CHAPTER II

GRASPING AND REGRASPING PRELIMINARIES

In this chapter, we describe necessary definitions and propositions which will be applied in the discussion on our regrasp planning problems.

2.1 Nomenclatures

Following the definitions in (Boyd and Vandenberghe, 2004), we denote by $\text{INT}(\cdot)$, $\text{RI}(\cdot)$ and $\text{CO}(\cdot)$ the interior, the relative interior[1] and the convex hull of a set. For an arbitrary vector $\boldsymbol{v}$, let us denote by $P_{\boldsymbol{v}}$ the plane containing the origin and orthogonal to $\boldsymbol{v}$, i.e., $P_{\boldsymbol{v}} = \{\boldsymbol{x} | \boldsymbol{x} \cdot \boldsymbol{v} = 0, x \in \mathbb{R}^3\}$. A point at $\boldsymbol{x}$ is said to lie in the positive side of, negative side of, or exactly on $P_{\boldsymbol{v}}$ when $\boldsymbol{x} \cdot \boldsymbol{v} > 0, \boldsymbol{x} \cdot \boldsymbol{v} < 0$ or $\boldsymbol{x} \cdot \boldsymbol{v} = 0$, respectively. A closed half space $\mathcal{H}(\boldsymbol{v})$ is the set of all points that lie exactly on $P_{\boldsymbol{v}}$ or in the positive side of $P_{\boldsymbol{v}}$. An open half space $\mathcal{H}^+(\boldsymbol{v})$ is simply $\mathcal{H}(\boldsymbol{v}) - P_{\boldsymbol{v}}$. We define $\mathcal{H}^{z+}$ to be $\mathcal{H}^+((0, 0, 1))$ and $\mathcal{H}^{z-}$ to be $\mathcal{H}^+((0, 0, -1))$.

2.2 Contact Model

In grasping, the most commonly used contact model are hard contact without friction, hard contact with friction and soft finger contact. Soft contact grasp is different from hard contact grasp with ability that soft finger can exert torque about the surface normal while hard finger can exert force at contact point only. For analysis of hard contact, the point contact without friction can only exert a unidirectional force normal to the surface. Tangential forces can be produced by a finger up to the friction coefficient when friction is considered.

Coulomb friction (Stewart, 2000) is usually applied for friction model. Coulomb's law of friction states that for a contact point exerting a force f_N along the contact normal, the friction force (the tangential contact force) is less than or equal to $f_t = \mu f_N$ where μ is the frictional coefficient. This equation indicates that when the contact is maintained without slip, the contact can exert any force in a cone C of which the half angle is equal to $\tan^{-1}(\mu)$. The cone is emanated from the contact point and the axis coincides with the contact normal n. This cone is commonly called a friction cone. Cone in 2D case can

[1] A relative interior of a set is the interior relative to the affine hull of the set. Intuitively speaking, a relative interior are all points not on the relative edge of the set, e.g., A relative interior of a line segment is the segment minus its endpoints, regardless of the dimension where the line is situated.

be expressed by two vectors as shown in Figure 2.1(a). In 3D case, a cone is described by quadratic function. Cone introduces complexity of nonlinearity to the problem. To simplify the problem, a cone can be replaced with an m-sided pyramid (Figure 2.1(b)). A pyramid has planar facets which avoid nonlinearity from the problem but at a price of lesser precision.

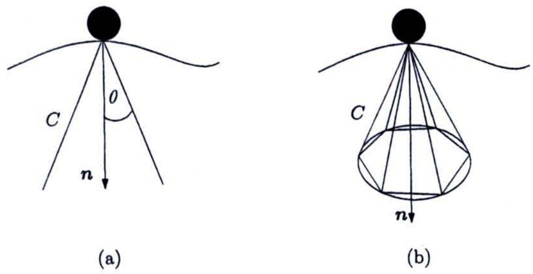

Figure 2.1: Coulomb friction: (a) is the friction cone for 2D grasps and (b) is the friction cone for 3D grasps and its approximating pyramid cone.

2.3 Grasp and Wrenches

Force closure is a property of a grasp which is defined by a set of contacts. Each contact can be defined by its position and inward normal direction. In this work, it is assumed that every contact of the same object is represented by the same contact model.

Definition 2.1 (Grasp) *A grasp G is defined by a set of ordered pairs* $\{(\boldsymbol{p}_1, \boldsymbol{n}_1), \ldots, (\boldsymbol{p}_n, \boldsymbol{n}_n)\}$ *where* $\boldsymbol{p}_i$ *and* $\boldsymbol{n}_i$ *are the position vector and the inward normal vector of* i^{th} *contact.*

A grasp achieves force closure when the grasp is able to counterbalance any external disturbance to the object being grasped. The external disturbance and the effect of contact points are represented as a force $\boldsymbol{f}$ and a torque $\boldsymbol{\tau}$. In 2D, it is conventional to combine a force $\boldsymbol{f} = (f_x, f_y)$ and a torque $\boldsymbol{\tau}$ into an entity called a *wrench* $\boldsymbol{w} = (f_x, f_y, \boldsymbol{\tau})$. A wrench is a vector of force concatenated with a vector of torque. In 2D space, force can be described by a 2D vector while torque is described by a 1D vector, hence, a wrench in 2D space is a 3D vector. Likewise, a wrench in 3D space is 6D vector formed by a 3D force vector concatenated with a 3D torque vector. Formally, a wrench $\boldsymbol{w}$ is denoted by $(\boldsymbol{f}, \boldsymbol{t})$ where $\boldsymbol{f}$ is a force vector and $\boldsymbol{t}$ is a torque vector.

Combining force and torque into wrench makes it simpler to consider the force closure property. An effect of a contact point or external disturbance can be easily described as a wrench. For example, let us consider an equilibrium in terms of wrenches. An object is said to be under equilibrium when the summation of all force and torque acting on the object is zero. Using wrench notation, an object achieves equilibrium when the summation of acting wrenches is the zero vectors.

Analysis on force closure concerns wrenches that can be exerted by a grasp. A contact is associated with a set of wrenches that it can exert. The set of wrenches that can be exert by a contact and by a grasp are referred as *a contact wrench set* and *a grasp wrench set*, respectively. In force closure analysis, a contact wrench is allowed to take arbitrarily large magnitude[2]. Since wrenches can be added up linearly, the set of wrenches exertable by the grasp is the positive combination of wrenches of its contacts. Let us refer to a positive combination of a set of vectors as a *linear positive span*, or positive span for short. Exertable wrenches of a grasp is a positive span of a contact wrench set of each contact.

Definition 2.2 (Positive Span) *Let W be a set of vectors. A positive span of W, denoted by* $\text{SPAN}^+(W)$, *is a set* $\{\alpha_i w_i | \alpha_i \geq 0, w_i \in W\}$.

2.3.1 Primitive Contact Wrenches

A contact wrench set can also be conveniently represented using positive span notation. A frictionless contact can only exert force in one direction and its contact wrench set is a ray in its respective wrench space. The ray can be represented as a positive span of a single wrench with arbitrary length lying in the same direction. For a frictional contact, a friction cone of which can be represented by positive span of its boundary force vectors. These vectors corresponds to boundary wrenches and the whole contact wrench set can be represented by a positive span of these boundary wrenches, using one single arbitrary length for each direction.

We refer to unit length boundary wrenches as *primitive contact wrenches*. A contact wrench set is a positive span of primitive contact wrenches. Similarly, a grasp wrench set is a positive span of its contact wrench sets which is also equal the to positive span all of primitive contact wrenches (from all contact points). Let $w_1, \ldots, w_n$ be primitive contact wrenches of a grasp. The grasp wrench set of a grasp whose primitive contact wrenches are $w_1, \ldots, w_n$ can be represented as follows.

[2]In practice, a magnitude of a wrench is limited by the realization of the contact, e.g., the actuator of finger, the size of motor, etc. This detail is unrelated to the contact position and hence is neglected.

$$\{\Sigma_{i=1}^{n}\alpha_i \boldsymbol{w}_i | \alpha_i \geq 0\} \tag{2.1}$$

2.3.2 Grasp Wrench Set

Primitive contact wrenches and positive span represent a grasp wrench set in a compact form. It is necessary to understand the properties of a grasp wrench set when it is represented as a positive span of the primitive contact wrenches. A key feature of a positive span is its convexity. Convexity of a grasp wrench set is an important property exploited by most grasping works.

Other than convexity, a grasp wrench set also has other interesting properties. In the 3D frictional contact case, a friction cone is bounded by a quadratic surface, not a finite number of wrenches. A prominent difference is that a 3D friction cone, though it still maintains convexity, is no longer a linear structure. This implies that the corresponding grasp wrench set itself is nonlinear as well. In many works, a circular friction cone is simplified by an m-sided pyramid. Each boundary force vector of the pyramid yields one primitive contact wrench. Since m is finite, the number of primitive contact wrenches is also finite and thus the grasp wrench set can now be represented by linear surfaces allowing several tools in linear algebra to be applicable for analysis.

2.4 Force Closure

A grasp achieves force closure when its grasp wrench set covers the entire wrench space. A property called *positively spanning* is defined to describe that the positive span of a vector set covers the entire space.

Definition 2.3 (Positively Span) *We say that a set V of n-dimensional vector* positively spans $\mathbb{R}^n$ *when* $\text{SPAN}^+(V) = \mathbb{R}^n$

The force closure property can be formally defined using the notion of positively spanning, namely, a grasp achieves force closure when its associated wrenches, i.e., the polyhedral convex cone generated from the primitive contact wrenches, positively span their respective wrench space (3D wrench space in case of planar grasp and 6D wrench space in case of 3D grasp).

Definition 2.4 (Force Closure) *A grasp, whose primitive contact wrenches form the set* W *in* $\mathbb{R}^n$*, is said to achieve force closure when* $\text{SPAN}^+(W)$ *positively span* $\mathbb{R}^n$.

Since the force closure property is defined over a set of vector (wrenches) associated with a grasp, it is more convenient to say that a set of vector achieves force closure, even though a set of vector cannot literally achieve force closure. Hereafter, saying that a set of wrenches achieves force closure is a short hand of saying that a grasp whose associated set of wrenches positively span $\mathbb{R}^n$.

2.5 Condition of Force Closure

The force closure property is defined using the notion of positively spanning. However, it is still indefinite to assert whether a set of vectors positively span a space. In this section we recite some of the well known conditions that assert on positively spanning of a set of vectors.

Mishra et al. related positively spanning of a set of vectors with a convex hull of the vectors. It is shown in (Mishra et al., 1987b) that a set of vectors W positively span a space when the origin of the space lies strictly inside the convex hull of W.

Proposition 2.5 *A set of wrenches* W *in* $\mathbb{R}^n$ *achieve force closure when the origin lies in the interior of the convex hull of* $\text{INT}(\text{CO}(W))$.

Proposition 2.5 transforms the force closure testing problem into a well defined computational geometry problem. A straightforward approach to solve the problem is to compute the convex hull of the primitive contact wrenches and directly whether the origin lies inside the interior. From this approach, it comes directly that if we can identify a half space through the origin that contains all primitive contact wrenches, the primitive contact wrenches cannot positively span the space.

Proposition 2.6 *A set of wrenches* W *do not positively span* $\mathbb{R}^3$ *if there exists a vector* v *such that the closed half space* $\mathcal{H}(v)$ *contains every wrench in* W.

A closely related property of force closure is equilibrium. Equilibrium indicates that the net resultant wrench of the system is a zero vector. A grasp is said to achieve equilibrium when it is possible for some contacts of the grasp to exert wrenches such that the net resultant wrench is zero vector. Formally, a grasp is an equilibrium grasp when Equation (2.2) has a non-trivial solution.

$$\Sigma_{i=1}^{n} \alpha_i w_i = 0 \tag{2.2}$$

Apparently, a grasp that achieves force closure also is an equilibrium grasp. However, the inverse is not necessary true. In the case of frictional contact, there exists a special class of equilibrium grasp called *non-marginal equilibrium.* A grasp achieves non-marginal equilibrium when the wrenches achieving equilibrium are not the wrenches associated with the boundary of a force cone. In practice, it means that any equilibrium grasp is also a force closure grasp under any arbitrarily greater frictional coefficient.

Nguyen (1988a) shows that a 2D 2-finger non-marginal equilibrium grasp is also a force closure grasp. Ponce and Faverjon (1995a) show the same implication in the case of 2D 3-finger grasp and also in the case of 3D 4-finger grasp (Ponce et al., 1997). Care should be taken not to take this implication into general. Though it might seems that non-marginal equilibrium implies force closure, this is not always true for any number of fingers. For example a 3D two finger non-marginal equilibrium grasp *does not* achieve force closure.

Proposition 2.7 *A sufficient condition for 2- and 3-finger force closure in 2D and 4-finger force closure in 3D is non-marginal equilibrium*

2.6 Regrasping

Regrasping is a process of repositioning contact points of robot fingers. Two primitive forms of repositioning are *finger switching* and *finger sliding*. To determine an appropriate sequence of these two processes, we introduce a structure called a switching graph. A node in a switching graph represents a connected set of force closure grasps on three(four) particular polygonal edges(faces) in 2D(3D). An edge connecting two nodes indicates that there exist a grasp associated with one node that can be switched to a grasp associated with the other by finger switching. By using a switching graph, the regrasp problem can be formulated into a graph search problem. A path from the graph search

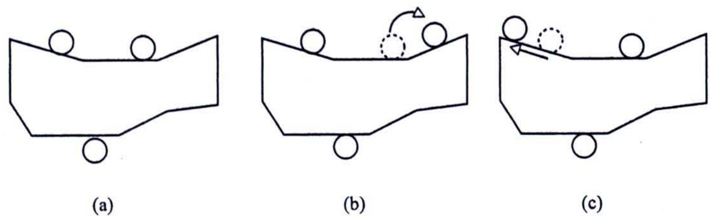

Figure 2.2: Regrasping overview: (a) Initial grasping configuration (b) A result of finger Switching. (c) a result of a finger sliding

determines a sequence of actions – switching and sliding to be executed in order to traverse from the initial to the final grasp. The following sections will describe the finger switching and sliding primitives and the switching graph in detail.

2.6.1 Finger Switching and Finger Sliding

Regrasping process which changes grasping configuration by placing an additional finger on desired contact point and then releasing one finger of the initial grasp is called finger switching. For example, let us assume that a starting grasp holds a polygonal object on points p_a, p_b and p_c and we want to switch to a grasp holding points p_b, p_c and p_d. A finger switching process starts by placing an additional finger on p_d and then releasing the finger at p_a. If both grasps satisfy the force closure property, the entire process still holds the force closure property. For the case of 4(5)-finger hand grasping a polygonal(polyhedral) object, finger switching requires that two(three) grasping configurations must have two contact points in common and both of them achieve force closure.

Finger sliding is a process for repositioning fingers by sliding them along edges(faces) of a polygon(polyhedron) while maintaining a force closure grasp during the sliding process. Using this process, we can change grasping configuration with in the same set of force closure grasps. This means the relation between finger sliding and a node of switching graph. However, finger sliding may be hard to implement mechanically since it is required that fingers must always touch the edge during sliding. Finger switching can imitate finger sliding by switching fingers from the initial to the final position of the sliding. Examples of finger switching and sliding are shown in Figure 2.2.