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APPENDICES

APPENDIX A

Constraint Matrices Formulation

The constraints matrices are the crucial parameters to define the region of the state in a polyhedral partition, to perform the PWA stability analysis and the controller synthesis. This section will show the summary how to construct the constraint matrices $\bar{G}_i, \bar{E}_i, \bar{F}_i, \bar{S}_i$. It is instructive to first formulate $\bar{H}$, $\bar{F}_i$, $\bar{G}_i$, and $\bar{E}_i$, respectively.

Polyhedral Hyperplane

From the definition of the hyperplane $\partial\mathcal{H}_k$ (2.9) and the hyperplane matrix $\bar{H}$ (2.10), it is obvious to obtain $\partial\mathcal{H}_k$ from the linear equation that separates any regions and all of them are collected in $\bar{H}$. Each hyperplane induced two closed half-spaces

$$\partial\mathcal{H}_k^+ = \{x \mid H_k x + h_k \geq 0\} \tag{1}$$

$$\partial\mathcal{H}_k^- = \{x \mid H_k x + h_k \leq 0\} \tag{2}$$

with the convention $h_k \leq 0$ that implies I_0 is always in $\partial\mathcal{H}_k^-$ for all $k \in K$.

Continuity Matrix

$$k\text{th row of } \bar{F}_i = \begin{cases} k\text{th row of } \bar{H}, & X_i \subseteq \partial\mathcal{H}_k^+ \\ \mathbf{0}, & \text{otherwise} \end{cases} \tag{3}$$

In order to make the continuity matrices full column rank, we can augment them according to

$$\bar{F}_i = \begin{bmatrix} F_i & f_i \\ I & 0 \end{bmatrix} \tag{4}$$

Cell Identifier

$$k\text{th row of } \bar{G}_i = \begin{cases} (-1) \times k\text{th row of } \bar{H}, & X_i \subseteq \partial\mathcal{H}_k^- \\ (+1) \times k\text{th row of } \bar{H}, & X_i \subseteq \partial\mathcal{H}_k^+ \end{cases}$$

Cell Bounding

The cell boundings $\bar{E}_i$ can be obtained by

- If $i \in I_0$, delete all rows of $\bar{G}_i$ whose the last entry is non-zero.
- If $i \in I_1$, and X_i is unbound, augment $\bar{G}_i$ with the row $\begin{bmatrix} 0_{1\times n} & 1 \end{bmatrix}$
- Otherwise, $\bar{E}_i = \bar{G}_i$.

The Constraint Matrices for PWA Bicycle model

Region	$\bar{G}_i$	$\bar{E}_i$	$\bar{F}_i$
X_1	$\begin{bmatrix} -1 & 0 & 0 & 0 & -0.1745 \\ 0 & 1 & 0 & 0 & -0.1745 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 & 0 & -0.1745 \\ 0 & 1 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -0.1745 \\ 0 & 1 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
X_2	$\begin{bmatrix} -1 & 0 & 0 & 0 & 0.1745 \\ 1 & 0 & 0 & 0 & 0.1745 \\ 0 & 1 & 0 & 0 & -0.1745 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 & 0 & 0.1745 \\ 1 & 0 & 0 & 0 & 0.1745 \\ 0 & 1 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
X_3	$\begin{bmatrix} 1 & 0 & 0 & 0 & -0.1745 \\ 0 & 1 & 0 & 0 & -0.1745 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 & -0.1745 \\ 0 & 1 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
X_4	$\begin{bmatrix} -1 & 0 & 0 & 0 & -0.1745 \\ 0 & -1 & 0 & 0 & 0.1745 \\ 0 & 1 & 0 & 0 & 0.1745 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 & 0 & -0.1745 \\ 0 & -1 & 0 & 0 & 0.1745 \\ 0 & 1 & 0 & 0 & 0.1745 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
X_5	$\begin{bmatrix} -1 & 0 & 0 & 0 & 0.1745 \\ 1 & 0 & 0 & 0 & 0.1745 \\ 0 & -1 & 0 & 0 & 0.1745 \\ 0 & 1 & 0 & 0 & 0.1745 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
X_6	$\begin{bmatrix} 1 & 0 & 0 & 0 & -0.1745 \\ 0 & -1 & 0 & 0 & 0.1745 \\ 0 & 1 & 0 & 0 & 0.1745 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 & -0.1745 \\ 0 & -1 & 0 & 0 & 0.1745 \\ 0 & 1 & 0 & 0 & 0.1745 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
X_7	$\begin{bmatrix} -1 & 0 & 0 & 0 & -0.1745 \\ 0 & -1 & 0 & 0 & -0.1745 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 & 0 & -0.1745 \\ 0 & -1 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -0.1745 \end{bmatrix}$
X_8	$\begin{bmatrix} -1 & 0 & 0 & 0 & 0.1745 \\ 1 & 0 & 0 & 0 & 0.1745 \\ 0 & -1 & 0 & 0 & -0.1745 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 & 0 & 0.1745 \\ 1 & 0 & 0 & 0 & 0.1745 \\ 0 & -1 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -0.1745 \end{bmatrix}$
X_9	$\begin{bmatrix} 1 & 0 & 0 & 0 & -0.1745 \\ 0 & -1 & 0 & 0 & -0.1745 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 & -0.1745 \\ 0 & -1 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 & -0.1745 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -0.1745 \end{bmatrix}$

APPENDIX B

Ellipsoid Cell Boundings

In mathematics, the ellipsoid can be written in different ways, e.g. the quadratic set, the shape matrix with uncertainty, etc. We will not go further to those topics. The minimum volume ellipsoid that cover each polyhedral cell in this thesis is suitable to define in this form

$$\mathcal{E}_{mve} = \{x \in \mathbb{R}^n \mid \|Sx + s\|_2 \leq 1\}$$

Our interested parameter of polyhedral cell is its m vertices v_i. The minimum volume ellipsoid is obtained by solving the following convex optimization problem

$$\begin{array}{ll} \text{minimize} & \log\det S^{-1} \\ \text{subject to} & \begin{bmatrix} I & Sv_i + s \\ v_i^T S^T + s^T & 1 \end{bmatrix} \geq 0, \quad i = 1, \ldots, m \\ & S = S^T > 0 \end{array} \tag{5}$$

We call S an ellipsoid cell bounding. It is useful for deriving the control law as shown in Theorem 2.2.

The Ellipsoid Cell Bounding for PWA Bicycle model

The computed parameter, polyhedral vertices, and the resulted ellipsoid cell bounding in all 9 regions are listed below.

$$\begin{array}{lll} x_a = 0.1745 & y_a = 0.1745 & z_a = 100 \\ x_b = 1.0472 & y_b = 1.0472 & z_b = 100 \end{array}$$

x_a and x_b denote the bounding point of parameter φ $(10^\circ, 60^\circ)$.

y_a and y_b denote the bounding point of parameter α $(10^\circ, 60^\circ)$.

z_b denotes the bounding point of parameter $\dot{\varphi}$, $\dot{\alpha}$ (no bounding, so we assign a sufficiently high value).

Polytope X_1

$$v_{11}^1 = \begin{bmatrix} -x_a \\ y_b \\ -z_b \\ -z_b \end{bmatrix} \quad v_{12}^1 = \begin{bmatrix} -x_a \\ y_b \\ -z_b \\ z_b \end{bmatrix} \quad v_{13}^1 = \begin{bmatrix} -x_a \\ y_b \\ z_b \\ -z_b \end{bmatrix} \quad v_{14}^1 = \begin{bmatrix} -x_a \\ y_b \\ z_b \\ z_b \end{bmatrix}$$

$$v_{21}^1 = \begin{bmatrix} -x_b \\ y_b \\ -z_b \\ -z_b \end{bmatrix} \quad v_{22}^1 = \begin{bmatrix} -x_b \\ y_b \\ -z_b \\ z_b \end{bmatrix} \quad v_{23}^1 = \begin{bmatrix} -x_b \\ y_b \\ z_b \\ -z_b \end{bmatrix} \quad v_{24}^1 = \begin{bmatrix} -x_b \\ y_b \\ z_b \\ z_b \end{bmatrix}$$

$$v_{31}^1 = \begin{bmatrix} -x_b \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{32}^1 = \begin{bmatrix} -x_b \\ y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{33}^1 = \begin{bmatrix} -x_b \\ y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{34}^1 = \begin{bmatrix} -x_b \\ y_a \\ z_b \\ z_b \end{bmatrix}$$

$$v_{41}^1 = \begin{bmatrix} -x_a \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{42}^1 = \begin{bmatrix} -x_a \\ y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{43}^1 = \begin{bmatrix} -x_a \\ y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{44}^1 = \begin{bmatrix} -x_a \\ y_a \\ z_b \\ z_b \end{bmatrix}$$

Ellipsoid $\mathcal{E}_1$

$$S_1 = \begin{bmatrix} 1.1459 & 0 & 0 & 0 \\ 0 & 1.1459 & 0 & 0 \\ 0 & 0 & 0.0050 & 0 \\ 0 & 0 & 0 & 0.0050 \end{bmatrix} \quad s_1 = \begin{bmatrix} 0.7 \\ -0.7 \\ 0 \\ 0 \end{bmatrix}$$

Polytope X_2

$$v_{11}^2 = \begin{bmatrix} x_a \\ y_b \\ -z_b \\ -z_b \end{bmatrix} \quad v_{12}^2 = \begin{bmatrix} x_a \\ y_b \\ -z_b \\ z_b \end{bmatrix} \quad v_{13}^2 = \begin{bmatrix} x_a \\ y_b \\ z_b \\ -z_b \end{bmatrix} \quad v_{14}^2 = \begin{bmatrix} x_a \\ y_b \\ z_b \\ z_b \end{bmatrix}$$

$$v_{21}^2 = \begin{bmatrix} -x_a \\ y_b \\ -z_b \\ -z_b \end{bmatrix} \quad v_{22}^2 = \begin{bmatrix} -x_a \\ y_b \\ -z_b \\ z_b \end{bmatrix} \quad v_{23}^2 = \begin{bmatrix} -x_a \\ y_b \\ z_b \\ -z_b \end{bmatrix} \quad v_{24}^2 = \begin{bmatrix} -x_a \\ y_b \\ z_b \\ z_b \end{bmatrix}$$

$$v_{31}^2 = \begin{bmatrix} -x_a \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{32}^2 = \begin{bmatrix} -x_a \\ y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{33}^2 = \begin{bmatrix} -x_a \\ y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{34}^2 = \begin{bmatrix} -x_a \\ y_a \\ z_b \\ z_b \end{bmatrix}$$

$$v_{41}^2 = \begin{bmatrix} x_a \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{42}^2 = \begin{bmatrix} x_a \\ y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{43}^2 = \begin{bmatrix} x_a \\ y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{44}^2 = \begin{bmatrix} x_a \\ y_a \\ z_b \\ z_b \end{bmatrix}$$

Ellipsoid $\mathcal{E}_2$

$$S_2 = \begin{bmatrix} 2.8647 & 0 & 0 & 0 \\ 0 & 1.1459 & 0 & 0 \\ 0 & 0 & 0.0050 & 0 \\ 0 & 0 & 0 & 0.0050 \end{bmatrix} \quad s_2` = \begin{bmatrix} 0 \\ -0.7 \\ 0 \\ 0 \end{bmatrix}$$

Polytope X_3

$$v_{11}^3 = \begin{bmatrix} x_b \\ y_b \\ -z_b \\ -z_b \end{bmatrix} \quad v_{12}^3 = \begin{bmatrix} x_b \\ y_b \\ -z_b \\ z_b \end{bmatrix} \quad v_{13}^3 = \begin{bmatrix} x_b \\ y_b \\ z_b \\ -z_b \end{bmatrix} \quad v_{14}^3 = \begin{bmatrix} x_b \\ y_b \\ z_b \\ z_b \end{bmatrix}$$

$$v_{21}^3 = \begin{bmatrix} -x_a \\ y_b \\ -z_b \\ -z_b \end{bmatrix} \quad v_{22}^3 = \begin{bmatrix} -x_a \\ y_b \\ -z_b \\ z_b \end{bmatrix} \quad v_{23}^3 = \begin{bmatrix} -x_a \\ y_b \\ z_b \\ -z_b \end{bmatrix} \quad v_{24}^3 = \begin{bmatrix} -x_a \\ y_b \\ z_b \\ z_b \end{bmatrix}$$

$$v_{31}^3 = \begin{bmatrix} -x_a \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{32}^3 = \begin{bmatrix} -x_a \\ y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{33}^3 = \begin{bmatrix} -x_a \\ y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{34}^3 = \begin{bmatrix} -x_a \\ y_a \\ z_b \\ z_b \end{bmatrix}$$

$$v_{41}^3 = \begin{bmatrix} x_b \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{42}^3 = \begin{bmatrix} x_b \\ y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{43}^3 = \begin{bmatrix} x_b \\ y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{44}^3 = \begin{bmatrix} x_b \\ y_a \\ z_b \\ z_b \end{bmatrix}$$

Ellipsoid $\mathcal{E}_3$

$$S_3 = \begin{bmatrix} 1.1459 & 0 & 0 & 0 \\ 0 & 1.1459 & 0 & 0 \\ 0 & 0 & 0.0050 & 0 \\ 0 & 0 & 0 & 0.0050 \end{bmatrix} \qquad s_3 = \begin{bmatrix} -0.7 \\ -0.7 \\ 0 \\ 0 \end{bmatrix}$$

Polytope X_4

$$v_{11}^4 = \begin{bmatrix} -x_a \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{12}^4 = \begin{bmatrix} -x_a \\ y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{13}^4 = \begin{bmatrix} -x_a \\ y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{14}^4 = \begin{bmatrix} -x_a \\ y_a \\ z_b \\ z_b \end{bmatrix}$$

$$v_{21}^4 = \begin{bmatrix} -x_b \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{22}^4 = \begin{bmatrix} -x_b \\ y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{23}^4 = \begin{bmatrix} -x_b \\ y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{24}^4 = \begin{bmatrix} -x_b \\ -y_a \\ z_b \\ z_b \end{bmatrix}$$

$$v_{31}^4 = \begin{bmatrix} -x_b \\ -y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{32}^4 = \begin{bmatrix} -x_b \\ -y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{33}^4 = \begin{bmatrix} -x_b \\ -y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{34}^4 = \begin{bmatrix} -x_b \\ -y_a \\ z_b \\ z_b \end{bmatrix}$$

$$v_{41}^4 = \begin{bmatrix} -x_a \\ -y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{42}^4 = \begin{bmatrix} -x_a \\ -y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{43}^4 = \begin{bmatrix} -x_a \\ -y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{44}^4 = \begin{bmatrix} -x_a \\ -y_a \\ z_b \\ z_b \end{bmatrix}$$

Ellipsoid $\mathcal{E}_4$

$$S_4 = \begin{bmatrix} 1.1459 & 0 & 0 & 0 \\ 0 & 2.8647 & 0 & 0 \\ 0 & 0 & 0.0050 & 0 \\ 0 & 0 & 0 & 0.0050 \end{bmatrix} \qquad s_4 = \begin{bmatrix} 0.7 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Polytope X_5

$$v_{11}^5 = \begin{bmatrix} x_a \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{12}^5 = \begin{bmatrix} x_a \\ y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{13}^5 = \begin{bmatrix} x_a \\ y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{14}^5 = \begin{bmatrix} x_a \\ y_a \\ z_b \\ z_b \end{bmatrix}$$

$$v_{21}^5 = \begin{bmatrix} -x_a \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{22}^5 = \begin{bmatrix} -x_a \\ y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{23}^5 = \begin{bmatrix} -x_a \\ y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{24}^5 = \begin{bmatrix} -x_a \\ y_a \\ z_b \\ z_b \end{bmatrix}$$

$$v_{31}^5 = \begin{bmatrix} -x_a \\ -y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{32}^5 = \begin{bmatrix} -x_a \\ -y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{33}^5 = \begin{bmatrix} -x_a \\ -y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{34}^5 = \begin{bmatrix} x_a \\ -y_a \\ z_b \\ z_b \end{bmatrix}$$

$$v_{41}^5 = \begin{bmatrix} x_a \\ -y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{42}^5 = \begin{bmatrix} x_a \\ -y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{43}^5 = \begin{bmatrix} x_a \\ -y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{44}^5 = \begin{bmatrix} x_a \\ -y_a \\ z_b \\ z_b \end{bmatrix}$$

Ellipsoid $\mathcal{E}_5$

$$S_5 = \begin{bmatrix} 2.8648 & 0 & 0 & 0 \\ 0 & 2.8648 & 0 & 0 \\ 0 & 0 & 0.0050 & 0 \\ 0 & 0 & 0 & 0.0050 \end{bmatrix} \qquad s_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Polytope X_6

$$v_{11}^6 = \begin{bmatrix} x_b \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{12}^6 = \begin{bmatrix} x_b \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{13}^6 = \begin{bmatrix} x_b \\ y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{14}^6 = \begin{bmatrix} x_b \\ y_a \\ -z_b \\ -z_b \end{bmatrix}$$

$$v_{21}^6 = \begin{bmatrix} x_a \\ y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{22}^6 = \begin{bmatrix} x_a \\ y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{23}^6 = \begin{bmatrix} x_a \\ y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{24}^6 = \begin{bmatrix} x_a \\ y_a \\ z_b \\ -z_b \end{bmatrix}$$

$$v_{31}^6 = \begin{bmatrix} x_a \\ -y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{32}^6 = \begin{bmatrix} x_a \\ -y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{33}^6 = \begin{bmatrix} x_a \\ -y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{34}^6 = \begin{bmatrix} x_a \\ -y_a \\ -z_b \\ z_b \end{bmatrix}$$

$$v_{41}^6 = \begin{bmatrix} x_b \\ -y_a \\ z_b \\ z_b \end{bmatrix} \quad v_{42}^6 = \begin{bmatrix} x_b \\ -y_a \\ z_b \\ z_b \end{bmatrix} \quad v_{43}^6 = \begin{bmatrix} x_b \\ -y_a \\ z_b \\ z_b \end{bmatrix} \quad v_{44}^6 = \begin{bmatrix} x_b \\ -y_a \\ z_b \\ z_b \end{bmatrix}$$

Ellipsoid $\mathcal{E}_6$

$$S_6 = \begin{bmatrix} 1.1459 & 0 & 0 & 0 \\ 0 & 2.8647 & 0 & 0 \\ 0 & 0 & 0.0050 & 0 \\ 0 & 0 & 0 & 0.0050 \end{bmatrix} \qquad s_6 = \begin{bmatrix} -0.7 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Polytope X_7

$$v_{11}^7 = \begin{bmatrix} -x_a \\ -y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{12}^7 = \begin{bmatrix} -x_a \\ -y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{13}^7 = \begin{bmatrix} -x_a \\ -y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{14}^{=} \begin{bmatrix} -x_a \\ -y_a \\ z_b \\ z_b \end{bmatrix}$$

$$v_{21}^7 = \begin{bmatrix} -x_b \\ -y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{22}^7 = \begin{bmatrix} -x_b \\ -y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{23}^7 = \begin{bmatrix} -x_b \\ -y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{24}^7 = \begin{bmatrix} -x_b \\ -y_a \\ z_b \\ z_b \end{bmatrix}$$

$$v_{31}^7 = \begin{bmatrix} -x_b \\ -y_b \\ -z_b \\ -z_b \end{bmatrix} \quad v_{32}^7 = \begin{bmatrix} -x_b \\ -y_b \\ -z_b \\ z_b \end{bmatrix} \quad v_{33}^7 = \begin{bmatrix} -x_b \\ -y_b \\ z_b \\ -z_b \end{bmatrix} \quad v_{34}^7 = \begin{bmatrix} -x_b \\ -y_b \\ z_b \\ z_b \end{bmatrix}$$

$$v_{41}^7 = \begin{bmatrix} -x_a \\ -y_b \\ -z_b \\ -z_b \end{bmatrix} \quad v_{42}^7 = \begin{bmatrix} -x_a \\ -y_b \\ -z_b \\ z_b \end{bmatrix} \quad v_{43}^7 = \begin{bmatrix} -x_a \\ -y_b \\ z_b \\ -z_b \end{bmatrix} \quad v_{44}^7 = \begin{bmatrix} -x_a \\ -y_b \\ z_b \\ z_b \end{bmatrix}$$

Ellipsoid $\mathcal{E}_7$

$$S_7 = \begin{bmatrix} 1.1459 & 0 & 0 & 0 \\ 0 & 1.1459 & 0 & 0 \\ 0 & 0 & 0.0050 & 0 \\ 0 & 0 & 0 & 0.0050 \end{bmatrix} \qquad s_7 = \begin{bmatrix} 0.7 \\ 0.7 \\ 0 \\ 0 \end{bmatrix}$$

Polytope X_8

$$v_{11}^8 = \begin{bmatrix} x_a \\ -y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{12}^8 = \begin{bmatrix} x_a \\ -y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{13}^8 = \begin{bmatrix} x_a \\ -y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{14}^8 = \begin{bmatrix} x_a \\ -y_a \\ -z_b \\ -z_b \end{bmatrix}$$

$$v_{21}^8 = \begin{bmatrix} -x_a \\ -y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{22}^8 = \begin{bmatrix} -x_a \\ -y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{23}^8 = \begin{bmatrix} -x_a \\ -y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{24}^8 = \begin{bmatrix} -x_a \\ -y_a \\ z_b \\ -z_b \end{bmatrix}$$

$$v_{31}^8 = \begin{bmatrix} -x_a \\ -y_b \\ z_b \\ -z_b \end{bmatrix} \quad v_{32}^8 = \begin{bmatrix} -x_a \\ -y_b \\ -z_b \\ z_b \end{bmatrix} \quad v_{33}^8 = \begin{bmatrix} -x_a \\ -y_b \\ -z_b \\ z_b \end{bmatrix} \quad v_{34}^8 = \begin{bmatrix} -x_a \\ -y_b \\ -z_b \\ z_b \end{bmatrix}$$

$$v_{41}^8 = \begin{bmatrix} x_a \\ -y_b \\ z_b \\ z_b \end{bmatrix} \quad v_{42}^8 = \begin{bmatrix} x_a \\ -y_b \\ z_b \\ z_b \end{bmatrix} \quad v_{43}^8 = \begin{bmatrix} x_a \\ -y_b \\ z_b \\ z_b \end{bmatrix} \quad v_{44}^8 = \begin{bmatrix} x_a \\ -y_b \\ z_b \\ z_b \end{bmatrix}$$

Ellipsoid $\mathcal{E}_8$

$$S_8 = \begin{bmatrix} 2.8647 & 0 & 0 & 0 \\ 0 & 1.1459 & 0 & 0 \\ 0 & 0 & 0.0050 & 0 \\ 0 & 0 & 0 & 0.0050 \end{bmatrix} \qquad s_8 = \begin{bmatrix} 0 \\ 0.7 \\ 0 \\ 0 \end{bmatrix}$$

Polytope X_9

$$v_{11}^9 = \begin{bmatrix} x_b \\ -y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{12}^9 = \begin{bmatrix} x_b \\ -y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{13}^9 = \begin{bmatrix} x_b \\ -y_a \\ -z_b \\ -z_b \end{bmatrix} \quad v_{14}^9 = \begin{bmatrix} x_b \\ -y_a \\ -z_b \\ -z_b \end{bmatrix}$$

$$v_{21}^9 = \begin{bmatrix} x_a \\ -y_a \\ -z_b \\ z_b \end{bmatrix} \quad v_{22}^9 = \begin{bmatrix} x_a \\ -y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{23}^9 = \begin{bmatrix} x_a \\ -y_a \\ z_b \\ -z_b \end{bmatrix} \quad v_{24}^9 = \begin{bmatrix} x_a \\ -y_a \\ z_b \\ -z_b \end{bmatrix}$$

$$v_{31}^9 = \begin{bmatrix} x_a \\ -y_b \\ z_b \\ -z_b \end{bmatrix} \quad v_{32}^9 = \begin{bmatrix} x_a \\ -y_b \\ -z_b \\ z_b \end{bmatrix} \quad v_{33}^9 = \begin{bmatrix} x_a \\ -y_b \\ -z_b \\ z_b \end{bmatrix} \quad v_{34}^9 = \begin{bmatrix} x_a \\ -y_b \\ -z_b \\ z_b \end{bmatrix}$$

$$v_{41}^9 = \begin{bmatrix} x_b \\ -y_b \\ z_b \\ z_b \end{bmatrix} \quad v_{42}^9 = \begin{bmatrix} x_b \\ -y_b \\ z_b \\ z_b \end{bmatrix} \quad v_{43}^9 = \begin{bmatrix} x_b \\ -y_b \\ z_b \\ z_b \end{bmatrix} \quad v_{44}^9 = \begin{bmatrix} x_b \\ -y_b \\ z_b \\ z_b \end{bmatrix}$$

Ellipsoid $\mathcal{E}_9$

$$S_1 = \begin{bmatrix} 1.1459 & 0 & 0 & 0 \\ 0 & 1.1459 & 0 & 0 \\ 0 & 0 & 0.0050 & 0 \\ 0 & 0 & 0 & 0.0050 \end{bmatrix} \qquad s_1 = \begin{bmatrix} -0.7 \\ 0.7 \\ 0 \\ 0 \end{bmatrix}$$

Biography

Born in Ratchaburi, Thailand, in 1987. Sompol Suntharasantic finished his high school, Phichit Pittayakom, in 2005 and passed the entrance examination to Chulalongkorn University. He obtained his Bachelor's Degree in Electrical Engineering in 2009. He was granted "Sitkonkuti" Scholarship from Electrical Enginnering Department to pursue his Master's degree in electrical engineering at Chulalongkorn University, Thailand, since 2009. He studied and did his research in Control Systems Research Laboratory.

Throughout the graduate studies, Sompol's research was under the supervision of Assistant Professor Manop Wongsaisuwan. His field of interest includes nonlinear control, piecewise-affine control, linear matrix inequalities, electronics, and robotics.

List of Publications

1. S. Suntharasantic, and M. Wongsaisuwan. Piecewise Affine Model and Control of Bicycle by Gyroscopic Stabilization. in *Proc. of ECTI-CON conference*. (2011): accepted for publication.

2. S. Suntharasantic, P. Rungtweesuk, and M. Wongsaisuwan. Piecewise Affine Model Approximation for Unmanned Bicycle. in *SICE Annual Conference*. (2011): paper submitted.