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Sophisticated Derivatives, Simple Hedging*

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ABSTRACT

I review how static hedges for Thailand's popular sophisticated derivatives are constructed from portfolios of vanilla options. The hedges are simple, and rebalancing is not needed or rare. Although the hedges are designed under restrictive assumptions, and sometimes the replication is approximate, previous empirical tests showed that the hedges outperformed the conventional dynamic hedges for most derivatives and practical market conditions.

Keywords: Derivatives, Hedging

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บทคัดย่อ

ผู้เขียนประมวลวิธีถัวความเสี่ยงแบบสถิตสำหรับอนุพันธ์ชนิดซับซ้อนซึ่งเป็นที่นิยมในประเทศไทยโดยใช้กลุ่มออปชันชนิดเรียบง่าย การถัวความเสี่ยงทำได้ง่าย การปรับกลุ่มออปชันที่ใช้ไม่มีความจำเป็นหรือทำเพียงน้อยครั้ง แม้วิธีที่ใช้จะได้รับการออกแบบภายใต้สมมติฐานที่เคร่งครัด และการลดความเสี่ยงเป็นเพียงการประมาณ การศึกษาเชิงประจักษ์ในอดีตพบว่า ส่วนมากสำหรับอนุพันธ์ชนิดซับซ้อน วิธีถัวแบบสถิตให้ผลลัพธ์ที่เหนือกว่าวิธีถัวแบบพลวัตภายใต้ภาวะตลาดทั่วไป

คำสำคัญ : อนุพันธ์ การถัวความเสี่ยง

INTRODUCTION

The Thai market grows, evolves, and becomes more connected with the world's capital markets. As a result, financial technologies improve, investors and fundraisers are better educated and informed, and regulations and taxes continually change to accommodate market efficiency. Competition in these business environments pressures financial engineers (FEs) to design products even more effectively to accommodate target investors and fundraisers' needs. The products must be innovative. They must improve on or make a significant contribution to existing ones.

Despite the innovative designs, from an FE's viewpoint, innovative products are structured products. It is likely that the risks of these products are extremely high. Together with the product designs, the FE will have to propose effective hedging strategies to decrease the risks to acceptable levels.

Structured products can be decomposed into two components. The first component is the core. It is a straight bond whose function is for the second component to embed into it. The second component is the performance component. It pays returns with respect to certain market conditions being demanded by investors or fundraisers. The FE embeds sophisticated, exotic derivatives for the performance. Because the risks associated with the straight bond are low and easy to manage, it is the risks from the sophisticated derivatives with which the FE is concerned.

Conventional approaches to manage the derivatives' risks are dynamic delta and delta-gamma hedging. In the dynamic delta-hedging strategy, the hedged portfolio consists of the derivatives and hedging assets. Their deltas are equal and of opposite signs so that the resulting profit and loss from the underlying price movement cancel each other out. As the underlying price is moving with time, the portfolio is dynamically rebalanced to maintain a delta-neutral position. The dynamic delta-hedging strategy fails when the underlying price jumps. The dynamic delta-gamma-hedging strategy, in which both the deltas and the gammas of the derivatives and hedging assets are equal, is proposed to mitigate the problem (Raju, 2012).

Continuous, dynamic rebalancing is time-consuming, incurs large transaction costs, and suffers large losses from extreme price reversals. It is not very practical. Meanwhile, discrete, dynamic rebalancing risks under- or overhedging. Bowie and Carr (1994) and Derman, Ergener, and Kani (1995) added that dynamic hedging, especially for sophisticated derivatives, required an extensive knowledge of advanced mathematics, the strategies were not very intuitive, and the hedges were difficult. Static hedging is preferred. Its construction makes no assumptions beyond those of standard options theories. Once the hedging structure is selected, portfolio rebalancing is no longer needed or rarely adjusted.

Static Hedging

To construct a static hedging strategy, the FE designs a portfolio of vanilla call or put options with varying exercise prices and expirations but fixed member options and weights. This portfolio replicates the values of the target derivatives under all possible underlying prices and times from today onward to the expiration. In addition to being applied for hedging, the static strategy can be applied for valuation. Premiums are sometimes reduced and traders are more competitive due to lower transaction costs and times.

According to Thailand's Securities and Exchange Commission, from January 1, 2018, to July 31, 2018, commercial banks and securities companies issued structured notes amounting to 32.03 billion baht. Approximately 30 billion baht worth of these notes had embedded options on one underlying price, while the remainder had rainbow features. Sophisticated, exotic features, e.g. barrier, lookback, and digital features, or their combination or modification were sold by leading issuers such as Phatra Securities. In this article, I review the static hedges for the barrier, lookback, and digital features. I will discuss rebates because some issuers bundle them with barrier features. The underlying price is the stock-futures price. The specification is made with respect to the common practices in the Thai market.

Barrier Option Features

Barrier options can be knock-in or knock-out. The knock-in options start their lives when the underlying price crosses predetermined barriers. Otherwise, the options expire as worthless. The knock-out options are vanilla options. However, if the underlying price crosses the barrier, the options expire immediately.

Dynamic hedging strategies are difficult. Despite being closed formed, the pricing formulas for barrier options are highly complicated (Rubinstein & Reiner, 1991). Closed-formed deltas and gammas are unknown. For hedging, these Greeks are estimated numerically. Most importantly, when the underlying price is at the barrier, the gammas are infinite. The risk cannot be hedged.

To construct a static hedge, turn first to the simplest, down-and-in call feature with equal barrier and exercise prices. A down-and-in call is alive only when the underlying price falls from its current level and crosses the barrier before the expiration date. The option is equivalent to a vanilla put if the underlying price finishes higher than the exercise price. The two options pay nothing at expiration. This fact suggests to the hedger to hedge the down-and-in call using the vanilla put.

It is important to note that when the barrier is crossed, the down-and-in call becomes an active vanilla call, while the hedging vanilla put is not equivalent to the vanilla call. The static hedge is imperfect.

Bowie and Carr (1994) recalled the put-call parity (Stoll, 1969) in equation (1).

$$C(X) - P(X) = e^{-rT}(F - X) \tag{1}$$

C(X) and P(X) are the call and put option prices, respectively. The exercise price is X. F is the underlying futures price, r is the riskless interest rate, and T is the time to expiration. When F crosses X, F = X. The call and put are equal in value. The hedger can sell the hedging put, and then use the receipt to buy exactly the target call. The hedge now becomes perfect.

In the market, the barrier and exercise price can differ. If this is the case, the static hedge that relies on the put-call parity no longer works. The hedge must be redesigned. To proceed, put-call symmetry is introduced.

The put-call symmetry relates the prices of vanilla puts and calls at exercise prices X and K on opposite sides of the futures price F, as in equation (2) (Carr, 1994; Carr, Ellis, & Gupta, 1998).

$$C(X)\frac{1}{\sqrt{X}} = P(K)\frac{1}{\sqrt{K}} \tag{2}$$

The relationship of the exercise prices with the underlying futures price is imposed by $\sqrt{XK} = F$. Bowie and Carr (1994) applied the relationship for the redesign. Let X be the exercise price of the down-and-in call, and $H \le X$ is its barrier. When the underlying price crosses the barrier, H = F. The hedging put's exercise price K is $\frac{H^2}{X}$, thus resulting in the put price of $P(\frac{H^2}{X})$. From the put-call symmetry, the hedger will have to hold $\frac{X}{H}$ units of the hedging put so that the sale receipt $\frac{X}{H}P(\frac{H^2}{X})$ can buy the emergent call for C(X).

To conclude, the static hedge is to hold $\frac{X}{H}$ units of the hedging put whose exercise price is $\frac{H^2}{X}$. The hedger sells the put when the underlying price crosses the barrier and then buys the call for C(X) to continue a perfect hedge.

The two examples from Bowie and Carr (1994) demonstrated that hedging puts gave the same cash flows as the target barrier calls. The prices must equal, hence constituting a simple pricing approach alternative to that of Rubinstein and Reiner (1991). For the static hedging of other types of barrier options, readers may consult Bowie and Carr (1994), Reiner et al. (1995), Carr et al. (1998), and Jun and Ku (2015).

Lookback Option Features

Lookback options set the exercise prices in retrospect based on the most beneficial underlying prices over their lives. To construct the static hedge, note that the call gives the same cash flow $F_T - F_0$ as does the futures contract if the underlying price never falls below the current futures price. F_T and F_0 are the futures prices on the option's expiration date and today's date, respectively. However, if the lowest underlying price is $F_{t \le T} < F_0$, the option pays $F_T - F_t = (F_T - F_0) + (F_0 - F_t)$. $F_T - F_0$ is the payoff from the futures contract, while $F_0 - F_t > 0$ is the contribution from the option's lookback feature. A long position in the futures contract alone cannot hedge the lookback call perfectly.

To hedge for the $F_0 - F_t$ part, Bowie and Carr (1994) introduced a down-and-in bond (DIB). This derivative is hypothetical. It pays 1 baht on the expiration date T if $F_{t \le T}$ touches the barrier $H_i < F_0$.

Let us assume for the moment that $F_{t \le T} = F_0 - 1$, where 1 is the assumed tick size of 1 baht. The option's payoff is $(F_T - F_0) + 1$. To hedge for the additional 1-baht payout, the hedger must add the DIB, whose exercise price is $H_1 = F_0 - 1$, to the hedge. There are F_0 possible values for the exercise price $F_{t \le T}$, ranging from $F_0 - 1$ to $F_0 - F_0$. This fact leads the hedger to hold one unit each of F_0 DIB(H_i)s with barriers $H_i = F_0 - i$. $i = 1,\ldots,F_0$. The static hedge by a portfolio of a futures contract and F_0 DIBs is a perfect hedge. Because the value of the futures contract is zero, the replication portfolio for the target lookback call is the summed values of the F_0 DIBs.

Readers may argue that the replication portfolio does not fit the static-hedging definition (Loucks, 2010). It does not hold vanilla options. To ensure that the strategy is static, recall the put-call parity. A futures contract is equivalent to a portfolio that longs one unit of vanilla call and shorts one unit of vanilla put with the same F_0 exercise price. Moreover, a DIB(H_i) with barrier H_i is equivalent to a portfolio that longs 2 units of digital put DP(H_i) with exercise price H_i and shorts $\frac{1}{H_i}$ units of vanilla put P(H_i) with exercise price H_i (Bowie & Carr, 1994).

$$DIB(H_i) \;=\; 2DP(H_i) - \frac{1}{H_i} P(H_i) \qquad (2)$$

A DP (digital call or DC) pays 1 baht on the expiration day if the underlying price finishes at or below (above) the exercise price H_i. Otherwise, it pays nothing. Furthermore, Reiner and Rubinstein (1991) and Chriss and Ong (1995) showed that a DP (DC) could be replicated by a portfolio of vanilla puts (calls).

Digital Option Features

Although the pricing formulas and the Greeks of digital options are closed-formed and simple (Reiner & Rubinstein, 1991), the dynamic hedging is difficult and very risky, especially when the underlying price fluctuates around the exercise price and the expiration date is near.

The options can be hedged by static portfolios of vanilla options. For a digital put and call with exercise price H_i, the replication portfolios are the vertical put spread $lim_{n\to\infty} \frac{n}{2}\{P(H_i + \frac{1}{n}) - P(H_i - \frac{1}{n})\}$ and the vertical call spread $lim_{n\to\infty} \frac{n}{2}\{C(H_i - \frac{1}{n}) - C(H_i + \frac{1}{n})\}$, respectively (Reiner & Rubinstein, 1995). This replication converges faster than that of Chriss and Ong (1995) (Bowie & Carr, 1994).

The replication by these limit spreads is impractical. Carr et al. (1998) applied the Richardson extrapolation to construct portfolios that could replicate the spreads approximately. I use the modified Geske-Johnson formula over the original formula used by Carr et al. (1998). Not only does the modified formula overcome the nonuniform convergence found in the original formula, but it also gives more accurate results (Chang, Chung, & Stapleton, 2007).

The $DC(H_i)$ and $DP(H_i)$ with the exercise price H_i are replicated by the portfolios of vanilla calls $C(H_i \pm \frac{1}{n})$ and puts $P(H_i \pm \frac{1}{n})$ in equations (4) and (5), respectively.

$$DC(H_i) = 10\frac{2}{3}C(H_i - \frac{1}{4}) - 10\frac{2}{3}C(H_i + \frac{1}{4}) - 4C(H_i - \frac{1}{2}) + 4C(H_i + \frac{1}{2}) + \frac{1}{3}C(H_i - 1) - \frac{1}{3}C(H_i + 1), \qquad (4)$$

$$DP(H_i) = 10\frac{2}{3}P(H_i + \frac{1}{4}) - 10\frac{2}{3}P(H_i - \frac{1}{4}) - 4P(H_i + \frac{1}{2}) + 4P(H_i - \frac{1}{2}) + \frac{1}{3}P(H_i + 1) - \frac{1}{3}P(H_i - 1), \qquad (5)$$

Although the approximation is accurate (Chang et al., 2007), Carr et al. (1998) cautioned that it deteriorated near expiration when the underlying price was near the exercise price.

Other Option Features

The exotic option features that can be statically hedged are not limited to the barrier, lookback, and digital features. The features that I did not cover are not popular in the Thai market, or the analyses of the features do not lead to useful theoretical and practical extensions. However, interested readers may see Rubinstein (1991) for chooser options; Levy (1996) for Asian options; Thomas (1996) for compound options; Su (2005) for basket options; Chung and Shih (2009) for American vanilla options; Chung, Shih, and Tsai (2009) for American exotic options; and Molchanov and Schmultz (2010) for rainbow options.

Rebates

Sometimes, knock-out options offer rebates when they are knocked out, and knocked-in options pay rebates at expiration when the options expire out of the money. Rebates are valuable derivatives.

Down-and-In Rebates

Readers can apply the DIB(H_i) to the price and statically hedge the rebates for knock-in options. Suppose the payoff of a down-and-in rebate DIR(H_i) with barrier H_i is R baht. It is paid at expiration only if the futures price F never crosses H_i. The hedger can construct a static replication portfolio by first longing R units of a zero-coupon bond that pays 1 baht at expiration. Next, he shorts R units of the DIB(H_i). The value of this portfolio is

$$\mathrm{DIR}(H_i) = Re^{-rT} - R \times \mathrm{DIB}(H_i) \tag{6}$$

For this portfolio, if the option expires out of the money, the bond pays R baht and the DIB(H_i) is worth nothing. Otherwise, the bond and DIB(H_i) are worth R baht and therefore cancel each other out. The rebate for these two cases are R and 0.00, respectively, thus satisfying the obligation of the DIR(H_i).

Up-and-In Rebates

I introduce an up-and-in *bond*, UIB(H_i), with the exercise price H_i to analyze an up-and-in rebate UIR(H_i) with respect to barrier H_i. The UIB(H_i) is a derivative that pays 1 baht at expiration if F rises and reaches H_i prior to the expiration. Otherwise, it pays nothing. Carr et al. (1998) showed that the UIB(H_i) can be priced and statically replicated by the hedger longing two units of DC(H_i) and $\frac{1}{H_i}$ units of C(H_i). The static portfolio is

$$\mathrm{UIB}(H_i) = 2\mathrm{DC}(H_i) + \frac{1}{H_i}\mathrm{C}(H_i) \tag{7}$$

To price and hedge the UIR(H_i), the hedge may follow similar steps to construct the replication DIR(H_i) to obtain

$$\mathrm{UIR}(H_i) = Re^{-rT} - R \times \mathrm{UIB}(H_i) \tag{8}$$

Even Simpler Hedging

Static Hedging of Vanilla Options

It should be clear to readers now that static hedging is simple. Despite its static simplicity, at times it is difficult to find the prescribed vanilla options. The question is whether it is possible to use readily available vanilla options to substitute for the prescribed ones?

Carr and Wu (2014) showed that the substitution was possible. A European call can be replicated by a portfolio of calls with a shorter expiration. Suppose that the target call C(K,T) has the exercise price K and expiration T, while the M calls, $C_{j=1,\ldots,M}(K_j,\tau)$, in the replication portfolio have the exercise prices K_j and the same expiration $\tau < T$. The portfolio is

$$C(K,T) = \sum_{j=1}^{M} W_j C_j(K_j,\tau) \tag{9}$$

W_j is the portfolio weight, which is equal to

$$W_j = \frac{\omega(K_j)K_j\sigma\sqrt{2(T-\tau)}}{e^{-x_j^2}} w_j, \tag{10}$$

where $\omega(K_j) = \dfrac{N(d_j)}{K_j\sigma\sqrt{(T-\tau)}}$, $d_j = \dfrac{Ln\left(\dfrac{K_j}{K}\right)+\left(r+\dfrac{\sigma^2}{2}\right)(T-\tau)}{\sigma\sqrt{(T-\tau)}}$, and $K_j = Ke^{x_j\sigma\sqrt{2(T-\tau)}-\left(r+\frac{\sigma^2}{2}\right)(T-\tau)}$. σ^2 is the variance of the log underlying return. $N(d_j)$ denotes the probability density of a standard normal variable evaluated at d_j. Finally, $(x_j, w_j)_{j=1,\ldots,M}$ are the node-and-weight pairs of the Gauss-Hermite quadrature. The pairs for $M = 2,\ldots,6$ are shown in Table 1.

Table 1: Nodes (x_j) and Weights (w_j) of the Gauss-Hermite Quadrature

j	m = 2		m = 3		m = 4		m = 5		m = 6	
	x_j	w_j	x_j	w_j	x_j	w_j	x_j	w_j	x_j	w_j
1	−0.7071	0.8862	−1.2247	0.2954	−1.6507	0.0813	−2.0202	0.0200	−2.3506	0.0045
2	0.7071	0.8862	0.0000	1.1816	−0.5246	0.8049	−0.9586	0.3936	−1.3358	0.1571
3			1.2247	0.2954	0.5246	0.8049	0.0000	0.9453	−0.4361	0.7246
4					1.6507	0.0813	0.9586	0.3936	0.4361	0.7246
5							2.0202	0.0200	1.3358	0.1571
6									2.3506	0.0045

Source: https://keisan.casio.com/exec/system/1281195844

The Original Geske-Johnson Formula

Digital options are important options. From both theoretical and practical perspectives, the options can be used to replicate, hedge, and price various sophisticated derivatives (Ingersoll, 2000). I propose that the readers use the modified Geske-Johnson formula together with the Reiner-Rubinstein portfolio to replicate the $DC(H_i)$ and $DP(H_i)$ because the approach is more accurate. However, the replication requires 6 options, while the less accurate, original Geske-Johnson formula together with the Chriss-Ong portfolio requires only 4 options. Readers who prefer simpler hedging may choose the original formula, which gives

$$DC(H_i) = 6C(H_i) - 0.5C(H_i + 1) + 8C(H_i + \frac{1}{2}) - 13.5C(H_i + \frac{1}{3}), \tag{11}$$

$$DP(H_i) = 6P(H_i) - 0.5P(H_i - 1) + 8P(H_i - \frac{1}{2}) - 13.5P(H_i - \frac{1}{3}). \tag{12}$$

Performance Comparison of Static and Dynamic Hedging Strategies

Static hedging needs no or few rebalancing routines. Its performance should be superior to that of dynamic hedging due to low transaction costs and no under- or overhedging (e.g., Bowie & Carr, 1994). Tompkins (2002) argued that this claim was not necessarily true. A static-hedging portfolio is constructed under certain restrictive assumptions. In reality, these assumptions may not be satisfied. Examples include hedging costs, bid-ask spreads, stochastic volatility, and model risks.

In the literature, researchers compared the performance of static hedging against that of dynamic hedging for various types of exotic options. In most studies, static hedging showed superior performance.

Using a Monte-Carlo simulation approach, Tompkins (2002) made the comparison under the positive-transaction-costs and stochastic-volatility conditions for the compound, digital, chooser, Asian, barrier, and lookback options. The researcher reported that static hedging performed better than dynamic hedging in most cases. Engleman, Fengler, Nelholm, and Schwender (2006), Nalholm and Poulsen (2006), and Jun and Ku (2015) consistently reported the superior performance of static hedging over dynamic hedging for barrier options of different types and under various market conditions. Chung and Shih (2009) examined the static hedging performance for American vanilla options and reported that the numerical efficiency of the static hedge portfolio was comparable to advanced numerical methods. Moreover, the resulting delta and gamma calculations were very accurate. Finally, the static hedging of vanilla options is superior to the conventional dynamic hedging in practical situations when the analysis was based on the S&P 500 index options data (Carr & Wu, 2012).

CONCLUSION

To compete successfully in today's market, financial engineers are under great pressure to design products in innovative ways to best satisfy investors' and fundraisers' needs. The innovative products almost always exhibit the features of high-risk, sophisticated, exotic derivatives. The high risk must be managed.

The derivatives can be hedged statically by a portfolio of vanilla options. The portfolio needs no or few rebalancing routines over the life of its target. The hedge is simple and intuitive. It does not require advanced mathematical knowledge and skills. Moreover, it can save many transaction costs over the conventional dynamic hedge.

In this article, I reviewed the static hedging strategies for barrier, lookback, and digital options. In the Thai market, these options are sold over the counter, and the option features are popular and embedded in structured notes.

Although static hedging strategies are designed under restrictive assumptions that are hardly satisfied in reality, previous studies tested and found that the strategies outperformed the dynamic hedging strategies for most options and practical market conditions.

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