CHAPTER III

EXPERIMENTAL BICYCLE

In this chapter, we focus on the parameter measurement of the prototype autonomous bicycle with gyroscopic flywheel. the critical issues are the bicycle dimensions and the gyroscopic flywheel parameters.

3.1 Bicycle

This bicycle is the adult size bicycle and meets the criteria of the BicyRobo Thailand competition. The wheel base length is more than 50 cm, the diameter of each wheel is more than 50 cm, and the tire width is less than 5 cm. We end up with the our used bicycle in Figure 3.1. The body is a rigid frame without suspension.

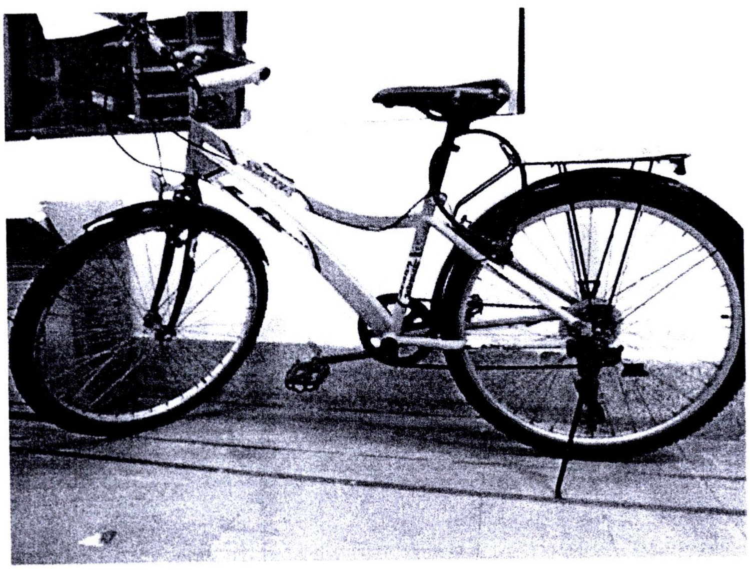

Figure 3.1: The selected bicycle before modifying.

To design the new features and estimate the parameters from the real world model, we draw the 3D CAD graphic in CATIA[1]. Figure 3.2 shows the 3D CAD of bicycle robot with the actual measured dimension. The pedal, saddle, barrel adjuster and rear deraileur will be removed from this original bike.

The measured bicycle parameters are collected in Table 2.1. Some parameters such as the moment of inertia is needed to calculated indirectly. Here, we let the CATIA software to calculate them all by inputing the mass that we can simply measure it and the type of part material (to figure out the mass density). These data are used for the whole simulation in this project.

[1]CAD software for designing mechanical part.

Figure 3.2: The 3D CAD drawing of the bicycle before modifying.

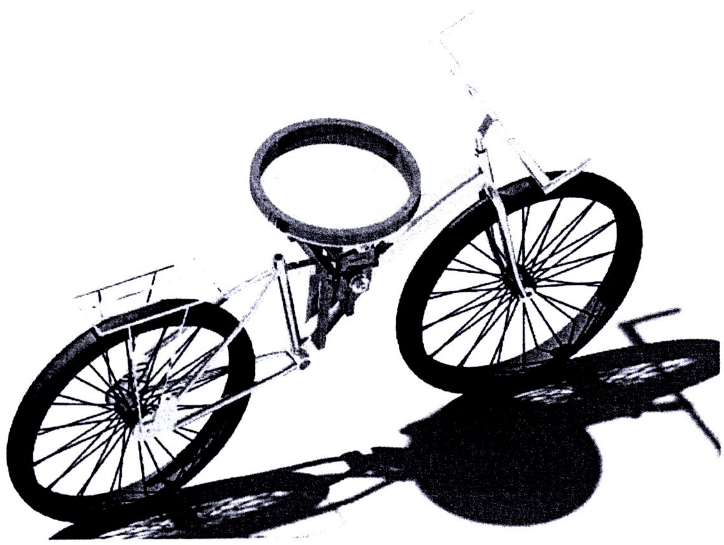

Figure 3.3: Bicycle robot attached with gyroscopic flywheel.

3.2 Gyroscopic Flywheel

The flywheel is treated as an actuator for controlling the bicycle rolling angle. We consider the critical case that this actuator can generate the moment to resist the moment produced by the gravitational torque when the bicycle is tilting. While the bicycle rolling angle is larger, the gravitational moment becomes larger too.

From Figure 3.4, $\mathbf{xyz}$ is the global axes and $\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3$ is the principal axes of the flywheel. The basis vector $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ rotate together with the gyroscopic flywheel at the angular velocity $\omega_{\mathbf{e}}$.

$$\omega_{\mathbf{e}} = \dot{\varphi}\mathbf{e}_1 + \dot{\alpha}\mathbf{e}_2 \tag{3.1}$$

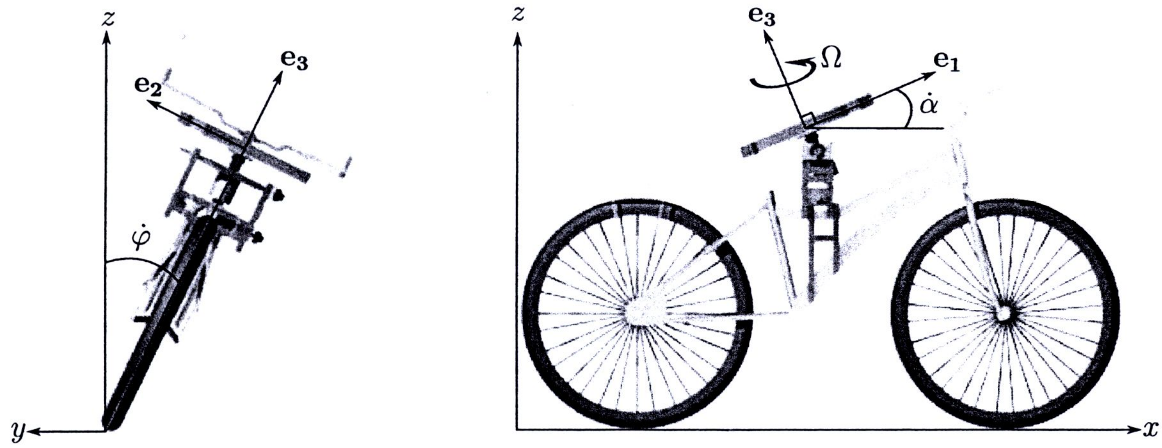

Figure 3.4: The bicycle configuration for sizing the flywheel (mass and dimension).

Let the bicycle initially stands at the rolling angle of φ radian with zero forward moving velocity and the flywheel spin at a constant speed Ω rad/s about $\mathbf{e}_3$-axis and precessing at $\dot{\alpha}$ rad/s about $\mathbf{e}_2$ axis. The angular momentum of the flywheel about O can be written as

$$\mathbf{H}_o = I_{1o}\dot{\varphi}\mathbf{e}_1 + I_{2o}\dot{\alpha}\mathbf{e}_2 + I_{3o}\Omega\mathbf{e}_3 \tag{3.2}$$

where I_{1o}, I_{2o}, I_{3o} are the moment of intertia with respect to the point O. We assign the point O to be the midpoint between the ground contact point of front and rear wheel. The vertical principal $\mathbf{e}_3$-axis of of the flywheel pass through this point. In our case $\dot{\varphi} = 0$ because we assume that the gravitational moment is equal to the moment generated by the precession torque from the gyroscopic effect. By the way, we should note that the flywheel will precess to generate the moment with magnitude greater than the gravitational torque to pull the bicycle back to stand upright at $\varphi = 0$ rad. Yet, we calculate the least moment that the flywheel must be able to generate. Thus, it reduces to

$$\mathbf{H}_o = I_2\dot{\alpha}\mathbf{e}_{2o} + I_{3o}\Omega\mathbf{e}_3 \tag{3.3}$$

Carrying out the details for the change of $\mathbf{H}_o$, we write

$$\begin{aligned}
\dot{\mathbf{H}}_o &= I_{2o}\ddot{\alpha}\mathbf{e}_2 + I_{2o}\dot{\alpha}\dot{\mathbf{e}}_2 + I_{3o}\dot{\Omega}\mathbf{e}_3 + I_{3o}\Omega\dot{\mathbf{e}}_3 \\
&= I_{2o}\ddot{\alpha}\mathbf{e}_2 + I_{2o}\dot{\alpha}(\omega_{\mathbf{e}} \times \mathbf{e}_2) + I_{3o}\dot{\Omega}\mathbf{e}_3 + I_{3o}\Omega(\omega_{\mathbf{e}} \times \mathbf{e}_3) \\
&= I_{3o}\Omega\dot{\alpha}\mathbf{e}_1 + (I_{2o}\ddot{\alpha} - I_{3o}\Omega\dot{\varphi})\mathbf{e}_2 + I_{2o}\dot{\alpha}\dot{\varphi}\mathbf{e}_3
\end{aligned}$$

Take $\dot{\varphi} = 0$, we have

$$\dot{\mathbf{H}}_o = I_{3o}\Omega\dot{\alpha}\mathbf{e}_1 + I_{2o}\ddot{\alpha}\mathbf{e}_2 \tag{3.4}$$

This change in $\mathbf{H}_{o1}$ must equal to the gravitational moment $\mathbf{M}_{o1}$ around x-axis. The produced moment is given by

$$\begin{aligned}
\mathbf{M}_{o1} &= z_B\mathbf{e}_3 \times (-m_B g\mathbf{k}) + z_G\mathbf{e}_3 \times (-m_G g\mathbf{k}) \\
&= (m_B z_B + m_G z_G)g\sin\varphi\mathbf{e}_1
\end{aligned} \tag{3.5}$$

From the Euler's equation for rigid-body dynamics $\mathbf{M}_{o1} = \dot{\mathbf{H}}_{o1}$, (3.4), and (3.5), we have

$$I_{3o}\Omega\dot{\alpha} = (m_B z_B + m_G z_G)g\sin\varphi \tag{3.6}$$

$$I_{2o}\ddot{\alpha} - I_{3o}\Omega\dot{\varphi} = 0 \tag{3.7}$$

Torque component $\mathbf{e}_2$ of $\dot{\mathbf{H}}_o$ will result at the ground contact point of front and rear wheel and it is resisted by the reaction torque of the ground contact. Therefore, there is no rotational motion for this axis (The bicycle does not tip over to the front or back). The important role to stabilize the bicycle is at the $\mathbf{e}_1$ axis. Its relationship is shown in (3.6). We need an excessive moment to pull the bicycle in the reverse direction. The gyroscopic flywheel should be designed in order to satisfy the equation below

$$(m_B z_B + m_G z_G) g \sin\varphi < I_{3o} \Omega \dot{\alpha} \tag{3.8}$$

Note that $I_{3o} = I_3$ where I_3 is the moment of inertia of the flywheel about its principal axis. For simplicity to manage the calculation, we introduce

$$M_{req} = (m_B z_B + m_G z_G) g \sin\varphi \tag{3.9}$$

$$M_{gen} = I_{3G} \Omega \dot{\alpha} \tag{3.10}$$

Take the parameter value in Table 3.1 and the formula in Table 3.2 to find M_{req} and M_{gen}, we finally get $M_{\text{req}} = 20.5481$ kg·m and $M_{\text{gen}} = 29.8311$ kg·m. The DIY[2] Gyroscopic Flywheel can produce the moment in which its magnitude is greater than the required value with the factor of 1.4518.

Table 3.1: Parameters for Gyroscopic Flywheel Design Calculation.

Parameter	**Symbol**	**Value**	**Unit**
Disk mass	m_d	3.3929	kg
Circular tube mass	m_c	5.6400	kg
Flywheel mass	m_G	9.0329	kg
Bicycle body mass	m_B	30	kg
Outer radius	r_d	0.20	m
Inner radius	r_c	0.18	m
Disk thickness	h_d	0.01	m
Circular tube thickness	h_c	0.03	m
Height of bicycle center of mass	z_B	0.50	m
Height of flywheel center of mass	z_G	1.00	m
Rolling angle	φ	5	degree
Spinning angular velocity	Ω	3000	rpm
Precessing angular velocity	$\dot{\alpha}$	20	degree/s
Gravitational acceleration	g	9.81	m/s^2
Iron mass density	ρ_{Fe}	7874	kg/m^3
Aluminium mass density	ρ_{Al}	2700	kg/m^3

[2]Do It Yourself

Moment of Inertia of Flywheel

The Flywheel is assigned to spin about $\mathbf{e}_3$ axis and precess about $\mathbf{e}_2$ axis. Figure 3.5 shows the dimension and other description for the calculation of the flywheel moment of inertia. Refer to the "List of moments of inertia" from [51], we obtain the moment of inertia in two parts - Disk and Cylindrical tube about the point D and C. Then, we take them to rotate about O using Parallel axis theorem and combine them together by addition. The summary of the Flywheel moment of inertia is in Table 3.2. The mass calculation here are $m_d = \rho_{Al} V_d$, $m_c = \rho_{Fe} V_c$, and $m_G = m_d + m_c$ where ρ_{Al} is the density of Aluminium and ρ_{Fe} is the density of iron.

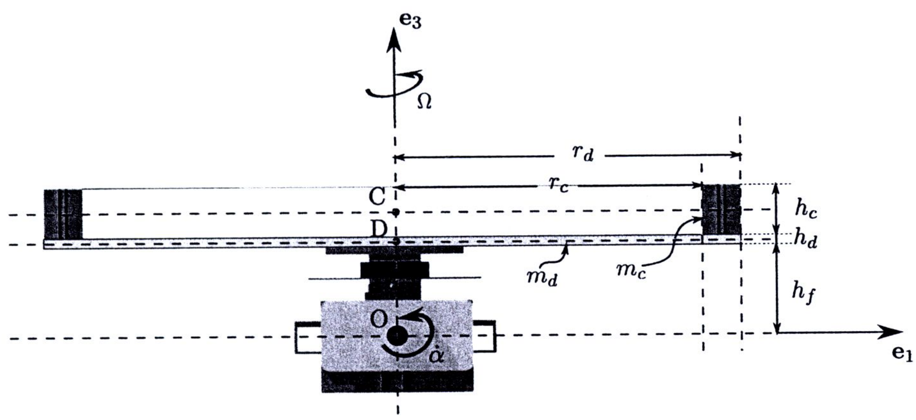

Figure 3.5: Side View Cross-section of Flywheel configuration.

Table 3.2: Summary of Flywheel Moment of Inertia.

Object	I_3	I_2	Volume
Disk (about D)	$I_{3D} = \frac{1}{2} m_d r_d^2$	$I_{2D} = \frac{1}{12} m_d (3r_d^2 + h_d^2)$	$V_D = \pi r_d^2 h_d$
Cylindrical tube (about C)	$I_{3C} = \frac{1}{2} m_c (r_c^2 + r_d^2)$	$I_{2C} = \frac{1}{12} m_c [3(r_c^2 + r_d^2) + h_c^2]$	$V_C = \pi (r_d^2 - r_c^2) h_c$
Gyroscopic Flywheel (about O)	$I_{3G} = I_{3D} + I_{3C}$	$I_{2G} = I_{2D} + m_d h_f^2 + I_{2C} + m_c (h_f + \frac{h_d}{2} + \frac{h_c}{2})^2$	$V_G = V_D + V_C$