ABSTRACT

In this research, we consider the sufficient conditions for the existence of solutions of a discrete fractional boundary value problem of fractional difference equations In this paper we consider a Caputo fractional sum-difference equations with fractional sum boundary value conditions of the form

$$\begin{aligned} &\Delta_C^{\alpha} u(t) = f(t+\alpha-1, u(t+\alpha-1), (\Psi^{\beta} u)(t+\alpha-2)), \quad t \in \mathbb{N}_{0,T}, \\ &u(\alpha-2) = y(u), \\ &u(T+\alpha) = \Delta^{-\gamma} g(T+\alpha+\gamma-3)\, u(T+\alpha+\gamma-3), \end{aligned}$$

where $1 < \alpha \leq 2$, $0 < \beta \leq 1,\ 2 < \gamma \leq 3$, Δ_C^{α} is the Caputo fractional difference operater order α, $y \in C(U, U)$ and $g \in C(\mathbb{N}_{\alpha-2,T+\alpha}, \mathbb{R}^{+} \cap U)$ are given functions, $f : \mathbb{N}_{\alpha-\not{\vDash},\mathbb{T}+\alpha} \times \mathbb{U} \times \mathbb{U} \to \mathbb{U}$, for $\varphi : \mathbb{N}_{\alpha-2,T+\alpha} \times \mathbb{N}_{\alpha-2,T+\alpha} \to [0, \infty)$,

$$(\Psi^{\beta} u)(t) := [\Delta^{-\beta} \varphi\, u](t+\beta) = \frac{1}{\Gamma(\beta)} \sum_{s=\alpha-\beta-2}^{t-\beta} (t-\sigma(s))^{\underline{\beta-1}} \varphi(t, s+\beta)\, u(s+\beta).$$

Our goal is to establish some criteria of existence for the boundary problems with nonlocal-sum boundary condition, using the Banach fixed point theorem and the Schaefer's fixed point theorem. Finally, we present some examples to show the importance of these results.