CHAPTER V

ADAPTIVE FINITE ELEMENT ALGORITHM

In this Chapter, we designed an adaptive algorithm for finite element methods for obtaining sequences of approximate solution $\{U_h^n\}$. The algorithm in this Chapter is designed based on the a posteriori error estimations obtained in the previous Chapter.

5.1 Time Error Control

Let TOL_{time} be the tolerance for the error control on time discretization. We control the error on time discretization according to

$$\sum_{n=1}^{m} \tau_n (\eta_{time}^n)^2 \leq TOL_{time}. \tag{5.1}$$

We used equi-distribute technique (equally distribution of errors on all elements) in order to control the time error indicators η_{time}^n by checking

$$(\eta_{time}^n)^2 < \frac{TOL_{time}}{T} \tag{5.2}$$

for all $n = 1, 2, ..., m$, to guarantee (5.1).

Given $\delta_1 \in (0, 1)$ and $\delta_2 > 1$, we use δ_1 to shorten the time-step size τ_n in order to reduce the time error indicator, and if the error indicator is too small, we expand the time-step size with δ_2 in order to improve the performance.

Typically, the smaller time-step size, the more accuracy we get. But if the time-step size is too small, it will reduce the performance of the program, namely, more loops in the program. So we may control the time error indicator in such a way that

$$\frac{\theta_{time} TOL_{time}}{T} \leq (\eta^n_{time})^2 \leq \frac{TOL_{time}}{T} \tag{5.3}$$

where $\theta_{time} \in (0,1)$ is a chosen parameter. (Typically, the value of θ_{time} is 0.5)

The following is an algorithm for obtaining a suitable time-step size τ_n with given parameters TOL_{time}, δ_1, δ_2 and τ_{n-1}, where $\delta_1 \delta_2 < 1$.

Time Step Control Algorithm

1. Set $\tau_n = \tau_{n-1}$.
2. Solve for U^n_h and compute the error time indicators η^n_{time}.
3. If (5.3) is satisfy, then exit the loop,

   else go to the next step.
4. If $(\eta^n_{time})^2 > \frac{TOL_{time}}{T}$ do $\tau_n = \delta_1 \tau_n$ and go to step 2,

   else $\tau_n = \delta_2 \tau_n$ and go to step 2.

Remark 5.1. *This algorithm guarantees that* (5.3) *is satisfied in finite steps.*

5.2 Space Error Control

We balance between accuracy and performance by the controlling parameter $\theta_{space} \in (0,1)$ by checking the condition

$$\frac{\theta_{space} TOL_{space}}{T} \leq (\eta^n_{space})^2 \leq \frac{TOL_{space}}{T}. \tag{5.4}$$

With mesh $\mathcal{M}^n$, we refine the mesh in order to increase accuracy and coarsen the mesh for maintaining performance. Let TOL_{space} be the tolerance for the space error control,

Adaptive Finite Element Algorithm

1. Set $\mathcal{M}^n = \mathcal{M}^{n-1}$.

2. Find the suitable τ_n using **Time Step Control Algorithm.**

   Compute the error indicators η_K^n and estimator η_{space}^n.

3. $t^n = t^{n-1} + \tau_n$.

4. While (5.4) is not satisfied do

   (a) Refine/Coarsen the mesh $\mathcal{M}^n$ to obtain a new $\mathcal{M}^n$.

   (b) Solve for U_h^n.

   (c) Compute the error indicators η_K^n for all $K \in \mathcal{M}^n$.

5. Check τ_n. If η_{time}^n is satisfied then exited this loop, else go to step 2.

From the algorithm, we will be looping in steps 2 to 4 until the error estimates are in the ranges we set, then exit the loops in step 5.

Refine/Coarsen Algorithm

With a given $\theta_{refine}, \theta_{coarsen} \in (0, 0.5]$, we compute error indicator for each element $K \in \mathcal{M}^n$, we sort the element by value of error indicator. We refine the first $\theta_{refine}N$ elements and coarsen the last $\theta_{coarsen}N$ elements to obtain a new mesh where N is the total number of element in $\mathcal{M}^n$.