CHAPTER I

INTRODUCTION

Finite element method is a standard numerical technique for obtaining approximate solutions which are based on variational formulation of partial differential equations(PDEs). The finite element method is widely used in many applications in science and engineering, for example, mechanical engineering, structural simulation, aeronautical, biomechanical, automotive industries, etc.

Adaptivity is one of the key idea for improving accuracy and performance for finite element methods in an efficient way. Adaptive finite element method was first introduced in late 70's by I. Babuska[2]. Adaptive finite element method is more efficient and less work than finite element method if high accuracy is required especially in the presence of singularities or boundary layers, for examples.

A posteriori error analysis is the main idea for designing adaptive algorithm for finite element methods. In the adaptive algorithm, we use a posteriori error estimates as indicators, which are computable quantities of known data. The adaptive algorithm solves for finite element solutions and selects some elements for refinement and some elements for coarsening depending on the error indicators on each element.

An adaptive finite element method will loop the following procedure

$$\ldots \longrightarrow Solve \longrightarrow Estimate \longrightarrow Refine/Coarsen \longrightarrow \ldots$$

With a given initial mesh,

Solve finds finite element solution based on current mesh.

Estimate computes the error indicators on each element based on known data and solution.

Refine/Coarsen repartitions the current mesh to maintain the accuracy and performance in the system based on the error indicators.

The analysis and convergence results about adaptive finite element method is begun by the work of W. Dorfler[8] in 1996 for Poisson's equation. In 2002, P. Morin et al[11] extended [8] to elliptic PDEs with piecewise constant coefficient A. They also introduced the concept of oscillators. K. Mekchay and R. H. Nochetto[10] worked on general second order linear elliptic PDEs in 2005.

For parabolic PDE, Z. Chen and F. Jia[5] derived a posteriori error estimates for linear parabolic PDEs in 2004. Here, the considered the model problem,

$$\frac{\partial u}{\partial t} - \nabla \cdot (a(x)\nabla u) = f(x,t) \qquad \text{in } \Omega \times (0,T)$$

$$u = 0 \qquad \text{on } \partial\Omega \times (0,T), \qquad u(x,0) = u_0(x) \quad \text{in } \Omega,$$

where $u \in L^2(\Omega)$, $a(x)$ is a piecewise constant function and $f \in L^2(0,T;L^2(\Omega))$, i.e., $f : (0,T) \to L^2(\Omega)$.

In this thesis, we extended the work from Z. Chen and F. Jia by considering a semi-linear parabolic problem:

$$\frac{\partial u}{\partial t} - \nabla \cdot (a(x)\nabla u) = f(u) \qquad \text{in } \Omega \times (0,T)$$

$$u = 0 \qquad \text{on } \partial\Omega \times (0,T), \qquad u(x,0) = u_0(x) \quad \text{in } \Omega,$$

where $a(x)$ is now a positive function in $L^\infty(\Omega)$ and f is non-linear Lipschitz function of u.

We derived the upper and local lower bounds based on the standard residual technique to show that a posteriori error estimators are reliable and efficiency, and also constructed an adaptive algorithm for the finite element methods.