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Technology-enhanced Flipped Mathematics Education in Secondary Schools: A Synopsis of Theory and Practice

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Abstract

This article focuses on the technology-supported Flipped Classroom Approach (FCA) in Mathematics education and how this form of teaching and learning can be implemented in secondary schools. In the German-speaking countries in particular, research on the FCA focuses on the individual components of this form of education and how this approach can be used and implemented in tertiary education. Our research contributions to the scholastic and practical application of the FCA in Mathematics still show gaps, which should be reduced with this work. In addition, the new possibilities that can arise in a flipped classroom setting through the targeted use of modern educational technologies will be highlighted. The findings and contents of this article can also be used as a support for teachers who would like to design and conduct their mathematics lessons in the near future according to the Flipped Classroom Approach.

Keywords: flipped classroom, inverted classroom, mathematics education, student-centred learning, practical teaching experiences

Introduction

"The students deal independently in small groups with self-chosen topics or questions and try to solve the problems with mathematical concept and adapted technological tools." This is an excerpt from a typical flipped classroom in mathematics. Since the beginning of the new millennium, this form of education has become increasingly popular and important, especially in the natural sciences. In addition to new teaching and learning methods, modern educational technologies have become increasingly popular in schools since 2000. The trend in recent years that more and more high-end products are also made available open access has contributed to the fact that more and more high-quality educational technologies are to be found in schools. In Austria, it was the mathematics school-leaving examination from the 2017/18 school year and the obligatory use of higher-value technologies [2], which led to the fact that educational technologies are used almost everywhere in upper secondary schools. However, not only the use of modern technologies is crucial, but above all the way in which modern technologies are integrated into school learning and how a synthesis of contemporary didactics and high-quality technologies can be produced.

Our teaching records and post-processing for the use of a technology-supported FCA in Mathematics lessons at an Austrian High-School in the academic years 2013/14 to 2016/17 have been investigated in accordance with the Action Research (Capobianco & Feldman, 2010; Mamlok-Naaman & Eilks, 2012; Zambo, 2011) and the findings from this have been combined with the theoretical elements of the FCA and contemporary mathematics education. This link is intended to provide new insights for both theory and practice.

The Central Elements of a Technology-Enhanced Flipped Classroom Approach

Especially since the beginning of the new millennium, the Flipped Classroom Approach (FCA) as well as technology-enhanced education have been increasingly mentioned in articles in educational journals or at conferences on modern teaching. As popular as these forms of education may be, experts and practitioners differ in defining and describing these forms of teaching and learning (Bishop & Verleger, 2013). If one is looking for the largest common divider in most of the FCA's descriptions, it can be helpful to eliminate all the elements or components of the approach that are not necessarily required to characterize learning in a flipped classroom. It is a learning setting in which students construct knowledge and competences themselves in an active and collaborative way by tackling their own chosen real-world questions or tasks, and the teachers provide an environment for the acquisition of basic knowledge and competences (for the pre-class phase) as well as for the application and deepening of this knowledge and these competences (for the in-class phase). In a similar way, Wasserman et al. (2015) and Galway et al. (2014) describe learning according to the FCA. Here, learning in a flipped classroom is identified by the fact that passive learning activities take place outside of the classrooms with lower cognitive goals and filling the in-class time with active and pupil-centred actions that pursue higher cognitive goals. Additionally, in an educational and didactic manner, this approach can be counted as blended learning and has components of both independent and technology-enhanced learning (Staker & Horn, 2012; Sureka et al., 2013).

Likewise, the use of modern educational technologies also offers a very broad spectrum. In this context, educational technologies include communication and presentation technologies (e.g., the ePortfolio software Mahara) as well as technologies for the representation and processing of mathematical-technical tasks (e.g., GeoGebra). On the one hand, these technologies can be used for the distribution and exchange of teaching and learning materials as well as for the application of drill-and-practice programs. The focus here is on the behavioural learning paradigm. On the other hand, the targeted use of modern technologies can increase the scope for both teachers and learners, which corresponds to the constructivist approach to education. This depends to a large extent on the role a teacher attributes to the technologies. Goos et al. (2000) have worked out a fourfold division of possible applications. Again, a transition from a behavioural use of technologies to a constructivist-connectivism (Baumgartner & Kalz, 2004) use of technologies can be recognized. According to Goos et al. (2000), technologies can be applied as masters, servants, partners, and/or extension of self in education.

Technological devices in a Flipped Classroom

As already described above, learning in a flipped classroom should achieve higher learning goals. In order to reach this goal, technological tools are needed for both teachers and students. In recent years, educational technologies have developed very rapidly. This concerns not only the quality of technological devices but also access to technologies. It is particularly important for education that many technologies have developed towards the open access principle. Because, if and only if technologies are available for free, it is possible that these technologies can be made available to a large number of students and thus have an impact on their learning (Chao et al., 2016; Goerres et al., 2015; Orlando & Attard, 2016; Samuelsson, 2006). For this reason, the following section only covers technologies that also provide open access.

In order to offer a 21st century Flipped Classroom Education, it is necessary to provide technologies for the following areas: 1) technologies to export direct instruction from the classroom, 2) technologies to mathematically deal with real-world problems, and 3) technologies to communicate with each other outside the classroom and present the learning products.

In most FCA applications, videos are used to outsource direct instructions from the classroom (Wasserman et al., 2015). Existing and freely accessible videos can be used for this purpose. An example of this is the Khan Academy [3]. On this platform, among other things, learning videos on mathematics and other scientific areas are provided. If one wants to create videos himself, screen recording programs or simple and cheap document cameras are the right choice. The screen recording programs Jing [4] and Screencast-O-Matic [5] have proven useful for school purposes. These programs are free, they are easy to use, and they allow the videos generated to be uploaded to the Internet for easy sharing.

The program GeoGebra [6] proved to be very helpful and purposeful when real-world problems were to be mathematically modelled and then solved. Another advantage of this program is that it is intuitive to use, is provided free of charge and the areas of algebra, (dynamic) geometry, statistics and numeric can be processed with this program. A further benefit of GeoGebra is that it also provides a digital learning environment [7] in which learning artefacts can be shared. Further applications and teaching examples of GeoGebra are presented and discussed, for example by Hall and Lingefjärd (2016).

Learning is a social process. This is all the more true when real problems are dealt with in the learning process (Herreid & Schiller, 2013). So that collaboration can also take place outside the classroom, the use of communication programs is necessary. In order to be able to share documents and learning products as well as to exchange information about the recovery process of these products with co-learners or teachers, the program Slack [8] is used in our case. However, it must be borne in mind that the majority of today's students have grown up with digital media, are therefore familiar with these communication technologies and in many cases, create a communication space without the help of teachers.

When real problems are dealt with, pupils have to work more than when they solve didactic tasks in a textbook. In order to reward this additional work, it is important that the resulting learning products can be made visible. It makes sense to use the ePortfolio software Mahara [9] for this purpose. The use of Mahara has the advantages that own learning products can be presented online, these learning products can also be linked with other content of the Internet, and that also foreign elements of the Internet can be easily integrated into the own portfolio. It is possible that the portfolio software can be used both openly for others and closed (i.e. only within the class), which predestines Mahara especially for use in schools (Treeck, Himpsl-Gutermann, & Robes, 2013).

Methods and Goals

The insights into flipped mathematics lessons as well as the practical aspects are based on our involvement as a secondary school teacher, head of professional development for mathematics teachers and university professor for prospective mathematics teachers. The experiences gained in the academic years 2015/16, 2016/17 (only in the area of teacher training) and 2017/18 were documented and analysed (2015/16), and then reintegrated into the lessons (2017/18) as well as re-examined according to the Action Research Method as described by Capobianco & Feldman (2010), Mamlok-Naaman & Eilks (2012) and Zambo (2011). Our goal is to work out how a practical implementation of a technology-enhanced Flipped Mathematics Education can be achieved and how this method of facilitating mathematics learning can be conveyed to in-service and prospective teachers.

The Common Components of a Technology-Enhanced FCA and Mathematics Education

The following subchapters illustrate in which fields of learning mathematical content a synthesis with the FCA and modern technologies is possible. First, scientific descriptions are discussed, and then practical insights and recommendations are given. The authors see the practical suggestions as just one of many potential modes of implementing a technology-enhanced Flipped Mathematics Education. In any case, it will have to be adapted and adjusted to one's own framework conditions, if the examples are to be realized in the classroom.

Motivation

According to Galway et al. (2014), a learner-friendly environment in which students should be active is the heart of the Flipped Classroom Approach. A characteristic feature of this learning environment is that it is both possible and easy to individualise the learning process. This gives students more control over the learning process and allows them to perform their learning activities at their own pace. In particular, the new possibilities of modern, digital educational technologies significantly simplified the individualization of learning environments by students. By increasing the participation and involvement in the learning process, the motivation and self-confidence of the pupils is also to be increased, which plays a decisive role for the success of a learning process.

Mathematics in the school context creates different memories and feelings in different people. Often one is pleased when one thinks of mathematics and remembers which problems have been solved. However, in many cases it is also the case that mathematics causes discomfort and anxiety. This is a problem as fear and other negative feelings make learning mathematics difficult or even impossible (Hung et al., 2014; Lee & Johnston-Wilder, 2013). The elimination of anxiety and unease in mathematics education is in most situations only a necessary however not a sufficient condition, so that learning can actually take place. In order to achieve sustainable learning of mathematical knowledge and skills, it is usually necessary to have an intrinsic interest and motivation (Chao et al., 2016).

How to motivate students.

It turned out that the motivation and self-confidence of the students in learning mathematics can be increased if the learners themselves are allowed to define the problems, issues and solution strategies. Especially at the beginning of this process, it is helpful to provide students with prototypical templates so that the newly acquired freedom does not overwhelm them. This is particularly important when students use higher-value technologies in the learning process. In the same way, students tend in most cases to name too complex questions and tasks. The teachers should act as moderators, so that the achievement of success experiences is not impossible. In connection with the independent handling of own questions, it is helpful if the students are provided with rich learning resources (textbooks and worksheets) or if the use of their own devices (e.g., notebooks) is permitted. The formation of learning groups and the permitting and promoting of exchange on the subject turned out to be valuable tools. In this case, the teachers should pay attention to an adequate volume after initiation, so that other groups of students are not disturbed. It was also possible to identify the use of smartphones as a beneficial learning resource. On the one hand, students use specific apps to model or illustrate mathematical facts and on the other hand, the smartphone is used as a research tool. In addition to this subject-oriented approach, this also leads to improvements in the student-teacher relationship, as learners feel more trust in the teacher, if the learners are allowed to use smartphones a tool for learning.

Teachers can also contribute to a student-friendly learning environment and increase motivation by being available to students when help is needed, but otherwise play a passive to invisible role in the classroom. Above all, the supervising wandering of the teachers through the classrooms should be avoided, as the students feel controlled and constantly evaluated and this leads to a reduction of their well-being.

A concrete example of how to motivate students in mathematics is the research on, the construction of and then the technology-supported investigation of a mathematical or Leonardo bridge. This education experiment can be meaningfully implemented from a mathematical perspective in the 9th grade according to the Austrian curriculum [1]. Here, the learning processes can be divided into three parts, which the students have to carry out in groups (4-5 persons).

As a first step, the student groups have to do research on the Leonardo bridge. It is recommended that sub-groups of two to three students be formed and that each sub-group has the opportunity to conduct Internet-based research. The aim is to find out what is behind the term "Leonardo bridge," what the special features of this bridge are, how this bridge type works, where in reality such bridges occur and what the students themselves need for the construction of such a bridge. The learning product of this first phase should be an ePortfolio view, on which the following information can be found: 1) general information about Leonardo bridges, 2) an individually adapted construction manual for a Leonardo bridge and, 3) a resource list for the construction of an own Leonardo bridge. According to a technology-enhanced Flipped Classroom Approach, large parts of this first phase can already be performed before the teaching unit and/or can be deepened and expanded at home afterwards. This allows each group of students to adjust the intensity of the research activities according to their own needs and interests. However, it is recommended that the objectives and the learning products to be produced in this phase are described very precisely by the teacher.

In the second phase of this learning sequence, the students will build their own Leonardo bridge. This requires the material from phase one as well as sufficient time (double hour if possible) and space. In our case, the attic of the school building could be used. The construction process and the final product are to be documented photographically by the students. Photos with the students' smartphones are sufficient for this purpose.

The third phase deals with the technology-supported mathematical investigation of the bridge. According to the curriculum for Mathematics in Austria, trigonometry, vectors, and functions are offered in the 9th grade. For example, the course of the bridge can be described with the aid of a quadratic function and examined in more detail based on this. This can also lead to a deepening. This means that the topic of the chain line is discussed or that an approximation of the length of a curve is made. To make these investigations possible, the use of modern educational technologies is indispensable. According to the FCA, depending on the availability of time, parts of it can be treated in class or carried out as individual or group work at home.

The interlocking of the real world and the school world, as well as the hands-on actions of the students, can lead to a significant increase in motivation among the majority of students. The example just explained can also be used in the areas real-world problems and tasks, technologies, and collaboration described below.

Cognitive Workload

Learning, in general, is a demanding process and when mathematics is at the centre of learning, it rarely becomes easier. The following arguments about cognitive demands in learning are based on Clarke et al. (2005): The central element in the learning process, from a medical psychological perspective, is the working memory. A problem with this is, that this memory is limited. If the working memory is overloaded, this makes the learning process more difficult or hinders it. For this reason, unnecessary demands should be reduced as much as possible. This can be done by providing the learner with documents containing information on current and past knowledge and competences. This is especially important in mathematics, as it is an interlinked and interacting building of knowledge and competence. In addition, in the area of more recent mathematics education, it is implemented with a variety of technical devices. On the one hand, this makes it possible to expand and deepen mathematics; on the other hand, in addition to mathematical knowledge and skills, it is also necessary to acquire meta-competencies, e. g. the use of software, in order to master mathematical problems and challenges. Therefore, the learning process should be split into several periods. One time with a focus on technological tools and another time with the aim of acquiring new mathematical skills. If these conditions are fulfilled, a synthesis of mathematical content and technological tools is to be achieved, which are necessary for the achievement of higher-level learning goals as well as for dealing with real-world issues. This three-step approach is especially recommended for students with lower mathematical skills.

It is also the splitting of the learning process into different learning locations and learning time, which is characteristic of a technology-enhanced Flipped Classroom Approach. The exploration and acquisition of basic knowledge and competences should already take place at home before the lessons (see above). In this case, mathematical content can be the focus of learning as well as the acquisition of basic functions of new technologies or the operation of software. In the classroom there should be a linking of basic knowledge (pre-phase) with higher-level learning goals and real-world problems. Concerning the cognitive load (see above) this means that the students deal with mathematical or technological basics in the pre-class phase and in the classroom, mathematics and technologies interact, which is necessary for the expansion of knowledge and skills. If students experience problems during this process, the teacher and classmates form a safety net, so that the learning process is not interrupted by this cognitive multiple load.

How to handle the cognitive workload.

Relief of the working memory as well as the division of the learning process can occur and be helpful for purely mathematical knowledge contents as well as for the combination of technologies and mathematics. If the focus is on the acquisition of purely mathematical content, it is advisable to deal with simpler actions such as representations of mathematical facts and operations in the pre-class phase. The gained knowledge and the acquired competences must be internalized by means of worksheets and/or work orders. The important thing here is that the learning products are mobile, i.e., they can be taken to school.

If the interplay of mathematics and educational technologies is the goal of a teaching sequence, it was useful to focus on getting to know the new technologies and its functions in the pre-class phase. This means that students learn how to handle the technologies and acquire the skills needed to carry out certain operations with technologies or software. This approach and knowledge must be recorded and memorized, so that it can be used at a later date.

In the in-class phase the basic knowledge from the pre-class phase is then applied and deepened. If in the pre-class phase there were purely mathematical aspects in the centre, then in the in-class phase for this subject area arguments and justifications and/or mathematical models are to be created. If technologies were the subject of the pre-class phase, this technological knowledge is to be applied first to mathematical and then to real-world questions and problems.

Real-world Problems and Tasks

It is problems and tasks that are an integral part of most mathematics lessons and this can also trigger a fruitful learning process. But, not any problems and tasks can do this. Coles & Brown (2016), Lingefjärd & Kilpatrick (1998), Bishop & Verleger (2013) and Tait-McCutcheon & Loveridge (2016) advocate open tasks and non-routine problems such as the mathematical-technological exploration of a Leonardo bridge (see above). On the one hand, these tasks and problems are intended to increase the demand and research of the pupils, and on the other hand, these tasks and problems are supposed to place greater challenges on the pupils. However, with this approach to mathematics, there is no unambiguously correct problem-solving-strategy and no definitively correct result. Both the learners and teachers must be aware of this in advance. When such types of tasks and problems are used for the first time in mathematics education, it is of great importance and relevance for the pupils to receive guidance and support from the teacher. Leonard et al. (2014) add that these tasks should concern the cultures and environments of learners. The aim is to develop an authentic context and therefore increase the motivation and interest of the students as well. Bell & Pape (2012), Clarke et al. (2005) and Ngware (2015) also argue that learners must do mathematics. This means that the learners become active participants in mathematics education and this can be achieved by means of the tasks and problems outlined above.

In order for students to achieve higher level learning goals, the pursuit of real-world problems in the in-class phase is a central element of the Flipped Classroom Approach. But, there cannot be the single one or the right real problem for all students. Therefore, students or groups of students must formulate their own problem and then specify how to deal with and solve the problem. Then, these strategies are presented to the teacher and the other students, and it has to be argued why they have chosen the proposed approach. When implementing the problem-solving strategy, learners should be able to act autonomously and use the aids and technologies they consider appropriate. The teacher only intervenes in the learning process if the student asks for support or if it is obvious that the learning process is failing. The activities of problem solving should always lead to a learning product or artefact. For example, these artefacts can be contributions in an ePortfolio, an article for a (fictional) journal or a learning poster – the limits are only the creativity of the students.

How to implement real-world problems and tasks.

If real-world problems are to be at the heart of the in-class time, it is initially advisable if the creation of real-world relations is very simple. This means that the topic should be chosen carefully so that the majority of students can recognize the mathematical content in everyday life. The principle of "from near to far" has proved to be a promising one in the first student-centred work on real-world issues. This means that at the beginning of the application of the FCA, topics that can already be found in or around the school building and which require few additional resources should be dealt with. A concrete example of this is working with square functions in the 9th grade. The students can throw paper balls out of the classroom window, hold their impact points to the ground and thus describe the flight parabola by cleverly selecting the axis. A further possibility is that the students bite into a (coloured) sheet of paper, then trace their dental impressions with a pencil and create a model of their "dentition parabola" by a clever choice of axes. In this context, it is important that students write strategy papers or work plans at the beginning and receive peer feedback. In this way, flagrant mistakes or misconception can be pointed out and avoided in advance, which can keep the motivation of the students in the later course of their work high. Finally, the products of real-world problem solving are to be recorded and made visible. Also, here the use of mathematical technologies is recommended. On the one hand, this makes it easier to change the models or adapt them to new circumstances, and on the other hand, the use of educational technologies simplifies the visualisation of learning outcomes and in many cases, makes it possible to perform complex mathematical operations.

Technologies

Chao et al. (2016) and Fogarty et al. (2001) emphasize that many students appreciate the possibility of using new technologies in mathematics lessons. Technologies make it easier to visualize students' learning outcomes and, in general, it can often be observed that the use of new technologies increase the motivation of learners. But, the mere use of technologies in the classroom does not yet lead to learning with technologies, which requires much more. (Orlando & Attard, 2016). The spectrum of how new media and technologies can be used in mathematics education is very broad: On the one hand, technological advances can be used for electronic drill programs. Students must solve standard problems and apply the same pattern over and over again. By using technologies in this way, the number of problems and the duration can be increased at will. This use of technologies follows a behaviourist approach to learning. On the other hand, the new technologies can also be utilized as a space of opportunities for learning Mathematics. This means that the pupils explore and learn Mathematics themselves with technological tools (constructivist approach) and/or apply mathematical findings to real-life situations by means of the technology devices (Samuelsson, 2006). Regardless of the type of application, Lingefjärd & Kilpatrick (1998) were able to recognize 20 years ago that schoolchildren are very uncritical of the results provided by technical aids. This credulity is all the more noticeable the less experience students have with the new technological devices. Although this realisation is already 20 years old, it should still be considered, especially with inexperienced students and under the aspect that technologies are a very innovative field, which constantly produces novelties.

Learning and teaching in a modern flipped classroom is only made feasible through the targeted use of new educational technologies. New technologies have a central role in both the FCA's pre-class and in-class phase. But, similar to learning mathematics, technological tools should never be used for their own sake in a flipped classroom but must always serve the learning process and should make learning easier for the students and increase their possibilities.

In the pre-class phase of the FCA, educational technologies are used so that learners can acquire a basic knowledge and skills of the new subject matter or new educational software at home. It is the application of modern technologies, which makes it possible for the learners to be presented with different representations of the new knowledge and to make different learning paths possible. The use of technologies in the pre-class phase also makes it possible to individualize the learning process. This means that students can choose the forms of presentations that meet their needs and personalize the pace and intensity of their exploration.

In the in-class phase, new technologies enable students to deal with real-world questions and issues. These technological aids are often dynamic mathematical programs (e.g., GeoGebra), spreadsheet programs, but also internet-based searches, which are needed for realistic models and their investigation. New technologies also make it possible to expand the learning space and to embed and link one's own learning activities and products in a larger context.

How to introduce and use technologies.

When (first) a combination of technologies and Mathematics occurs, it makes sense to use technologies that students are allowed to use in future examinations or tests.

Through this, the interest in technologies can be increased and the importance and meaningfulness of learning how to use technologies can be understood easily by the pupils. It also turned out to be useful to plan a certain "play phase" after the first introduction of new technologies or new functions of a technology. In this case, the pupils should apply the newly acquired competences in partner work without having to pursue a specific curriculum goal. The first problems or limitations of technologies can--and should--arise, which can then be discussed with learning partners or the teacher. This should be done free of learning pressure or learning stress. Furthermore, it was effective when technologies were used primarily in those areas where its added value over paper and pencil work is quickly and clearly recognizable. Typical possibilities for this are the use of dynamic geometry programs for examining figures and bodies, the use of spreadsheet programs for algorithmic calculations or the processing of large amounts of data in connection with demanding operations such as regression analysis.

Collaboration

The above-mentioned cornerstones of the synthesis of modern Mathematics education and a technology-enhanced FCA lead to the fact that individual work is a very rare form of learning. In most cases, students form small groups (2 to 4 people) and try to define and analyse real-world problems with the help of mathematics and modern technologies. This form of cooperation takes place mainly in the in-class time of flipped mathematics lessons. But, also in the pre-class phase, a collaboration can be advantageous and support the learning process. This can occur when working on learning tasks together in the pre-class phase, or when there are ambiguities or problems, and one is looking for help. It is important that the collaboration can take place between the pupils, but also between the pupils and the teacher. This means that the teacher must be able to assume different roles (depending on the students' needs) and that communication in the mathematical learning process is becoming increasingly important (Chen et al., 2016; Choi, 2013; Goerres, Kärger, & Lambach, 2015; Sureka et al., 2013).

How to foster collaboration.

At the beginning of Mathematics education according to a technology-enhanced FCA, students are the quickest to get enthusiastic about collaboration and teamwork. So that this positive flow is not immediately dampened, the students should be allowed to choose the group members themselves during their first work. A flexible group size between three and five students was useful, so that no student would be left behind, and the group would still be so small that everyone could participate in the solution process. If the students are familiar with this form of learning, the groups should also be assembled thematically or randomly. This is to prevent the same students from forming a group all the time. In order to ensure that the performance to be provided are clear and comprehensible for all students in a group and to minimise any possible tensions within a group, the actions and learning achievements of the group and its members should be clearly described and assigned. It was shown that this also simplifies the assessment of student performances by the teacher and makes it easier for the pupils to understand them.

Conclusions

The main part of this article demonstrated that learning according to a technology-enhanced FCA as well as contemporary mathematics education have many similarities. Through this interaction, a learning environment can be created in which the students are the main protagonists and work on real-world problems. This form of learning makes it possible to increase the motivation and active participation of the students and thus facilitate sustainable learning.

The results of the action research revealed that in mathematics education according to a technology-enhanced FCA, the activity and freedom of the pupils is of great importance. Freedom in this context means that students are largely free to choose their own learning objectives, learning paths and methods/media to achieve them. The teachers are only supposed to give the framework conditions and the direction. However, this increase in freedom also means that a (social, subject-based and technological) safety net is needed for the pupils so that freedom does not lead to the fear of losing one's way.

Notated Online Resources

1 Mathematik. Retrieved from: https://bildung.bmbwf.gv.at/schulen/unterricht/lp/lp_neu_ahs_07_11859.pdf 22.05.2018

2 Verordnung der Bundesministerin für Unterricht, Kunst und Kultur über die Reifeprüfung in den allgemein bildenden höheren Schulen. Retrieved from: https://www.ris.bka.gv.at/GeltendeFassung.wxe?Abfrage=Bundesnormen&Gesetzesnummer=20007845 22.05.2018

3 Khan Academy. Retrieved from: https://www.khanacademy.org/ 08.06.2018

4 TechSmith Jing. Retrieved from: https://www.techsmith.de/jing-werkzeug.html 08.06.2018

5 Screencast-O-Matic. Retrieved from: https://screencast-o-matic.com/ 08.06.2018

6 GeoGebra. Retrieved from: https://www.geogebra.org/ 08.06.2018

7 GeoGebra Unterrichtsmaterialien. Retrieved from: https://www.geogebra.org/materials 08.06.2018

8 Slack. Retrieved from: https://slack.com/ 08.06.2018

9 Mahara. Retrieved from: https://mahara.org/ 08.06.2018

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References

Baumgartner, P., & Kalz, M. (2004). Content Management Systeme für den Bildungsbereich. In *Baumgartner, P., Häfele, H. und Maier-Häfele K: Content-Management-Systeme in e-Education: Auswahl, Potenziale und Einsatzmöglichkeiten*. Innsbruck; Wien; München; Bozen.

Bell, C. V., & Pape, S. J. (2012). Scaffolding students' opportunities to learn mathematics through social interactions. *Mathematics Education Research Journal, 24*(4), 423–445. https://doi.org/10.1007/s13394-012-0048-1

Bishop, J. L., & Verleger, M. A. (2013). The flipped classroom: A survey of the research (Vol. 30). Presented at the ASEE National Conference Proceedings, Atlanta, GA.

Capobianco, B. M., & Feldman, A. (2010). Repositioning Teacher Action Research in Science Teacher Education. *Journal of Science Teacher Education, 21*(8), 909–915. https://doi.org/10.1007/s10972-010-9219-7

Chao, T., Chen, J., Star, J. R., & Dede, C. (2016). Using Digital Resources for Motivation and Engagement in Learning Mathematics: Reflections from Teachers and Students. *Digital Experiences in Mathematics Education, 2*(3), 253–277. https://doi.org/10.1007/s40751-016-0024-6

Chen, L., Wang, X., Li, J., Bao, H., & Ren, G. (2016). Promoting Students' Engagement? Flipped Classroom Matters a Lot – An Empirical Research in College. In S. K. S. Cheung, L. Kwok, J. Shang, A. Wang, & R. Kwan (Eds.), *Blended Learning: Aligning Theory with Practices : 9th International Conference, ICBL 2016, Beijing, China, July 19-21, 2016, Proceedings* (pp. 196–206). Cham: Springer International Publishing. Retrieved from http://dx.doi.org/10.1007/978-3-319-41165-1_18

Choi, E. M. (2013). Applying inverted classroom to software engineering education. *International Journal of E-Education, E-Business, E-Management and E-Learning, 3*(2), 121.

Clarke, T., Ayres, P., & Sweller, J. (2005). The impact of sequencing and prior knowledge on learning mathematics through spreadsheet applications. *Educational Technology Research and Development, 53*(3), 15–24. https://doi.org/10.1007/BF02504794

Coles, A., & Brown, L. (2016). Task design for ways of working: making distinctions in teaching and learning mathematics. *Journal of Mathematics Teacher Education, 19*(2), 149–168. https://doi.org/10.1007/s10857-015-9337-4

Fogarty, G., Cretchley, P., Harman, C., Ellerton, N., & Konki, N. (2001). Validation of a questionnaire to measure mathematics confidence, computer confidence, and attitudes towards the use of technology for learning mathematics. *Mathematics Education Research Journal, 13*(2), 154–160. https://doi.org/10.1007/BF03217104

Galway, L. P., Corbett, K. K., Takaro, T. K., Tairyan, K., & Frank, E. (2014). A novel integration of online and flipped classroom instructional models in public health higher education. *BMC Medical Education, 14*(1), 181. https://doi.org/10.1186/1472-6920-14-181

Goerres, A., Kärger, C., & Lambach, D. (2015). Aktives Lernen in der Massenveranstaltung: Flipped-Classroom-Lehre als Alternative zur klassischen Vorlesung in der Politikwissenschaft. *ZPol Zeitschrift für Politikwissenschaft, 25*(1), 135–152. https://doi.org/10.5771/1430-6387-2015-1-135

Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2000). Reshaping teacher and student roles in technology-enriched classrooms. *Mathematics Education Research Journal, 12*(3), 303–320. https://doi.org/10.1007/BF03217091

Hall, J., & LingefjÃ, T. (2016). *Mathematical Modeling: Applications with GeoGebra*. John Wiley & Sons.

Herreid, C. F., & Schiller, N. A. (2013). Case studies and the flipped classroom. *Journal of College Science Teaching, 42*(5), 62–66.

Hung, C.-M., Huang, I., & Hwang, G.-J. (2014). Effects of digital game-based learning on students' self-efficacy, motivation, anxiety, and achievements in learning mathematics. *Journal of Computers in Education, 1*(2), 151–166. https://doi.org/10.1007/s40692-014-0008-8

Lee, C., & Johnston-Wilder, S. (2013). Learning mathematics—letting the pupils have their say. *Educational Studies in Mathematics, 83*(2), 163–180. https://doi.org/10.1007/s10649-012-9445-3

Leonard, J., Moore, C. M., & Brooks, W. (2014). Multicultural Children's Literature as a Context for Teaching Mathematics for Cultural Relevance in Urban Schools. *The Urban Review, 46*(3), 325–348. https://doi.org/10.1007/s11256-013-0264-3

Lingefjärd, T., & Kilpatrick, J. (1998). Authority and responsibility when learning mathematics in a technology-enhanced environment. In D. Tinsley & D. C. Johnson (Eds.), *Information and Communications Technologies in School Mathematics: IFIP TC3 / WG3.1 Working Conference on Secondary School Mathematics in the World of Communication Technology: Learning, Teaching and the Curriculum, 26–31 October 1997, Grenoble, France* (pp. 233–236). Boston, MA: Springer US. https://doi.org/10.1007/978-0-387-35287-9_28

Mamlok-Naaman, R., & Eilks, I. (2012). Different Types of Action Research to Promote Chemistry Teachers' Professional Development—A Joined Theoretical Reflection on Two Cases from Israel and Germany. *International Journal of Science and Mathematics Education, 10*(3), 581–610. https://doi.org/10.1007/s10763-011-9306-z

Ngware, M. W., Ciera, J., Musyoka, P. K., & Oketch, M. (2015). Quality of teaching mathematics and learning achievement gains: evidence from primary schools in Kenya. *Educational Studies in Mathematics, 89*(1), 111–131. https://doi.org/10.1007/s10649-015-9594-2

Orlando, J., & Attard, C. (2016). Digital natives come of age: the reality of today's early career teachers using mobile devices to teach mathematics. *Mathematics Education Research Journal, 28*(1), 107–121. https://doi.org/10.1007/s13394-015-0159-6

Samuelsson, J. (2006). ICT as a Change Agent of Mathematics Teaching in Swedish Secondary School. *Education and Information Technologies, 11*(1), 71–81. https://doi.org/10.1007/s10639-005-5713-5

Staker, H., & Horn, M. B. (2012). Classifying K-12 Blended Learning. *Innosight Institute*.

Sureka, A., Gupta, M., Sarkar, D., & Chaudhary, V. (2013). A case-study on teaching undergraduate-level software engineering course using inverted-classroom, large-group, real-client and studio-based instruction model. *arXiv Preprint arXiv:1309.0714*. Retrieved from http://arxiv.org/abs/1309.0714

Tait-McCutcheon, S. L., & Loveridge, J. (2016). Examining equity of opportunities for learning mathematics through positioning theory. *Mathematics Education Research Journal, 28*(2), 327–348. https://doi.org/10.1007/s13394-016-0169-z

Treeck, T. van, Himpsl-Gutermann, K., & Robes, J. (2013). *Offene und partizipative Lernkonzepte. E-Portfolios, MOOCs und Flipped Classrooms*. Retrieved from http://www.pedocs.de/volltexte/2013/8354/

Wasserman, N. H., Quint, C., Norris, S. A., & Carr, T. (2015). Exploring Flipped Classroom Instruction in Calculus III. *International Journal of Science and Mathematics Education*, 1–24. https://doi.org/10.1007/s10763-015-9704-8

Zambo, D. (2011). Action Research as Signature Pedagogy in an Education Doctorate Program: The Reality and Hope. *Innovative Higher Education, 36*(4), 261–271. https://doi.org/10.1007/s10755-010-9171-7