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ภาคผนวก ก

ผลผลิตของโครงการ

ก.1 ผลงานวิจัยตีพิมพ์ในวารสารวิชาการระดับนานาชาติ

ผลการวิจัยได้ถูกเสนอในบทความ

- **N. Yongram**, "Spin Correlations in e^+e^- Pair Creation by Two-Photons and Entanglement in QED", *International Journal of Theoretical Physics* (2010) [submitted]

- **N. Yongram**, "Spin correlations in elastic e^+e^- scattering in QED", *The European Physical Journal D* 47(8) (2008) บทความหมายเลข 8 ในฉบับที่ 47 ของปี ค.ศ. 2008 Journal Impact Factor (2007) = 1.294 ปรากฏบนฐานข้อมูลวิจัยนานาชาติ arXiv:quant-ph/0806.2244

ก.2 การนำผลงานวิจัยไปใช้ประโยชน์ในเชิงวิชาการ

ก.2.1 การสร้างนักวิจัยใหม่

ในการดำเนินงานตามโครงการวิจัยนี้ได้มีการให้นิสิตวิจัยของหน่วยวิจัยฟิสิกส์รากฐานและ จักรวาล วิทยาร่วมศึกษาการคำนวณ แอมปลิจูดของการเปลี่ยนสถานะสุญญากาศไปยังสุญญากาศ (the vacuum-to-vacuum transition amplitude) อันดับที่ 4 ของ q_0 และวาดภาพฟายน์แมนของกระบวนการการกระเจิงของอิเล็คตรอน-อิเล็คตรอน ในพลศาสตร์ไฟฟ้าเชิงควอนตัมแบบสมมาตรยวดยิ่ง

- นิสิตระดับปริญญาโทที่ได้ช่วยทำงานในโครงการนี้คือ นางสาวพัฒชญา มณีคำ ได้ศึกษาการคำนวณ the vacuum-to-vacuum transition amplitude อันดับที่ 4 ของ q_0และวาดภาพฟายน์แมนของกระบวนการการกระเจิงของ อิเล็คตรอน-อิเล็คตรอน ในพลศาสตร์ไฟฟ้าเชิงควอนตัมแบบสมมาตรยวดยิ่ง

ก.2.2 การพัฒนาการเรียนการสอนและการสร้างกลุ่มวิจัย

ผลจากโครงการนี้ทำให้เริ่มมีแนวคิดที่จะสอดแทรกเนื้อหาวิชาเกี่ยวกับทฤษฎีสนามควอนตัม
และสหสัมพันธ์ของโพรไรเซชันของคู่กำเนิดอนุภาคในทฤษฎีสนามควอนตัมเข้าไปในราย
วิชาอนุภาคและสนาม1ของมหาวิทยาลัยนเรศวรการทำงานในโครงการนี้ได้เหนี่ยวนำให้โครง
งานวิจัยระดับปริญญาตรีและปริญญาโทของนิสิตในหน่วยวิจัยเป็นไปในแนวทางค้นคว้าและ
หาความรู้พื้นฐานอันมุ่งจะก่อให้เกิดประโยชน์ต่อแนวทางหลักของโครงการคือการศึกษาพล
วัตรของพลังงานมืดและฟิสิกส์รากฐานของเอกภพนิสิตได้ถูกถ่ายทอดความรู้พื้นฐานและถูก
กำหนดให้อ่านและวิเคราะห์บทความที่เกี่ยวข้องในประเด็นที่เป็นที่สนใจของโครงการโครงการ
นี้ทำให้กิจกรรมทางการวิจัยและการสอนบูรณาการเข้าด้วยกันและทำให้กลุ่มวิจัยมีความเข้ม
แข็งขึ้นอย่างมาก

ก.3 ผลผลิตของโครงการด้านอื่น ๆ

ก.3.1 การเสนอผลงานในที่ประชุมวิชาการและการบรรยายสัมมนาภายนอก

มีการนำเสนอผลงานในการประชุมทางวิชาการดังนี้

- The 4th Order $e^- e^- \rightarrow e^- e^-$ in Suppersymmetric QED การเสนอผลงานแบบโปส
  เตอร์โดย หัวหน้าโครงการนี้ในการประชุมวิชาการ สกว พ.ศ. 2552 "เมธีวิจัยอาวุโส สก
  ว. พบนักวิจัยรุ่นใหม่" วันที่ 15 ถึง 17 ตุลาคม 2552

ภาคผนวก ข

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Spin correlations in elastic e$^+$e$^-$ scattering in QED

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Abstract. Spin correlations are carefully investigated in elastic e^+e^- scattering in QED, for initially *polarized* as well as *unpolarized* particles, with emphasis placed on energy or speed of the underlying particles involved in the process. An explicit expression is derived for the corresponding transition probabilities in closed form to the leading order. These expressions differ from those obtained from simply combining the spins of the relevant particles, which are of kinematic nature. It is remarkable that these explicit results obtained from quantum field theory show a clear violation of Bell's inequality at *all* energies, in support of quantum theory in the relativistic regime. We hope that our explicit expression will lead to experiments, of the type described in the bulk of this paper, that monitor speed.

PACS. 12.20.Ds Specific calculations – 12.20.Fv Experimental tests – 12.20.-m Quantum electrodynamics

1 Introduction

In recent years, we have been interested in studying joint polarization correlations of fundamental processes in quantum electrodynamics (QED) and in the electro-weak theory [1–4], for initially *polarized* and *unpolarized* particles. Our main conclusion, based on explicit computations in quantum field theory, is that the mere fact that particles emerging from a process have non-zero speeds upon reaching the detectors, implies, in general, that their spin polarization correlation probabilities *depend on speed* [1–4] and *may also depend on the underlying couplings* [4]. The explicit expressions of polarization correlations follow from these *dynamical* computations, and are non-speculative, involving no arbitrary input assumptions, and are seen to depend on speed, and possibly on the couplings as well. These are unlike those from the rather naïve method of simply combining the spins of the particles in question, which are of kinematical nature. Such a method is, in general, not applicable to relativistic particles, and the regularly used formal arguments (based on combining spins only) completely fail. In the limit of low energies, our earlier expressions [1–4] for the polarization correlations were shown to be reduced to the naïve ones just mentioned by simply combining spins. In our previous investigation [4], in which a process for the creation of a $\mu^+\mu^-$ pair (from e^+e^- scattering, for example) was considered under the electro-weak theory, it was noted that, due to the threshold needed to create such a pair, the zero-energy limit may not be taken, and that the study of polarization correlations by simply combining spins (without recourse to

quantum field theory) has no meaning. The focus of this paper is the derivation of the *explicit* polarization correlation probabilities in elastic e^+e^- scattering in QED, for initially *polarized* as well as *unpolarized* particles, with emphasis put on the *energy* available in the process so that a detailed study can be carried out in the relativistic regime as well. The reasons for our present investigation are twofold. Firstly, several experiments on $e^+e^- \rightarrow e^+e^-$ have been carried out over the years [5–9], and it is expected that our explicit new expression for the polarization correlations, depending on speeds, may lead to new experiments on polarization correlations that monitor the speed of the underlying particles. Secondly, such a study may be relevant to experiments in the light of Bell's theorem (monitoring speed), as discussed below.

The relevant quantity of interest here in testing Bell's inequality [10,11] is, in a standard notation,

$$S = \frac{p_{12}(a_1, a_2)}{p_{12}(\infty, \infty)} - \frac{p_{12}(a_1, a_2')}{p_{12}(\infty, \infty)} + \frac{p_{12}(a_1', a_2)}{p_{12}(\infty, \infty)} + \frac{p_{12}(a_1', a_2')}{p_{12}(\infty, \infty)}$$
$$- \frac{p_{12}(a_1', \infty)}{p_{12}(\infty, \infty)} - \frac{p_{12}(\infty, a_2)}{p_{12}(\infty, \infty)} \quad (1)$$

as is *computed from* QED. Here a_1, a_2 (a_1', a_2') specify directions along which the polarizations of two particles are measured, with $p_{12}(a_1, a_2)/p_{12}(\infty, \infty)$ denoting the joint probability, and $p_{12}(a_1, \infty)/p_{12}(\infty, \infty)$ and $p_{12}(\infty, a_2)/p_{12}(\infty, \infty)$ denoting the probabilities when the polarization of only one of the particles is measured ($p_{12}(\infty, \infty)$ is normalization factor). The corresponding probabilities, as computed from QED, will be denoted by $P[\chi_1, \chi_2]$, $P[\chi_1, -]$, and $P[-, \chi_2]$, with χ_1 and χ_2 denoting
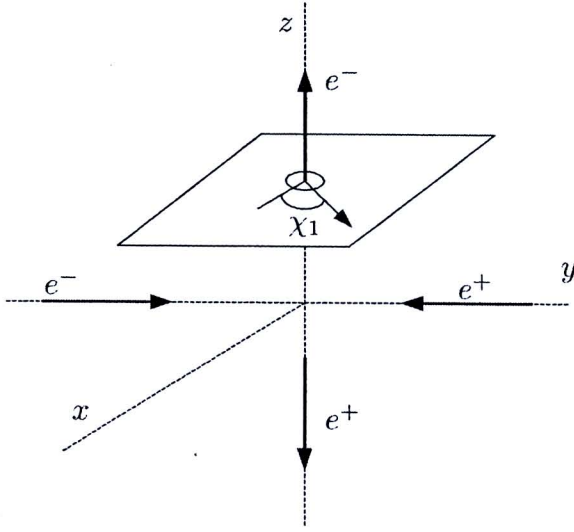
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Fig. 1. The figure depicts e^+e^- scattering, with the electron and positron initially moving the y-axis, while the emerging electron and positron moving along the z-axis. The angle χ_1, measured relative to the x-axis, denotes the orientation of spin of the emerging electron.

angles specifying directions along which spin measurements are carried out with respect to certain axes spelled out in the bulk of the paper. To show that the QED process is in violation with Bell's inequality of local hidden variables (LHV), it is sufficient to find one set of angles χ_1, χ_2, χ_1', and χ_2', such that S, as computed in QED, has a value outside of the interval $[-1, 0]$. In this work, it is implicitly assumed that the polarization parameters in the particle states are directly observable and may be used for Bell-type measurements (as discussed). We show a clear violation of Bell's inequality for *all* speeds, in support of quantum theory in the relativistic regime, i.e. of quantum field theory.

2 Spin correlations; initially polarized particles

We consider the process $e^+e^- \rightarrow e^+e^-$ in the center of mass frame (see Fig. 1), with an electron-positron pair initially polarized with one spin up (along z-axis) and one spin down. With $\boldsymbol{p}_1 = \gamma m \beta(0,1,0) = -\boldsymbol{p}_2$ denoting the momenta of initial electron and positron, respectively, and $\gamma = 1/\sqrt{1 - \beta^2}$, we consider the momenta of the emerging electron and positron as

$$\boldsymbol{k}_1 = \gamma m \beta(0,0,1) = -\boldsymbol{k}_2. \qquad (2)$$

For four-spinors of the initial electron and positron, respectively, we have

$$u(p_1) \sim \begin{pmatrix} \uparrow \\ i\rho \downarrow \end{pmatrix} \quad \text{and} \quad \overline{v}(p_2) \sim \begin{pmatrix} i\rho \uparrow^\dagger & -\downarrow^\dagger \end{pmatrix} \qquad (3)$$

$$\rho = \frac{\gamma\beta}{\gamma + 1} = \frac{\beta}{1 + \sqrt{1 - \beta^2}} \qquad (4)$$

where $\uparrow \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is a spin up, $\downarrow \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is a spin down, $\uparrow^\dagger \equiv \begin{pmatrix} 1 & 0 \end{pmatrix}$ is a transpose matrix of spin up, and $\downarrow^\dagger \equiv \begin{pmatrix} 0 & 1 \end{pmatrix}$ is a transpose matrix of spin down. For four-spinors of the emerging electron and positron, respectively, we have

$$\overline{u}(k_1) \sim \begin{pmatrix} \zeta_1^\dagger & \rho\sigma_3\zeta_1^\dagger \end{pmatrix} \quad \text{and} \quad v(k_2) \sim \begin{pmatrix} \rho\sigma_3\zeta_2 \\ \zeta_2 \end{pmatrix} \qquad (5)$$

$$\rho = \frac{\gamma\beta}{\gamma + 1} = \frac{\beta}{1 + \sqrt{1 - \beta^2}} \qquad (6)$$

where the two-spinors ζ_1, ζ_2 will be specified later.

The well-known expression for the amplitude of the process is [12,13]

$$\mathcal{M} \propto \overline{v}(p_2)\gamma^\mu u(p_1)\overline{u}(k_1)\gamma_\mu v(k_2)\frac{1}{(p_1 + p_2)^2}$$
$$- \overline{u}(k_1)\gamma^\mu u(p_1)\overline{v}(p_2)\gamma_\mu v(k_2)\frac{1}{(p_1 - k_1)^2}. \qquad (7)$$

Given the amplitude of the process above, we compute the conditional joint probability of spin measurements of e^+, e^- along directions specified by the angles χ_1 and χ_2, as shown in Figure 1. We then have two-spinors as follows:

$$\zeta_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\chi_1/2} \\ e^{i\chi_1/2} \end{pmatrix} \quad \text{and} \quad \zeta_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\chi_2/2} \\ e^{i\chi_2/2} \end{pmatrix}. \qquad (8)$$

Here we have considered only the single state (cf. [2,4]). Using the four-spinors of initial and emerging e^+, e^-, and the two-spinors in term of the angles χ_1 and χ_2 to calculate the invariant amplitude of the process in Figure 1, gives

$$\mathcal{M} \propto \left[A(\beta)\cos\left(\frac{\chi_1 + \chi_2}{2}\right) + B(\beta)\sin\left(\frac{\chi_1 - \chi_2}{2}\right) \right]$$
$$+ i\left[C(\beta)\sin\left(\frac{\chi_1 + \chi_2}{2}\right) + D(\beta)\cos\left(\frac{\chi_1 - \chi_2}{2}\right) \right] \qquad (9)$$

where

$$A(\beta) = 1 - \rho^2(1 - \rho) + 2\beta^2(1 - \rho^2)^2$$
$$B(\beta) = \rho(1 + \rho) + 8\beta^2\rho^2$$
$$C(\beta) = 1 + \rho^2(1 - \rho) + 2\beta(1 - \rho^4)$$
$$D(\beta) = \rho(1 + \rho).$$

Using the notation $F[\chi_1, \chi_2]$ for the absolute value square of the right-hand side of (9), the conditional joint probability distribution of spin measurements along the directions specified by angles χ_1 and χ_2 is given by

$$P[\chi_1, \chi_2] = \frac{F[\chi_1, \chi_2]}{N(\beta)}. \qquad (10)$$

The normalization factor $N(\beta)$ is obtained by summing over all the polarizations of the emerging particles. This is

equivalent to the summing of $F[\chi_1, \chi_2]$ over the pairs of angles

$$(\chi_1, \chi_2), \quad (\chi_1, \chi_2), \quad (\chi_1 + \pi, \chi_2), \quad (\chi_1, \chi_2 + \pi) \quad (11)$$

which leads to

$$
\begin{aligned}
N(\beta) &= F[\chi_1, \chi_2] + F[\chi_1 + \pi, \chi_2] \\
&\quad + F[\chi_1, \chi_2 + \pi] + P[\chi_1 + \pi, \chi_2 + \pi] \\
&= 2[A^2(\beta) + B^2(\beta) + C^2(\beta) + D^2(\beta)] \quad (12)
\end{aligned}
$$

giving

$$
\begin{aligned}
P[\chi_1, \chi_2] &= \frac{\left[A(\beta)\cos\left(\frac{\chi_1+\chi_2}{2}\right) + B(\beta)\sin\left(\frac{\chi_1-\chi_2}{2}\right)\right]^2}{2[A^2(\beta) + B^2(\beta) + C^2(\beta) + D^2(\beta)]} \\
&\quad + \frac{\left[C(\beta)\sin\left(\frac{\chi_1+\chi_2}{2}\right) + D(\beta)\cos\left(\frac{\chi_1-\chi_2}{2}\right)\right]^2}{2[A^2(\beta) + B^2(\beta) + C^2(\beta) + D^2(\beta)]}. \quad (13)
\end{aligned}
$$

If only one of the spins is measured, say, corresponding to χ_1, the probability can be written as

$$
\begin{aligned}
P[\chi_1, -] &= P[\chi_1, \chi_2] + P[\chi_1, \chi_2 + \pi] \\
&= \frac{1}{2} + \frac{2[A(\beta)B(\beta) + C(\beta)D(\beta)]\sin\chi_1}{2[A^2(\beta) + B^2(\beta) + C^2(\beta) + D^2(\beta)]}. \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (14)
\end{aligned}
$$

Similarly, if only one of the spins is measured corresponding to χ_2, the probability can be written as

$$
\begin{aligned}
P[-, \chi_2] &= P[\chi_1, \chi_2] + P[\chi_1 + \pi, \chi_2] \\
&= \frac{1}{2} + \frac{2[C(\beta)D(\beta) - A(\beta)B(\beta)]\sin\chi_2}{2[A^2(\beta) + B^2(\beta) + C^2(\beta) + D^2(\beta)]}. \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (15)
\end{aligned}
$$

For all $0 \le \beta \le 1$, angles χ_1, χ_2, χ_1' and χ_2' are readily found to lead to a violation of Bell's inequality of LHV theories. For example, for $\beta = 0.9$, $\chi_1 = 0°$, $\chi_2 = 45°$, $\chi_1' = 69°$, and $\chi_2' = 200°$, $S = -1.311$, violating the inequality from below.

3 Spin correlations; initially unpolarized particles

For the process $e^+e^- \rightarrow e^+e^-$, in the center of mass (c.m.), with initially unpolarized spins, with momenta $\boldsymbol{p}_1 = \gamma m \beta(0, 1, 0) = -\boldsymbol{p}_2$, we take for the final electron and positron

$$\boldsymbol{k}_1 = \gamma m \beta(1, 0, 0) = -\boldsymbol{k}_2 \quad (16)$$

and for the four-spinors

$$u(k_1) = \left(\frac{k^0 + m}{2m}\right)^{1/2}\left(\begin{array}{c} \xi_1 \\ \frac{\boldsymbol{k}_1 \cdot \boldsymbol{\sigma}}{k^0 + m}\xi_1 \end{array}\right), \quad \xi_1 = \left(\begin{array}{c} -i\cos\chi_1/2 \\ \sin\chi_1/2 \end{array}\right) \quad (17)$$

$$v(k_2) = \left(\frac{k^0 + m}{2m}\right)^{1/2}\left(\begin{array}{c} \frac{\boldsymbol{k}_2 \cdot \boldsymbol{\sigma}}{k^0 + m}\xi_2 \\ \xi_2 \end{array}\right), \quad \xi_2 = \left(\begin{array}{c} -i\cos\chi_2/2 \\ \sin\chi_2/2 \end{array}\right). \quad (18)$$
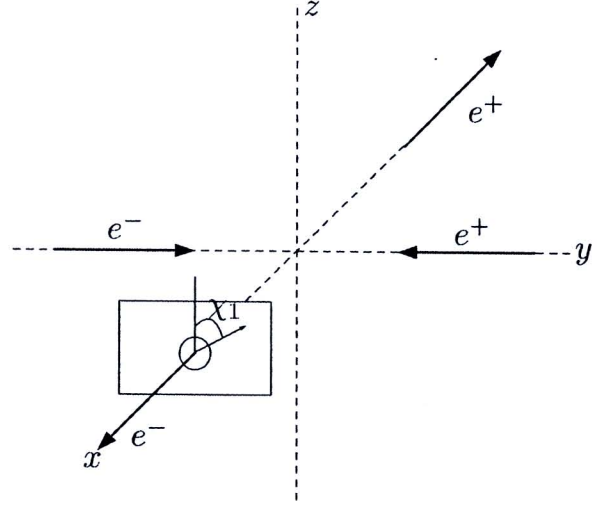

Fig. 2. The figure depicts e^+e^- scattering, with e^+, e^- moving along the y-axis, and the emerging electron and positron moving along the x-axis. The angle χ_1, measured relative to the z-axis, denotes the orientation of spin of the emerging electron.

The absolute value square of the right-hand side of (9), with initially unpolarized electrons and positron, leads to

Prob $\propto$

$$
\begin{aligned}
&\frac{\text{Tr}\left[\gamma^\sigma(\gamma p_2 + m)\gamma^\mu(-\gamma p_1 + m)\right]\overline{u}(k_1)\gamma_\mu v(k_2)\overline{v}(k_2)\gamma_\sigma u(k_1)}{(p_1 + p_2)^4} \\
&- \frac{\text{Tr}\left[(\gamma p_2 + m)\gamma^\mu(-\gamma p_1 + m)\gamma^\sigma\right]\overline{v}(k_2)\gamma_\sigma u(k_1)\overline{u}(k_1)\gamma_\mu v(k_2)}{(p_1 + p_2)^2(p_1 - k_1)^2} \\
&- \frac{\text{Tr}\left[\gamma^\mu(-\gamma p_1 + m)\gamma^\sigma(\gamma p_2 + m)\right]\overline{u}(k_1)\gamma_\mu v(k_2)\overline{v}(k_2)\gamma_\sigma u(k_1)}{(p_1 + p_2)^2(p_1 - k_1)^2} \\
&+ \frac{\overline{u}(k_1)\gamma^\mu(-\gamma p_1 + m)\gamma^\sigma u(k_1)\overline{v}(k_2)\gamma_\sigma(\gamma p_2 + m)\gamma_\mu v(k_2)}{(p_1 - k_1)^4} \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (19)
\end{aligned}
$$

which, after simplification and collecting terms, reduces to

$$
\begin{aligned}
\text{Prob} \;\propto\; & [2\beta^4(1 + 2\beta^2) - 3(1 + \beta^2)]\sin^2\left(\frac{\chi_1 - \chi_2}{2}\right) \\
& + (1 + \beta^2 + 2\beta^4)\cos^2\left(\frac{\chi_1 + \chi_2}{2}\right) + 5(1 - \beta^2) \\
\equiv\; & F[\chi_1, \chi_2] \qquad\qquad\qquad\qquad\qquad (20)
\end{aligned}
$$

where we have used the expressions for the spinors in (17) and (18).

Given that the process has occurred, the conditional probability that the spins of the emerging electron and positron make angles χ_1 and χ_2, respectively, with the z-axis is obtained directly from (20) as

$$P[\chi_1, \chi_2] = \frac{F[\chi_1, \chi_2]}{C}. \quad (21)$$

The normalization constant C is obtained by summing over the polarizations of the emerging electrons. This is

equivalent to the summing of $F[\chi_1, \chi_2]$ over the pairs of angles in (11) for any arbitrarily chosen fixed χ_1 and χ_2, corresponding to the orthonormal spinors

$$
\begin{pmatrix} -i\cos\chi_j/2 \\ \sin\chi_j/2 \end{pmatrix}, \quad \begin{pmatrix} -i\cos(\chi_j+\pi)/2 \\ \sin(\chi_j+\pi)/2 \end{pmatrix} = \begin{pmatrix} i\sin\chi_j/2 \\ \cos\chi_j/2 \end{pmatrix} \tag{22}
$$

providing a complete set, for each $j = 1, 2$, in reference to (17) and (18). This is,

$$
\begin{aligned}
C &= F[\chi_1, \chi_2] + F[\chi_1 + \pi, \chi_2] \\
&\quad + F[\chi_1, \chi_2 + \pi] + F[\chi_1 + \pi, \chi_2 + \pi] \\
&= 8[2 - 3\beta^2 + \beta^4 + \beta^6]
\end{aligned} \tag{23}
$$

which as expected is independent of χ_1 and χ_2. This gives

$$
\begin{aligned}
P[\chi_1, \chi_2] &= \frac{[2\beta^4(1+2\beta^2) - 3(1+\beta^2)]\sin^2\left(\frac{\chi_1-\chi_2}{2}\right)}{8[2 - 3\beta^2 + \beta^4 + \beta^6]} \\
&\quad + \frac{[1 + \beta^2 + 2\beta^4]\cos^2\left(\frac{\chi_1+\chi_2}{2}\right) + 5(1 - \beta^2)}{8[2 - 3\beta^2 + \beta^4 + \beta^6]}.
\end{aligned} \tag{24}
$$

By summing over

$$
\chi_2, \quad \chi_2 + \pi \tag{25}
$$

for any arbitrarily fixed χ_2, we obtain

$$
P[\chi_1, -] = \frac{1}{2} \tag{26}
$$

and similarly,

$$
P[-, \chi_2] = \frac{1}{2} \tag{27}
$$

for the probabilities when only one of the photons polarizations is measured.

A clear violation of Bell's inequality of LHV theories was obtained for all $0 \leq \beta \leq 1$. For example, for $\beta = 0.8$, with $\chi_1 = 0°$, $\chi_2 = 45°$, $\chi_1' = 210°$, and $\chi_2' = 15°$, $S = -1.167$, violating the inequality from below.

4 Conclusion

A critical study of polarization correlation probabilities in elastic e^+e^- scattering was carried out for initially *polarized* as well as *unpolarized* particles, emphasizing their dependence on speed, and an explicit expression was obtained for them in QED. The necessity of such a study within the realm of quantum field theory cannot be overemphasized, as estimates of such correlations from simply combining spins (as is often done), have no meaning, as they do not involve dynamical considerations. The relevant dynamics is, of course, dictated directly from quantum field theory. The explicit expression for the polarization correlation obtained is interesting in its own right, but may also lead to experiments that investigate such correlations by monitoring speed, not only for initially polarized particles, but also for unpolarized ones. Our results may also be relevant in the realm of Bell's inequality with emphasis put on relativistic aspects of quant! um theory, that is, of quantum field theory. Our expressions have shown clear violations of Bell's inequality of LHV theories, in support of quantum theory in the relativistic regime. In recent years, several experiments have been already performed (cf. [14–18]) on particles' polarization correlations. It is expected that the novel properties recorded here by explicit calculations following directly from field theory (which is based on the principle of relativity and quantum theory) will lead to new experiments on polarization correlations monitoring speed in the light of Bell's Theorem. We hope that these computations, within the general setting of quantum field theory, will also be useful in areas of physics such as quantum teleportation and quantum information.

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Spin Correlations in e^+e^- Pair Creation by Two-Photons and Entanglement in QED

N. Yongram

Abstract Spin correlations of e^+e^- pair productions of two colliding photons are investigated and explicit expressions for their corresponding probabilities are derived and found to be *energy* (speed) dependent, for initially *linearly* and *circularly polarized* photons, difference from those obtained by simply combining the spins of the relevant particles, for initially *polarized* photons. These expressions also depend on the angles of spin of e^+ (and/or of e^-), for initially *linearly polarized* photons, but not for *circularly polarized* photons, as a function of the energy. It is remarkable that these explicit results obtained from quantum field theory show a clear violation of Bell's inequality of Local Hidden Variables theories at all *energies* beyond that of the threshold one for particle production, in support of quantum field theory in the relativistic regime. We hope that our explicit expression will lead to experiments, of the type described in the bulk of this paper, which can monitor energy (and speed) in polarization correlation experiments.

1 INTRODUCTION

There have been many investigations over the years of particles's polarizations correlations [1–5] in the light of Bell's inequality, and Bell-like experiments have been proposed recently in high energy physics context [6–12].

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We have been interested in studying joint polarization correlation probabilities of particles produced by initially *polarized* as well as *unpolarized* particles in fundamental processes of quantum electrodynamics (QED) and the electro-weak theory [13–17]. Such studies, based on explicit computations in quantum field theory, show that the mere fact that particles emerging from a process have non-zero speeds upon reaching the detectors, imply, in general, that their polarization correlations *depend on speed* [13–17] and *may also depend on the underlying couplings* [17]. The explicit expressions of polarization correlations based on *dynamical* computations following from field theory, are non-speculative, involving no arbitrary input assumptions, and depend on speed, and possibly on the couplings as well. These are unlike formal arguments based simply on combining the spins of the particles in question, which are of kinematical nature. In the limit of low energies, our earlier expressions [13–17] for the polarization correlations were shown to be reduced to the simple method just mentioned by combining spins. In one of our previous investigations [17], in which $\mu^+\mu^-$ pair production in e^+e^- scattering was considered in the electro-weak theory. It was noted that, due to the threshold needed to create such a pair, the zero-energy limit may not be taken, and that the study of polarization correlations by simply combining spins (without recourse to quantum field theory) has no meaning. The focus of this paper is the derivation of the explicit polarization correlation probabilities of simultaneous measurements of spin of e^+e^- pair production of two colliding photons, as well as the corresponding probabilities when only one of the polarization of emerging e^+ (or e^-) is measured, in QED, for initially *linearly polarized* as well as *circularly polarized* photons, and particulary with initially *unpolarized* photons, with emphasis put on the *energy* available in the process so that a detailed study can be carried out in the relativistic regime as well. Since the production of e^+e^- pair by the collision of two photons ($\gamma\gamma \to e^+e^-$) was originally proposed by Breit and Wheeler [18], is so called "*Breit-Wheeler process*". In the present paper, in the spirit of Bell type experiments, we are interested in joint *conditional* probability distributions of spin measurements rather than of cross sections [19]. Conditional in the sense that *given* such a process has occurred, *then* we ask as to what is the probability distribution of spin measurements correlations. It has still not being directly observed because of a relatively high energy threshold which is about 1.022 MeV. With the continuous effort of the experimental verification on Earth-based experiments, using the technology based on free electron X-ray laser and its numerous applications, some first indications of its possible verification, have been reached. it also is one of most relevant elementary processes in high-energy astrophysics as well as in cosmology [20]. The QED calculations have been performed in stimulated Breit-Wheeler cross section [21,22] as well. The reasons for our present investigation are two fold. First several theoretical [23–25] and experiments on $\gamma\gamma \to e^+e^-$ have been carried out over the years [26], and it is expected that our explicit new expression for the polarization correlations obtained, depending on energies, may lead to new experiments on spin correlations which monitor the energy (speed) of the underlying particles. Second,

such a study may be relevant to experiments in the light of Bell's theorem (monitoring speed) as mentioned above and discussed below.

The relevant quantity of interest here in testing Bell's inequality [6,7] is, in a standard notation,

$$S = \frac{p_{12}(a_1, a_2)}{p_{12}(\infty, \infty)} - \frac{p_{12}(a_1, a_2')}{p_{12}(\infty, \infty)} + \frac{p_{12}(a_1', a_2)}{p_{12}(\infty, \infty)} + \frac{p_{12}(a_1', a_2')}{p_{12}(\infty, \infty)}$$
$$- \frac{p_{12}(a_1', \infty)}{p_{12}(\infty, \infty)} - \frac{p_{12}(\infty, a_2)}{p_{12}(\infty, \infty)} \tag{1}$$

as is *computed from* QED. Here a_1, a_2 (a_1', a_2') specify directions along which the polarizations of two particles are measured, with $p_{12}(a_1, a_2)/p_{12}(\infty, \infty)$ denoting the joint probability, and $p_{12}(a_1, \infty)/p_{12}(\infty, \infty)$, $p_{12}(\infty, a_2)/p_{12}(\infty, \infty)$ denoting the probabilities when the polarization of only one of the particles is measured. [$p_{12}(\infty, \infty)$ is normalization factor.] The corresponding probabilities as computed from QED will be denoted by $P[\chi_1, \chi_2]$, $P[\chi_1, -]$, $P[-, \chi_2]$ with χ_1, χ_2 denoting angles specifying directions along which spin measurements are carried out with respect to certain axes spelled out in the bulk of the paper. To show that the QED process is in violation with Bell's inequality of LHV, it is sufficient to find one set of angles χ_1, χ_2, χ_1', χ_2', such that S, as computed in QED, leads to a value of S outside the interval $[-1, 0]$. In this work, it is implicitly assumed that the polarization parameters in the particle states are directly observable and may be used for Bell-type measurements as discussed. We show a clear violating of Bell's inequality for *all* speeds in support of quantum theory in the relativistic regime, i.e., of quantum field theory.

We consider the process of $e^+ e^-$ pair production of the collision of two photons, $\gamma(k_1)\gamma(k_2) \to e^+(p_1)e^-(p_2)$, in the center of mass frame of the process (see Fig. 1), given by the amplitude of the process is well known[27], up to an overall multiplicative factor irrelevant for the problem at hand.

$$\mathcal{A} \propto \overline{u}(p_2)\left[\frac{\gamma^\mu \gamma k_1 \gamma^\nu}{2p_1 k_1} + \frac{\gamma^\nu \gamma k_2 \gamma^\mu}{2p_1 k_2} + \frac{\gamma^\mu p_1^\nu}{p_1 k_1} + \frac{\gamma^\nu p_1^\mu}{p_1 k_2}\right]v(p_1)e_1^\nu e_2^\mu \tag{2}$$

where $e_1^\mu = (0, \mathbf{e}_1)$, $e_2^\mu = (0, -\mathbf{e}_2)$ are the polarizations of two photons $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively. It is convenient to compute the amplitude of the process above, be rewritten as $(j = 1, 2)$

$$\mathbf{e}_j \equiv (e_j^{(1)}, e_j^{(2)}, e_j^{(3)}) \tag{3}$$

and these polarizations will be specified later. For the four momenta of the initially photons and the emerging e^+ (e^-), respectively, given by

$$\mathbf{k}_1 = \omega(0, 0, 1) = -\mathbf{k}_2 \tag{4}$$

$$\mathbf{p}_1 = \omega\sqrt{1 - \left(\frac{m_e}{\omega}\right)^2}(1, 0, 0) = -\mathbf{p}_2 \tag{5}$$

where ω is the energy of the photons, m_e denote the mass of an electron and positron. The measurement of the spin projection of the positron is taken along an axis making an angle χ_1 with the z-axis and lying in parallel to the $x - z$ plane, as shown in Fig 1,

$$v(p_1) = \sqrt{\frac{\omega}{m_e}} \begin{pmatrix} \rho\sigma_1\xi_1 \\ \xi_1 \end{pmatrix} \text{ and } u(p_2) = \sqrt{\frac{\omega}{m_e}} \begin{pmatrix} \xi_2 \\ -\rho\sigma_1\xi_2 \end{pmatrix} \tag{6}$$

where

$$\rho = \frac{\sqrt{1 - (m_e/\omega)}}{\sqrt{1 + (m_e/\omega)}} \tag{7}$$

and the direction of the spin of the electron makes an angle χ_2 with the z-axis. For the two-spinors, we have

$$\xi_1 = \begin{pmatrix} -\mathrm{i}\cos\chi_1/2 \\ \sin\chi_1/2 \end{pmatrix} \quad \text{and} \quad \xi_2 = \begin{pmatrix} -\mathrm{i}\cos\chi_2/2 \\ \sin\chi_2/2 \end{pmatrix} \tag{8}$$

A straightforward computation gives the matrix elements as follows:

$$\bar{u}\left(\gamma^i\gamma^0\gamma^j\right)v \sim 2\rho\mathrm{i}\varepsilon_{ijk}[\xi_2^\dagger\sigma_k\sigma_1\xi_1] \tag{9}$$

$$\bar{u}\gamma^i v \sim (1-\rho^2)[\xi_2^\dagger\sigma_1\xi_1]\delta^{i1} + (1+\rho^2)[\xi_2^\dagger\sigma_2\xi_1]\delta^{i2}$$
$$+ (1+\rho^2)[\xi_2^\dagger\sigma_3\xi_1]\delta^{i3} \tag{10}$$

$$\bar{u}\left(\gamma^i\gamma^m\gamma^j\right)v \sim \left(-\delta^{mj}\delta^{i1} - \delta^{mi}\delta^{j1} + \delta^{ji}\delta^{m1}\right)(1-\rho^2)[\xi_2^\dagger\sigma_1\xi_1]$$
$$+ \left(-\delta^{mj}\delta^{i2} - \delta^{mi}\delta^{j2} + \delta^{ji}\delta^{m2}\right)(1+\rho^2)[\xi_2^\dagger\sigma_2\xi_1]$$
$$+ \left(-\delta^{mj}\delta^{i3} - \delta^{mi}\delta^{j3} + \delta^{ji}\delta^{m3}\right)(1+\rho^2)[\xi_2^\dagger\sigma_3\xi_1]$$
$$- \mathrm{i}(1-\rho^2)\varepsilon_{mji}[\xi_2^\dagger\xi_1] \tag{11}$$

where $k = 1, 2, 3$. The matrix element above and Eq. (3) give the amplitude $\mathcal{A}$ in Eq. (2) as

$$\mathcal{A} \propto -\mathrm{i}(1-\rho^2)\mathbf{k} \cdot (\mathbf{e}_1 \times \mathbf{e}_2)[\xi_2^\dagger\xi_1]$$
$$+ (1-\rho^2)\sqrt{1 - \left(\frac{m_e}{\omega}\right)^2}\left(e_1^{(1)}e_2^{(1)} + e_2^{(1)}e_1^{(1)}\right)[\xi_2^\dagger\sigma_1\xi_1]$$
$$+ (1+\rho^2)\sqrt{1 - \left(\frac{m_e}{\omega}\right)^2}\left(e_1^{(2)}e_2^{(1)} + e_2^{(2)}e_1^{(1)}\right)[\xi_2^\dagger\sigma_2\xi_1] \tag{12}$$

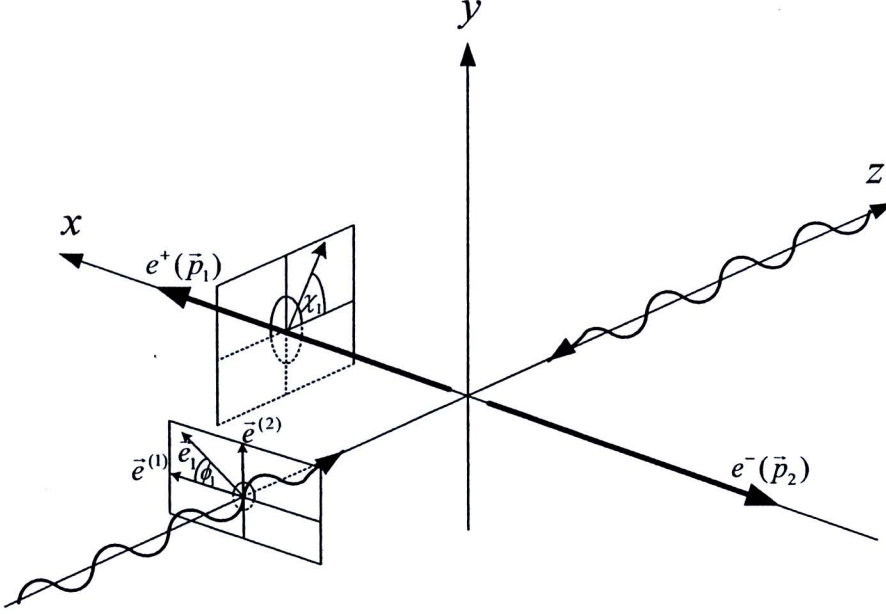which depends on the specification of the polarization of the initial photons in the process.

Fig. 1 The figure depicts e^+e^- pair production by two-photons, with the initially photons moving along the z-axis, while the emerging electron and positron moving along the x-axis. The angle χ_1 measured relative to the z-axis, denotes the orientation of spin of one of the emerging positron may make. The unit vectors $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are in x- and y-axis, respectively.

In Sec. 2, the amplitude in Eq. (12) is applied to compute exact joint probabilities of simultaneous measurement of spins of e^+e^- pair productions of two colliding linear and circular polarized, as well as unpolarized photons. It is also applied to compute of the corresponding probabilities of one of spin of e^+ (or e^-). We then simulated our computation to show violation of Bell's inequality of LHV theories, which may lead to new experiments on spin correlations which monitor the energy (speed) of the underlying particles in Sec. 3. Finally, we conclude our results in Sec. 4.

2 POLARIZATION CORRELATIONS IN BREIT-WHEELER PROCESS

2.1 initially linearly polarized photons

Now we consider the process, $\gamma\gamma \to e^+e^-$, with linear polarization of initial photons. The photon polarization vector $\mathbf{e}_j$ is given by $(j = 1, 2)$

$$\mathbf{e}_j = \cos\phi_j \mathbf{e}^{(1)} + \sin\phi_j \mathbf{e}^{(2)} \tag{13}$$

where $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are unit vectors in the plane of interaction and perpendicular to it, respectively. These photon linear polarization vectors are given by

$$\mathbf{e}^{(1)} = (1,0,0) \text{ and } \mathbf{e}^{(2)} = (0,1,0) \tag{14}$$

and ϕ is the angle between $\mathbf{e}$ and the $\mathbf{e}^{(1)}$ axis, as shown in Fig. 1. In this case, the planes of photon polarization at an angle of ϕ_1 and ϕ_2 are equal to $\pi/4$ with x-axis, then Eq. (13) gives the polarization of the initial photons as $\mathbf{e}_1 = \frac{1}{\sqrt{2}}(1,1,0) = -\mathbf{e}_2$. By substituting the polarization state of initial photons in Eq. (12), we have the amplitude of the process of the initial linear polarized photons, given by

$$\mathcal{A}_{\text{linear}} \propto \frac{i m_e}{\omega \left[1 + \left(\frac{m_e}{\omega}\right)\right]} \sin\left(\frac{\chi_1 - \chi_2}{2}\right) - \frac{1}{\left[1 + \left(\frac{m_e}{\omega}\right)\right]} \sin\left(\frac{\chi_1 + \chi_2}{2}\right) \tag{15}$$

Using notation $F[\chi_1, \chi_2]$ for the absolute value square of the right-hand side of Eq. (15), the conditional joint probability distribution of spin measurements along the directions specified by angle χ_1, χ_2 is given by

$$P[\chi_1, \chi_2] = \frac{F[\chi_1, \chi_2]}{N(\omega)} \tag{16}$$

The normalization factor $N(\omega)$ is obtained by summing over all the polarizations of the emerging particles. This is equivalent to summing of $F[\chi_1, \chi_2]$ over the pairs of angles

$$(\chi_1, \chi_2), \quad (\chi_1 + \pi, \chi_2), \quad (\chi_1, \chi_2 + \pi), \quad (\chi_1 + \pi, \chi_2 + \pi) \tag{17}$$

This leads to

$$N(\omega) = F[\chi_1, \chi_2] + F[\chi_1 + \pi, \chi_2] + F[\chi_1, \chi_2 + \pi] + P[\chi_1 + \pi, \chi_2 + \pi]$$

$$= \frac{2\left[1 + \left(\frac{m_e}{\omega}\right)^2\right]}{\left[1 + \left(\frac{m_e}{\omega}\right)\right]^2} \tag{18}$$

giving

$$P[\chi_1, \chi_2] = \frac{m_e^2}{2\omega^2 \left[1 + \left(\frac{m_e}{\omega}\right)^2\right]} \sin^2\left(\frac{\chi_1 - \chi_2}{2}\right) + \frac{1}{2\left[1 + \left(\frac{m_e}{\omega}\right)^2\right]} \sin^2\left(\frac{\chi_1 + \chi_2}{2}\right) \tag{19}$$

If only one of the spins is measured corresponding to χ_1, the probability can be written as

$$P[\chi_1, -] = P[\chi_1, \chi_2] + P[\chi_1, \chi_2 + \pi]$$

$$= \frac{1}{2} \tag{20}$$

Similarly, if only one of the spins is measured corresponding to χ_2, the probability can be written as

$$P[-,\chi_2] = P[\chi_1,\chi_2] + P[\chi_1 + \pi,\chi_2]$$

$$= \frac{1}{2} \qquad (21)$$

2.2 initially circularly polarized photons

Here we turn to consider the production of two initially photons $\mathbf{k_1}$ and $\mathbf{k_2}$ in the process, with the right-handed and left-handed circular polarization, are specified by the vectors

$$\mathbf{e_1} = \frac{1}{\sqrt{2}}(1,i,0) \text{ and } \mathbf{e_2} = \frac{1}{\sqrt{2}}(1,-i,0) \qquad (22)$$

respectively, producing e^+e^- pairs, and place detectors for the latter at opposite end of the x-axis, as in Fig. 1, this gives

$$\mathcal{A}_{\text{cir}} \propto \omega \cos\left(\frac{\chi_1 - \chi_2}{2}\right) + i\sqrt{1 - \left(\frac{m_e}{\omega}\right)^2}\sin\left(\frac{\chi_1 - \chi_2}{2}\right) \qquad (23)$$

By using notations in Eqs. (16)–(18) to compute the joint probability of spin correlations of e^+e^- produced by two circularly polarized photons, this gives

$$P[\chi_1,\chi_2] = \frac{\omega^2 + \left[(1-\omega^2) - \left(\frac{m_e}{\omega}\right)^2\right]\sin^2\left(\frac{\chi_1 - \chi_2}{2}\right)}{2\left[2\omega^2 + \left[(1-\omega^2) - \left(\frac{m_e}{\omega}\right)^2\right]\right]} \qquad (24)$$

$$P[\chi_1, -] = \frac{1}{2} \qquad (25)$$

$$P[-,\chi_2] = \frac{1}{2} \qquad (26)$$

2.3 initially unpolarized photons

Finally, we consider the process $\gamma\gamma \to e^+e^-$, in the c.m., with initially unpolarized photons with some over all polarization of the photons

$$\sum_{\text{pol}} e_2^i e_1^j = \delta^{ij} - n_2^i n_1^j \qquad (27)$$

where $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$ and $i,j = 1,2$, using the identity $\sum_{\text{pol}} \mathbf{k} \cdot (\mathbf{e_1} \times \mathbf{e_2}) = 0$, this gives

$$\mathcal{A}_{\text{unpol}} \propto \xi_2^\dagger \xi_1 \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 (0\ 1)_1 + \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2 (1\ 0)_1 \right\} \qquad (28)$$

8

(see Fig. 1), generating an energy independent (normalized) entangled state for e^+e^- given by

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_2 (0\ 1)_1 + \begin{pmatrix}0\\1\end{pmatrix}_2 (1\ 0)_1\right] \tag{29}$$

Therefore the joint probability of spin correlations of e^+e^- is given by

$$P[\chi_1,\chi_2] = \|\xi_2^\dagger \xi_1 |\psi\rangle\|^2$$

$$= \frac{1}{2}\sin^2\left(\frac{\chi_1-\chi_2}{2}\right) \tag{30}$$

and

$$P[\chi_1,-] = \|\xi_1|\psi\rangle\|^2 = \frac{1}{2} \tag{31}$$

$$P[-,\chi_2] = \|\xi_2^\dagger|\psi\rangle\|^2 = \frac{1}{2} \tag{32}$$

$P[\chi_1,-]$ is also *equivalently* obtained by summing $P[\chi_1,\chi_2]$ over

$$\chi_1, \quad \chi_2+\pi \tag{33}$$

for any arbitrarily chosen χ_2, i.e., as in Eq. (20) and similarly for $P[-,\chi_2]$. As the result of our calculation, we found that the joint polarization correlation probabilities of the initial unpolarized photons are independent of energy, as in the combining spin of the kinematic considerable.

3 SIMULATIONS AND DISCUSSIONS

In this section, we simulate the joint polarization correlation probabilities of e^+e^- pair productions of two colliding photons, for initially linearly and circularly polarized photons, computed in the previous section to show a clear violation of the relevant Bell-like inequality as a function of the energy of initially photons. We note the important statical property that

$$P[\chi_1,\chi_2] \neq P[\chi_1,-]P[-,\chi_2] \tag{34}$$

in general, showing obvious correlations occurring between the two spins. The indicator S in Eq. (1) computed according to the probabilities of simultaneous measurement of the spins of e^+e^- $P[\chi_1,\chi_2]$ in Eqs. (19), (24), (30), and the corresponding probabilities when only one of the spins of e^+ (or e^-) $P[\chi_1,-]$ in Eqs. (20), (25), (31), as well as $P[-,\chi_2]$ in Eqs. (21), (26), (32), may be readily evaluated. To show violation of Bell's inequality of LHV theories, it is sufficient to find four angles χ_1, χ_2, χ_1', χ_2' at accessible energies, for which S falls outside the interval $[-1,0]$. We perform such a simulation fitting, presenting our results, and discussing their physical implications.

Table 1 The indicator S computed according to the probabilities of spin correlations of e^+e^- pair productions of two-colliding linearly polarized photons are simulated. For example, the four angles $\chi_1 = 0°$, and χ_2, χ_1', χ_2' are shown in a following table. R, B, G and O denotes Red, Blue, Green and Orange, respectively.

line	ω(MeV)	χ_2(degree)	χ_1'(degree)	χ_2'(degree)	S
-	1.05	45	15	180	-1.37576
R	1.05	45	30	140	-1.36279
B	1.05	45	30	153.5	-1.35814
G	1.05	45	67	213	-1.30592
O	1.05	45	90	270	-1.03585
-	5.00	45	15	180	-1.39234
-	10.00	45	15	180	-1.39304
-	35.00	45	15	180	-1.39326
-	46.60×10^3	45	15	180	-1.39328

As shown in Table 1, we found that the deriving probabilities in Sec. 2.1, for initially linearly polarized photons, give S outside the interval $[-1, 0]$ from below for four angles χ_1, χ_2, χ_1', χ_2' at accessible energies. The plotting in Fig. 2 show that the probability $P[\chi_1', \chi_2']$ in Eq. (19) is a varying function of the energy of initial photons. These probabilities, for the set of angles $(180° < \chi_1' < 360°, 0° \leq \chi_2' \leq 360°)$ and $(0° \leq \chi_1' \leq 360°, 180° < \chi_2' < 360°)$, are rapidly increasing, $\omega \leq 3.5$ MeV, after that they are slightly increasing with energy of the initially photons. On the other hand, for the set of angles $(0° < \chi_1' < 180°, 0° \leq \chi_2' \leq 360°)$ and $(0° \leq \chi_1' \leq 360°, 0° < \chi_2' < 180°)$, they are slightly decreasing with energy of the initially photons. However, we found that S are varying functions of the set of angles χ_1', χ_2' when angles $\chi_1 = 0°$, $\chi_2 = 45°$ and the energy are fixed which can see are decreasing functions of the energy of the initially photons when the angles are fixed (as shown in Fig. 4). Such the results as the probabilities and the indicator S depend on the angles of spin of e^+e^- and the energy of initially photons in processes.

As shown in Table 2 we found that the deriving probabilities in Sec. 2.2, for initially circularly polarized photons, gives S outside the interval $[-1, 0]$ from below for four angles χ_1, χ_2, χ_1', χ_2' at accessible energies. The plotting in Fig. 3 show that the probability $P[\chi_1', \chi_2']$ in Eq. (24) is a varying function of the energy of initially photons. These probabilities, for four set of angles χ_1' and χ_2' (see in Table 2), are rapidly increasing, $\omega \leq 5$ MeV, after that they are slightly increasing with the energy of the initially photons. These plotting lies on the same line, even though the set of angles χ_1' and χ_2' were changed. We also found that S are varying functions of angles χ_1', χ_2' when angles $\chi_1 = 0°$, $\chi_2 = 155°$ and the energy are fixed. These S are a rapidly decreasing with the increasing energy of initially photons when angles $\chi_1 = 0°$, $\chi_2 = 155°$, $\chi_1' = 15°$ and $\chi_2' = 50°$ are fixed as shown in Table 3, and plotted in Fig. 4. Such the results as the indicator S depend on the angles of the measurement of spin of e^+e^- and the energy of initially photons, but the probabilities only depend on the energy of initially photons in processes.
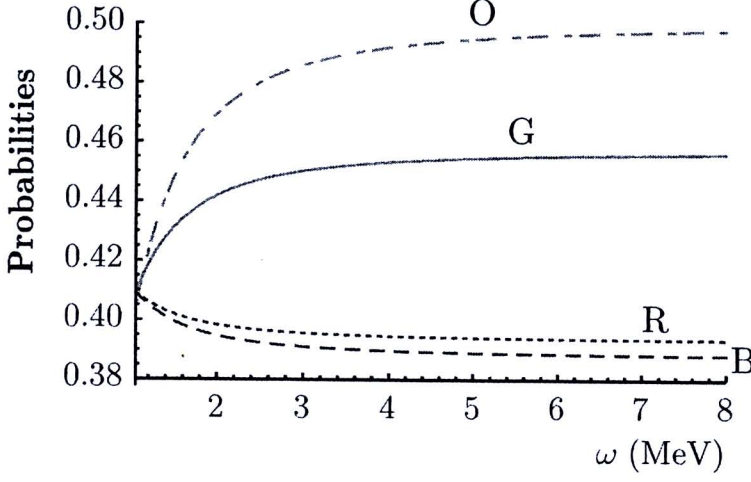
Fig. 2 The probabilities of simultaneous measurement of spin correlations of e^+e^- pair productions of two-colliding linearly polarized photons, $P[\chi_1', \chi_2']$, as a function of the energies of photons. The dashed-dotted line (O) is for the set of angles $\chi_1' = 67°$, $\chi_2' = 213°$. The solid line (G) is for the set of angles $\chi_1' = 90°$, $\chi_2' = 270°$. The dotted line (R) is for the set of angles $\chi_1' = 30°$, $\chi_2' = 140°$. The dashed line (B) is for the set of angles $\chi_1' = 30°$, $\chi_2' = 153.5°$.

Table 2 The indicator S computed according to the probabilities of spin correlations of e^+e^- pair productions of two-colliding circularly polarized photons are simulated. For example, the four angles $\chi_1 = 0°$, and χ_2, χ_1', χ_2' are shown in a following table. R, B, G and O denotes Red, Blue, Green and Orange, respectively.

line	ω(MeV)	χ_2(degree)	χ_1'(degree)	χ_2'(degree)	S
R	1.05	155	15	50	-1.32878
G	1.05	155	45	10	-1.30828
B	1.05	155	85	50	-1.05177
O	1.05	155	90	55	-1.01432

Table 3 The indicator S computed according to the probabilities of spin correlations of e^+e^- pair productions of two-colliding circularly polarized photons are simulated. For example, the four angles $\chi_1 = 0°$, and $\chi_2 = 155°$, $\chi_1' = 15°$, $\chi_2' = 50°$ and varying photon energies are shown in a following table.

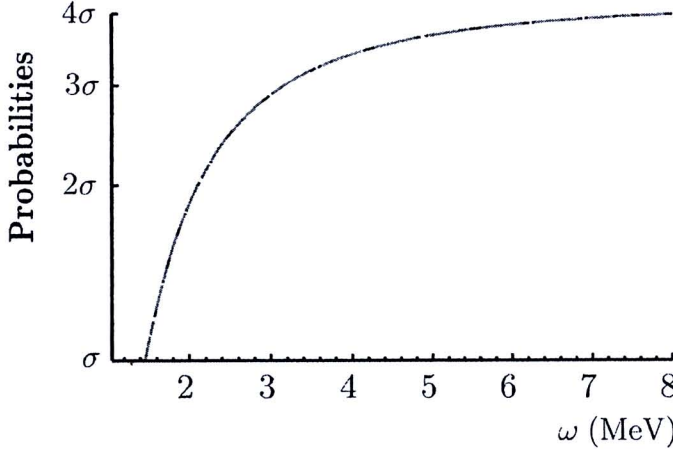| ω(MeV)	S
1.05	-1.328 784 959 959 7406
5.00	-1.328 784 959 960 4962
10.00	-1.328 784 959 960 5301
35.00	-1.328 784 959 960 5410
46.60×10^3	-1.328 784 959 960 5420

Fig. 3 The probabilities of simultaneous measurement of spin correlations of e^+e^- pair productions of two-colliding circularly polarized photons, $P[\chi_1', \chi_2']$, as a function of the energies of photons. We have taken the angles as in Table 2.

Table 4 The value of the probabilities of spin correlations of e^+e^- pair productions of two-colliding circularly polarized photons are shown in Fig. 3.

variables	Probabilities
σ	0.454 788 011 072 0782
2σ	0.454 788 011 072 1606
3σ	0.454 788 011 072 2077
4σ	0.454 788 011 072 2416

Table 5 The indicator S computed according to the probabilities of spin correlations of e^+e^- pair productions of two-colliding unpolarized photons are simulated. For example, the four angles $\chi_1 = 0°$, and χ_2, χ_1', χ_2' are shown in a following table.

ω(MeV)	χ_2(degree)	χ_1'(degree)	χ_2'(degree)	S
independent	85	25	181	-1.14675
independent	67	55	181	-1.34218
independent	23	45	180	-1.46192

In Table 4 we found that the deriving probabilities in Sec. 2.3, for initially unpolarized photons, also give S outside the $[-1, 0]$ from below for four angles χ_1, χ_2, χ_1', χ_2', and *independent on energies* (shown in Table 5). These S are plotted in Fig. 4 as well. The indicator S and the probability $P[\chi_1, \chi_2] = (1/2)\sin^2[(\chi_1 - \chi_2)/2]$ are *independent of the energy* of initially photons, but they only depend on the angles χ_1, χ_2 of the measurement of e^+e^- spins respectively. Finally, we have plotted the indicator S as a functions of the energy of initially photons, for four set of angles χ_1, χ_2, χ_1', χ_2', as shown in Fig. 4. Such the result as the indicator S for initially *linearly polarized* photons
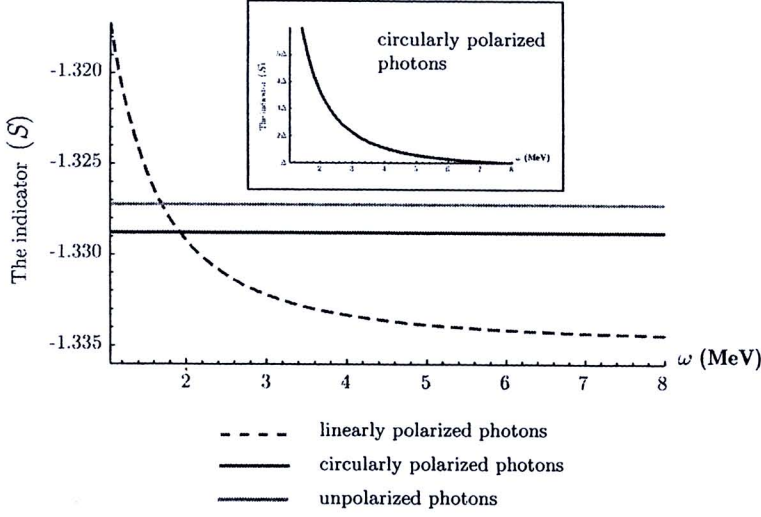
Fig. 4 The plotting of the indicator S as a functions of the energy of initially photons, for four set of angles $\chi_1 = 0°$, $\chi_2 = 45°$, $\chi_1' = 15°$, $\chi_2' = 140°$ of spin of e^+e^- for initially linearly polarized photon, as well as for four set of angles $\chi_1 = 140°$, $\chi_2 = 155°$, $\chi_1' = 15°$, $\chi_2' = 50°$ of spin of e^+e^- for initially circularly polarized photon. In particular for four set of angles $\chi_1 = 140°$, $\chi_2 = 23°$, $\chi_1' = 67°$, $\chi_2' = 132°$ of spin of e^+e^- for initially unpolarized photon.

Table 6 The value of the probabilities of spin correlations of e^+e^- pair productions of two-colliding circularly polarized photons are shown in Fig. 4.

variables	The indicator (S)
$\triangle$	-1.328 784 959 960 5240
$2\triangle$	-1.328 784 959 960 4287
$3\triangle$	-1.328 784 959 960 3334
$4\triangle$	-1.328 784 959 960 2382
$5\triangle$	-1.328 784 959 960 1430

is faster decreasing with the energy than that of initially *circularly polarized* photons when set of angles is fixed.

4 CONCLUSIONS

We have investigated and derived, in detail, the explicit polarization correlation probabilities of simultaneous measurement of spins of e^+e^- pair productions by two colliding photons for initially *linearly* and *circularly polarized* photons, emphasizing their dependence on energy for initially *polarized* photons. The expressions were obtained in the (relativistic) QED setting. The necessity of such a study within the realm of quantum field theory cannot be overemphasized, as estimates of such correlations from simply combining spins

(as is often done), have no meaning, as they do not involve dynamical considerations. The relevant dynamics is, of course, dictated directly from quantum field theory. The explicit expression for the polarization correlation obtained is interesting in its own right, and they may also lead to experiments that investigate such correlations by monitoring energy (speed), for initially *linearly* and *circularly polarized* photons, but not for *unpolarized* ones. The simulation study carried out in Sec. 3, shows that these expressions also depend on the angles of spin of e^+ (and/or e^-), for initially *linearly polarized* photons, but not for *circularly polarized* photons, as a function of the energy. Our results may also be relevant in the realm of Bell's inequality with emphasis on relativistic aspects of quantum theory, that is, of quantum field theory. Our expressions have shown clear violations of Bell's inequality of LHV theories, in support of quantum theory in the relativistic regime. In recent years, several experiments have been already performed (cf. [8–12]) on particles' polarization correlations. It is expected that the novel properties recorded here by explicit calculations following directly from field theory (which is based on the principle of relativity and quantum theory) will lead to new experiments on polarization correlations for monitoring speed in the light of Bell's theorem. We hope that these computations, within the general setting of quantum field theory, will also be useful in areas of physics such as quantum teleportation and quantum information.

ACKNOWLEDGMENTS

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