ภาคผนวก

Quadratic Polynomials with integer coefficients in Arithmetic Progression

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Abstract

In this paper we will show the following theorem. Let $\mathcal{A}$ be the set of all (a, b, c) where $a, b, c \in \mathbb{N}$ and (a, b, c) is an arithmetic progression with the common difference $d \in \mathbb{N}$. If $\gcd(a, b, c)$ is divisible by d, then $p := ax^2 + bx - c = 0$ has rational roots if and only if $b = dF_{2k-1}$ for some k where F_{2k-1} is a $2k - 1$th Fibonacci number. Moreover, if we consider $q := ax^2 + bx + c = 0$ then the set

$$P_d = \{(a, b, c) \in \mathcal{A} \mid q(x) = ax^2 + bx + c = 0 \text{ has rational roots } \}$$

is finite.

Keywords: arithmetic progression , quadratic polynomial, rational root