันไม่จำเป็นต้องหาอนุพันธ์ได้ โดยอาศัยความรู้พื้นฐานจากอสมการโคชี อสมการโคชีที่ถูกปรับปรุง การใช้ประโยชน์จากสมการศูนย์ รูปแบบของไลบ์นิซ-นิวตัน เมทริกซ์ถ่วงน้ำหนักอิสระ อสมการเจนเซนและฟังก์ชันนอลไลปูนอฟ-คราซอฟสกี ทำให้เราได้หลักเกณฑ์เสถียรภาพ และการทำให้มีเสถียรภาพที่ขึ้นกับตัวหน่วงเป็นช่วงรูปแบบใหม่สำหรับระบบควบคุมไม่เชิงเส้นที่ซึ่งตัวหน่วงแปรผันตามเวลาเป็นช่วง อยู่ในรูปอสมการเมทริกซ์เชิงเส้นตัวอย่างเชิงตัวเลขที่ให้มาแสดงให้เห็นถึงศักยภาพของวิถีดังกล่าวที่ปรับปรุงผลลัพธ์ที่มีอยู่แล้ว

คำหลัก : หลักเกณฑ์เสถียรภาพที่ขึ้นกับตัวหน่วงเป็นช่วง, ระบบควบคุมไม่เชิงเส้น, อสมการเมตริกซ์เชิงเส้น, ฟังก์ชันนอลไลปูนอฟ-คราซอฟสกี, ตัวหน่วงที่ขึ้นกับเวลาเป็นช่วง

Abstract

Project Code: TRG5780069

Project Title: Stability and stabilization for nonlinear control systems with interval non-differentiable time-varying delay and applications

Investigator: Asst. Prof. Dr. Kanit Mukdasai
Department of Mathematics, Faculty of Science, Khon Kaen University

Mentor: Assoc. Prof. Dr. Piyapong Niamsup
Department of Mathematics, Faculty of Science, Chiang Mai University

E-mail Address: kanit@kku.ac.th (Kanit Mukdasai)
piyapong.n@cmu.ac.th (Piyapong Niamsup)

Project Period: June 2, 2014 - June 1, 2016

This research proposes the stability and stabilization analysis of interval time-varying delay nonlinear systems. The lower and upper bounds for the time-varying delay are available, but the delay function is not necessary to be differentiable. Based on Cauchy's inequality, modified version of Cauchy's inequality, utilization of zero equation, Leibniz-Newton formula, free-weighting matrices, Jensen's inequality and Lyapunov-Krasovskii functional, new delay-range-dependent stability and stabilization criteria for nonlinear control systems with interval time-varying delay are established in terms of linear matrix inequalities (LMIs). Numerical examples show that the proposed criteria improve the existing results significantly with much less computational effort.

Keywords: Delay-range-dependent stability criteria, nonlinear control systems, Linear matrix inequality, Lyapunov–Krasovskii functional, interval time-varying delay.

กิตติกรรมประกาศ

ผู้วิจัยขอขอบพระคุณ

สำนักงานกองทุนสนับสนุนการวิจัยและมหาวิทยาลัยขอนแก่น ที่ได้ให้โอกาสผู้วิจัยได้รับทุนเพื่อเป็นการพัฒนาศักยภาพในการทำงานวิจัยอาจารย์รุ่นใหม่ในการทำงานวิจัยครั้งนี้

รองศาสตราจารย์ ดร.ปิยะพงศ์ เนียมทรัพย์ นักวิจัยที่ปรึกษาให้กับโครงการนี้ผู้ซึ่งอบรม สั่งสอนและถ่ายทอดความรู้ด้านต่างๆ จนผู้วิจัยสามารถทำงานวิจัยได้สำเร็จตามเป้าหมาย

คณะผู้ประเมินของวารสารวิชาการต่างๆ ที่ได้ให้คำแนะนำ ตลอดทั้งปรับปรุงต้นฉบับของบทความที่ส่งไปตีพิมพ์ในวารสารนั้นๆ

คณาจารย์ นักศึกษาและเจ้าหน้าที่ฝ่ายสนับสนุน ภาควิชาคณิตศาสตร์ คณะวิทยาศาสตร์ มหาวิทยาลัยขอนแก่น ได้ร่วมศึกษาวิจัยและช่วยเหลือโครงการวิจัยในครั้งนี้

ผศ.ดร.คณิต มุกดาใส
หัวหน้าโครงการวิจัย

Contents

Chapter 1

Executive Summary

The following notations will be used in this research : N denotes the set of all natural numbers; R^+ denotes the set of all real non-negative numbers; R^n denotes the n-dimensional Euclidean space equipped with the Euclidean norm $\|\cdot\|$; $R^{n\times r}$ denotes the space of all matrices of $(n \times r)$-dimensions; A^T denotes the transpose of the matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; matrix A is called semi-positive definite $(A \geq 0)$ if $x^TAx \geq 0$, for all $x \in R^n$; A is positive definite $(A > 0)$ if $x^TAx > 0$ for all $x \in R^n - \{0\}$; matrix B is called semi-negative definite $(B \leq 0)$ if $x^TBx \leq 0$, for all $x \in R^n$; B is negative definite $(B < 0)$ if $x^TBx < 0$ for all $x \in R^n - \{0\}$; $A > B$ means $A - B > 0$ $(B - A < 0)$; $A \geq B$ means $A - B \geq 0$ $(B - A \leq 0)$; $\bar{h} = \max\{h_2, r_2\}$, $h_2, r_2 \in R^+$; $x_t = x(t+s), s \in [-\bar{h}, 0]$; $*$ represents the elements below the main diagonal of a symmetric matrix.

Consider the following uncertain impulsive switched linear control system with time delays

$$\begin{cases} \dot{x}(t) = A_{i_k}(t)x(t) + B_{i_k}(t)x(t - h_{i_k}(t)) + C_{i_k}(t)x(t - r_{i_k}(t)) \\ \qquad\quad + f_{i_k}(t, x(t)) + g_{i_k}(t, x(t - h_{i_k}(t))) + w_{i_k}(t, x(t - r_{i_k}(t))) \\ \qquad\quad + D_{i_k}u(t), \quad t \neq t_k, \\ \Delta x(t) = x(t) - x(t^-) = G_k x(t^- - h_{i_k}(t^-)), \ t = t_k, \\ x(t_0 + s) = \phi(s), \ \forall s \in [-\bar{h}, 0], \\ A_{i_k}(t) = A_{i_k} + \Delta A_{i_k}(t), \quad B_{i_k}(t) = B_{i_k} + \Delta B_{i_k}(t), \\ C_{i_k}(t) = C_{i_k} + \Delta C_{i_k}(t), \end{cases} \tag{1.1}$$

where $x(t) \in R^n$ denotes the state variable, $u(t) \in R^s$ denotes the control input, $i_k \in \{1, 2, ..., m\}$, $k, m \in N$. A_{i_k}, B_{i_k}, C_{i_k}, D_{i_k} and G_k are given constant matrices of appropriate dimensions. The delays $h_{i_k}(t)$ and $r_{i_k}(t)$ are interval time-varying bounded continuous functions satisfying

$$0 \leq h_1 \leq h_{i_k}(t) \leq h_2,$$

$$0 \leq r_1 \leq r_{i_k}(t) \leq r_2,$$

where h_1, h_2, r_1 and r_2 are given positive real constants. The uncertainties $f_{i_k}(.)$, $g_{i_k}(.)$ and $w_{i_k}(.)$ represent the nonlinear parameter perturbations with respect to the current state $x(t)$, the delayed state $x(t-h_{i_k}(t))$ and delayed state $x(t-r_{i_k}(t))$, respectively. They satisfy that $f_{i_k}(t,0)=0$, $g_{i_k}(t,0)=0$, $w_{i_k}(t,0)=0$ and

$$f_{i_k}^T(t,x(t))f_{i_k}(t,x(t)) \leq \eta^2 x^T(t)x(t),$$

$$g_{i_k}^T(t,x(t-h_{i_k}(t)))g_{i_k}(t,x(t-h_{i_k}(t))) \leq \rho^2 x^T(t-h_{i_k}(t))x(t-h_{i_k}(t)),$$

$$w_{i_k}^T(t,x(t-r_{i_k}(t)))w_{i_k}(t,x(t-r_{i_k}(t))) \leq \zeta^2 x^T(t-r_{i_k}(t))x(t-r_{i_k}(t)),$$

where η, ρ and ζ are given positive real constants. The uncertain matrices $\Delta A_{i_k}(t)$, $\Delta B_{i_k}(t)$, $\Delta C_{i_k}(t)$ and $\Delta D_{i_k}(t)$ are norm bounded and can be described as

$$\begin{bmatrix}\Delta A_{i_k}(t) & \Delta B_{i_k}(t) & \Delta C_{i_k}(t)\end{bmatrix} = K_{i_k}\Delta_{i_k}(t)\begin{bmatrix}L_{i_k}^1 & L_{i_k}^2 & L_{i_k}^3\end{bmatrix}, \tag{1.2}$$

where $K_{i_k}, L_{i_k}^1, L_{i_k}^2$ and $L_{i_k}^3$ are given constant matrices of appropriate dimensions. The class of parametric uncertainties $\Delta_{i_k}(t)$ which satisfies

$$\Delta_{i_k}(t) = F_{i_k}(t)[I - JF_{i_k}(t)]^{-1}, \tag{1.3}$$

is said to be admissible where J is a known matrix satisfying

$$I - JJ^T > 0, \tag{1.4}$$

and $F_{i_k}(t)$ is uncertain matrix satisfying

$$F_{i_k}(t)^T F_{i_k}(t) \leq I. \tag{1.5}$$

$\Delta x(t) = x(t_k^+) - x(t_k^-)$, $\lim\limits_{\nu \to 0^+} x(t_k+\nu) = x(t_k^+)$, $x(t_k^-) = \lim\limits_{\nu \to 0^+} x(t-\nu)$. $\phi(t)$ is the initial function with the norm $\|\phi\| = \sup_{\theta\in[-\bar{h},0]}\|\phi(\theta)\|$. We assume that the solution of the impulsive switched system (1.1) is right continuous i.e., $x(t_k^+) =$

$x(t_k)$. t_k is an impulsive switching time point and $t_0 < t_1 < t_2 < \cdots < t_k < \cdots, t_k \to +\infty$ as $k \to +\infty$ and introduce the quantity

$$\tau = \inf\{t_{i+1} - t_i : i = 1, 2, 3, ...\}.$$

This τ is called the dwell time of the system (1.1). Under the switching law of system (1.1), at the time t_k, the system switches to the i_k subsystem from the i_{k-1} subsystem.

Remark The conditions (1.4) and (1.5) guarantee that $I - JF_{i_k}(t)$ is invertible. It is easy to show that when $J = 0$, the parametric uncertainty of linear fractional form reduces to a norm bounded one.

The objectives of this research are: (i) to establish new delay-range-dependent sufficient conditions for robust exponential stability of system (1.1) when $u(t) = 0$ and (ii) to design a robust state feedback controller

$$u(t) = Kx(t), \tag{1.6}$$

which robustly exponentially stabilizes (1.1), where K is a constant gain matrix of appropriate dimensions to be designed.

Definition 1.0.1 *Given $\beta > 0$. The system (1.1) is exponentially stable, if there exist switching function i_k and positive real constant M such that any solution $x(t, \phi)$ of the system satisfies*

$$\|x(t, \phi)\| \leq M\|\phi\|e^{-\beta t}, \quad \forall t \in R^+.$$

Lemma 1.0.2 *(Halanay Lemma) Let $m(t)$ be a positive scalar function and assume that the following condition holds:*

$$D^+m(t) \leq -am(t) + b\bar{m}(t), \quad t \geq t_0,$$

where $D^+m(t) = \limsup_{\Delta t \to 0^+} \dfrac{m(t + \Delta t) - m(t)}{\Delta t}$, $0 < b < a$. Then, there exists $\beta > 0$ such that for all $t \geq t_0$,

$$m(t) \leq \bar{m}(t_0)e^{-\beta(t-t_0)}.$$

Here, $\bar{m}(t) = \sup_{t-\bar{h} \leq s \leq t} \{m(s)\}$ and β satisfies $\beta - a + be^{\beta\bar{h}} = 0$.

Lemma 1.0.3 *(Schur complement lemma) Given constant sysmetric matrices* X Y, Z *where* $Y > 0$. *Then* $X + Z^T Y^{-1} Z < 0$ *if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \text{ or } \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0.$$

Lemma 1.0.4 *For given matrices* $Q = Q^T$, H, E, $R = R^T > 0$ *of appropriate dimension, then*

$$Q + HFE + E^T F^T H^T < 0,$$

for all F *satisfies* $F^T F \leq R$, *if and only if there exist a positive number* $\epsilon > 0$, *such that*

$$Q + \epsilon^{-1} H H^T + \epsilon E^T R E < 0.$$

Lemma 1.0.5 *Suppose that* $\Delta(t)$ *is given by* (1.3)-(1.5). *Let* M, S *and* N *be real matrices of appropriate dimensions with* $M = M^T$. *Then, the inequality*

$$M + S\Delta(t)N + N^T \Delta^T(t) S^T < 0,$$

holds, if and only if, for any scalar $\delta > 0$,

$$\begin{pmatrix} M & S & \delta N^T \\ S^T & -\delta I & \delta J^T \\ \delta N & \delta J & -\delta I \end{pmatrix} < 0.$$

Lemma 1.0.6 *Let* G_k *be given matrices as in (1.1). Let* P *be symmetric positive definite matrix. Then*

$$\begin{pmatrix} P & PG_k \\ G_k^T P & G_k^T P G_k \end{pmatrix} \leq \delta_k I, \tag{1.7}$$

if and only if

$$\begin{pmatrix} -\delta_k I & 0 & P \\ 0 & -\delta_k I & G_k^T P \\ P & PG_k & -P \end{pmatrix} \leq 0, \tag{1.8}$$

for δ_k *is positive real constant,* $k \in N$.

Proof. Consider inequality (1.7), we have

$$\begin{pmatrix} P & PG_k \\ G_k^T P & G_k^T P G_k \end{pmatrix} \leq \delta_k I.$$

Equivalently,

$$\begin{pmatrix} -\delta_k I & 0 \\ 0 & -\delta_k I \end{pmatrix} + \begin{pmatrix} I \\ G_k^T \end{pmatrix} P \begin{pmatrix} I & G_k \end{pmatrix} \leq 0.$$

By using Schur complement Lemma in the above inequality, we get

$$\begin{pmatrix} -\delta_k I & 0 & I \\ 0 & -\delta_k I & G_k^T \\ I & G_k & -P^{-1} \end{pmatrix} \leq 0. \tag{1.9}$$

Pre-multiplying (1.9) by $diag\{I, I, P\}$ and post-multiplying by $diag\{I, I, P\}$, we obtain

$$\begin{pmatrix} -\delta_k I & 0 & P \\ 0 & -\delta_k I & G_k^T P \\ P & PG_k & -P \end{pmatrix} \leq 0. \tag{1.10}$$

The proof of the lemma is complete. □

Then, we consider system described by the following state equation of the form

$$\begin{cases} \dot{x}(t) = A(t)x(t) + B(t)x(t-h(t)) + f(t, x(t)) \\ \qquad +g(t, x(t-h(t))) + D(t)\int_{t-\delta(t)}^{t} x(s)ds, & t > 0; \\ x(t_0+\theta) = \phi(\theta), \quad \dot{x}(t_0+\theta) = \varphi(\theta), & \forall\theta \in [-\max\{h_2, \delta\}, 0], \\ A(t) = [A + \Delta A(t)], \quad B(t) = [B + \Delta B(t)], \\ D(t) = [D + \Delta D(t)], \end{cases} \tag{1.11}$$

where $x(t) \in R^n$ is the state variable, A, B and $D \in R^{n\times n}$ are known real constant matrices. $h(t)$, $\delta(t)$ are discrete and distributed time-varying delays, respectively,

$$0 \leq h_1 \leq h(t) \leq h_2, \quad \dot{h}(t) \leq h_d \leq \infty,$$
$$0 \leq \delta(t) \leq \delta,$$

where h_1, h_2 are positive real constants representing lower and upper bounds of the delay, respectively, h_d, δ are positive real constant. Consider the initial function $\phi(t), \varphi(t) \in C([-\max\{h_2, \delta\}, 0], R^n)$ denote continuous vector-valued initial function of t $\in [-\max\{h_2, \delta\}, 0]$ with the norm $\|\phi\| = \sup_{\theta\in[-\max\{h_2,\delta\},0]} \|\phi(\theta)\|$, $\|\varphi\| = \sup_{\theta\in[-\max\{h_2,\delta\},0]} \|\varphi(\theta)\|$. The uncertain matrices $\Delta A(t)$, $\Delta B(t)$ and $\Delta D(t)$ are norm bounded and can be described as

$$\begin{bmatrix}\Delta A(t) & \Delta B(t) & \Delta D(t)\end{bmatrix} = EF(t)\begin{bmatrix}G_1 & G_2 & G_3\end{bmatrix}$$

where E, G_1, G_2 are know constant matrices with appropriate dimensions. The uncertain matrix $F(t)$ satisfies

$$F(t)^T F(t) \leq I.$$

The uncertainties $f(t, x(t)), g(t, x(t - h(t)))$ represent the nonlinear parameter perturbations with respect to the current state $x(t)$ and the delayed state $x(t - h(t))$, respectively, and are bounded in magnitude of the form

$$f^T(t, x(t))f(t, x(t)) \leq \eta^2 x^T(t)x(t),$$
$$g^T(t, x(t - h(t)))g(t, x(t - h(t))) \leq \rho^2 x^T(t - h(t))x(t - h(t)),$$

where η, ρ are given real constants.where η, ρ are given real constants.

Definition 1.0.7 The system (1.11) is exponentially stable, if there exist positive real numbers α and M such that for each $\phi(t), \varphi(t) \in C([-h_2, 0], R^n)$, the solution $x(t, \phi, \varphi)$ of the system (1.11) satisfies

$$\|x(t, \phi, \varphi)\| \leq M \max\{\|\phi\|, \|\varphi\|\}e^{-\alpha t}, \quad \forall t \in R^+.$$

Lemma 1.0.8 (Jensen's inequality) *For any constant matrix* $Q \in R^{n\times n}, Q = Q^T > 0$*, scalar* $h > 0$*, vector function* $\dot{x}(t) : [-h_2, 0] \to R^n$ *such that the integrations concerned are well defined, then*

$$-h\int_{-h}^{0} \dot{x}^T(s + t)Q\dot{x}(s + t)ds \leq -\Big(\int_{-h}^{0} \dot{x}(s + t)ds\Big)^T Q\Big(\int_{-h}^{0} \dot{x}(s + t)ds\Big).$$

Rearranging the term $\int_{-h}^{0} \dot{x}(s+t)ds$ *with* $x(t)-x(t-h)$, *we can yield the following inequality:*

$$-h\int_{-h}^{0} \dot{x}^T(s+t)Q\dot{x}(s+t)ds \leq \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -Q & Q \\ Q & -Q \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}.$$

Lemma 1.0.9 *For any constant matrices* $Q_{11}, Q_{22}, Q_{12} \in R^n, Q_{11} \geq 0, Q_{22} \geq 0, \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \geq 0$ *scalar* $h_1 \leq h(t) \leq h_2$, *and vector function* $\dot{x} : [-h_2, 0] \to R^n$ *such that the following integration is well defined, then*

$$\begin{aligned} &-(h_2-h_1)\int_{t-h_2}^{t-h_1} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} dt \\ &\leq \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ \int_{t-h(t)}^{t-h_1} x(t)dt \end{bmatrix}^T \begin{bmatrix} -Q_{22} & Q_{22} & -Q_{12}^T \\ * & -Q_{22} & Q_{12}^T \\ * & * & -Q_{11} \end{bmatrix} \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ \int_{t-h(t)}^{t-h_1} x(t)dt \end{bmatrix}. \end{aligned}$$

Lemma 1.0.10 *Let* $x(t) \in R^n$ *be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any matrices* $X, M_i \in R^{n\times n}, i = 1, 2, \ldots, 5$ *and a scalar function* $h := h(t) \geq 0$:

$$\begin{aligned} -\int_{t-h}^{t} \dot{x}^T(s)X\dot{x}(s)ds \leq &\begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} M_1^T + M_1 & -M_1^T + M_2 \\ -M_1 + M_2^T & -M_2^T - M_2 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix} \\ &+h \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} M_3 & M_4 \\ M_4^T & M_5 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}, \end{aligned}$$

where

$$\begin{bmatrix} X & M_1 & M_2 \\ M_1^T & M_3 & M_4 \\ M_2^T & M_4^T & M_5 \end{bmatrix} \geq 0.$$

Lemma 1.0.11 *For given matrices* $Q = Q^T$, H, E, *and* $R = R^T > 0$ *of appropriate dimemsion, then*

$$Q + HFE + E^T F^T H^T < 0$$

for all F satisfies $F^TF \leq R$, if and only if there exist a positive number $\epsilon > 0$, such that

$$Q + \epsilon^{-1}HH^T + \epsilon E^T RE < 0.$$

Lemma 1.0.12 *(**Schur complement lemma**) Given constant sysmetric matrices X Y, Z where $Y > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \ \text{ or } \ \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0.$$

Chapter 2

Main Results

In this research, we investigate the problems of robust exponential stability and stabilization analysis for uncertain impulsive switched linear control systems with discrete interval time-varying delays and nonlinear perturbations. The time delays are continuous functions belonging to the given interval delays, which mean that the lower and upper bounds for the time-varying delays are available, but the delay functions are not necessary to be differentiable. Based on the combination of mixed model transformation, Halanay inequality, utilization of zero equations, decomposition technique of coefficient matrices and a common Lyapunov functional, new delay-range-dependent robust exponential stability and stabilization criteria are established in terms of linear matrix inequalities (LMIs) for considered systems. Moreover, The problem of robust exponential stability criteria for uncertain linear system with interval discrete time-varying delay, distributed time-varying delay and nonlinear perturbations is studied in this research. Based on constructing an augmented Lyapunov-Krasovskii functional, decomposition technique of coefficient matrix, combination of descriptor model transformation and utilization of zero equation, new delay-range-dependent robust exponential stability criteria are derived in terms of LMIs.

2.1 Stabilization for Impulsive Switched Nonlinear Control System with Delays

We now introduce the following notations for later use:

$$\sum\nolimits_{i_k} = \left(\ \Sigma_{i,j}^{i_k} \ \right)_{9\times 9}, \tag{2.12}$$

where $\Sigma_{i,j}^{i_k} = {\Sigma_{j,i}^{i_k}}^T, \quad i,j = 1,2,3,...,9,$

$$U = PM, \quad W = PN,$$

$$\Sigma_{1,1}^{i_k} = U + W + U^T + W^T + Q_1^T(A_{i_k} + B_{i_k}^1 + C_{i_k}^1) + (A_{i_k} + B_{i_k}^1 + C_{i_k}^1)^T Q_1 + \epsilon_1\eta^2 I + aP,$$

$$\Sigma_{1,2}^{i_k} = P - Q_1^T + (A_{i_k} + B_{i_k}^1 + C_{i_k}^1)^T Q_2, \quad \Sigma_{1,3}^{i_k} = -U + Q_1^T B_{i_k}^2 + (h_2 - h_1)Q_3,$$

$$\Sigma_{1,4}^{i_k} = -W + Q_1^T C_{i_k}^2 + (r_2 - r_1)Q_4, \quad \Sigma_{1,5}^{i_k} = Q_1^T, \quad \Sigma_{1,6}^{i_k} = Q_1^T, \quad \Sigma_{1,7}^{i_k} = Q_1^T,$$

$$\Sigma_{1,8}^{i_k} = -U - Q_1^T B_{i_k}^1 + (h_2 - h_1)Q_5, \quad \Sigma_{1,9}^{i_k} = -W - Q_1^T C_{i_k}^1 + (r_2 - r_1)Q_6,$$

$$\Sigma_{2,2}^{i_k} = -Q_2^T - Q_2, \quad \Sigma_{2,3}^{i_k} = Q_2^T B_{i_k}^2, \quad \Sigma_{2,4}^{i_k} = Q_2^T C_{i_k}^2, \quad \Sigma_{2,5}^{i_k} = Q_2^T, \quad \Sigma_{2,6}^{i_k} = Q_2^T,$$

$$\Sigma_{2,7}^{i_k} = Q_2^T, \quad \Sigma_{2,8}^{i_k} = -Q_2^T B_{i_k}^1, \quad \Sigma_{2,9}^{i_k} = -Q_2^T C_{i_k}^1,$$

$$\Sigma_{3,3}^{i_k} = -(h_2 - h_1)Q_3 - (h_2 - h_1)Q_3^T + \epsilon_2\rho^2 I - bP,$$

$$\Sigma_{3,4}^{i_k} = \Sigma_{3,5}^{i_k} = \Sigma_{3,6}^{i_k} = \Sigma_{3,7}^{i_k} = 0, \quad \Sigma_{3,8}^{i_k} = -(h_2 - h_1)Q_3^T - (h_2 - h_1)Q_5,$$

$$\Sigma_{3,9}^{i_k} = 0, \Sigma_{4,4}^{i_k} = -(r_2 - r_1)Q_4 - (r_2 - r_1)Q_4^T + \epsilon_3\zeta^2 I - cP,$$

$$\Sigma_{4,5}^{i_k} = \Sigma_{4,6}^{i_k} = \Sigma_{4,7}^{i_k} = \Sigma_{4,8}^{i_k} = 0, \quad \Sigma_{4,9}^{i_k} = -(r_2 - r_1)Q_4^T - (r_2 - r_1)Q_6,$$

$$\Sigma_{5,5}^{i_k} = -\epsilon_1 I, \quad \Sigma_{5,6}^{i_k} = \Sigma_{5,7}^{i_k} = \Sigma_{5,8}^{i_k} = \Sigma_{5,9}^{i_k} = 0, \quad \Sigma_{6,6}^{i_k} = -\epsilon_2 I,$$

$$\Sigma_{6,7}^{i_k} = \Sigma_{6,8}^{i_k} = \Sigma_{6,9}^{i_k} = 0, \quad \Sigma_{7,7}^{i_k} = -\epsilon_3 I, \quad \Sigma_{7,8}^{i_k} = \Sigma_{7,9}^{i_k} = 0,$$

$$\Sigma_{8,8}^{i_k} = -(h_2 - h_1)Q_5 - (h_2 - h_1)Q_5^T, \quad \Sigma_{8,9}^{i_k} = 0,$$

$$\Sigma_{9,9}^{i_k} = -(r_2 - r_1)Q_6 - (r_2 - r_1)Q_6^T.$$

Theorem 2.1.1 *The nominal system (1.1) when $u(t) = 0$ is exponentially stable, if there exist symmetric positive definite matrix P, any appropriate dimensional matrices M, N, Q_i, $i = 1, 2, \ldots, 7$, and positive real constants $\mu, \lambda, \eta, \rho, \zeta, \epsilon_1, \epsilon_2, \epsilon_3, a, b, c$ with $a > b + c$ and $\delta_k > 0$ for all $k \in N$ such that the following LMIs hold:*

$$\textstyle\sum_{i_k} < 0, \tag{2.13}$$

$$\begin{pmatrix} -\delta_k I & 0 & P \\ 0 & -\delta_k I & G_k^T P \\ P & PG_k & -P \end{pmatrix} \leq 0, \tag{2.14}$$

$$\mu\bar{h} \leq \inf_{k\in N}\{t_k - t_{k-1}\}, \tag{2.15}$$

$$\max\{\bar{\delta}_k + \bar{\delta}_k e^{\lambda\bar{h}}\} \leq M < e^{\lambda\mu\bar{h}}, \tag{2.16}$$

where $\bar{\delta}_k = \frac{\delta_k}{\lambda_{\min}(P)}$, $k \in N$ *and* λ *is the unique positive root of the equation* $\lambda - a + (b+c)e^{\lambda \bar{h}} = 0$.

We introduce the following notations for later use:

$$\begin{aligned} \Gamma_{i_k}^T &= \begin{bmatrix} K_{i_k}^T Q_1 & K_{i_k}^T Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & (v_2 - v_1) K_{i_k}^T Q_3 \end{bmatrix}, \\ \Upsilon_{i_k} &= \begin{bmatrix} L_{i_k}^1 & 0 & L_{i_k}^2 & L_{i_k}^3 & 0 & 0 & 0 & 0 & L_{i_k}^4 \end{bmatrix}, \\ \widetilde{\sum}_{i_k} &= \begin{pmatrix} \sum_{i_k} & \Gamma_{i_k} & \sigma \Upsilon_{i_k}^T \\ \Gamma_{i_k}^T & -\sigma I & \sigma J^T \\ \sigma \Upsilon_{i_k} & \sigma J & -\sigma I \end{pmatrix}. \end{aligned}$$

Theorem 2.1.2 *The system (1.1) when* $u(t) = 0$ *is robustly exponentially stable, if there exist symmetric positive definite matrix* P*, any appropriate dimensional matrices* M, N, Q_i*,* $i = 1, 2, \ldots, 7$*, and positive real constants* $\sigma, \mu, \lambda, \eta, \rho, \zeta, \epsilon_1, \epsilon_2, \epsilon_3, a, b, c$ *with* $a > b + c$ *and* $\delta_k > 0$ *for all* $k \in N$ *such that the following LMIs hold:*

$$\widetilde{\sum}_{i_k} < 0, \tag{2.17}$$

$$\begin{pmatrix} -\delta_k I & 0 & P \\ 0 & -\delta_k I & G_k^T P \\ P & P G_k & -P \end{pmatrix} \leq 0, \tag{2.18}$$

$$\mu \bar{h} \leq \inf_{k \in N} \{t_k - t_{k-1}\}, \tag{2.19}$$

$$\max\{\bar{\delta}_k + \bar{\delta}_k e^{\lambda \bar{h}}\} \leq M < e^{\lambda \mu \bar{h}}, \tag{2.20}$$

where $\bar{\delta}_k = \frac{\delta_k}{\lambda_{\min}(P)}$, $k \in N$ *and* λ *is the unique positive root of the equation* $\lambda - a + (b+c)e^{\lambda \bar{h}} = 0$.

We introduce the following notations for later use:

$$\begin{aligned} \Pi_{i_k}^T &= \begin{bmatrix} D_{i_k}^T Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \Psi_{i_k} &= \begin{bmatrix} D_{i_k}^T Q_1 & D_{i_k}^T Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

Theorem 2.1.3 *If there exist symmetric positive definite matrix* P*, any appropriate dimensional matrices* M, N, Q_i*,* $i = 1, 2, \ldots, 7$*, and positive real constants* $\mu, \lambda, \eta, \rho, \zeta, \tau, \epsilon_1, \epsilon_2, \epsilon_3, a, b, c$ *with* $a > b + c$ *and* $\delta_k > 0$ *for all* $k \in N$ *such that the* *following LMIs hold:*

$$\begin{pmatrix} \sum_{i_k} & \Psi_{i_k}^T & \tau\Pi_{i_k} \\ \Psi_{i_k} & -\tau I & 0 \\ \tau\Pi_{i_k}^T & 0 & -\tau I \end{pmatrix} < 0, \tag{2.21}$$

$$\begin{pmatrix} -\delta_k I & 0 & P \\ 0 & -\delta_k I & G_k^T P \\ P & PG_k & -P \end{pmatrix} \leq 0, \tag{2.22}$$

$$\mu\bar{h} \leq \inf_{k\in N}\{t_k - t_{k-1}\}, \tag{2.23}$$

$$\max\{\bar{\delta}_k + \bar{\delta}_k e^{\lambda\bar{h}}\} \leq M < e^{\lambda\mu\bar{h}}, \tag{2.24}$$

where $\bar{\delta}_k = \frac{\delta_k}{\lambda_{\min}(P)}$, $k \in N$ *and* λ *is the unique positive root of the equation* $\lambda - a + (b+c)e^{\lambda\bar{h}} = 0$, *then closed-loop nominal system (1.1) is exponentially stable and the state feedback control law is given by*

$$u(t) = D_{i_k}^T Q_1 x(t).$$

We introduce the following notations for later use:

$$\begin{aligned} \widetilde{\Pi}_{i_k}^T &= \begin{bmatrix} D_{i_k}^T Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \widetilde{\Psi}_{i_k} &= \begin{bmatrix} D_{i_k}^T Q_1 & D_{i_k}^T Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

Theorem 2.1.4 *If there exist symmetric positive definite matrix* P, *any appropriate dimensional matrices* M, N, Q_i, $i = 1, 2, \ldots, 7$, *and positive real constants* $\sigma, \mu, \lambda, \eta, \rho, \zeta, \xi, \epsilon_1, \epsilon_2, \epsilon_3, a, b, c$ *with* $a > b + c$ *and* $\delta_k > 0$ *for all* $k \in N$ *such that the following LMIs hold:*

$$\begin{pmatrix} \widetilde{\sum}_{i_k} & \widetilde{\Psi}_{i_k}^T & \xi\widetilde{\Pi}_{i_k} \\ \widetilde{\Psi}_{i_k} & -\xi I & 0 \\ \xi\widetilde{\Pi}_{i_k}^T & 0 & -\xi I \end{pmatrix} < 0, \tag{2.25}$$

$$\begin{pmatrix} -\delta_k I & 0 & P \\ 0 & -\delta_k I & G_k^T P \\ P & PG_k & -P \end{pmatrix} \leq 0, \tag{2.26}$$

$$\mu\bar{h} \leq \inf_{k\in N}\{t_k - t_{k-1}\}, \tag{2.27}$$

$$\max\{\bar{\delta}_k + \bar{\delta}_k e^{\lambda\bar{h}}\} \leq M < e^{\lambda\mu\bar{h}}, \tag{2.28}$$

where $\bar{\delta}_k = \frac{\delta_k}{\lambda_{\min}(P)}$, $k \in N$ *and* λ *is the unique positive root of the equation* $\lambda - a + (b+c)e^{\lambda \bar{h}} = 0$, *then closed-loop system (1.1) is robustly exponentially stable and the state feedback control law is given by*

$$u(t) = D_{i_k}^T Q_1 x(t).$$

2.2 Stability for Nonlinear System with Delays

We introduce the following notations for later use.

$$\sum = \left[\Sigma_{i,j}\right]_{17\times 17}, \tag{2.29}$$

where $\Sigma_{i,j} = \Sigma_{j,i}^T, \quad i,j = 1,2,3,...,17,$

$$\begin{aligned}
U_1 &= P_1 C,\\
U_2 &= P_1 H,\\
\Sigma_{1,1} &= U_1 + U_1^T + U_2 + U_2^T + Q_1^T A_1 + A_1^T Q_1 + 2\alpha P_1 + P_2 + P_3 - e^{-2\alpha h_2} P_5\\
&\quad - e^{-2\alpha\beta h_2} P_8 + h_2^2 P_{10} - e^{-2\alpha h_2} P_{11} + (\beta h_2)^2 P_{12} - e^{-2\alpha\beta h_2} P_{13} + P_{14} + P_{15}\\
&\quad + P_{16} + P_{17} + h_2[W_1^T + W_1] + h_2^2 W_3 + \beta h_2 [W_6^T + W_6] + (\beta h_2)^2 W_8\\
&\quad + \epsilon_1 \eta^2 + \delta^2 P_{22} + M_1^T + M_1 + N_1^T + N_1 + Z_1^T A_1 + A_1^T Z_1,\\
\Sigma_{1,2} &= P_1 - Q_1^T + A_1^T Q_2 + h_2^2 Q_3 + (\beta h_2)^2 Q_4 + M_2 + N_2 - Z_1^T + A_1^T Z_2,\\
\Sigma_{1,3} &= -U_1 + Q_1^T B_2 + e^{-2\alpha h_2} P_5 + h_2 R_2^T + e^{-2\alpha h_2} P_{11} - M_1^T + M_3 + N_3\\
&\quad + Z_1^T B_2 + A_1^T Z_3,\\
\Sigma_{1,4} &= -U_2 + e^{-2\alpha\beta h_2} P_8 + \beta h_2 R_5^T + e^{-2\alpha\beta h_2} P_{13} + M_4 - N_1^T + N_4 + A_1^T Z_4,\\
\Sigma_{1,5} &= -U_1 - Q_1^T B_1 - M_1^T + M_5 + N_5 - Z_1^T B_1 + A_1^T Z_5,\\
\Sigma_{1,6} &= -U_2 + M_6 - N_1^T + N_6 + A_1^T Z_6,\\
\Sigma_{1,7} &= h_2[-W_1^T + W_2] + h_2^2 W_4 + M_7 + N_7 + A_1^T Z_7,\\
\Sigma_{1,8} &= \beta h_2[-W_6^T + W_7] + (\beta h_2)^2 W_9 + M_8 + N_8 + A_1^T Z_8,\\
\Sigma_{1,9} &= -e^{-2\alpha h_2} Q_3^T + M_9 + N_9 + A_1^T Z_9,
\end{aligned}$$

$$\Sigma_{1,10} = -e^{-2\alpha\beta h_2} Q_4^T + M_{10} + N_{10} + A_1^T Z_{10},$$

$$\Sigma_{1,11} = M_{11} + N_{11} + A_1^T Z_{11},$$

$$\Sigma_{1,12} = M_{12} + N_{12} + A_1^T Z_{12},$$

$$\Sigma_{1,13} = M_{13} + N_{13} + A_1^T Z_{13},$$

$$\Sigma_{1,14} = M_{14} + N_{14} + A_1^T Z_{14},$$

$$\Sigma_{1,15} = Q_1^T + M_{15} + N_{15} + Z_1^T + A_1^T Z_{15},$$

$$\Sigma_{1,16} = Q_1^T + M_{16} + N_{16} + Z_1^T + A_1^T Z_{16},$$

$$\Sigma_{1,17} = Q_1^T D + M_{17} + N_{17} + Z_1^T D + A_1^T Z_{17},$$

$$\begin{aligned}\Sigma_{2,2} = &-Q_2^T - Q_2 + h_2^2[P_4 + P_5 + P_6] + (\beta h_2)^2[P_7 + P_8 + P_9] + h_2^2 P_{11} \\ &+(\beta h_2) P_{13} + (h_2 - h_1)^2[P_{18} + P_{19}] + (\beta h_2 - \beta h_1)^2[P_{20} + P_{21}] \\ &-Z_2^T - Z_2,\end{aligned}$$

$$\Sigma_{2,3} = Q_2^T B_2 - M_2^T - Z_2^T B_2 - Z_3,$$

$$\Sigma_{2,4} = -N_2^T - Z_4,$$

$$\Sigma_{2,5} = -Q_2^T B_1 - M_2^T - Z_2^T B_1 - Z_5,$$

$$\Sigma_{2,6} = -N_2^T - Z_6,$$

$$\Sigma_{2,7} = -Z_7,$$

$$\Sigma_{2,8} = -Z_8,$$

$$\Sigma_{2,9} = -Z_9,$$

$$\Sigma_{2,10} = -Z_{10},$$

$$\Sigma_{2,11} = -Z_{11},$$

$$\Sigma_{2,12} = -Z_{12},$$

$$\Sigma_{2,13} = -Z_{13},$$

$$\Sigma_{2,14} = -Z_{14},$$

$$\Sigma_{2,15} = Q_2^T + Z_2^T - Z_{15},$$

$$\Sigma_{2,16} = Q_2^T + Z_2^T - Z_{16},$$

$$\Sigma_{2,17} = Q_2^T D + Z_2^T D - Z_{17},$$

$$\Sigma_{3,3} = -e^{-2\alpha h_2} P_2 + h_d P_2 - e^{-2\alpha h_2} P_5 + h_2^2 R_1 - h_2 R_2^T - h_2 R_2 - e^{-2\alpha h_2} P_{11}$$
$$-e^{-2\alpha h_2} P_{19} + \epsilon_2 \rho^2 - M_3^T - M_3 + Z_3^T B_2 + B_2^T Z_3,$$

$$\Sigma_{3,4} = -M_4 - N_3^T + B_2^T Z_4,$$

$$\Sigma_{3,5} = -M_3^T - N_5 - Z_3^T B_1 + B_2^T Z_5,$$

$$\Sigma_{3,6} = -M_6 - N_3^T + B_2^T Z_6,$$

$$\Sigma_{3,7} = -M_7 + B_2^T Z_7,$$

$$\Sigma_{3,8} = -M_8 + B_2^T Z_8,$$

$$\Sigma_{3,9} = e^{-2\alpha h_2} Q_3 - M_9 + B_2^T Z_9,$$

$$\Sigma_{3,10} = -M_{10} + B_2^T Z_{10},$$

$$\Sigma_{3,11} = e^{-2\alpha h_2} P_{19} - M_{11} + B_2^T Z_{11},$$

$$\Sigma_{3,12} = -M_{12} + B_2^T Z_{12},$$

$$\Sigma_{3,13} = -M_{13} + B_2^T Z_{13},$$

$$\Sigma_{3,14} = -M_{14} + B_2^T Z_{14},$$

$$\Sigma_{3,15} = -M_{15} + Z_3^T + B_2^T Z_{15},$$

$$\Sigma_{3,16} = -M_{16} + Z_3^T + B_2^T Z_{16},$$

$$\Sigma_{3,17} = -M_{17} + Z_3^T D + B_2^T Z_{17},$$

$$\Sigma_{4,4} = -e^{-2\alpha\beta h_2} P_3 + \beta h_d P_3 - e^{-2\alpha\beta h_2} P_8 + (\beta h_2)^2 R_4 - \beta h_2 R_5^T - \beta h_2 R_5$$
$$-e^{-2\alpha\beta h_2} P_{13} - e^{-2\alpha\beta h_2} P_{21} - N_4^T - N_4,$$

$$\Sigma_{4,5} = -M_4^T - N_5 - Z_4^T B_1,$$

$$\Sigma_{4,6} = -N_4^T - N_6,$$

$$\Sigma_{4,7} = -N_7,$$

$$\Sigma_{4,8} = -N_8,$$

$$\Sigma_{4,9} = -N_9,$$

$$\Sigma_{4,10} = e^{-2\alpha\beta h_2} Q_4 - N_{10},$$

$$\Sigma_{4,11} = -N_{11},$$

$$\Sigma_{4,12} = e^{-2\alpha\beta h_2} P_{21} - N_{12},$$

$$\Sigma_{4,13} = -N_{13},$$

$$\Sigma_{4,14} = -N_{14}$$

$$\Sigma_{4,15} = -N_{15} + Z_4^T,$$

$$\Sigma_{4,16} = -N_{16} + Z_4^T,$$

$$\Sigma_{4,17} = -N_{17} + Z_4^T D,$$

$$\Sigma_{5,5} = -e^{-2\alpha h_2} P_4 - M_5^T - M_5 - Z_5^T B_1 - B_1^T Z_5,$$

$$\Sigma_{5,6} = -M_6 - N_5^T - B_1^T Z_6,$$

$$\Sigma_{5,7} = -M_7 - B_1^T Z_7,$$

$$\Sigma_{5,8} = -M_8 - B_1^T Z_8,$$

$$\Sigma_{5,9} = -M_9 - B_1^T Z_9,$$

$$\Sigma_{5,10} = -M_{10} - B_1^T Z_{10},$$

$$\Sigma_{5,11} = -M_{11} - B_1^T Z_{11},$$

$$\Sigma_{5,12} = -M_{12} - B_1^T Z_{12},$$

$$\Sigma_{5,13} = -M_{13} - B_1^T Z_{13},$$

$$\Sigma_{5,14} = -M_{14} - B_1^T Z_{14},$$

$$\Sigma_{5,15} = -M_{15} + Z_5^T - B_1^T Z_{15},$$

$$\Sigma_{5,16} = -M_{16} + Z_5^T - B_1^T Z_{16},$$

$$\Sigma_{5,17} = -M_{17} + Z_5^T D - B_1^T Z_{17},$$

$$\Sigma_{6,6} = -e^{-2\alpha\beta h_2} P_7 - N_6^T - N_6,$$

$$\Sigma_{6,7} = -N_7,$$

$$\Sigma_{6,8} = -N_8,$$

$$\Sigma_{6,9} = -N_9,$$

$$\Sigma_{6,10} = -N_{10},$$

$$\Sigma_{6,11} = -N_{11},$$

$$\Sigma_{6,12} = -N_{12},$$

$$\Sigma_{6,13} = -N_{13},$$

$$\Sigma_{6,14} = -N_{14},$$

$$\Sigma_{6,15} = Z_6^T - N_{15},$$

$$\Sigma_{6,16} = Z_6^T - N_{16},$$

$$\Sigma_{6,17} = Z_6^T D - N_{17},$$

$$\Sigma_{7,7} = -e^{-2\alpha h_2} P_{14} + h_2[-W_2^T - W_2] + h_2^2 W_5,$$

$$\Sigma_{7,8} = \Sigma_{7,9} = \Sigma_{7,10} = \Sigma_{7,11} = \Sigma_{7,12} = \Sigma_{7,13} = \Sigma_{7,14} = 0,$$

$$\Sigma_{7,15} = Z_7^T,$$

$$\Sigma_{7,16} = Z_7^T,$$

$$\Sigma_{7,17} = Z_7^T D,$$

$$\Sigma_{8,8} = -e^{-2\alpha\beta h_2} P_{15} + \beta h_2[-W_7^T - W_7] + (\beta h_2)^2 W_1 0,$$

$$\Sigma_{8,9} = \Sigma_{8,10} = \Sigma_{8,11} = \Sigma_{8,12} = \Sigma_{8,13} = \Sigma_{8,14} = 0,$$

$$\Sigma_{8,15} = Z_8^T,$$

$$\Sigma_{8,16} = Z_8^T,$$

$$\Sigma_{8,17} = Z_8^T D,$$

$$\Sigma_{9,9} = -e^{-2\alpha h_2} P_{10},$$

$$\Sigma_{9,10} = \Sigma_{9,11} = \Sigma_{9,12} = \Sigma_{9,13} = \Sigma_{9,14} = 0,$$

$$\Sigma_{9,15} = Z_9^T,$$

$$\Sigma_{9,16} = Z_9^T,$$

$$\Sigma_{9,17} = Z_9^T D,$$

$$\Sigma_{10,10} = -e^{-2\alpha\beta h_2} P_{12},$$

$$\Sigma_{10,11} = \Sigma_{10,12} = \Sigma_{10,13} = \Sigma_{10,14} = 0,$$

$$\Sigma_{10,15} = Z_{10}^T,$$

$$\Sigma_{10,16} = Z_{10}^T,$$

$$\Sigma_{10,17} = Z_{10}^T D,$$

$$\Sigma_{11,11} = -e^{-2\alpha h_1} P_{16} - e^{-2\alpha h_2} P_{19},$$

$\Sigma_{11,12} = \Sigma_{11,13} = \Sigma_{11,14} = 0,$

$\Sigma_{11,15} = Z_{11}^T,$

$\Sigma_{11,16} = Z_{11}^T,$

$\Sigma_{11,17} = Z_{11}^T D,$

$\Sigma_{12,12} = -e^{-2\alpha\beta h_1} P_{17} - e^{-2\alpha\beta h_2} P_{21},$

$\Sigma_{12,13} = \Sigma_{12,14} = 0,$

$\Sigma_{12,15} = Z_{12}^T,$

$\Sigma_{12,16} = Z_{12}^T,$

$\Sigma_{12,17} = Z_{12}^T D,$

$\Sigma_{13,13} = -e^{-2\alpha h_2} P_{18},$

$\Sigma_{13,14} = 0,$

$\Sigma_{13,15} = Z_{13}^T,$

$\Sigma_{13,16} = Z_{13}^T,$

$\Sigma_{13,17} = Z_{13}^T D,$

$\Sigma_{14,14} = -e^{-2\alpha\beta h_2} P_{20},$

$\Sigma_{14,15} = Z_{14}^T,$

$\Sigma_{14,16} = Z_{14}^T,$

$\Sigma_{14,17} = Z_{14}^T D,$

$\Sigma_{15,15} = -\epsilon_1 I + Z_{15}^T + Z_{15},$

$\Sigma_{15,16} = Z_{15}^T + Z_{16},$

$\Sigma_{15,17} = Z_{15}^T D + Z_{17},$

$\Sigma_{16,16} = -\epsilon_2 I + Z_{16}^T + Z_{16},$

$\Sigma_{16,17} = Z_{16}^T D + Z_{17},$

$\Sigma_{17,17} = -e^{-2\alpha\delta} P_{22} + Z_{17}^T D + D^T Z_{17}.$

Theorem 2.2.1 *For given positive real constants h_1, h_2, h_d, α and β, system* (1.11) *is exponentially stable if there exist positive definite symmetric matrices P_i, $i = 1, 2, \ldots, 22$, any appropriate dimensional matrices $C, H, Q_j, W_k, R_s, M_m, N_n, Z_t$,*

$j = 1, 2, \ldots, 4,\ k = 1, 2, \ldots, 10,\ s = 1, 2, \ldots, 6,\ m = 1, 2, \ldots, 17,\ n = 1, 2, \ldots, 17,$ $t = 1, 2, \ldots, 17$ *and positive real constants* ϵ_1 *and* ϵ_2 *such that the following symmetric linear matrix inequalities hold:*

$$\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0, \tag{2.30}$$

$$\begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} > 0, \tag{2.31}$$

$$\begin{bmatrix} P_{10} & Q_3 \\ * & P_{11} \end{bmatrix} \geq 0, \tag{2.32}$$

$$\begin{bmatrix} P_{12} & Q_4 \\ * & P_{13} \end{bmatrix} \geq 0, \tag{2.33}$$

$$\begin{bmatrix} e^{-2\alpha h_2} P_6 - R_3 & W_1 & W_2 \\ * & W_3 & W_4 \\ * & * & W_5 \end{bmatrix} \geq 0, \tag{2.34}$$

$$\begin{bmatrix} e^{-2\alpha\beta h_2} P_9 - R_6 & W_6 & W_7 \\ * & W_8 & W_9 \\ * & * & W_{10} \end{bmatrix} \geq 0, \tag{2.35}$$

$$\sum < 0. \tag{2.36}$$

Moreover, the solution $x(t, \phi, \varphi)$ *satisfies the inequality*

$$\|x(t, \phi, \varphi)\| \leq \sqrt{\frac{N}{\lambda_{\min}(P_1)}} \max[\|\phi\|, \|\varphi\|] e^{-\alpha t}, \quad t \in \mathbb{R}^+,$$

where

$$\begin{aligned} N = {} & \lambda_{\max}(P_1) + h_2 \lambda_{\max}(P_2 + P_{14} + P_{16}) + \beta h_2 \lambda_{\max}(P_3 + P_{15} + P_{17}) \\ & + h_2^3 \lambda_{\max}(P_4 + P_5 + P_6 + P_{18} + P_{19}) + \delta^3 \lambda_{\max}(P_{22}) \\ & + (\beta h_2)^3 \lambda_{\max}(P_7 + P_8 + P_9 + P_{20} + P_{21}) \\ & + h_2^3 \lambda_{\max}\Big(\begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} \Big) + (\beta h_2)^3 \lambda_{\max}\Big(\begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \Big) \\ & + h_2^3 \lambda_{\max}\Big(\begin{bmatrix} P_{10} & Q_3 \\ Q_3^T & P_{11} \end{bmatrix} \Big) + (\beta h_2)^3 \lambda_{\max}\Big(\begin{bmatrix} P_{12} & Q_4 \\ Q_4^T & P_{13} \end{bmatrix} \Big). \end{aligned}$$

We introduce the following notations for later use.

$$\begin{aligned}\Gamma^T &= \big[E^T(Q_1+Z_1) \quad E^T(Q_2+Z_2) \quad E^TZ_3 \quad E^TZ_4 \quad E^TZ_5 \quad E^TZ_6 \quad E^TZ_7 \\ &\quad E^TZ_8 \quad E^TZ_9 \quad E^TZ_{10} \quad E^TZ_{11} \quad E^TZ_{12} \quad E^TZ_{13} \quad E^TZ_{14} \quad E^TZ_{15} \\ &\quad E^TZ_{16} \quad E^TZ_{17}\big].\end{aligned}$$

$$\widetilde{\sum} = \left[\widetilde{\Sigma}_{i,j}\right]_{17\times 17}, \tag{2.37}$$

where $\widetilde{\Sigma}_{i,j} = \widetilde{\Sigma}^T_{j,i} = \Sigma_{i,j}, \quad i,j = 1,2,3,...,17,$ except

$$\begin{aligned}\widetilde{\Sigma}_{1,1} &= U_1+U_1^T+U_2+U_2^T+Q_1^TA_1+A_1^TQ_1+2\alpha P_1+P_2+P_3-e^{-2\alpha h_2}P_5 \\ &\quad +h_2^2P_{10}-e^{-2\alpha h_2}P_{11}+(\beta h_2)^2P_{12}-e^{-2\alpha\beta h_2}P_{13}+P_{14}+P_{15}+P_{16}+P_{17} \\ &\quad +h_2[W_1^T+W_1]+h_2^2W_3+\beta h_2[W_6^T+W_6]+(\beta h_2)^2W_8+\epsilon_1\eta^2+\delta^2P_{22} \\ &\quad -e^{-2\alpha\beta h_2}P_8+M_1^T+M_1+N_1^T+N_1+Z_1^TA_1+A_1^TZ_1+\varepsilon G_1^TG_1, \\ \widetilde{\Sigma}_{1,3} &= -U_1+Q_1^TB_2+e^{-2\alpha h_2}P_5+h_2R_2^T+e^{-2\alpha h_2}P_{11}-M_1^T+M_3+N_3+Z_1^TB_2 \\ &\quad +A_1^TZ_3+\varepsilon G_1^TG_2, \\ \widetilde{\Sigma}_{1,17} &= Q_1^TD+M_{17}+N_{17}+Z_1^TD+A_1^TZ_{17}+\varepsilon G_1^TG_3, \\ \widetilde{\Sigma}_{3,3} &= -e^{-2\alpha h_2}P_2+h_dP_2-e^{-2\alpha h_2}P_5+h_2^2R_1-h_2R_2^T-h_2R_2-e^{-2\alpha h_2}P_{11} \\ &\quad +\epsilon_2\rho^2-M_3^T-M_3+Z_3^TB_2+B_2^TZ_3+\varepsilon G_2^TG_2, \quad \widetilde{\Sigma}_{3,17} = -M_{17}+Z_3^TD \\ &\quad -e^{-2\alpha h_2}P_{19}+B_2^TZ_{17}+\varepsilon G_2^TG_3, \\ \widetilde{\Sigma}_{17,17} &= -e^{-2\alpha\delta}P_{22}+Z_{17}^TD+D^TZ_{17}+\varepsilon G_3^TG_3.\end{aligned}$$

Theorem 2.2.2 *For given positive real constants h_1, h_2, h_d, α and β, system* (1.11) *is robust exponentially stable if there exist positive definite symmetric matrices P_i, $i = 1,2,\ldots,22$, any appropriate dimensional matrices $C, H, Q_j, W_k, R_s, M_m, N_n, Z_t$, $j = 1,2,\ldots,4$, $k = 1,2,\ldots,10$, $s = 1,2,\ldots,6$, $m = 1,2,\ldots,17$, $n = 1,2,\ldots,17$, $t = 1,2,\ldots,17$ and positive real constants ϵ_1, ϵ_2 and ε such that the following* *symmetric linear matrix inequalities hold:*

$$\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0, \tag{2.38}$$

$$\begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} > 0, \tag{2.39}$$

$$\begin{bmatrix} P_{10} & Q_3 \\ * & P_{11} \end{bmatrix} \geq 0, \tag{2.40}$$

$$\begin{bmatrix} P_{12} & Q_4 \\ * & P_{13} \end{bmatrix} \geq 0, \tag{2.41}$$

$$\begin{bmatrix} e^{-2\alpha h_2}P_6 - R_3 & W_1 & W_2 \\ * & W_3 & W_4 \\ * & * & W_5 \end{bmatrix} \geq 0, \tag{2.42}$$

$$\begin{bmatrix} e^{-2\alpha\beta h_2}P_9 - R_6 & W_6 & W_7 \\ * & W_8 & W_9 \\ * & * & W_{10} \end{bmatrix} \geq 0, \tag{2.43}$$

$$\begin{bmatrix} \widetilde{\sum} & \Gamma \\ \Gamma^T & -\varepsilon I \end{bmatrix} < 0. \tag{2.44}$$

Moreover, the solution $x(t, \phi, \varphi)$ satisfies the inequality

$$\|x(t, \phi, \varphi)\| \leq \sqrt{\frac{N}{\lambda_{\min}(P_1)}} \max[\|\phi\|, \|\varphi\|] e^{-\alpha t}, \quad t \in \mathbb{R}^+,$$

where

$$\begin{aligned} N = {} & \lambda_{\max}(P_1) + h_2\lambda_{\max}(P_2 + P_{14} + P_{16}) + \beta h_2 \lambda_{\max}(P_3 + P_{15} + P_{17}) \\ & + h_2^3 \lambda_{\max}(P_4 + P_5 + P_6 + P_{18} + P_{19}) + \delta^3 \lambda_{\max}(P_{22}) \\ & + (\beta h_2)^3 \lambda_{\max}(P_7 + P_8 + P_9 + P_{20} + P_{21}) \\ & + h_2^3 \lambda_{\max}\Big(\begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} \Big) + (\beta h_2)^3 \lambda_{\max}\Big(\begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \Big) \\ & + h_2^3 \lambda_{\max}\Big(\begin{bmatrix} P_{10} & Q_3 \\ Q_3^T & P_{11} \end{bmatrix} \Big) + (\beta h_2)^3 \lambda_{\max}\Big(\begin{bmatrix} P_{12} & Q_4 \\ Q_4^T & P_{13} \end{bmatrix} \Big). \end{aligned}$$

Appendix

A1 P. Niamsup, N. Yotha, **K. Mukdasai**, New Delay-Range-Dependent Robust Exponential Stability Criteria of Uncertain Impulsive Switched Linear Systems with Mixed Interval Nondifferentiable Time-Varying Delays and Nonlinear Perturbations, Discrete Dynamics in Nature and Society, Article ID 406420, 2015, 10 pages.

A2 P. Somchai, **K. Mukdasai**, New delay-range-dependent robust exponential stability criteria for uncertain linear systems with discrete interval and distributed time-varying delays and nonlinear perturbations, Asian-European Journal of Mathematics, Vol. 8, no. 3, 2015, 1550061, 28 pages.

Output

1. ผลงานตีพิมพ์ในวารสารวิชาการนานาชาติ

1.1 P. Niamsup, N. Yotha, **K. Mukdasai**, New delay-range-dependent robust exponential stability criteria of uncertain impulsive switched linear systems with mixed interval nondifferentiable time-varying delays and nonlinear perturbations, **Discrete Dynamics in Nature and Society,** Article ID 406420, 2015, 10 pages.

1.2 P. Somchai, **K. Mukdasai**, New delay-range-dependent robust exponential stability criteria for uncertain linear systems with discrete interval and distributed time-varying delays and nonlinear perturbations, **Asian-European Journal of Mathematics**, Vol. 8, no. 3, 2015, 1550061, 28 pages.

2. การนำผลงานวิจัยไปใช้ประโยชน์

ผลงานวิจัยที่ได้มามีการนำไปใช้ประโยชน์ทั้งเชิงวิชาการ และเชิงสาธารณะโดยทำให้มีการพัฒนาการเรียนการสอนและมีเครือข่ายความร่วมมือสร้างกระแสความสนใจในวงกว้าง

3. อื่น ๆ: การเสนอผลงานในที่ประชุมวิชาการ

A1. P. Niamsup, N. Yotha, **K. Mukdasai**, New Delay-Range-Dependent Robust Exponential Stability Criteria of Uncertain Impulsive Switched Linear Systems with Mixed Interval Nondifferentiable Time-Varying Delays and Nonlinear Perturbations, **Discrete Dynamics in Nature and Society,** Article ID 406420, 2015, 10 pages.

A2. P. Somchai, **K. Mukdasai**, New delayrangedependent robust exponential stability criteria for uncertain linear systems with discrete interval and distributed time-varying delays and nonlinear perturbations, **Asian-European Journal of Mathematics**, Vol. 8, no. 3, 2015, 1550061, 28 pages.