Songklanakarin J. Sci. Technol.
41 (6), 1376-1380, Nov. - Dec. 2019

http://www.sjst.psu.ac.th

Original Article

Effect of fraction of demand backordered in inventory management while obtaining optimum order quantity and reorder point

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Received: 4 January 2018; Revised: 20 June 2018; Accepted: 20 August 2018

Abstract

In this paper a continuous inventory model is studied with a mixture of back orders and lost sales, in which both order quantity and reorder point, are decision variables, when demand per unit time is normally distributed and lead time follows Erlang distribution. The objectives of this study is to estimate the optimal order quantity *(Q)* and reorder point *(r)*, when the annual expected cost is minimum. An algorithm is developed to estimate the optimum solutions. A real example based on primary data is cited to estimate the optimal order quantity and reorder point with a mixture of back orders and lost sales, when annual expected cost is at minimum. This study reveals that the reorder point decreases, when the fraction of demand backordered during stock out period increases to get the almost same amount of optimal order quantity for different values of the fraction of demand backordered.

Keywords: inventory modeling, reorder point, optimal order quantity, fraction of demand backordered

1. Introduction

Almost all the industries and business sectors in India may suffer from some common problems, namely inadequacy of finance, difficulties of marketing, low level technology, competition from large scale industries, unskilled manpower, poor transportation system, inadequate credit assistance, inadequate infrastructure, poor inventory management, and others (Bharathi, 2011). Among the problems, stated here, some are beyond the control of researcher, but some of them can be controlled, i.e., if it is possible to manage the inventory problem, then some of the stated problems viz. financial problem, marketing problem, transportation problem, etc., can be reduced and controlled. Waters (2003) states that to maintain the customer service, it is required to maintain the buffer stock due to the uncertainty in supply and demand. Mercado (2008) says that the objective of inventory management is to meet the customer needs, keeping inventory costs at a reasonable level to produce a profit. Soni (2012) and Mercado (2008) state that a huge investment in inventory introduces overstocking and such a situation will increase holding cost and holding period. So proper review of inventory level is required in order to facilitate uninterrupted production and to guard against the risk of unpredictable changes in demand and supply (Fichtinfer & Arikan, 2011).

The fundamental problems of inventory management can be described by two questions, 1) when should an order be placed and 2) how much should be ordered? The principal goal of inventory management is to make the goods available as per demand of customers, avoiding spoilage, pilferage and obsolesces, maintaining an optimum stock (Clark, 1957).

Inventory management is vital for a business, because an inventory incurs the biggest expenses and so it needs to be carefully controlled in order to run the business effectively. Basically inventory represents 45% - 90% of all expenses for a business and a small scale industry does not have a huge capital, so it is required to ensure that the business has the right goods on hand to avoid spoilage, obsolesce and high holding cost (Haller, 1991; Kansal, 2009). Also it is required to maintain the buffer inventory, to avoid stock out which should be of such extent that the inventory is optimum, investing minimum cost and so that while maintaining buffer inventory

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it is required to avoid goods which are old, worn, shop-worn, obsolete or of wrong color and size, as per the demand of consumers, with respect to the time and location (Kumar, 2011; Suresh & Shashidhar, 2007). To avoid the stock out situation, to fulfill the customers demand in small scale industries, it is required to maintain the safety stock. It is required because replenishment order needs lead time in transit inventory (Stulman, 1989).

2. Review of Literature

Different authors develop different models to minimize the expected cost in different literatures. Soni (2012) established the relation between inventory and sales and to reduce the holding cost inventory model. Kopytov, Greenglaz, Muravyov, and Puzinkevich (2007), Cobb, Rum, and Salmer (2013), and Das (1976) develop a cost model to optimize the order quantity and reorder point, when both demand during lead time and lead time follows either gamma distribution or truncated normal distribution. Following the same avenue Burgin (1972), Moon and Choi (1998), and Wu (2000) estimate the optimal order quantity and re-order point, minimizing the annual expected cost, where demand during lead time is a normal variate and a comparison is made with the distribution free approach of demand during lead time. Lan, Chung, and Wun-Jung (1999), Ouyang, Chen, and Chang (1999) consider both demand and lead time as decision variables and estimate the optimal order quantity and reorder point, introducing a new factor - fraction of the demand back ordered during lead time. Liao and Shyu (1991) laid emphasis to reduce the lead time cost along with optimizing the order quantity and reorder point. Bagchi and Hayya, (1984) advocate demand as truncated normal variate and lead time as Erlang variate and their mixture that is demand follows weighted gamma distribution.

From the intensive literature survey, it is found that several expected cost models are introduced to estimate optimum order quantity and reorder point, but in these models the demand of the products are either normally distributed or a distribution free approach of the demand of products are considered. Also in some other studies, lead time is considered as constant.

In the study both demand of the products and lead time are considered as decision variables, where demand during lead time is normally distributed and lead time follows Erlang distribution. Considering demand is non-negative the marginal distribution of demand is a weighted Gamma variate and an approach is made to minimize the annual expected cost, when demand of the product is a weighted gamma va-riate.

3. Objectives of the Study

1) To compute the optimal order quantity with normal demand and Erlang lead time.
2) To compute the reorder point with normal demand and Erlang lead time.

4. Methodology

Moon and Choi (1998) and Wu (2000) represent the total expected annual cost as -

C(Q,r,L) = Ordering Cost + Holding Cost + Stock out Cost + Lead Time Cost

But avoiding the Lead Time Crashing Cost, the total annual expected cost is -

C(Q,r) = Ordering Cost + Holding Cost + Stock out Cost

$$= \frac{AD}{Q} + h\left[\frac{Q}{2} + r - E(X) + (1-\beta)B(r)\right] + \frac{D}{Q}\left[\pi + \pi_0(1-\beta)\right]B(r) \qquad (1)$$

Where,

A = Fixed ordering cost per order.
D = Average demand per year.
Q = Order quantity (A decision variable).
h = Inventory holding cost per item per year.
r = Reorder point (A decision variable).
π = Fixed penalty cost per unit short.
π_0 = Marginal profit per unit.
L = Length of Lead time.
β = fraction of the demand backordered during the stock out period, $0 \le \beta \le 1$.
E(X) = Expected (mean) demand of the products during lead time.
B(r) = *E(X-r)* = $\int_r^\infty (X-r)f(x)dx$ = Expected shortages per cycle and X is the demand of the product.

Now to minimize the annual expected cost (Equation (1)) with respect to the optimum values of order quantity (Q) and reorder point (r),

$$\frac{\partial C(Q,r)}{\partial Q} = 0$$

$$i.e., Q = \sqrt{\frac{2D[A + \{\pi + \pi_0(1-\beta)\}B(r)]}{h}} \qquad (2)$$

and $\frac{\partial C(Q,r)}{\partial r} = 0$

$$i.e., \frac{\partial}{\partial r}B(r) = -\frac{hQ}{hQ(1-\beta) + D[\pi + \pi_0(1-\beta)]}$$

$$i.e., \quad -[1 - F(r)] = -\frac{hQ}{hQ(1-\beta) + D[\pi + \pi_0(1-\beta)]}$$

$$i.e., \quad F(r) = 1 - \frac{hQ}{hQ(1-\beta) + D[\pi + \pi_0(1-\beta)]} \qquad (3)$$

4.1 Distribution of demand during lead time with normal demand and Erlang lead time

Moon and Choi (1998) and Wu (2000) advo-cates that the demand during lead time is a normal variate with mean μL and variance $\sigma^2 L$, L is the length of Lead Time. But Bagchi and Hayya (1984) and Burgin (1972) highlighted that the demand per unit time 'X' is a normal variate with mean μ and variance σ^2 and demand during lead time for fixed lead time $L=l$ *i.e.*, $X|L=l$ is a normal variate with mean μl and variance $\sigma^2 l$, where the lead time L follows the Erlang distribution with α and k. Also they stated that the marginal density of demand during lead time is –

$$h(x) = \int_{l=0}^{\infty} f(x|L=l)g(L=l)dl$$

In this study authors state that the demand per unit time *'X'* of a product is non-negative, so $0 \le x \le \infty$ and so –

$$h^{*}(x) = \frac{h(x)}{\int_{x=0}^{\infty} h(x)dx} \quad (4)$$

Now, $h(x) = \int_{l=0}^{\infty} \frac{1}{\sigma\sqrt{(2\pi l)}} e^{-\frac{1}{2}\left(\frac{x-\mu l}{\sigma\sqrt{l}}\right)^2} \frac{\alpha^k l^{k-1} e^{-\alpha l}}{(k-1)!} dl$

$$= \left(\frac{\alpha}{\theta}\right)^k \frac{1}{\Gamma(k)} e^{-\frac{x}{\sigma^2}(\theta-\mu)} \sum_{j=0}^{k-1} \frac{(k+j-1)!}{(k-j-1)!j!} \left(\frac{\sigma^2}{2\theta}\right)^j x^{k-j-1}$$

Bagchi and Hayya (1984) and Burgin (1972)

And $\int_0^{\infty} h(x)dx = \int_0^{\infty} \left(\frac{\alpha}{\theta}\right)^k \frac{1}{\Gamma(k)} e^{-\frac{x}{\sigma^2}(\theta-\mu)} \sum_{j=0}^{k-1} \frac{(k+j-1)!}{(k-j-1)!j!} \left(\frac{\sigma^2}{2\theta}\right)^j x^{k-j-1} dx$

$$= \left(\frac{\alpha}{\theta}\right)^k \frac{1}{\Gamma(k)} \sum_{j=0}^{k-1} \frac{(k+j-1)!}{(k-j-1)!j!} \left(\frac{\sigma^2}{2\theta}\right)^j \int_0^{\infty} x^{k-j-1} e^{-\frac{x}{\sigma^2}(\theta-\mu)} dx$$

$$= \left(\frac{\alpha}{\theta}\right)^k \frac{1}{\Gamma(k)} \sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\sigma^2}{\theta-\mu}\right)^{k-j} \left(\frac{\sigma^2}{2\theta}\right)^j, \quad \theta^2 = 2\alpha\sigma^2 + \mu^2$$

Bagchi and Hayya (1984)

So, from (2)

$$h^{*}(x) = \frac{\sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j \left[\frac{\lambda^{k-j}}{\Gamma(k-j)} x^{k-j-1} e^{-\lambda x}\right]}{\sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j}, \qquad \lambda = \frac{\theta-\mu}{\sigma^2} \quad (5)$$

Bagchi and Hayya (1984)

which is the probability density function of weighted gamma distribution.

So, here the marginal density of the joint distribution of normal and Erlang distribution is a weighted Gamma distribution, *i.e.* the distribution of demand is a weighted Gamma distribution, when demand during lead time follows normal and lead time follows Erlang distribution. In this case the parameters of the distribution can be estimated either by the method of moments or by the method of maximum likelihood estimator and the parameters k, σ^2, θ *and* λ can be estimated from the estimated parameters of Normal distribution and Erlang distribution separately.

Now

$B(r) = \int_r^{\infty} (x-r) h^{*}(x) dx$

$$= \int_r^{\infty} (x-r) \frac{\sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j \left[\frac{\lambda^{k-j}}{\Gamma(k-j)} x^{k-j-1} e^{-\lambda x}\right]}{\sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j} dx$$

$$= \int_r^{\infty} \frac{\sum_{j=0}^{k-1} \frac{(k-j)(k+j-1)!}{\lambda . j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j \left[\frac{\lambda^{k-j+1}}{\Gamma(k-j+1)} x^{(k-j+1)-1} e^{-\lambda x} dx\right]}{\sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j} - r\left[\int_r^{\infty} \frac{\sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j \left[\frac{\lambda^{k-j}}{\Gamma(k-j)} x^{k-j-1} e^{-\lambda x}\right]}{\sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j} dx\right]$$

$$= \frac{\sum_{j=0}^{k-1} \frac{(k-j)(k+j-1)!}{\lambda . j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j \int_r^{\infty} \frac{\lambda^{k-j+1}}{\Gamma(k-j+1)} x^{(k-j+1)-1} e^{-\lambda x} dx}{\sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j} - r\left[\frac{\sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j \int_r^{\infty} \frac{\lambda^{k-j}}{\Gamma(k-j)} x^{k-j-1} e^{-\lambda x}}{\sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j} dx\right] \quad (6)$$

and $B(r)\big|_{r=0} = \int_0^{\infty} x\, h^{*}(x) dx$

$$= \frac{\sum_{j=0}^{k-1} \frac{(k-j)(k+j-1)!}{\lambda . j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j \int_0^{\infty} \frac{\lambda^{k-j+1}}{\Gamma(k-j+1)} x^{(k-j+1)-1} e^{-\lambda x} dx}{\sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j}$$

$$= \frac{\sum_{j=0}^{k-1} \frac{(k-j)(k+j-1)!}{\lambda . j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j}{\sum_{j=0}^{k-1} \frac{(k+j-1)!}{j!} \left(\frac{\lambda\sigma^2}{2\theta}\right)^j} \qquad \text{[since, Total probability=1]} \quad (7)$$

Using the following iterative algorithm, the optimum value of order quantity (Q) and the reorder point (r) can be obtained -

a) Evaluate $B(r)\big|_{r=0}$ using (7), for the estimated values of k, σ^2, λ and θ.
b) Evaluate Q, using $B(r)\big|_{r=0}$ in (2)
c) Evaluate F(r), using the value of Q in (3)
d) Determine 'r' from F(r), F(r) is the distribution function of Gamma distribution.
e) Obtain $B(r)$ Using the value of 'r' in (6)
f) Again evaluate Q, using the value of $B(r)$ in (ii)
g) Repeat these steps until no change occurs in the values of Q and r, upto a certain decimal point, which is the optimum value of order quantity (Q) and reorder point (r).

5. Data Collection, Calculation and Result

To check the fitting of the distribution and the basic expected cost model, a primary data is collected from Sualkuchi, from 16th to 27th December, 2016. Here the data is collected on the demand of the *Mulberry* silk yarns, its demand and the required lead time for last six years (ie from 2011 to 2016). Sualkuchi is a multicast town, situated on the North bank of the river Brahmaputra, about 35 kilometers away from Guwahati, which is known as the 'Manchester of Assam'. Having a long tradition of silk weaving at least since the 17th century, Sualkuchi is the prime center of the silk hand loom industry of Assam. Originally Sualkuchi is a craft village, having several cottage industries, *viz.*- oil pressing in the traditional *ghani*, gold smithy, pottery, handloom industry, etc. But now, all most all the cottage industries except handloom industry are extinct gradually and the artisans have already taken up silk weaving as their profession.

And based on the sample data, the estimated value of the parameters of Erlang distribution (k and α) is obtained using the method of moments, where the estimated values are –

$$\hat{k} = 28 \quad \text{and} \quad \hat{\alpha} = 7.441122355$$

Here the probability density function is

$$h^*(x) = \frac{\sum_{j=0}^{27} \frac{(27+j)!}{j!} (0.067083562)^j \left[\frac{0.057493301^{28-j}}{\Gamma(28-j)} x^{27-j} e^{-0.057493301.x}\right]}{\sum_{j=0}^{27} \frac{(27+j)!}{j!} (0.067083562)^j}, x \geq 0$$

In this study it is found that $A=35600$, $D=1072$, $h=125.14$, $\pi=2066$, $\pi_0=1854$ and based on the collected information the values of reorder point and the values of optimal order quantity are obtained for different values of β -

From Table 1 and Figure 1, it is found that though the amount of order quantity are almost same, but the reorder level reduces, when the fraction of the demand backordered during the stock out period increases, *i.e.* the reorder point (r) and the fraction of the demand backordered during the stock out period (β) does not have any significant impact on the amount of order quantity (Q).

Table 1. Values of reorder point and optimal order quantity for different values of fraction of the demand backordered (β).

β	r	Q
β = 0	650.62	826.1582
β = 0.1	648.15	826.4353
β = 0.2	645.55	826.7205
β = 0.3	642.77	827.0492
β = 0.4	639.82	827.3955
β = 0.5	636.65	827.7894
β = 0.6	633.25	828.2179
β = 0.7	629.58	828.6923
β = 0.8	625.57	828.2473`
β = 0.9	621.19	829.8692
β = 1	616.35	830.5947

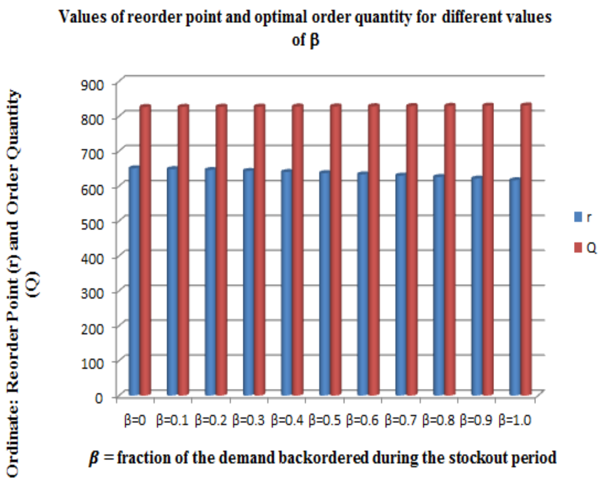

Figure 1. Values of reorder point and optimal order quantity for different values of fraction of the demand backordered (β).

6. Conclusions

This paper presents an inventory model with normal demand and Erlang lead time to minimize the total annual expected inventory cost, which is an aggregate of ordering cost, holding cost and stock out cost. In this study also backorder of demand during lead time is considered. Again considering the annual expected cost is at minimum, the optimal order quantity and reorder points are estimated and based on a sample data; it is found that the reorder point decreases, when the fraction of demand backordered during stock out period increases, to get an almost same amount of optimal order quantity.

In future the researcher can introduce different models with other different distributions, namely demand is a normal variate and lead time follows exponential distribution, demand follows geometric distribution and lead time is an exponential variate *etc.* Also these models can be applied in other different small scale industries, *viz*. bamboo and cane industries, bakery, candle industries, fruit juice making industries, jam jelly making industries and in different large scale industries, *viz*. biscuit industries, cement industries, chocolate industries, cosmetic industries, poultry firm *etc*., knowing the distribution of demand and lead time properly.

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