CHAPTER VI

PIECEWISE AFFINE CONTROL FOR BICYCLE ROBOT

The unstable nonlinear bicycle robot system has been already transformed to the PWA system defined by the state-space matrices and cell boundings. This made the stability analysis for the actual nonlinear system easier by searching for the PWQ Lyapunov candidate function of an approximated PWA model. The problem can be cast as a convex optimization problem which has a powerful tool for solving this kind of problem.

In this chapter, we gather all information so far from the beginning to derive the globally quadratic Lypunov function and thus to generate the feedback control laws for system stabilization.

6.1 Problem Formulation

The problem is formulated according to Theorem 2.2. In this problem, we use the discontinuous model which provides the smallest average error value. The system matrices will be brought from Chapter 4. The quadratic cell boundings are computed via the minimum volume outer ellipsoid covering polytopes (see Figure 6.1) problem see the detail in Appendix B. This is the feasibility SDP problem which will be solved using YALMIP [52], the modeling language for advanced modeling and solution of convex and nonconvex optimization problems, which is implemented in MATLAB. The selected solver is SDPT3 [53].

6.2 Main Result

The outcome parameters of solving the problem (2.26) are shown below:

$$Y = \begin{bmatrix} -62.444 & -116.74 & 569.69 & 6956.2 \end{bmatrix}$$

$$Q = Q^T = \begin{bmatrix} 0.33589 & 0.057914 & -0.98225 & 1.2957 \\ * & 0.56223 & -1.9066 & -3.7011 \\ * & * & 9.2882 & 12.037 \\ * & * & * & 315.32 \end{bmatrix} > 0$$

$$L = YQ^{-1} = \begin{bmatrix} -337.98 & -116.31 & -28.303 & 23.165 \end{bmatrix}$$

The globally quadratic Lyapunov function is $V(x) = x^T P x$ where

$$P = Q^{-1} = \begin{bmatrix} 9.7037 & 7.7934 & 2.6923 & -0.051174 \\ * & 12.29 & 3.3681 & -0.016343 \\ * & * & 1.1013 & -0.013572 \\ * & * & * & 0.003708 \end{bmatrix} > 0$$

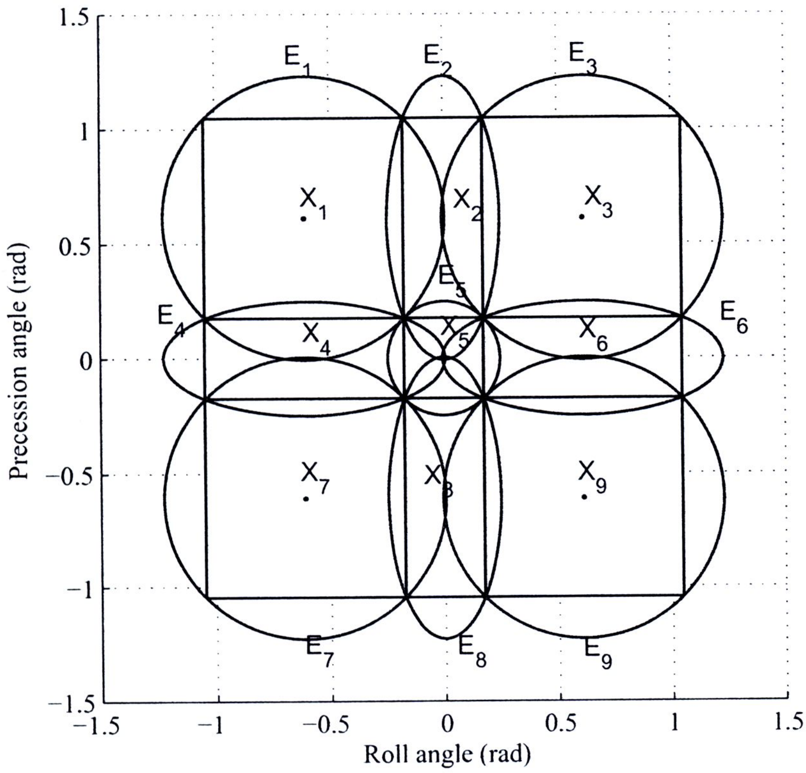

Figure 6.1: Polyhedral partition with its outer minimum volumn ellipsoid approximation.

The obtained gain L is used to feedback with $u = -Lx$ in the bicycle system. We show the simulation result of this control laws in the original nonlinear bicycle model (Figure 6.2) and the approximated PWA model (Figure 6.3).

From the series of resulting plots in Figures 6.4-6.11, we conclude that the gain L can perfectly stabilize the approximated PWA system and also the original nonlinear bicycle system. Moreover, the approximated PWA model yield a very good response as it travels quite close to the nonlinear trajectory for all partitioned regions.

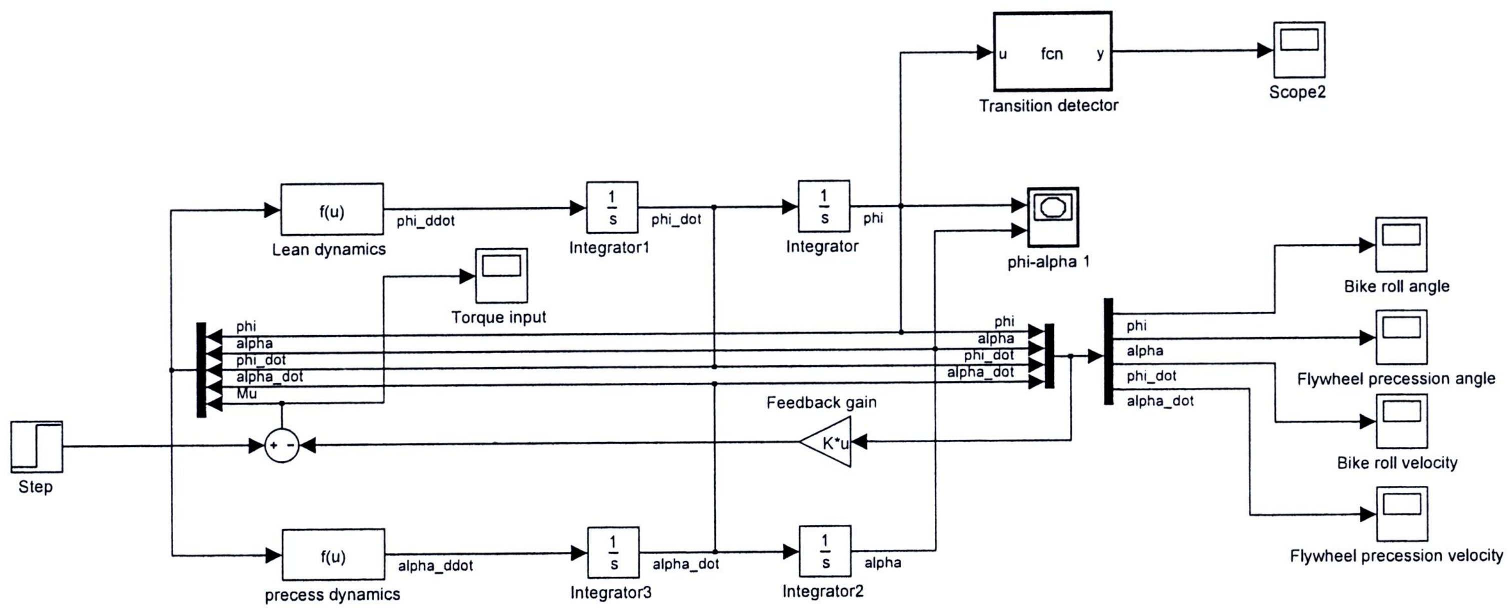

Figure 6.2: Simulink model of nonlinear bicycle model.

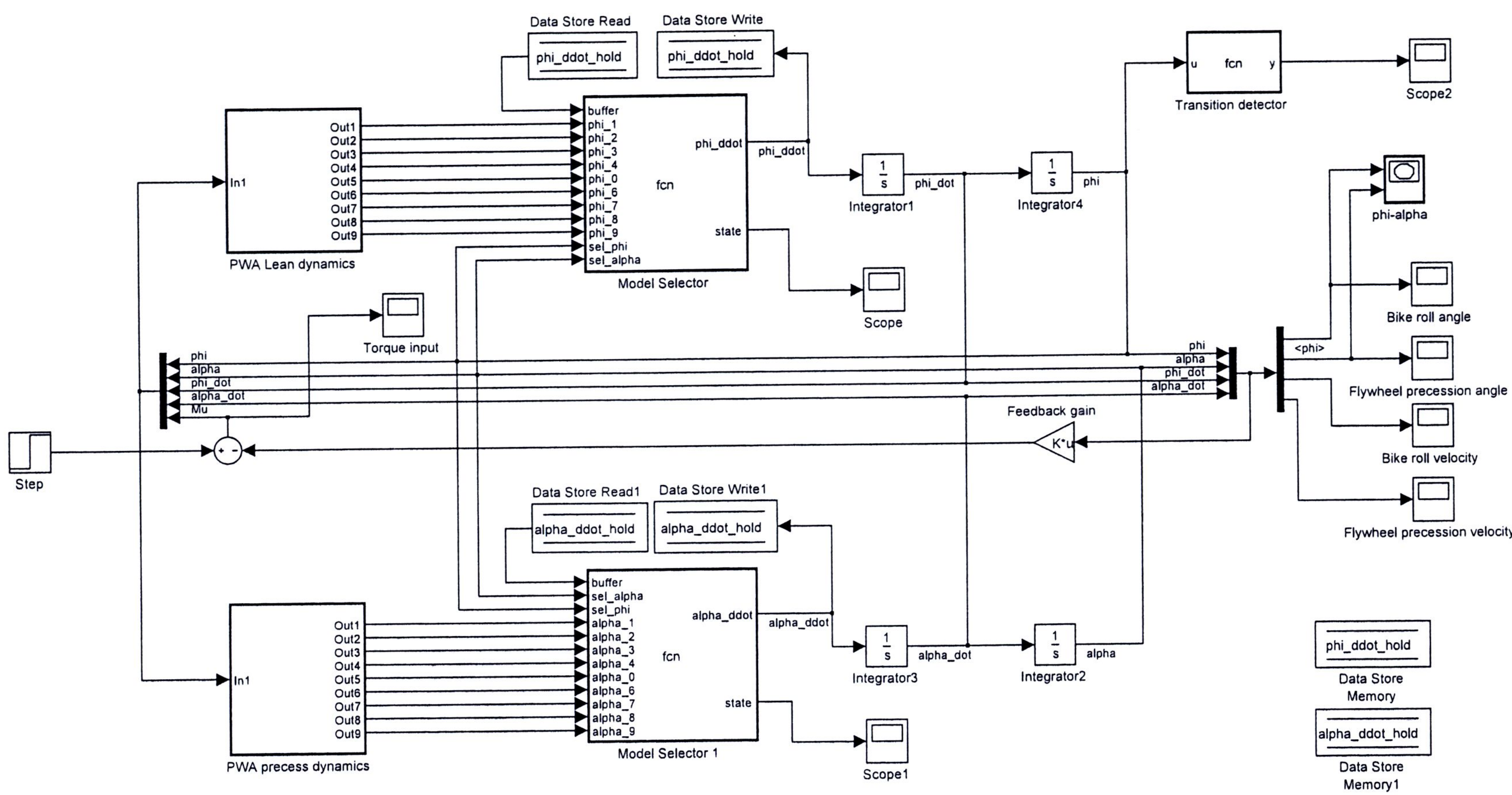

Figure 6.3: Simulink model - PWA bicycle model.

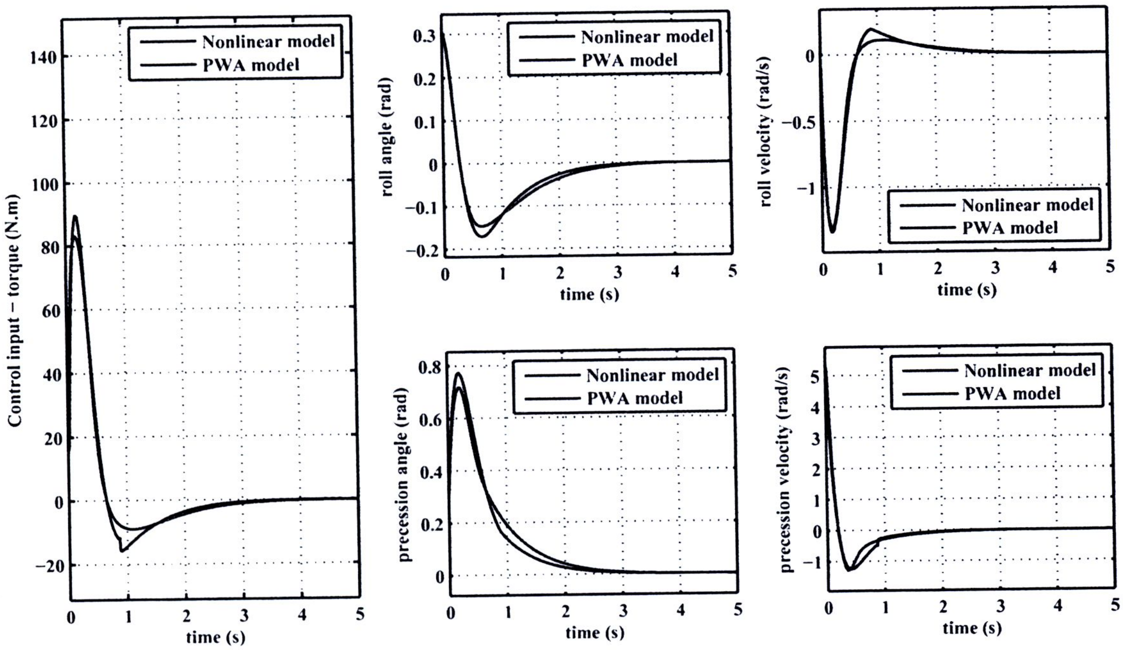

Figure 6.4: The response of roll angle, roll velocity, precession angle, and precession velocity of Nonlinear and PWA model with the initial condition $(\varphi(0), \alpha(0), \dot{\varphi}(0), \dot{\alpha}(0)) = (0.3, 0.3, 0, 0)$.

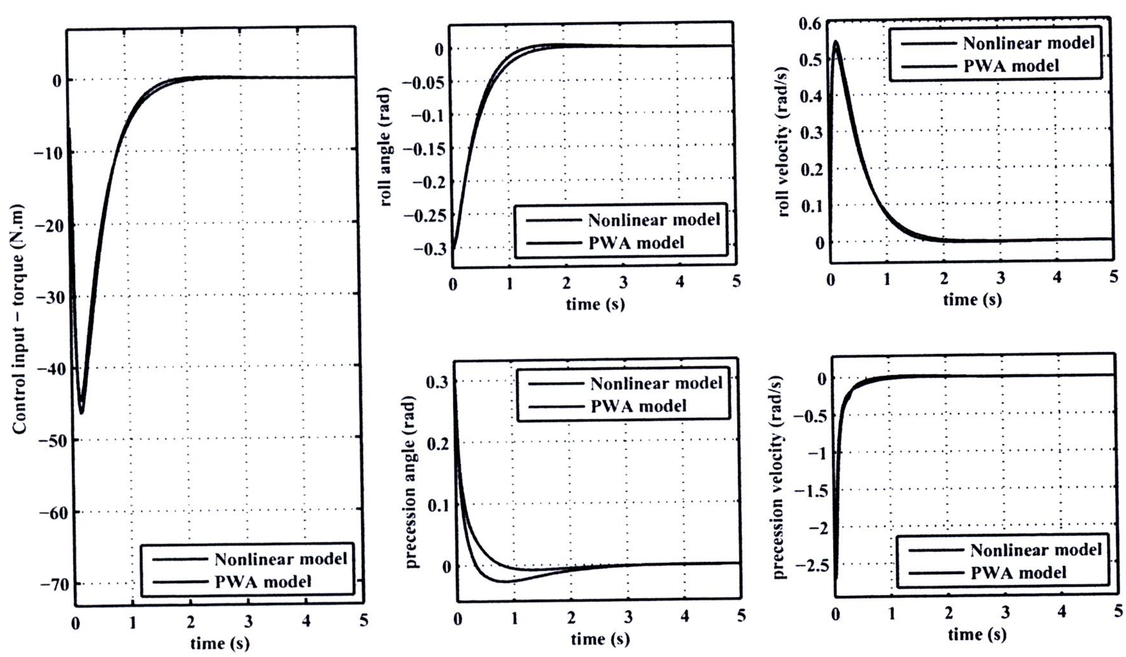

Figure 6.5: The response of roll angle, roll velocity, precession angle, and precession velocity of Nonlinear and PWA model with the initial condition $(\varphi(0), \alpha(0), \dot{\varphi}(0), \dot{\alpha}(0)) = (-0.3, 0.3, 0, 0)$.

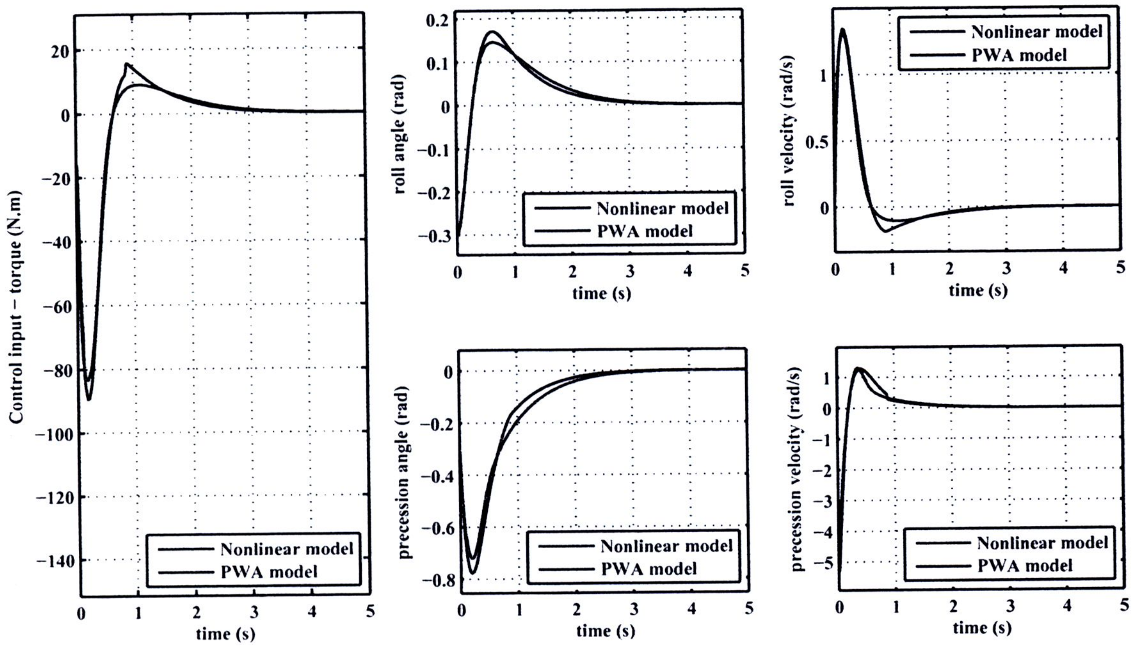

Figure 6.6: The response of roll angle, roll velocity, precession angle, and precession velocity of Nonlinear and PWA model with the initial condition $(\varphi(0), \alpha(0), \dot{\varphi}(0), \dot{\alpha}(0)) = (-0.3, -0.3, 0, 0)$.

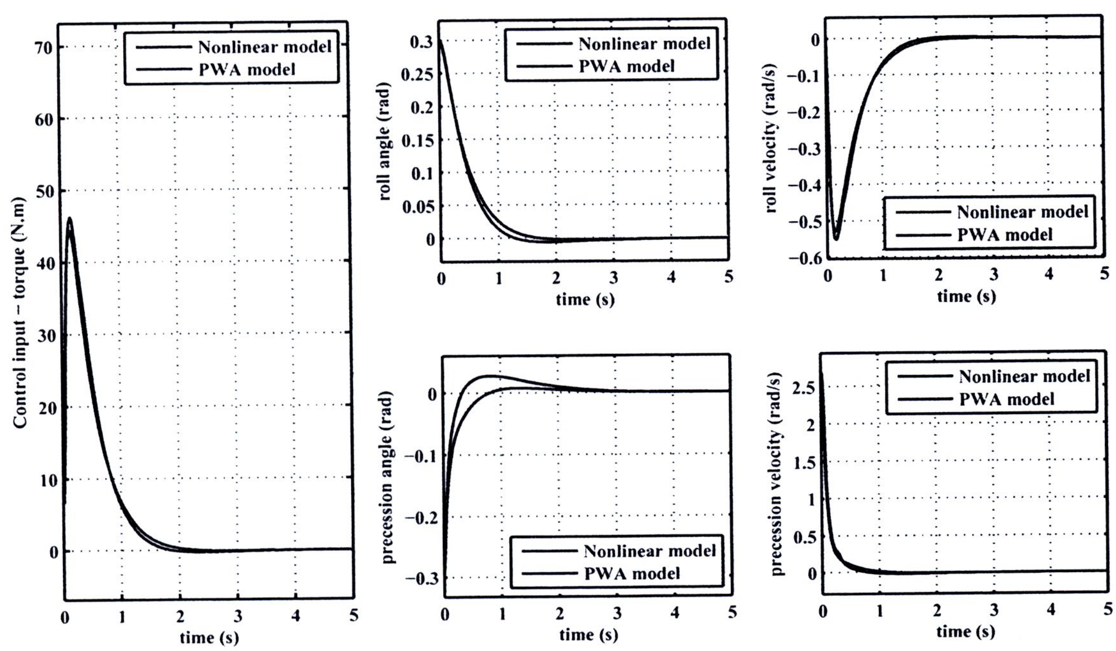

Figure 6.7: The response of roll angle, roll velocity, precession angle, and precession velocity of Nonlinear and PWA model with the initial condition $(\varphi(0), \alpha(0), \dot{\varphi}(0), \dot{\alpha}(0)) = (0.3, -0.3, 0, 0)$.

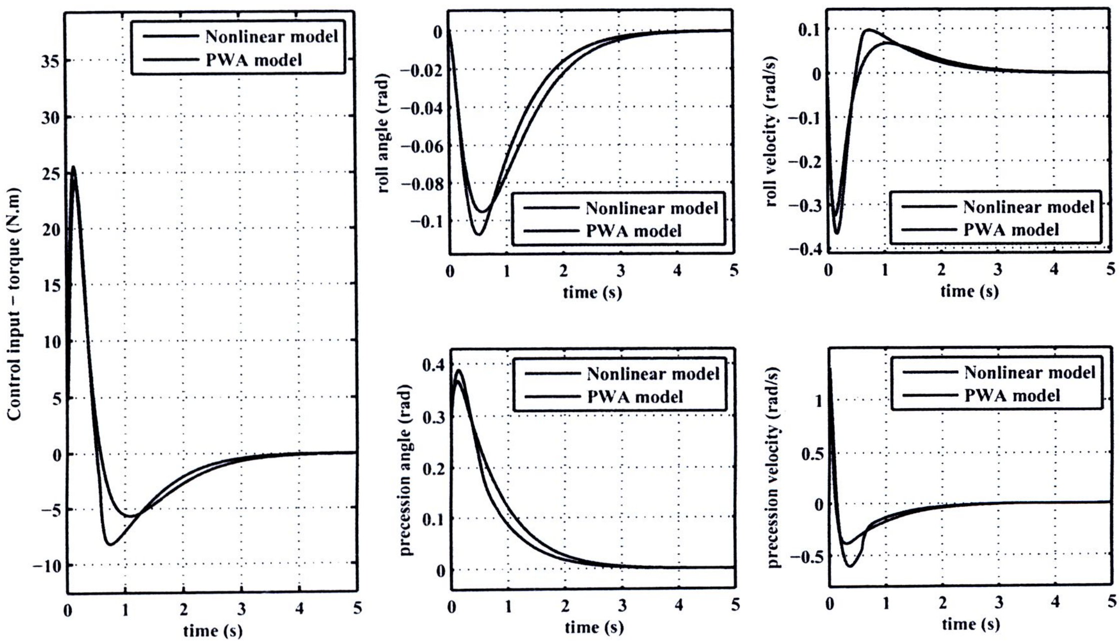

Figure 6.8: The response of roll angle, roll velocity, precession angle, and precession velocity of Nonlinear and PWA model with the initial condition $(\varphi(0), \alpha(0), \dot{\varphi}(0), \dot{\alpha}(0)) = (0, 0.3, 0, 0)$.

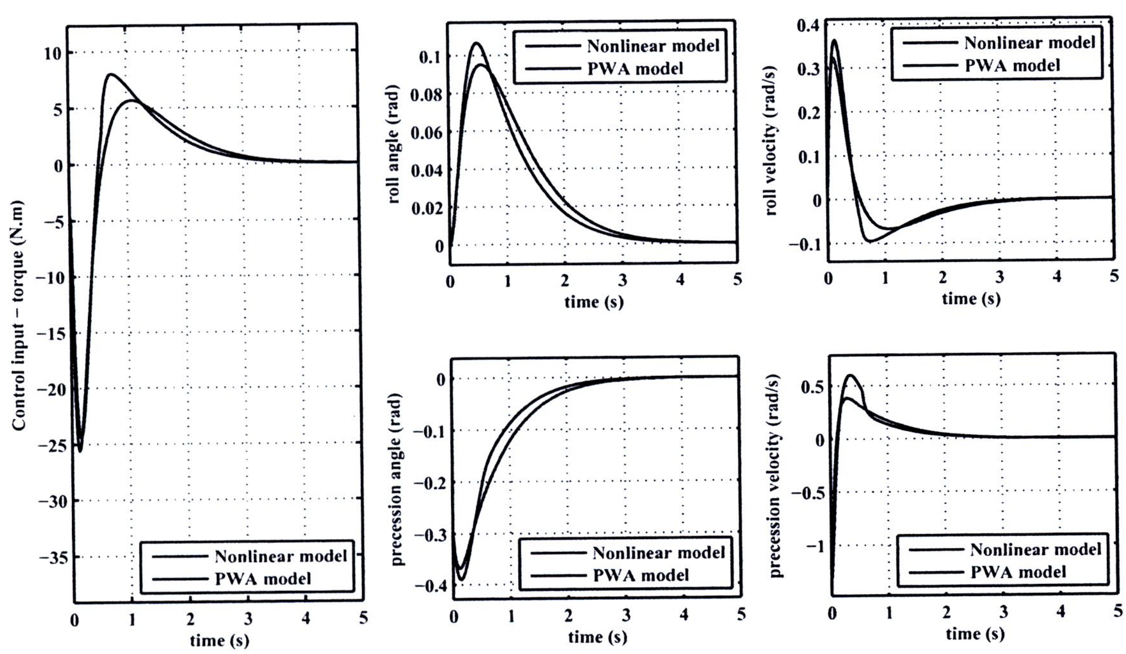

Figure 6.9: The response of roll angle, roll velocity, precession angle, and precession velocity of Nonlinear and PWA model with the initial condition $(\varphi(0), \alpha(0), \dot{\varphi}(0), \dot{\alpha}(0)) = (0, -0.3, 0, 0)$.

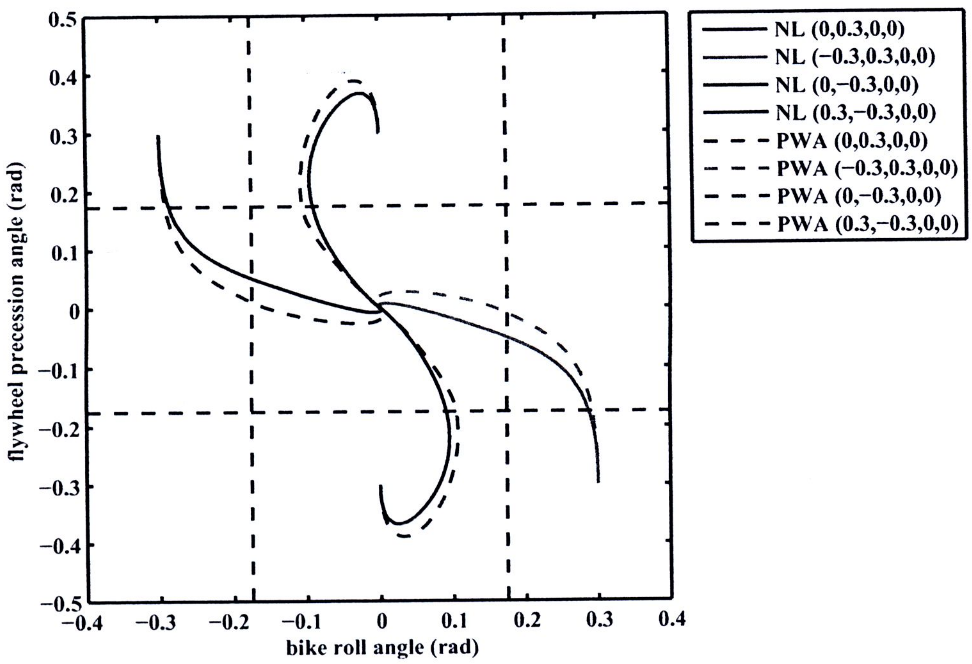

Figure 6.10: The trajectory of bike roll angle vs flywheel precession angle with 4 sets of initial conditions $(\varphi(0), \alpha(0), \dot{\varphi}(0), \dot{\alpha}(0))$.

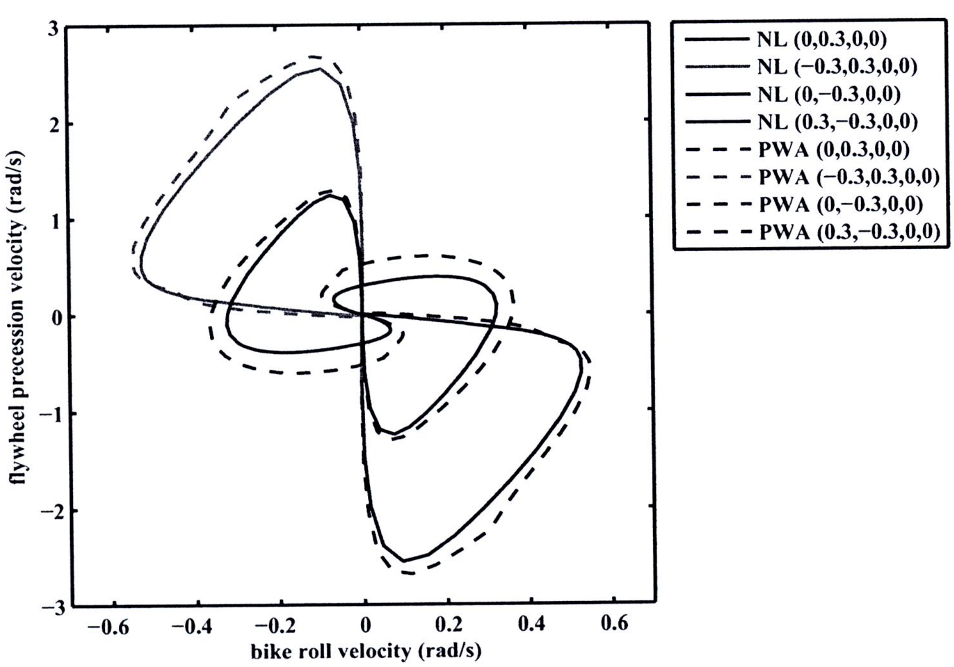

Figure 6.11: The trajectory of bike roll velocity vs flywheel precession velocity with 4 sets of initial conditions $(\varphi(0), \alpha(0), \dot{\varphi}(0), \dot{\alpha}(0))$.

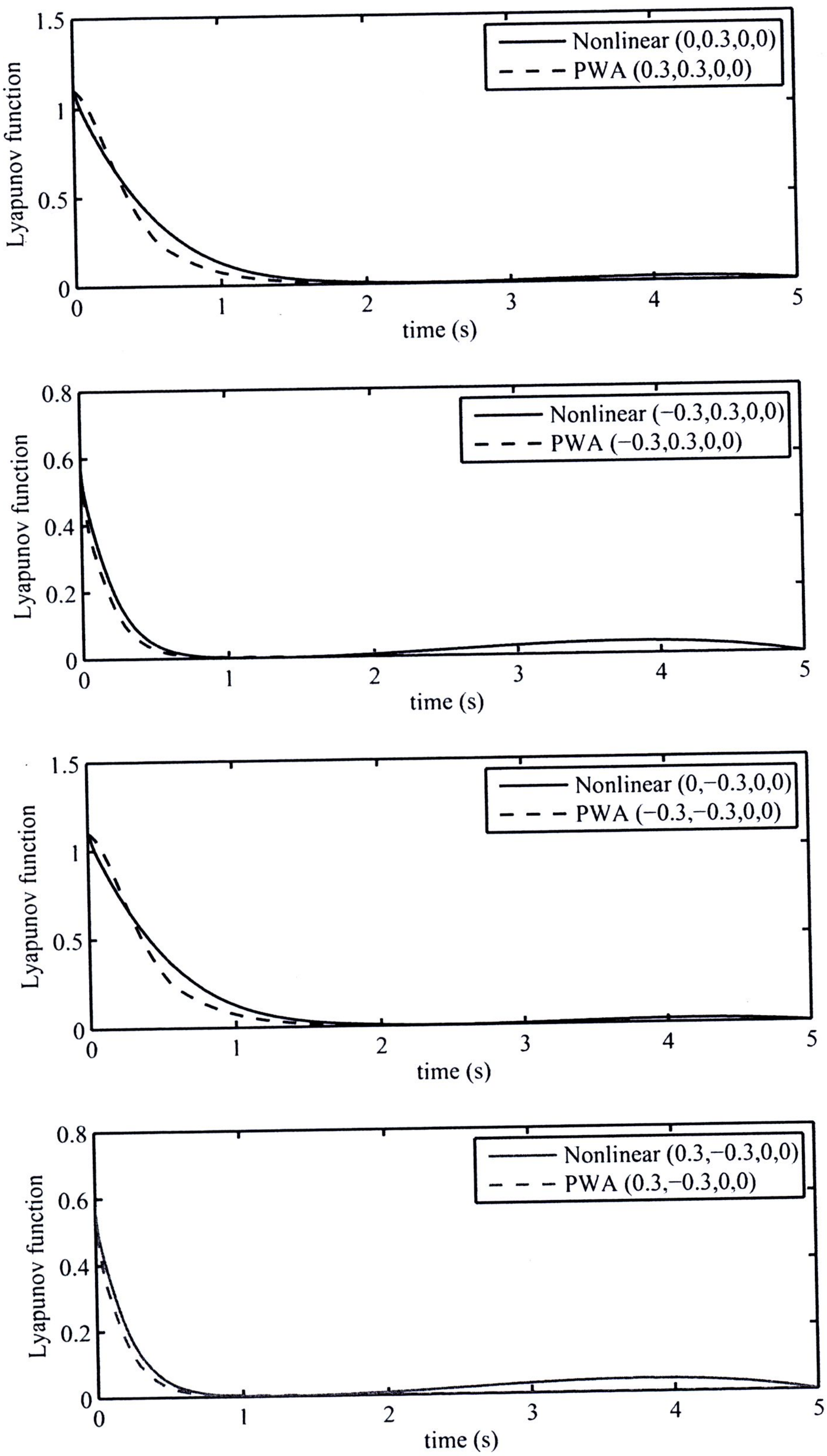

Figure 6.12: Lyapunov function plot of the bicycle dynamic system.