CHAPTER I

INTRODUCTION

1.1 General

Small defects and flaws are unavoidably introduced in components during the manufacturing and fabrication process or initiated during applications as a result of external excitations. Defects and flaws present in the component act as the stress riser and, in turn, the strength reducer due to the high stress concentration introduced in their neighborhood and this can eventually lead to progressive failure of such component once certain critical conditions have been attained. Fracture and fatigue analysis and assessment have therefore become essential ingredients in the design of engineering components to ensure their safety and integrity under service conditions. To aid such analysis, physically admissible, mathematical models and powerful solution techniques must be developed in order to capture and predict responses of interest with acceptable level of accuracy.

In general, fractures can be classified as either brittle fractures or ductile fractures depending on their dominant failure characteristic and behavior of fields in the vicinity of the fracture front. Behavior of the ductile fracture is dominated primarily by the significant plastic deformation induced around the front while the brittle fracture is characterized by a rapid rate of crack advance with relatively row energy release and a localized plastic deformation, i.e. small-scale yielding pertains (see for examples, Anderson, 2005; Gdoutos, 2005). Common mathematical models used in the modeling of the latter type of fractures are established within the context of linear elastic fracture mechanics (LEFM). For this particular case, a localized zone of plastic deformation is discarded and the entire body including a region of high stress concentration is assumed to be linearly elastic. A single parameter, either the stress intensity factor or the strain energy release rate, can be used to completely describe the dominant or asymptotic field in the neighborhood of the crack front (K-field). For ductile fractures, the size of a region with inelastic deformation is relatively large when compared with the crack dimension and the K-field predicted by LEFM does not exist. Various models based on elastic-plastic fracture mechanics (EPFM) have been proposed instead to improve the response prediction (see Anderson, 2005). For this type of fractures, two different parameters, one associated with the J-integral and the other corresponding to the crack opening displacement, have been widely used to measure the extent of fields around the crack front. It should be remarked that while LEFM-based analysis yields certain unrealistic aspects of the fields near the crack front (e.g. singularity of the stress and strain field at the crack front), results from such analysis have been used successfully in the response prediction for various engineering applications (e.g. fatigue analysis, prediction of crack growth initiation and propagation direction, etc.). The present study is focused only on brittle fractures.

The stress intensity factor is a fundamental quantity in linear elastic fracture mechanics that provides a complete measure of a dominant field in the vicinity of the crack front and, in particular, indicates the extent or magnitude of an asymptotic, singular stress field (e.g. Williams, 1957). In general, the stress intensity factor depends on several factors such as loading conditions, material properties, geometries of both bodies and cracks and the location along the crack front, and determination of such quantity requires solving a complete boundary value problem associated with the entire body. Supported by evidences from various experiments (e.g. Krishna Rao and Hasebe, 1995 and Xin *et al.*, 2010), a body containing pre-existing cracks, when loaded, only deforms without creating any new surface for a certain range of applied loads. Once the applied load reaches a critical value (i.e. the corresponding stress intensity factor reaches its critical value), crack growth initiation is observed. The critical value of the stress intensity factor at the onset of crack advance is termed the fracture toughness. This quantity, generally taken as a property of materials, indicates the ability of the constituting material to resist the formation of a new surface. Fracture toughness can be determined from experiments following various well-known standards such as the British standard (BS), the American Society for Testing and Materials (ASTM), a series of

International Standard (ISO) and the European Structural Integrity Society (ESIS). Among several types of specimens and crack configurations found in earlier experimental studies (e.g. single-edge notched bending (SEB), compact tension (CT), arc-shaped tension (AT) and disk-shaped compact tension (DCT) specimens), the two most common specimens widely used to determine the fracture toughness are the single-edge notched bending (SEB) specimen and the compact tension (CT) specimen as shown schematically in Figure 1.1 according to ASTM E399-90 (1997).

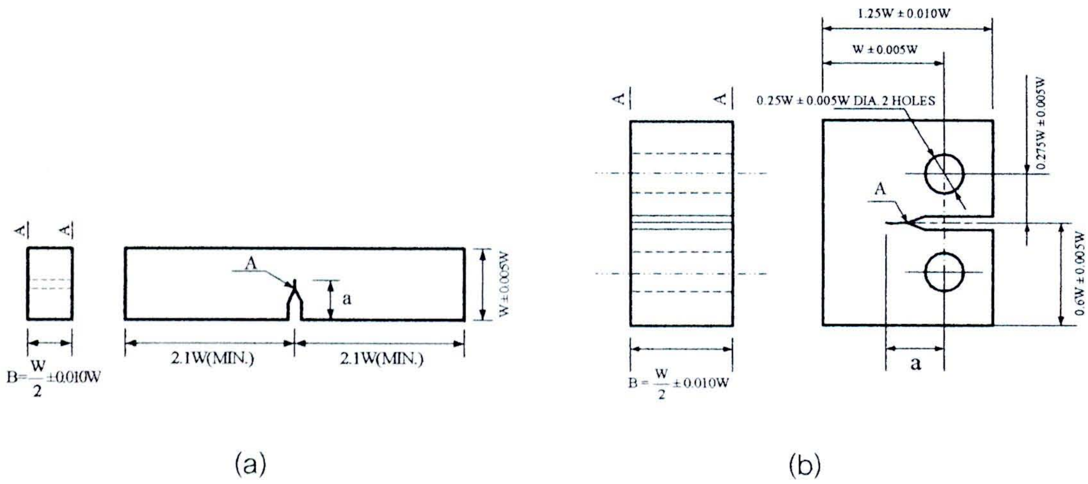

Figure 1.1 Schematic of specimen configuration and crack: (a) single-edge notched bending (SEB) specimen and (b) compact tension (CT) specimen

Results from many experiments revealed that the fracture toughness exhibits strong dependence on the thickness of a specimen used as shown in Figure 1.2 (e.g. Anderson, 2005). A thinner specimen generally yields larger fracture toughness. As the specimen thickness increases to a certain value, the fracture toughness converges to a constant value termed the plane strain fracture toughness and denoted by K_{IC}. The plane strain fracture toughness represents a true material property since it is independent of the specimen thickness and this situation can be achieved only when the specimen thickness is sufficiently large to render the behavior over the majority of the crack front dominated by a plane strain condition. The ASTM E399-90 (1997) recommended the minimum specimen thickness to obtain a valid K_{IC} by

$$B \geq 2.5\left(\frac{K_{IC}}{\sigma_y}\right)^2 \tag{1.1}$$

where B is the specimen thickness, K_{IC} is the plane strain fracture toughness to be determined, and σ_y is the yield strength of constituting material. Inequality (1.1) was developed based on the assumption that the plastic zone size must be relatively small compared to the specimen thickness (i.e. $r_p \leq B/50$ where r_p is the plastic zone size along the crack front) and its applicability is still restricted to fractures at room temperature and high-strength materials. In addition, the estimation of the specimen thickness by (1.1) involves K_{IC} which is unknown a priori.

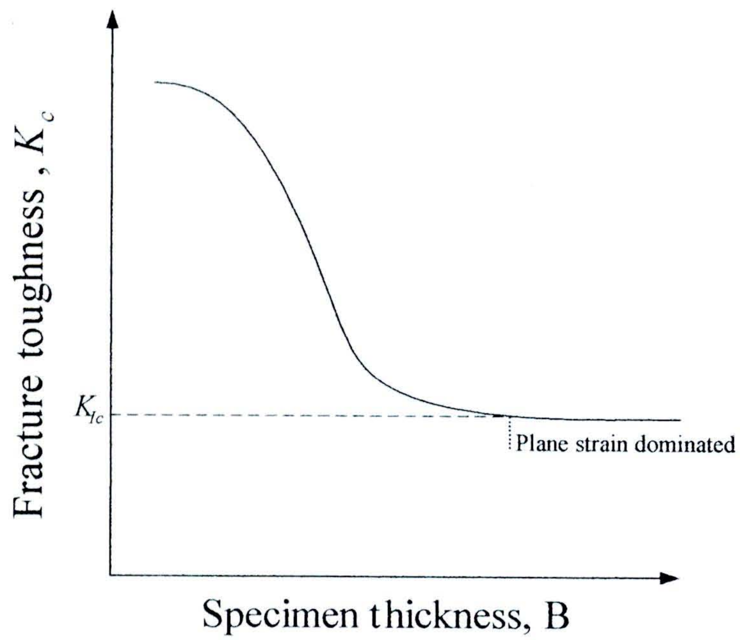

Figure 1.2 Relationship between fracture toughness K_c and specimen thickness B

An alternative to the above recommended empirical formula is to apply linear elastic fracture mechanics to perform comprehensive three-dimensional stress analysis along with an extensive parametric study on the specimen thickness. Knowledge of the distribution of the stress intensity factor along the entire crack front for various specimen thicknesses should at least provide useful information for selecting a proper specimen thickness to ensure that the plane strain condition is dominated and, as a result, the plane strain fracture toughness is obtained. More specifically, the thickness of the specimen can be increased in the simulation until the stress intensity factor over the majority of the crack front attains the value corresponding to the plane strain condition.

1.2 Background and review

A mathematical model formulated within the context of two-dimensional boundary value problems has been widely used in the stress analysis of a body containing defects and flaws due to its simplicity and cheap computational cost while still predicting results with an acceptable level of accuracy for several engineering applications. On the basis of an extensive literature survey, various studies on two-dimensional crack problems based on both the plain stress and plain strain assumptions were recorded (e.g. Chang and Mear, 1995; Dirgantara and Aliabadi, 2002; Khraishi and Demir, 2003; Freese and Baratta, 2006; Kutak *et al.*, 2007). It should be noted first that the former assumption is well-suited for modeling a body of a relatively small thickness in comparison with other dimensions whereas the latter is appropriate for components of comparatively large thickness. Here, details of some selected studies from this category are given. Dirgantara and Aliabadi (2002) applied the dual boundary element method to solve an elastic thin plate containing three types of cracks (i.e. centered crack, edge crack and cracks emanating from a hole). In their study, the crack surface displacement extrapolation (CSDE) technique was utilized along with the J-integral scheme to determine the stress intensity factors. Numerical results obtained were found to be in excellent agreement with existing benchmark solutions. Freese and Baratta (2006) thoroughly investigated a single edge-cracked specimen by using the weight function method and the modified mapping collocation technique. In their parametric study, the full range of crack length to specimen width ratio and various loading conditions such as three-point and four-point bending, pure bending, eccentrically loaded tension were considered. General explicit expressions for computing the stress intensity factors were also deduced. Recently, Kutak *et al.* (2007) proposed explicit formula for approximating the mode-I stress intensity factor for center crack, single edge crack, and double edge crack in a linearly elastic thin plate under the normal traction. In their work, the neural networks and the displacement extrapolation method (DEM) were employed to determine the stress intensity factor and a commercial FEM-package, ANSYS, was utilized to validate their numerical results. Expressions of the stress intensity factors for two-dimensional cracked bodies with various geometries, crack configurations, and loading conditions can also be found in many textbooks of fracture mechanics (e.g. Barsom and Rolfe, 1999; Anderson, 2005; Gdoutos, 2005). While use of a two-dimensional mathematical model to perform fracture analysis gains popularity from its simplicity, it still poses several drawbacks. As already been known, a two-dimensional model with the plane stress assumption is well-suited only for modeling a body of a relatively small thickness in comparison with other dimensions whereas that based on the plane strain assumption is appropriate for components of comparatively large thickness. In practices, there are various situations that both plane stress and plane strain assumptions do not apply (e.g. components with) and, for those cases, fracture data (viz. stress intensity factors) exhibits strong dependence on the component dimensions. To gain more insight into such complex boundary value problems, a comprehensive three-dimensional analysis must be performed.

In past decades, work related to three-dimensional linear fracture analysis has increasingly gained attention from various researchers due to the significant progress of powerful numerical techniques and personal computers and the need of more sophisticated mathematical models to better predict responses of complex physical problems encountered in practices. Here, we summarize a series of earlier studies relevant to the present study. Alam and Mendelsen (1983) utilized the method of line to study the mode-I stress intensity factor along the curved crack front of the compact tension specimen (according to the ASTM standard) under pure tension loading. Results from their study revealed that the distribution of the stress intensity along the crack front exhibits strong dependence on the difference between the crack length at the center of the crack front and the crack length at the boundary of the specimen, termed the crack tunnel depth. In particular, the stress intensity factor decreases in the central region of the crack front while increases over a small region near the intersection between the crack front and the outer boundary as the crack tunnel depth increases. Although such analysis was performed within the three-dimensional context, the specimen thickness was fixed and, as a result, the influence of this parameter on the distribution of the stress intensity factor cannot be addressed.

Later, Sukumar *et al.* (1997) developed a numerical technique based on the coupling between the finite element method and the element-free Galerkin method to solve an isotropic, linearly elastic, single edge cracked specimen subjected to the uniform normal traction at its top and bottom surfaces. The element-free Galerkin method was employed specially to model the crack surface and the stress intensity factor along the crack front is calculated using the volume and planar domain integral. It can be concluded from this study that the stress intensity factor attains its maximum value at the middle of the crack front and decreases very rapidly in the region where the crack front meets the boundary. In addition, the complete distribution of the stress intensity factor along the crack front obtained from such analysis shows significant discrepancy from the plane strain case and this additionally supports the need of the three-dimensional model. It should be noted however that this study is restricted only to a specimen of fixed thickness and an isotropic material with Poisson's ratio equal to 0.3. This is still insufficient to describe the influence of the thickness on the distribution of the stress intensity factor along the entire crack front. Next, Li *et al.* (1998) presented highly accurate numerical solutions of an identical problem by using a powerful numerical technique based on a weakly singular, symmetric Galerkin boundary element method (SGBEM). However, the main focus of this work was to develop the computational procedure and this particular problem chosen in their analysis was only for verification purpose.

Next, Wu (2006) explored the influence of thickness on the distribution of the mode-I stress intensity factor for the center-cracked specimen subjected to uniform normal traction at its top and bottom surfaces. In the analysis, a finite element software ANSYS was utilized to model the associated boundary value problem and the quarterpoint displacement method was employed to extract the stress intensity factor along the crack front. Results from this study indicated that the stress intensity factor in the central region of the crack front starts to attain a converged constant value (i.e. the plane strain condition) for a specimen with the thickness larger than four times of the crack length. It should be noted that their study is still restricted only to a center-cracked specimen made of an isotropic material with Poisson's ratio equal to 0.3. Later, Kotosov (2007) applied the first order plate theory to investigate the influence of the plate thickness on the value of the stress intensity factors for a through crack embedded in a linearly elastic infinite plate of finite thickness and subjected to both remote uniaxial tension and remote shear. Results from this study suggested that when the thickness of the plate increases, the stress intensity factors decreases and finally converges to the value associated with the plane strain condition. While the analysis has taken the thickness of the plate into account, use of the plate theory provides no information of the distribution of the stress intensity factors across the thickness. In particular, a single value of the stress intensity factor was obtained and it should represent only the average of such quantity over the entire crack front. Recently, Rungamornrat and Mear (2008b) revisited the same problem as that studied by Li *et al.* (1998) by incorporating material anisotropy. In their analysis, the weakly singular SGBEM based on a pair of weakly singular, weak-form integral equations for the displacement and traction proposed by Rungamornrat and Mear (2008a) was utilized. Similar to the work of Li *et al.* (1998), the key objective of this study was to develop an efficient and accurate numerical technique capable of performing three-dimensional linear fracture analysis. The distribution of the stress intensity factors along the crack front was reported and discussed for an isotropic material with Poisson's ratio equal to 1/3 and two types of transversely isotropic materials, i.e. zinc and graphite reinforced composite. However, the study did not consider the influence of the thickness on the value of the stress intensity factors.

From an extensive literature survey, the studies of the influence of the thickness of the body in the direction along the crack front on the value and distribution of the stress intensity factors are still restricted to certain geometries, crack configurations, loading conditions, and certain types of constituting materials. For instance, complete investigation in the case of isotropic materials is limited to certain values of Poisson's ratio and specimen geometries while results for anisotropic case are available only for a specimen with a fixed thickness.

1.3 Research Objective

The key objective of this research is to explore the influence of the thickness of specimens, commonly used in the determination of the fracture toughness, on the distribution of stress intensity factors along the crack front.

1.4 Research Scope

The main focus of this research is to perform a stress analysis of a compact tension (CT) specimen of various thicknesses and under pure mode-I loading condition. Dimensions of the specimen and loading characteristics are chosen to be consistent with those specified in ASTM E399-90. Two types of materials, one associated with an isotropic elastic material and the other corresponding to a transversely isotropic elastic material, are considered in this study. For the latter type of material, the axis of material symmetry is taken to be perpendicular to the crack surface.

1.5 Research Methodology

A computational procedure based on a weakly singular, symmetric Galerkin boundary element method (SGBEM) is utilized to perform the comprehensive stress analysis of a three-dimensional, homogeneous, generally anisotropic, linearly elastic medium containing cracks. The stress intensity factor is computed using a special formula in terms of the gradient of the relative crack-face displacement data along the crack front which is obtained directly from the SGBEM.

1.6 Research Significance

Results from the present study provide the profound understanding of the behavior of the mode-I stress intensity factor along the entire crack front of the CT specimen of various thicknesses for two important classes of linearly elastic materials, i.e. isotropic and transversely isotropic solids. The knowledge of the specimen thickness that yields the plane strain condition over the majority of the crack front has a direct application to the optimal design of a CT specimen used in the determination of the fracture toughness.