รหัสโครงการ SUT1-105-54-12-23

รายงานการวิจัย

การเกิดอนุภาคฟีในการประลัยนิวคลีออน-แอนตินิวคลีออน และความแปลกของนิวคลีออน

(ϕ production in nucleon-antinucleon annihilation and strangeness of nucleon)

ได้รับทุนอุดหนุนการวิจัยจาก

มหาวิทยาลัยเทคโนโลยีสุรนารี

ผลงานวิจัยเป็นความรับผิดชอบของหัวหน้าโครงการวิจัยแต่เพียงผู้เดียว

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ผลงานวิจัยเป็นความรับผิดชอบของหัวหน้าโครงการวิจัยแต่เพียงผู้เดียว

กันยายน 2558

ϕ production in nucleon-antinucleon annihilation and strangeness of nucleon

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This work was supported by Suranaree University of Technology in fiscal year 2554

September 2558

ACKNOWLEDGEMENTS

This work was supported by Suranaree University of Technology under Grant No. SUT1-105-54-12-23. The authors would like to thank Asst. Prof. Dr. Ayut Limphirat and Dr. Daris Samart for their consistent help.

บทคัดย่อ

โครงการวิจัยนี้ได้ทำการพิจารณาองค์ประกอบของควาร์กเอสที่มีอยู่ภายในนิวคลีออนจากการละเมิดกฎของโอเส็ดไอในที่พบในปฏิกิริยาแบบทำลายระหว่างนิวคลีออนกับแอนตินิวคลีออน ซึ่งการมีองค์ประกอบของคู่ควาร์กเอสกับแอนติควาร์กเอสในฟังก์ชันคลื่นนิวคลีออนจะสามารถก่อให้เกิดอนุภาคเมซอนชนิดฟีได้โดยตรงในปฏิกิริยาดังกล่าวโดยไม่เกิดการละเมิดกฎของโอเส็ดไอ แบบจำลองของโปรตอนที่มีองค์ประกอบของควาร์กเอสในฟังก์ชันคลื่นสามแบบจึงได้ถูกนำมาพิจารณา ได้แก่ ฟังก์ชันคลื่นที่ประกอบด้วยกลุ่มก้อนของควาร์กสองกลุ่มอันได้แก่กลุ่มที่ประกอบด้วยควาร์กยูสองตัวกับควาร์กดีหนึ่งตัว กับกลุ่มก้อนควาร์กเอสกับแอนติควาร์กเอส และแบบจำลองที่สองอยู่บนพื้นฐานของแบบจำลองไคแรลควาร์กซึ่งฟังก์ชันคลื่นมีลักษณะเป็นกลุ่มก้อนของอนุภาคเคออนกับไฮเปอร์รอน และแบบสุดท้ายซึ่งในรูปของเพนตะควาร์ก จากผลการคำนวณค่าโมเมนต์แม่เหล็ก และสปินของควาร์กเอสที่มีผลต่ออนุภาคโปรตรอน ในกรณีที่ไม่พิจารณาแบบจำลองของควาร์กแบบสัมพัทธภาพ พบว่าแบบจำลองของโปรตอนที่หนึ่งและสองนั้นให้ค่าโมเมนต์แม่เหล็กและสปินเป็นลบ สอดคล้องกับผลการทดลองที่ได้มีการรายงานออกเมื่อเร็วๆนี้ ในขณะที่แบบจำลองที่สามสามารถพบโมเมนต์แม่เหล็ก และสปินเป็นลบได้เฉพาะในกรณีที่โครงร่างของควาร์กสี่ตัวเป็น $[31]_{FS}[211]_F[22]_S$ และ $[31]_{FS}[31]_F[22]_S$ โดยอาศัยแบบจำลองทริปเปตพีศูนย์ร่วมกับแผนผังแบบเส้นของควาร์กยังผล เราได้คำนวณค่าสัดส่วนในการเกิดปฏิกิริยาของโปรตอนกับแอนติโปรตอนอยู่นิ่งซึ่งถูกประลัยไปเป็นอนุภาคฟีกับอนุภาคเมซอน ได้แก่ ไพ อีตา โรห์ และโอเมกา ผลลัพธ์ของสัดส่วนในการเกิดปฏิกิริยาของอนุภาคฟีกับอนุภาคเมซอนดังกล่าวจากสถานะอะตอมคลื่นเอสระหว่างโปรตอนกับแอนติโปรตอน พบว่าฟังก์ชันคลื่นตามแบบจำลองที่หนึ่งและสามนั้น ได้ค่าที่มีลักษณะเปลี่ยนแปลงไปตามแต่ละปฏิกิริยาอย่างมากและสอดคล้องเป็นอย่างดีกับผลที่ได้จากการทดลอง

ABSTRACT IN ENGLISH

Apparent channel-dependent violations of the OZI rule in nucleon-antinucleon annihilation reactions are discussed in the presence of an intrinsic strangeness component in the nucleon. Admixture of $s\bar{s}$ quark pairs in the nucleon wave function enables the direct coupling to the ϕ - meson in the annihilation channel without violating the OZI rule. Three models are considered in this work for the strangeness content of the proton wave function, namely, the uud cluster with a $s\bar{s}$ sea quark component, kaon-hyperon clusters based on a chiral quark model, and the pentaquark picture $uuds\bar{s}$. Nonrelativistic quark model calculations reveal that the strangeness magnetic moment μ_s and the strangeness contribution to the proton spin σ_s from the first two models are consistent with recent experimental data where μ_s and σ_s are negative. For the third model, the $uuds$ subsystem with the configurations $[31]_{FS}[211]_F[22]_S$ and $[31]_{FS}[31]_F[22]_S$ leads to negative values of μ_s and σ_s. With effective quark line diagrams incorporating the $^3P_0$ model we give estimates for the branching ratios of the annihilation reactions at rest $p\bar{p} \to \phi X$ $(X = \pi^0, \eta, \rho^0, \omega)$. Results for the branching ratios of ϕX production from atomic $p\bar{p}$ s-wave states are for the first and third model found to be strongly channel dependent and in good agreement with measured rates.

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Chapter 1

Introduction

In the simple constituent quark model, where the proton is made of two constituent u quarks and one d quark, a good explanation of static properties e.g. magnetic moment can be achieved. However, experimental results of the pion-nucleon sigma term value, strange magnetic moment μ_s, strangeness contribution to nucleon form factor [1] as well as the apparent violations in nucleon-antinucleon annihilation reactions involving ϕ meson [2] indicate that the proton might contain a substantial strange quark-antiquark ($s\bar{s}$) component. The strangeness sigma term appears to lie somewhere in the range of $2-7\%$ of the nucleon mass [3]. The substantial Okubo-Zwieg-Iizuka (OZI) rule violations in the $N\bar{N}$ annihilation reactions involving ϕ meson may suggest the presence of an intrinsic $s\bar{s}$ in nucleon wave function [4], for instance, the presence of a $q^3s\bar{s}(\bar{q}^3s\bar{s})$ piece in the $N(\bar{N})$ wave function. With such an assumption, the ϕ meson could be produced in $N\bar{N}$ annihilation reactions via a shake-out or rearrangement of the strange quarks already stored in the nucleon without the violation of the OZI rule. There are other explanations of the OZI rule violation without introducing strange component in the nucleon such as the resonance interpretation, instanton induced interaction [5], and rescattering [6].

The EMC spin experiment [7] on deep inelastic scattering of longitudinally polarised muons by longitudinally polarised protons revealed the first time that the polarization of the strange quark sea may contribute to the proton spin σ_s a significant negative value. This experimental result was confirmed by the subsequent deep inelastic double polarization experiments. Ref. [8] analyzed all the available data then in a systematic way and found $\sigma_s = -0.10 \pm 0.03$. Among a large number of theoretical works, Cheng and Li apply the chiral quark model (ChQM) to explain the spin and flavor structure of proton [9]. With the fluctuation of the proton into a kaon and a hyperon, they can explain the negative polarization of the strange quark sea and get other theoretical results consistent with the DIS experimental results.

However, the configuration of strange quarks in the nucleon is still an open question. The strangeness magnetic moment μ_s can be extrapolated from the strange magnetic form factor $G_M^s(Q^2)$ at the momentum transfer $Q^2 = 0$ measured in the parity violation experiments of electron scattering from a nucleon [10]. Most experimental measurements suggest a positive value for μ_s, in contrast to the recent experiment data [11] and most theoretical calculations which have obtained negative values for this observable [12, 13]. A recent work [14] has proposed a different form for the strangeness content of the proton which has the strange quark piece in terms of pentaquark configurations instead of the 5-quark component which consist of a uud cluster and a $s\bar{s}$ pair proposed for solving the puzzle of violation the OZI rule. Different pentaquark configurations that may be contained in the proton may yield both positive and negative values for the strangeness spin and magnetic moment of the proton.

The experimental results on μ_s, which is extracted from experimental data on $G_M^s(Q^2)$, are rather uncertain due to the large uncertainties in $G_M^s(Q^2)$ and the extrapolation approach. So it is believed that the proton-antiproton reactions involving ϕ production may be another platform to be applied to tackle the possible configuration of strange quarks in the proton. In the present work we consider the strange content in the proton wave function in three models, namely, the uud cluster with a $s\bar{s}$ sea quark component, kaon-hyperon clusters based on the chiral quark model, and the pentaquark picture $uuds\bar{s}$. The theoretical σ_s, μ_s and branching ratios of the reactions $p\bar{p} \to \phi X$ ($X = \pi^0, \eta, \rho^0, \omega$) will be compared to experimental data. We resort to the ${}^3P_0$ quark model [15] and the nearest threshold dominance model [16] to obtain quantitative predictions for the branching ratios of the annihilation reactions from atomic $p\bar{p}$ states with the relative orbital angular momentum $L = 0$ [17]. The paper is organized as follows. The proton wave functions are briefly described in Section 2 while σ_s and μ_s are calculated and discussed in Section 3 for various strangeness quark configurations. In Section 4 we evaluate the branching ratios for the reactions $p\bar{p} \to \phi X$ for the three forms of proton wave functions by using the ${}^3P_0$ quark model. Finally a summary and conclusion are given in Section 5.

Chapter 2

Proton wave functions

The proton wave function in the presence of strange quarks may include a 5-quark component $qqqs\bar{s}$ in addition to the uud quark component, taking generically the form

$$|p\rangle = A|uud\rangle + B|uuds\bar{s}\rangle \tag{2.1}$$

where A and B are the amplitudes for the 3- and 5-quark components in the proton, respectively [18]. The possible spin-flavor structures of the 5-quark components discussed in the $N\bar{N}$ annihilation process are considered in the next three subsections.

2.1 Proton wave function with an explicit $s\bar{s}$ sea-quark component

We consider the idea that strange quarks are present in the form of an $s\bar{s}$ sea-quark component in the proton state. This idea was proposed for describing the apparent violation of the OZI rule in the ϕNN production process [19] and in more general form used to discuss the ϕ meson production in $N\bar{N}$ annihilation reactions [4]. The corresponding 5-quark component for this model can be written in Fock space as

$$|uuds\bar{s}\rangle^{s\bar{s}} = a_0|(uud)_{1/2}(s\bar{s})_0\rangle_{1/2} + a_1|(uud)_{1/2}(s\bar{s})_1\rangle_{1/2} \tag{2.2}$$

where the subscripts denote the spin coupling of the quark clusters, a_0 and a_1 represent the amplitudes for the spin 0 and spin 1 components of the admixed $s\bar{s}$

pairs.

2.2 Proton wave function based on a chiral quark model

In the chiral quark model, the dominant process is the fluctuation of a valence quark q into a quark q' plus a Goldstone boson (GB) which in turn forms a $(q\bar{q}')$ system [20]. After the fluctuation of the u and d quarks in the proton, one of these quarks turns into a quark plus a quark-antiquark pair involving a strange quark. This idea was considered, for example, for calculating the flavor and spin content of the proton [9]. To obtain the proton wave function we consider the SU(3) invariant interaction Lagrangian of baryon octet with nonet of pseudoscalar mesons:

$$\mathcal{L}_I = -g_8\sqrt{2}\left(\alpha[\bar{B}BP]_F + (1-\alpha)[\bar{B}BP]_D\right) - g_1\frac{1}{\sqrt{3}}[\bar{B}BP]_S \tag{2.3}$$

where $g_8 = 3.8$ and $g_1 = 2.0$ are coupling constants [21] and α is known as the $F/(F+D)$ ratio with $F \simeq 0.51,\ D \simeq 0.76$ [22]. The square parentheses denote the SU(3) invariant combinations:

$$\begin{aligned}
[\bar{B}BP]_F &= \mathrm{Tr}(\bar{B}PB) - \mathrm{Tr}(\bar{B}BP)\,, && (2.4)\\
[\bar{B}BM]_D &= \mathrm{Tr}(\bar{B}PB) + \mathrm{Tr}(\bar{B}BP) - \frac{2}{3}\mathrm{Tr}(\bar{B}B)\mathrm{Tr}(P)\,, && (2.5)\\
[\bar{B}BP]_S &= \mathrm{Tr}(\bar{B}B)\mathrm{Tr}(P)\,, && (2.6)
\end{aligned}$$

where B and P are the baryon octet and pseudoscalar meson nonet matrices, respectively, given by

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}, \tag{2.7}$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}. \tag{2.8}$$

The part of the interaction Lagrangian which allows for a fluctuation of the proton into kaons and hyperons is contained in

$$\begin{aligned}\mathcal{L}_I &= -g_1\bar{p}\eta_1 p + g_8\left[\bar{p}\pi^0 + \frac{1-4\alpha}{\sqrt{3}}\bar{p}\eta_8 + \frac{1+2\alpha}{\sqrt{3}}\bar{\Lambda}K^- + (2\alpha-1)\bar{\Sigma}^0 K^-\right.\\ &- \left.\sqrt{2}\bar{n}\pi^- + \sqrt{2}(2\alpha-1)\bar{\Sigma}^- K^0\right]p + \cdots \end{aligned} \tag{2.9}$$

The final states resulting from pseudoscalar meson emission by the proton are summarized as

$$\begin{aligned}|\Psi\rangle &\sim -g_1|p\eta_1\rangle + g_8\left[\frac{1-4\alpha}{\sqrt{3}}|p\eta_8\rangle + |p\pi^0\rangle + \frac{1+2\alpha}{\sqrt{3}}|\Lambda K^+\rangle\right.\\ &+ \left.(2\alpha-1)|\Sigma^0 K^+\rangle - \sqrt{2}|n\pi^+\rangle + \sqrt{2}(2\alpha-1)|\Sigma^+ K^0\rangle\right]. \end{aligned} \tag{2.10}$$

In the absence of the fluctuation, the proton is made up of the conventional two u quarks and one d quark. Thus $\Psi(p)$ may be interpreted as the 5-quark component of the proton wave function which is given by

$$|uuds\bar{s}\rangle^{\mathrm{ChQM}} = G_1|\Sigma^0 K^+\rangle + G_2|\Sigma^+ K^0\rangle + G_3|\Lambda^0 K^+\rangle + G_4|p\eta_1\rangle + G_5|p\eta_8\rangle + \tag{2.11}$$

where the G_i are the coefficients corresponding to the respective factor in Eq. (2.10). Each component in the last equation can be represented in terms of quark cluster comnfigurations as

$$\begin{aligned}|p\eta_{1,8}\rangle &= |(uud)_{1/2}(s\bar{s})_0\rangle_{1/2}\,, \quad |\Sigma^0 K^+\rangle = |(uds)_{1/2}(u\bar{s})_0\rangle_{1/2}\,,\\ |\Sigma^+ K^0\rangle &= |(uus)_{1/2}(d\bar{s})_0\rangle_{1/2}\,, \quad |\Lambda^0 K^+\rangle = |(usd)_{1/2}(u\bar{s})_0\rangle_{1/2}\,. \end{aligned} \tag{2.12}$$

2.3 Proton wave function including general configurations of the *uuds* subsystem

Another, more general form of the 5-quark component was proposed and analyzed in Ref. [14]. Instead of first generating a meson coupling to a baryon cluster, they consider the genuine 5-quark or $q^4\bar{q}$ pentaquark component in the proton. In this model the 5-quark component in this model may be expressed in terms of the *uuds* and the $\bar{s}$ wave functions as

$$|uuds\bar{s}\rangle^{uuds} = |(uuds)\bar{s}\rangle_{1/2}. \tag{2.13}$$

The flavor wave functions for the $uuds\bar{s}$ components are usually constructed by coupling the $uuds$ to the $\bar{s}$ flavor wave function. The configurations studied in [14] include at most one unit of orbital angular momentum. The favored configurations are connected to a positive sign for the strangeness magnetic moment and a negative one for the strangeness contribution to the proton spin.

Chapter 3

Strangeness magnetic moment and spin of the proton

In the nonrelativistic quark model the strangeness magnetic moment operator $\vec{\mu}_s$ and the strangeness contribution to the proton spin operator $\vec{\sigma}_s$ are defined as

$$\vec{\mu}_s = \frac{e}{2m_s}\sum_i \widehat{S}_i(\widehat{\ell}_s + \widehat{\sigma}_s) \;, \tag{3.1}$$

$$\vec{\sigma}_s = \widehat{\sigma}_s + \widehat{\sigma}_{\bar{s}} \;. \tag{3.2}$$

$\widehat{S}_i$ is the strangeness counting operator with eigenvalue +1 for an s and -1 for an $\bar{s}$ quark and m_s is the constituent mass of the strange quark. To calculate the matrix elements of these operators explicit forms of the spin-flavor wave functions of the proton including orbital angular momentum are needed.

For the first model the spin-flavor wave function can be constructed by coupling the $|s\bar{s}\rangle_{j_s=0,1}$ configuration to the $|uud\rangle_{1/2}$ cluster. Since the admixed $s\bar{s}$ carries negative intrinsic parity, an orbital P-wave ($\ell = 1$) has to be introduced into the nucleon quark cluster wave function. The simplest configuration (see also Ref. [19]) corresponds to an 1S-state of the $s\bar{s}$ pair moving in a p-wave relative to the (uud) valence quark cluster of the nucleon. Then the 5-quark component with total angular momentum 1/2 can be written in the general form:

$$|uuds\bar{s}\rangle^{s\bar{s}}_{\frac{1}{2},m_{ps\bar{s}}=\frac{1}{2}} = \sum_{j_s,j_i=0,1} \alpha_{j_sj_i}|[(s\bar{s})_{j_s} \otimes \ell = 1]_{j_i} \otimes (uud)_{\frac{1}{2}}\rangle_{\frac{1}{2},m_{ps\bar{s}}=\frac{1}{2}} \tag{3.3}$$

with the normalization $\sum\limits_{j_s,j_i=0,1} |\alpha_{j_sj_i}|^2 = 1$.

Similarly, for the proton wave function in the ChQM, where the sea-quark contributions are embedded in the pseudoscalar mesons, a relative P-wave between the pseudoscalars and the uud or hyperon clusters has to be included. The spin-flavor wave function with spin $+1/2$ for each coupled meson-baryon state of Eq. (2.12) may be expressed as

$$|uuds\bar{s}\rangle^{\text{ChQM}}_{\frac{1}{2},\frac{1}{2}} = |[(q\bar{s})_{j_s=0} \otimes \ell = 1]_{j_i} \otimes (qqs)_s\rangle_{\frac{1}{2},m_{ps\bar{s}}=\frac{1}{2}}. \tag{3.4}$$

Wave functions of the pentaquark $uuds\bar{s}$ states employed in the third model are more complicated because no restrictions are set concerning the sub-clusters. One has to carefully consider the coupling of the color, spin, flavor and spatial parts to construct the total wave functions [14]. The color part of the antiquark in the pentaquark states is a [11] antitriplet, denoted by the Weyl tableau of the SU(3) group. Hence the color symmetry of all the $uuds$ configurations is limited to a [211] triplet in order to form a pentaquark color singlet labeled by the Weyl tableau [222]. Three flavor symmetry patterns exist for the $uuds$ system corresponding to the octet representation for the proton: $[31]_F$, $[22]_F$ and $[211]_F$ characterized by the S_4 Young tableau. However, the pentaquark should be antisymmetric under any permutation of the four quark configuration. If the spatial wave function is symmetric, the spin-flavor part of the $uuds$ component must be a [31] state in order to form the antisymmetric color-spin-flavor $uuds$ part of the pentaquark wave function. For instance, the flavor symmetry representations $[31]_F$ and $[211]_F$ may combine with the spin symmetry state $[22]_S$ to form the mixed symmetry spin-flavor states $[31]_{FS}$ (the explicit forms may be found in [14, 23, 24]). In this work we consider only the case that the $uuds$ component is in the ground state with the spin symmetry $[22]_S$ corresponding to spin zero, and the relative orbital angular momentum between the $uuds$ component and the $\bar{s}$ is of one unit to obtain the positive parity for the proton wave function.

$\vert uuds\bar{s}\rangle$	$\mu_s(\frac{eB^2}{2m_s})$	$\sigma_s(B^2)$
$s\bar{s}$	$-0.55\bar{\alpha}\alpha_{0,1}$	$-1.22\bar{\alpha}^2$ [17]
ChQM	$-1.1g_8^2$	$-0.31g_8^2$
$[31]_{FS}[211]_F[22]_S$	$-\frac{1}{3}[14]$	$-\frac{1}{3}[14]$
$[31]_{FS}[31]_F[22]_S$	$-\frac{1}{3}[14]$	$-\frac{1}{3}[14]$

Table 3.1: Strangeness magnetic moment and spin of the proton for the three models of the 5-quark component.

Theoretical results for the strangeness magnetic moment μ_s of the proton and the strangeness contribution to the proton spin σ_s are listed in Table I. In the first model we have fixed the configuration parameters as $\alpha_{1,0} = \alpha_{1,1} = \bar{\alpha}$. The strangeness magnetic moment μ_s depends explicitly on $\alpha_{0,1}$, which is related to the amplitude for the $s\bar{s}$ quark cluster with spin 0. Setting $\alpha_{0,1} = 0$ is equivalent to excluding the quantum number $J^{PC} = 0^{-+}$ for the $s\bar{s}$ admixture in the nucleon wave function connected to the the production of η and η' in $N\bar{N}$ annihilation as discussed in [18, 25]. The chiral quark model always gives results for μ_s and σ_s which are negative, the size of the strangeness contribution depends on the coupling g_8^2. For the third model, we show only the results for the cases where the *uuds* component is in the ground state with the spin-flavor configurations $[31]_{FS}[211]_F[22]_S$ and $[31]_{FS}[31]_F[22]_S$ and the relative motion between the *uuds* component and the $\bar{s}$ is a P-wave.

All the three models yield negative values for the strangeness contribution to the proton spin, which is consistent with present experimental results [7, 8]. Negative values for the strangeness magnetic moment also result from all three models. Note that we restricted the considerations of Ref. [14] to the pentaquark components with the *uuds* configurations $[31]_{FS}[211]_F[22]_S$ and $[31]_{FS}[31]_F[22]_S$, respectively.

Chapter 4

$N\bar{N}$ transition amplitude and branching ratios

To describe the annihilation reactions $N\bar{N} \to X\phi$ ($X = \pi^0, \eta, \rho^0, \omega$) we use an effective transition dynamics, which is evaluated in the context of a simple constituent quark model. In this specific process the ϕ meson couples to the intrinsic $s\bar{s}$ component of the nucleon, which is the leading order OZI allowed contribution. The process $p\overline{p}$ annihilation into ϕX involving the 5-quark components in the proton wave function can be described by the quark line diagrams of Fig. 1. In the hadronic transition the effective quark annihilation operator is taken with the quantum numbers of the vacuum ($^3P_0$, isospin $I = 0$ and color singlet). Meson decays and $N\bar{N}$ annihilation into two mesons are well described phenomenologically using such an effective quark-antiquark vertex. At least fro meson decay, this approximation has been given a rigorous basis in strong-coupling QCD. The nonperturbative $q\bar{q}\ {}^3P_0$ vertex is defined according to [26]

$$V^{ij} = \sum_{\mu} \sigma^{ij}_{-\mu} Y_{1\mu}(\vec{q}_i - \vec{q}_j)\delta^{(3)}(\vec{q}_i + \vec{q}_j)(-1)^{1+\mu} 1^{ij}_F 1^{ij}_C \ , \tag{4.1}$$

where $Y_{1\mu}(\vec{q}) = |\vec{q}|\mathcal{Y}_{1\mu}(\widehat{q})$ with $\mathcal{Y}_{1\mu}(\widehat{q})$ being the spherical harmonics in momentum space, and 1^{ij}_F and 1^{ij}_C are unit operators in flavor and color spaces, respectively. The spin operator $\sigma^{ij}_{-\mu}$ is part of the $^3P_0$ vertex, destroying or creating quark-antiquark pairs with spin 1.

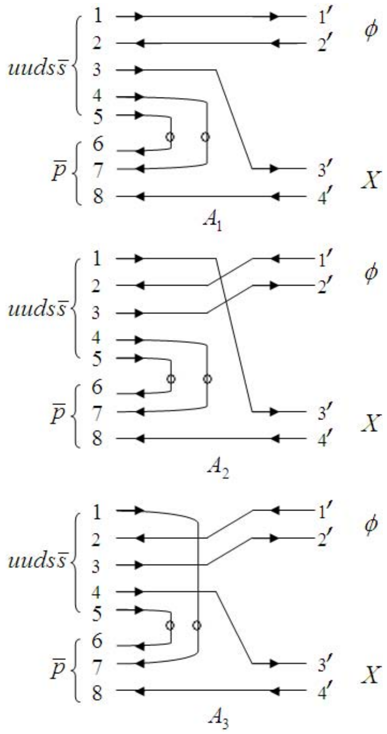

Figure 4.1: Quark line diagrams for the production of two meson final states in $p\bar{p}$ annihilation. Small circles refer to the effective vertex of the $^3P_0$ quark dynamics for $q\bar{q}$ annihilation. The first diagram corresponds to the shake-out of the intrinsic $s\bar{s}$ component of the proton wave function [4, 17].

In the momentum space representation the transition amplitudes for the quark diagrams of Fig. 1 are given by

$$\begin{aligned} T_{A_I} &= \int d^3q_1..d^3q_8 d^3q_{1'}..d^3q_{4'} \langle \phi X | \vec{q}_{1'}..\vec{q}_{4'} \rangle \\ &\quad \times \langle \vec{q}_{1'}..\vec{q}_{4'} | \mathcal{O}_{A_I} | \vec{q}_1..\vec{q}_8 \rangle \langle \vec{q}_1..\vec{q}_8 | (uuds\bar{s}) \otimes (\bar{u}\bar{u}\bar{d}) \rangle \end{aligned} \tag{4.2}$$

where $(\bar{u}\bar{u}\bar{d})$ stands for the antiproton wave function and $(uuds\bar{s})$ for the five quark component of the proton wave function. The effective operators $\mathcal{O}_{A_I}$ take the form

$$\mathcal{O}_{A_1} = \lambda_{A_1} \delta^{(3)}(\vec{q}_1 - \vec{q}_{1'}) \delta^{(3)}(\vec{q}_2 - \vec{q}_{2'}) \delta^{(3)}(\vec{q}_3 - \vec{q}_{3'}) \delta^{(3)}(\vec{q}_8 - \vec{q}_{4'}) V^{56} V^{47} \,, \tag{4.3}$$

$$\mathcal{O}_{A_2} = \lambda_{A_2} \delta^{(3)}(\vec{q}_2 - \vec{q}_{1'}) \delta^{(3)}(\vec{q}_3 - \vec{q}_{2'}) \delta^{(3)}(\vec{q}_1 - \vec{q}_{3'}) \delta^{(3)}(\vec{q}_8 - \vec{q}_{4'}) V^{56} V^{47} \,, \tag{4.4}$$

$$\mathcal{O}_{A_3} = \lambda_{A_3} \delta^{(3)}(\vec{q}_2 - \vec{q}_{1'}) \delta^{(3)}(\vec{q}_3 - \vec{q}_{2'}) \delta^{(3)}(\vec{q}_4 - \vec{q}_{3'}) \delta^{(3)}(\vec{q}_8 - \vec{q}_{4'}) V^{56} V^{17} \,. \tag{4.5}$$

The δ-functions represent the noninteracting and continuous quark-antiquark lines in the diagrams. The constants λ_{A_I} describe the effective strength of the transition topology and are considered to be overall fitting parameters in the phenomenological description of experimental data. Since the 5-quark component is treated as a small perturbative admixture in the proton ($B^2 << 1$), we ignore the transition amplitude with a term to $\langle \vec{q}_1..\vec{q}_8|(uuds\bar{s}) \otimes (\bar{u}\bar{u}\bar{d}\bar{s}s)\rangle$ or the rearrangement process [4].

In this work the internal spatial wave functions are taken in the harmonic oscillator approximation. For the mesons M (ϕ and X), the wave function can be expressed in terms of the quark momenta as

$$\langle M|\vec{q}_{i'}\vec{q}_{j'}\rangle \equiv \varphi_M(\vec{q}_{i'},\vec{q}_{j'})\chi_M = N_M \exp\left\{-\frac{R_M^2}{8}\left(\vec{q}_{i'}-\vec{q}_{j'}\right)^2\right\}\chi_M, \tag{4.6}$$

with $N_M = (R_M^2/\pi)^{3/4}$ and R_M is the meson radial parameter. The spin-color-flavor wave function is denoted by χ_M. The baryon wave functions are given by

$$\langle B|\vec{q}_i\vec{q}_j\vec{q}_k\rangle \equiv \varphi_B\chi_B = N_B \exp\left\{-\frac{R_B^2}{4}\left[(\vec{q}_j-\vec{q}_k)^2+\frac{(\vec{q}_j+\vec{q}_k-2\vec{q}_i)^2}{3}\right]\right\}\chi_B\,, \tag{4.7}$$

where $N_B = (3R_B^2/\pi)^{3/2}$ and R_B is the baryon radial parameter. For the first and the second model the full 5-quark component wave function, resulting from the coupling of a meson to a baryon, is given by

$$\begin{aligned}
\langle \vec{q}_1\cdot\vec{q}_5|uuds\bar{s}\rangle &= \varphi_{uuds\bar{s}}(\vec{q}_1,\cdots,\vec{q}_5)\chi_{uuds\bar{s}} \\
&= N_{uuds\bar{s}}\exp\left\{-\frac{R_B^2}{4}\left[(\vec{q}_4-\vec{q}_5)^2+\frac{(\vec{q}_4+\vec{q}_5-2\vec{q}_3)^2}{3}\right]\right\} \\
&\times \exp\left\{-\frac{R^2}{8}(\vec{q}_3+\vec{q}_4+\vec{q}_5-\vec{q}_1-\vec{q}_2)^2\right\}Y_{1\mu}(\vec{q}_3+\vec{q}_4+\vec{q}_5-\vec{q}_1-\vec{q}_2) \\
&\times \exp\left\{-\frac{R_M^2}{8}(\vec{q}_1-\vec{q}_2)^2\right\}\ (\chi_B\otimes\chi_M).
\end{aligned} \tag{4.8}$$

The exponential form with the radial parameter R and the spherical harmonics $Y_{1\mu}$ together represent the internal relative P-wave between the 3-quark and 2-quark clusters.

For the third model the proton wave function includes a pentaquark component $uuds\bar{s}$ with the $uuds$ part in the ground state and the P-wave internal relative orbital angular momentum between $uuds$ and the $\bar{s}$. One may write the spatial wave function of the pentaquark component $uuds\bar{s}$ as

$$\begin{aligned}\varphi_{uuds\bar{s}}(\vec{q}_1,\cdots,\vec{q}_5) &= N_{uuds\bar{s}}\exp\Bigg\{-\frac{R_B^2}{4}\Big[(\vec{q}_2-\vec{q}_3)^2+\frac{(\vec{q}_2+\vec{q}_3-2\vec{q}_4)^2}{3} \\ &+ \frac{(\vec{q}_2+\vec{q}_3+\vec{q}_4-3\vec{q}_5)^2}{6}+\frac{(\vec{q}_2+\vec{q}_3+\vec{q}_4+\vec{q}_5-4\vec{q}_1)^2}{10}\Big]\Bigg\} \\ &\times Y_{1\mu}\left(\frac{\vec{q}_2+\vec{q}_3+\vec{q}_4+\vec{q}_5-4\vec{q}_1}{\sqrt{20}}\right). \end{aligned} \tag{4.9}$$

By choosing the plane wave basis for the relative motion of the proton and antiproton, the initial state wave functions in the center of momentum system ($\vec{k}=\vec{q}_1+\vec{q}_2+\vec{q}_3+\vec{q}_4+\vec{q}_5$) are obtained as:

$$\langle \vec{q}_1\cdots\vec{q}_8|(uuds\bar{s})\otimes(\bar{u}\bar{u}\bar{d})\rangle = \varphi_{uuds\bar{s},\bar{p}}[\chi_{uuds\bar{s}}\otimes\chi_{\bar{p}}]_{S,S_z} \tag{4.10}$$

with

$$\varphi_{uuds\bar{s},\bar{p}} = \varphi_{uuds\bar{s}}\varphi_{\bar{p}}\delta^{(3)}(\vec{q}_1+\vec{q}_2+\vec{q}_3+\vec{q}_4+\vec{q}_5-\vec{k})\delta^{(3)}(\vec{q}_6+\vec{q}_7+\vec{q}_8+\vec{k})\ . \tag{4.11}$$

The spins of the $p\bar{p}$ system are coupled to the total spin S with projection S_z. Similarly, the final state ϕX wave functions in the center of momentum system are given by ($\vec{q}=\vec{q}_{1'}+\vec{q}_{2'}$):

$$\langle \phi X|\vec{q}_{1'}...\vec{q}_{4'}\rangle = \varphi_{\phi,X}[\chi_\phi\otimes\chi_X]_{j_i,m_\epsilon} \tag{4.12}$$

with

$$\varphi_{\phi,X} = \varphi_\phi\varphi_X\delta^{(3)}(\vec{q}-\vec{q}_{1'}-\vec{q}_{2'})\delta^{(3)}(\vec{q}+\vec{q}_{3'}+\vec{q}_{4'})\ . \tag{4.13}$$

The spins of the two meson states are coupled to j_i with projection m_ϵ.

In the low-momentum approximation, the transition amplitude T_{fi} of the annihilation reaction of the S-wave $\overline{p}p$ initial state i to the P-wave two-meson final state f with the quark line diagrams A_I as shown in Fig. 1 is derived as

$$T_{fi}(\vec{q},\vec{k}) = \lambda_{A_I}F_{L=0,\ell_f=1}q\exp\left\{-Q_q^2q^2-Q_k^2k^2\right\}\langle f|O_{A_I}|i\rangle \tag{4.14}$$

The index i represents the initial state ${}^{2I+1,2S+1}L_J$ where L is the orbital angular momentum, S is the total spin, J is the total angular momentum and I is the total isospin. The final state f is represented by the set of quantum numbers $f = \{\ell_f j J'\}$ where ℓ_f is the relative orbital angular momentum. The constants $F_{0,1}$, Q_q^2 and Q_k^2 are geometrical constants depending on the radial parameters. The matrix element $\langle f|O_{A_I}|i\rangle$ is the spin-flavor weight for a quark line diagram A_I. The detailed evaluation of the expression in Eq.(4.14) is given in Appendix A. Since the in the particle basis $p\bar{p}$ and $n\bar{n}$ give the same spin-flavor weight, the ϕ production from the nucleon-antinucleon annihilation at rest can be described by the transition amplitude Eq.(4.14) multiplied with a factor $\sqrt{2}$.

As we consider $p\bar{p}$ annihilations at rest where the strong interaction between the proton and antiproton may largely distort the $\overline{p}p$ hydrogen-like wave function at small distances [27], the effect of the initial state interaction is in general not negligible. The inclusion of the initial state interaction for the atomic state of the $p\bar{p}$ system results in the transition amplitude [28],

$$T_{f,LSJ}(\vec{q}) = \int d^3k \; T_{fi}(\vec{q},\vec{k})\phi^I_{LSJ}(\vec{k}), \tag{4.15}$$

where $\phi^I_{LSJ}(\vec{k})$ is the protonium wave function in momentum space for fixed isospin I. The partial decay width for the transition of the $p\bar{p}$ state to the two-meson state ϕX is given by

$$\Gamma_{p\bar{p}\to\phi X} = \int \frac{d^3p_\phi}{2E_\phi}\frac{d^3p_X}{2E_X}\delta^{(3)}(\vec{p}_\phi+\vec{p}_X)\delta(E-E_\phi-E_X)|T_{f,LSJ}(\vec{q})|^2 \tag{4.16}$$

where E is the total energy ($E = 1.876$ GeV) and $E_{\phi,X} = \sqrt{m^2_{\phi,X}+\vec{p}^{\,2}_{\phi,X}}$ is the energy of outgoing meson ϕ and X with mass $m_{\phi,X}$ and momentum $\vec{p}_{\phi,X}$. With the explicit form of the transition amplitude given by Eq. (4.14), the partial decay width for the S to P transition ($L = 0$, $\ell_f = 1$) is written as

$$\Gamma_{p\bar{p}\to\phi X} = \lambda^2_{A_I} f(\phi,X)\langle f|O_{A_I}|i\rangle^2\gamma(I,J), \tag{4.17}$$

with

$$\gamma(I,J) = |F_{0,1}\int d^3k \; \phi^I_{LSJ}(\vec{k})\exp\left\{-Z^2_\gamma k^2\right\}|^2 \tag{4.18}$$

Table 4.1: Spin-flavor matrix elements $\langle f|O_{A_I}|i\rangle$ for the transitions $p\bar{p}(L=0) \to \phi X(\ell_f = 1)$ which are described by the quark line diagram A_I. Here η_{ud} refers to the nonstrange flavor combination $\eta_{ud} = (u\bar{u} + d\bar{d})/\sqrt{2}$.

Transition	$s\bar{s}_{A_1}$	ChQM	$[31][31][22]_{A_1}$	$[31][211][22]_{A_1}$
$^{11}S_0 \to \omega\phi$	$\frac{5}{9\sqrt{6}}$	-0.097	$\frac{5}{36\sqrt{6}}$	$\frac{5}{36\sqrt{6}}$
$^{33}S_1 \to \pi^0\phi$	$\frac{5}{27\sqrt{2}}$	0.031	$\frac{5}{108\sqrt{2}}$	$\frac{5}{108\sqrt{2}}$
$^{31}S_0 \to \rho^0\phi$	$\frac{13}{27\sqrt{6}}$	0.040	$\frac{13}{108\sqrt{6}}$	$\frac{13}{108\sqrt{6}}$
$^{13}S_1 \to \eta_{ud}\phi$	$\frac{1}{9\sqrt{2}}$	0.013	$\frac{1}{36\sqrt{2}}$	$\frac{1}{36\sqrt{2}}$

and the kinematical phase-space factor defined by

$$f(\phi, X) = 2\pi \frac{E_\phi E_X}{E} q^3 \exp\left\{-2Z_\alpha^2 q^2\right\}. \tag{4.19}$$

The spin-flavor weights $\langle f|O_{A_I}|i\rangle$ for the transitions $N\bar{N} \to \phi X$ involving the different 5-quark components of the proton wave functions are listed in Table II. For the initial values of the total angular momentum J the statistical weights 1/4 and 3/4 have to be added for $J=0$ and $J=1$, respectively. Finally the branching ratio of S-wave $p\bar{p}$ annihilation to the final state ϕX is then given by

$$BR(\phi, X) = \frac{(2J+1)\Gamma_{p\bar{p}\to\phi X}}{4\Gamma_{tot}(J)}, \tag{4.20}$$

where $\Gamma_{tot}(J)$ is the total annihilation width of the $p\bar{p}$ atomic state with fixed principal quantum number [29].

The model dependence in Eq.(4.17) may be reduced by choosing a simplified phenomenological approach that has been applied in studies of two-meson branching ratios in nucleon-antinucleon [28] and radiative protonium annihilation [30]. Namely, instead of the phase space factor in Eq.(4.19) which depends on the relative momentum and the masses of ϕX system, we use a kinematical phase-space factor of the form

$$f(\phi, X) = q \cdot \exp\{-a_s\,(s - s_{\phi X})^{1/2}\} \tag{4.21}$$

where $a_s = 1.2$ GeV^{-1}, $s_{\phi X} = (m_\phi + m_X)^{1/2}$ and $\sqrt{s} = (m_\phi^2 + q^2)^{1/2} + (m_X^2 + q^2)^{1/2}$.

Last form is obtained from the fit to the momentum dependence of the cross section of various annihilation channels [16]. In addition, the functions $\gamma(I, J)$, depending on the initial-state interaction, are related to the probability for a protonium state to have isospin I and spin J with the normalization condition $\gamma(0, J) + \gamma(1, J) = 1$. Here we adopt for a protonium state the probability $\gamma(I, J)$ and the total decay width $\Gamma_{tot}(J)$ obtained in an optical potential calculation [31], where explicit values are listed in [29].

In Table III we give the theoretical results for the branching ratios of Eq. (4.20) compared with experimental data. The branching ratios $BR^{s\bar{s}}$, resulting from the first model where the proton wave function has an explicit $s\bar{s}$ admixture, have already been derived and studied in Ref. [17] by using the same approach. Annihilation processes in the first and third model are described by the quark line diagram A_1. Since the effective strength parameter λ_{A_1} is a priori unknown it has to be adjusted to data. For this purpose one entry (as indicated by $\star$) is normalized to the observed value.

For the second chiral model where the proton wave function contains a kaon-hyperon or eta-proton cluster component, all three quark line diagrams may have contributions to the $\overline{p}p$ annihilation process. However, the process proceeding by the diagram A_1 with the $|p\eta\rangle$ component in the proton wave function has no contribution to the transition because of orthogonality to the ϕ state. Therefore, the annihilation process in the second model can only be described by the quark line diagrams A_2 and A_3. Considering the same annihilation pattern in these two diagrams, for simplicity the two unknown strength parameters are of the same order with $\lambda_{A_2} = \lambda_{A_3}$. Model predictions are also normalized to experimental data (as indicated by $\star$). For final states with $X = \eta$, the physical η meson is produced by its nonstrange component η_{ud} with $\eta = \eta_{ud}(\sqrt{1/3}\cos\theta - \sqrt{2/3}\sin\theta)$ corresponding to a variation of the pseudoscalar mixing angle θ from $\theta = -10.7^o$ to $\theta = -20^o$.

As shown in Table III, the theoretical results of the first and third models, where the proton wave function possesses respectively a small kaon-hyperon component and a pentaquark, are in good agreement with the experimental data. Note that for these two cases the annihilation processes $p\bar{p} \to \phi X$ are described with the quark line diagram A_1.

Table 4.2: Branching ratio $BR(\times 10^4)$ for the transition $p\bar{p} \to \phi X$ $(X = \pi^0, \eta, \rho^0, \omega)$ in $p\bar{p}$ annihilation at rest. The results indicated by $\star$ are normalized to the experimental values.

Transition	$\mathrm{BR}^{\mathrm{exp}}$	$\mathrm{BR}^{s\bar{s}}$	$\mathrm{BR}^{\mathrm{ChQM}}$	$\mathrm{BR}^{[31][31][22]}$	$\mathrm{BR}^{[31][211][22]}$
${}^{11}S_0 \to \omega\phi$	6.3±2.3	6.3 $\star$	6.3 $\star$	6.3 $\star$	6.3 $\star$
${}^{33}S_1 \to \pi^0\phi$	5.5 ± 0.7	5.4	1.6	5.4	5.4
${}^{31}S_0 \to \rho^0\phi$	3.4 ± 1.0	3.8	0.87	3.8	3.8
${}^{13}S_1 \to \eta\phi$	0.9 ± 0.3	1.4−1.8	0.20−0.27	1.4−1.8	1.4−1.8

Chapter 5

Conclusions

Three models have been studied for the proton involving intrinsic strangeness in the form of a 5-quark component $qqqs\bar{s}$ in the wave function. In particular, the proton wave function is made up of a uud configuration and a uud cluster with a $s\bar{s}$ sea-quark component, kaon-hyperon clusters based on the simple chiral quark model, or a pentaquark component $uuds\bar{s}$. We have calculated the strangeness magnetic moment μ_s and spin σ_s for the first and second models and generate negative values in line with recent experimental indication. Similarly, for the third model we pick these configurations, where negative values for μ_s and σ_s result [14].

We further applied quark line diagrams supplemented by the $^3P_0$ vertex to study the annihilation reactions $p\bar{p} \to \phi X$ $(X = \pi^0, \eta, \rho^0, \omega)$ with the three types of proton wave functions. Excellent agreements of the model predictions in the first and third models with the experimental data are found for the branching ratios of the reactions of the $L = 0$ atomic $p\bar{p}$ state to ϕX $(X = \pi^0, \eta, \rho^0, \omega)$.

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Appendices

Appendix A

Transition amplitude of the proton wave function with $s\bar{s}$ sea quark

To describe the annihilation process of $p\bar{p} \to \phi X$ where $X = \pi^0, \eta, \rho^0, \omega$ with the proton wave function with $s\bar{s}$ sea quark we consider the shake-out of the intrinsic $s\bar{s}$ component of the proton wave function as indicated in the diagram A_1. With the operator $\mathcal{O}_{A_1}$ and the full account of the spin-flavor-color-orbital structure of the initial and final state, the transition amplitude can be written as

$$T_{if}^{s\bar{s}} = \lambda_{A_1} \langle f| \sum_{\nu,\lambda} (-1)^{\nu+\lambda} \sigma_{-\nu}^{56} \sigma_{-\lambda}^{47} 1_F^{56} 1_F^{47} 1_C^{56} 1_C^{47} I_{spatial}^{s\bar{s}} |i\rangle \tag{A.1}$$

where

$$|i\rangle = |\{\chi_{\frac{1}{2},m_{ps\bar{s}}}(uuds\bar{s}) \otimes \chi_{\frac{1}{2},m_{\bar{p}}}(\bar{u}\bar{u}\bar{d})\}_{S,S_z} \otimes (L,M)\rangle_{J,J_z}, \tag{A.2}$$

$$|f\rangle = |\{\chi_{1,m_\alpha}(\phi) \otimes \chi_{j_m,m_{3',4'}}(X)\}_{j,m_\epsilon} \otimes (\ell_f, m_f)\rangle_{J',J'_z}, \tag{A.3}$$

and the total angular momentum of the initial state $|i\rangle$ (the final state $|f\rangle$) are coupled to J with projection J_z (J' with projection J'_z). Subscripts refer to the corresponding orbital structure. The spin-flavor-color content of the clusters is denoted by $\chi(=\chi_\sigma \otimes \chi_F \otimes \chi_C)$. The 5-quark component $\chi_{\frac{1}{2},m_{ps\bar{s}}}(uuds\bar{s})$ is defined as

$$\chi_{\frac{1}{2},m_{ps\bar{s}}}(uuds\bar{s}) = |\{\chi_{j_s,m_s}(s\bar{s}) \otimes (\ell = 1, \mu)\}_{j_i,m_i} \otimes \chi_{\frac{1}{2},m_p}(uud)\rangle_{\frac{1}{2},m_{ps\bar{s}}}, \tag{A.4}$$

The spatial wave amplitude $I^{s\bar{s}}_{spatial}$ is explicitly given by

$$I^{s\bar{s}}_{spatial} = \int d^3q_1...d^3q_8 d^3q_{1'}...d^3q_{4'} \varphi_{\phi,X} \mathcal{O}^{spatial}_{A_1} \varphi_{uuds\bar{s},\bar{p}} \tag{A.5}$$

where

$$\begin{aligned}\mathcal{O}^{spatial}_{A_1} &= Y_{1\lambda}(\vec{q}_4 - \vec{q}_7)\delta^{(3)}(\vec{q}_4 + \vec{q}_7)Y_{1\nu}(\vec{q}_5 - \vec{q}_6)\delta^{(3)}(\vec{q}_5 + \vec{q}_6) \\ &\quad \delta^{(3)}(\vec{q}_1 - \vec{q}_{1'})\delta^{(3)}(\vec{q}_2 - \vec{q}_{2'})\delta^{(3)}(\vec{q}_3 - \vec{q}_{3'})\delta^{(3)}(\vec{q}_8 - \vec{q}_{4'}).\end{aligned} \tag{A.6}$$

By choosing a plane wave basis and the harmonic oscillator approximation for the relative motions and the internal spatial wave functions respectively, the spatial wave functions in $I^{s\bar{s}}_{spatial}$ are given by

$$\varphi_{uuds\bar{s},\bar{p}} = \varphi_{uuds\bar{s}}\varphi_{\bar{p}}\delta^{(3)}(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5 - \vec{k})\delta^{(3)}(\vec{q}_6 + \vec{q}_7 + \vec{q}_8 + \vec{k}), \tag{A.7}$$

$$\varphi_{\phi,X} = \varphi_\phi \varphi_X \delta^{(3)}(\vec{q} - \vec{q}_{1'} - \vec{q}_{2'})\delta^{(3)}(\vec{q} + \vec{q}_{3'} + \vec{q}_{4'}), \tag{A.8}$$

$$\begin{aligned}\varphi_{uuds\bar{s}} = N_{uuds\bar{s}}\mathrm{exp}&\left\{-\frac{R_B^2}{2}[(\frac{\vec{q}_4 - \vec{q}_5}{\sqrt{2}})^2 + (\frac{\vec{q}_4 + \vec{q}_5 - 2\vec{q}_3}{\sqrt{6}})^2]\right\} \\ &\mathrm{exp}\left\{-\frac{R^2}{8}(\vec{q}_3 + \vec{q}_4 + \vec{q}_5 - \vec{q}_1 - \vec{q}_2)^2\right\} \\ &Y_{1\mu}(\vec{q}_3 + \vec{q}_4 + \vec{q}_5 - \vec{q}_1 - \vec{q}_2) \\ &\mathrm{exp}\left\{-\frac{R_M^2}{8}(\vec{q}_1 - \vec{q}_2)^2\right\},\end{aligned} \tag{A.9}$$

$$\varphi_{\bar{p}} = N_{\bar{p}}\mathrm{exp}\left\{-\frac{R_B^2}{2}[(\frac{\vec{q}_6 - \vec{q}_7}{\sqrt{2}})^2 + (\frac{\vec{q}_6 + \vec{q}_7 - 2\vec{q}_8}{\sqrt{6}})^2]\right\}, \tag{A.10}$$

$$\varphi_\phi = N_\phi \mathrm{exp}\left\{-\frac{R_M^2}{8}(\vec{q}_{1'} - \vec{q}_{2'})^2\right\}, \tag{A.11}$$

$$\varphi_X = N_X \mathrm{exp}\left\{-\frac{R_M^2}{8}(\vec{q}_{3'} - \vec{q}_{4'})^2\right\}. \tag{A.12}$$

With $\vec{x}_2 = \vec{p}_2$, $\vec{x}_3 = \vec{p}_3$ and $\vec{x}_4 = b\vec{p}_3 + \vec{p}_4$ with $b = -1/2$, the spatial wave amplitude is reduced to

$$\begin{aligned} I^{s\bar{s}}_{spatial} = \int d^3p_2 d^3p_3 d^3p_4 N\delta^{(3)}(\vec{0})\exp(-Q_k^2\vec{k}^2 - Q_q^2\vec{q}^2 - Q_{kq}^2\vec{k}\cdot\vec{q} \\ -Q_{p_2}^2\vec{p}_2^2 - Q_{p_3}^2\vec{p}_3^2 - Q_{p_4}^2\vec{p}_4^2) \\ Y_{1,\lambda}(2(-\alpha_3\vec{k} - \alpha_4\vec{k} + \vec{k} - \vec{q} - \frac{\vec{p}_3}{2} - \vec{p}_4 - \vec{q}\beta_3 - \vec{q}\beta_4)) \\ Y_{1,\mu}(\vec{k} - 2\vec{q})Y_{1,\nu}(2(-\frac{\vec{p}_3}{2} + \vec{p}_4 + \vec{k}\alpha_4 + \vec{q}\beta_4)) \end{aligned} \tag{A.13}$$

where

$$\begin{aligned} Q_k^2 &= \frac{4R_M^2R_B^2 + 9R^2R_B^2 + 3R_M^2R^2}{24\,(R_M^2 + 3R_B^2)}, \\ Q_q^2 &= \frac{12R_B^4 + 5R_M^2R_B^2 + 36R^2R_B^2 + 12R_M^2R^2}{24\,(R_M^2 + 3R_B^2)}, \\ Q_{kq}^2 &= -\frac{R_M^2R_B^2 + 9R^2R_B^2 + 3R_M^2R^2}{6\,(R_M^2 + 3R_B^2)}, \\ Q_{p_2}^2 &= R_M^2, \\ Q_{p_3}^2 &= \frac{1}{2}\left(R_M^2 + 3R_B^2\right), \\ Q_{p_4}^2 &= 2R_B^2. \end{aligned} \tag{A.14}$$

To described the annihilation process at rest, the partial wave amplitude can be obtained by projecting the transition amplitude onto the partial wave corresponding to S to P transition. So the spatial wave amplitude for $L = 0$ to $\ell_f = 1$ transition is given by

$$I_{spatial,L=0,\ell_f=1} = \int d\Omega_k d\Omega_q \mathcal{Y}^*_{0,0}(\hat{k})\mathcal{Y}^*_{1,m_f}(\hat{q}) I_{spatial}\,. \tag{A.15}$$

The spatial wave amplitude corresponding to leading order in the external momenta q is given by

$$I^{s\bar{s}}_{spatial,0,1} = qF^{s\bar{s}}_{0,1} f^{s\bar{s}}_{0,1}(\nu,\lambda,\mu,m_f)\exp\left\{-Q_q^2q^2 - Q_k^2k^2\right\} \tag{A.16}$$

The geometrical constant $F_{0,1}^{s\bar{s}}$ and the spin-angular momentum function $f_{0,1}^{s\bar{s}}(\nu,\lambda,\mu,m_f)$ are given by

$$F_{0,1}^{s\bar{s}} = 2N\pi^2\left(\frac{1}{Q_{p_2}^2}\right)^{3/2}\left(\frac{3\sqrt{\pi}}{\left(Q_{p_4}^2\right)^{5/2}} - \frac{3\sqrt{\pi}}{4\left(Q_{p_3}^2\right)^{5/2}}\right)\delta^{(3)}(\vec{0}),$$
$$f_{0,1}^{s\bar{s}}(\nu,\lambda,\mu,m_f) = (-1)^{\nu}\delta_{\nu,-\lambda}\delta_{\mu,m_f}. \tag{A.17}$$

With the spatial wave amplitude $I_{spatial}^{s\bar{s}}$ the transition amplitude $T_{if}^{s\bar{s}}$ has the form as eq.(4.14) with the spin-flavor weight:

$$\langle f|O_{A_1}|i\rangle = \langle f|\sum_{\nu,\lambda}(-1)^{\nu+\lambda}\sigma_{-\nu}^{56}\sigma_{-\lambda}^{47}1_F^{56}1_F^{47}1_C^{56}1_C^{47}(-1)^{\nu}\delta_{\nu,-\lambda}\delta_{\mu,m_f}|i\rangle. \tag{A.18}$$

To evaluate the spin-color-flavor wight $\langle f|O_{A_1}|i\rangle$, for S to P transition, firstly we decouple the couple spins and angular momentums form the spin of quark clusters:

$$\begin{aligned}\langle f|O_{A_1}|i\rangle = \sum_{m_\epsilon,m_f}\sum_{m_\alpha,m_{3,8}}\sum_{m_{ps\bar{s}},m_{\bar{p}}}\sum_{m_i,m_p}\sum_{m_s,\mu}\sum_{\nu,\lambda}&\langle j,1,m_\epsilon,m_f|J',J_z'\rangle\\ \langle 1,j_m,m_\alpha,m_{3,8}|j,m_\epsilon\rangle&\langle\frac{1}{2},\frac{1}{2},m_{ps\bar{s}},m_{\bar{p}}|J,J_z\rangle\\ \langle j_i,\frac{1}{2},m_i,m_p|\frac{1}{2},m_{ps\bar{s}}\rangle&\langle j_s,1,m_s,\mu|j_i,m_i\rangle\\ (-1)^{\nu+\lambda}(-1)^{\nu}\delta_{\nu,-\lambda}\delta_{\mu,m_f}&\langle SCF\rangle^{s\bar{s}}\end{aligned} \tag{A.19}$$

with the matrix element

$$\begin{aligned}\langle SCF\rangle^{s\bar{s}} = &\langle\chi_{1,m_\alpha}(\phi)\chi_{j_m,m_{3',4'}}(X)|\\ &\sigma_{-\nu}^{56}\sigma_{-\lambda}^{47}1_F^{56}1_F^{47}1_C^{56}1_C^{47}|\chi_{j_s,m_s}(s\bar{s})\chi_{\frac{1}{2},m_p}(uud)\chi_{\frac{1}{2},m_{\bar{p}}}(\bar{u}\bar{u}\bar{d})\rangle.\end{aligned} \tag{A.20}$$

For color state wave function of meson and baryon that have to be singlet, the corresponding color state wave function for each $2q$ and $3q$ is given by

$$\chi_C(q\bar{q}) = \frac{1}{\sqrt{3}}(\mathrm{R\bar{R}} + \mathrm{G\bar{G}} + \mathrm{B\bar{B}}), \tag{A.21}$$

$$\chi_C(qqq) = \frac{1}{\sqrt{6}}(\mathrm{RGB} - \mathrm{RBG} - \mathrm{GRB} + \mathrm{GBR} - \mathrm{BGR} + \mathrm{BRG}). \tag{A.22}$$

Transition	$\langle f\|O_{A_1}\|i\rangle$
$^{11}S_0 \to \omega\phi$	$\frac{5}{3\sqrt{3}}$
$^{33}S_1 \to \pi^0\phi$	$\frac{5}{9}$
$^{31}S_0 \to \rho^0\phi$	$\frac{13}{9\sqrt{3}}$
$^{13}S_1 \to \eta\phi$	$\frac{1}{3}$

Table A.1: Spin-flavor matrix elements $\langle f|O_{A_1}|i\rangle$ for the transition $p\bar{p}(L=0) \to \phi X(\ell_f = 1)$ of the proton wave function with $s\bar{s}$ sea quark component. Here, η_{ud} refers to the nonstrange flavor combination $\eta_{ud} = (u\bar{u} + d\bar{d})/\sqrt{2}$.

With corresponding spin and isospin quantum number of each meson X

$$
\begin{aligned}
|\omega\rangle \equiv |j_m = 1, T_{38} = 0\rangle,\ |\pi^0\rangle \equiv |j_m = 0, T_{38} = 1\rangle, \\
|\rho^0\rangle \equiv |j_m = 1, T_{38} = 1\rangle,\ |\eta\rangle \equiv |j_m = 0, T_{38} = 0\rangle
\end{aligned}
\tag{A.23}
$$

the spin-flavor wight $\langle f|O_{A_1}|i\rangle$ of the different transitions are calculated as listed in table A.1

Appendix B

Transition amplitude of the proton wave function from the chiral quark model

In case of the ChQM, the annihilation process can be described by the quark line diagrams A_2 and A_3, we obtain the transition amplitude as

$$T_{if}^{ChQM} = T_{if}^{ChQM}(\mathcal{O}_{A_2}) + T_{if}^{ChQM}(\mathcal{O}_{A_3}). \tag{B.1}$$

The corresponding transition amplitude for the two quarks line diagrams are given by

$$T_{if}^{ChQM}(\mathcal{O}_{A_2}) = \lambda_{A_2}\langle f|\sum_{\nu,\lambda}(-1)^{\nu+\lambda}\sigma_{-\nu}^{56}\sigma_{-\lambda}^{47}1_F^{56}1_F^{47}1_C^{56}1_C^{47}I_{spatial,A_2}^{ChQM}|i\rangle \tag{B.2}$$

and

$$T_{if}^{ChQM}(\mathcal{O}_{A_3}) = \lambda_{A_3}\langle f|\sum_{\nu,\lambda}(-1)^{\nu+\lambda}\sigma_{-\nu}^{56}\sigma_{-\lambda}^{17}1_F^{56}1_F^{17}1_C^{56}1_C^{17}I_{spatial,A_3}^{ChQM}|i\rangle. \tag{B.3}$$

The initial stats $|i\rangle$ and final state $|f\rangle$ have the form as defined by eq.(A.2) and eq.(A.3), but the 5-quark component in this case is given by

$$\chi_{\frac{1}{2},m_{KY}}(uuds\bar{s}) = g_8 \sum_{i=1}^{3} b_i |\{\chi^i_{j_s,m_s}(q\bar{s}) \otimes (\ell = 1, \mu)\}_{j_i,m_i} \otimes \chi^i_{\frac{1}{2},m_Y}(qqs)\rangle_{\frac{1}{2},m_{KY}}, \tag{B.4}$$

where $i = 1, 2, 3$ represent the kaon-hyperon cluster $K^+\Sigma^0$, $K^0\Sigma^+$ and $K^+\Lambda^0$ respectively. The coefficients corresponding to each component are represented by b_i:

$$b_1 = 2\alpha - 1,\ b_2 = \frac{2\alpha + 1}{\sqrt{3}},\ b_3 = \sqrt{2}(2\alpha - 1). \tag{B.5}$$

The spatial wave amplitude corresponding to leading order in the external momenta q take the form

$$I^{ChQM}_{spatial,0,1,A_2} = qF^{A_2}_{0,1} f^{A_2}_{0,1}(\nu, \lambda, \mu, m_f) \mathrm{exp}\left\{-Q^2_q q^2 - Q^2_k k^2\right\}, \tag{B.6}$$

with the geometrical constant and the spin-angular momentum function are given by

$$\begin{aligned} F^{A_2}_{0,1} &= -6g_8 N\pi^4 (a_3 - 1)\, a_4 \left(\frac{1}{Q^2_{3_3}}\right)^{3/2} \sqrt{\frac{1}{Q^{10}_{p_1}}} \left(\frac{1}{Q^2_{p_4}}\right)^{3/2} (\beta_1 + \beta_4 + 1)\, \delta(\vec{0}), \\ f^{A_2}_{0,1}(\nu, \lambda, \mu, m_f) &= (-1)^\mu \delta_{\lambda,m_f} \delta_{\mu,-\nu} + (-1)^\lambda \delta_{\lambda,-\nu} \delta_{\mu,m_f} + (-1)^\mu \delta_{\mu,-\lambda} \delta_{\nu,m_f}. \end{aligned} \tag{B.7}$$

Substitute the obtained spatial wave amplitude (eq.(B.6)) into the transition amplitude eq.(B.2), we obtain

$$T^{ChQM}_{if}(\mathcal{O}_{A_2}) = \lambda_{A_2} q F^{A_2}_{0,1} \mathrm{exp}\left\{-Q^2_q q^2 - Q^2_k k^2\right\} \langle f|O_{A_2}|i\rangle \tag{B.8}$$

where

$$\langle f|O_{A_2}|i\rangle = \langle f| \sum_{\nu,\lambda} (-1)^{\nu+\lambda} \sigma^{56}_{-\nu} \sigma^{47}_{-\lambda} 1^{56}_F 1^{47}_F 1^{56}_C 1^{47}_C f^{A_2}_{0,1} |i\rangle. \tag{B.9}$$

The spin-orbital coupling in $\langle f|O_{A_2}|i\rangle$ can be decoupled to be

$$\begin{aligned}\langle f|O_{A_2}|i\rangle = \sum_{m_\epsilon,m_f}\sum_{m_\alpha,m_{1,8}}\sum_{m_\epsilon,m_f}\sum_{m_{KY},m_{\bar{p}}}\sum_{m_i,m_Y}\sum_{m_s,\mu}\sum_{\nu,\lambda}(-1)^{\nu+\lambda} \\ \langle j,1,m_\epsilon,m_f|J',J_z'\rangle\langle 1,j_m,m_\alpha,m_{1,8}|j,m_\epsilon\rangle \\ \langle\frac{1}{2},\frac{1}{2},m_{KY},m_{\bar{p}}|J,J_z\rangle\langle j_i,\frac{1}{2},m_i,m_Y|\frac{1}{2},m_{KY}\rangle \\ \langle j_s,1,m_s,\mu|j_i,m_i\rangle f_{0,1}^{A_2}\langle SCF\rangle_{A_2}^{ChQM}\end{aligned} \tag{B.10}$$

where $\langle SCF\rangle_{A_2}^{ChQM}$ is given by

$$\begin{aligned}\langle SCF\rangle_{A_2}^{ChQM} &= \sum_{i=1}^{3} b_i\langle\chi_{1,m_\alpha}(\phi)\chi_{j_m,m_{1,8}}(X)| \\ &\sigma_{-\nu}^{56}\sigma_{-\lambda}^{47}1_F^{56}1_F^{47}1_C^{56}1_C^{47}|\chi_{j_s,m_s}^i(q\bar{s})\chi_{\frac{1}{2},m_Y}^i(qqs)\chi_{\frac{1}{2},m_{\bar{p}}}(\bar{u}\bar{u}\bar{d})\rangle.\end{aligned} \tag{B.11}$$

The spin-color-flavor wave functions in above equation (χ) also take the form as eq.(**??**) with the spin-flavor wave function of the kaons $\chi_{j_s,m_s}^i(q\bar{s})$ and the hyperons $\chi_{\frac{1}{2},m_Y}^i(qqs)$ are given by

$$\chi_{0,0}^i(q\bar{s}) = \chi_F^{i=1,3(2)}(q\bar{s})\otimes\chi_\sigma(0,0) = |K^{+(0)}\rangle\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow), \tag{B.12}$$

$$\chi_{\frac{1}{2},m_Y}^i(qqs) = \frac{1}{\sqrt{2}}\{|\chi^i(qqs)\rangle_+|\frac{1}{2},m_Y\rangle_+ + |\chi^i(qqs)\rangle_-|\frac{1}{2},m_Y\rangle_-\}. \tag{B.13}$$

By using the color wave function (eq.(A.21) and eq.(A.22)) and the two-body matrix elements with the quark labeling as defined in the spatial part wave functions, the spin-color-flavor wight $\langle f|O_{A_2}|i\rangle$ are calculated as listed in Table B.1.

For the transition amplitude of the quark line diagram A_3 given by eq.(B.3), the corresponding spatial wave amplitude for S to P, in the low-momentum approximation, of this diagram is given by

$$I_{spatial,0,1,A_3}^{ChQM} = qF_{0,1}^{A_3}f_{0,1}^{A_3}(\nu,\lambda,\mu,m_f)\text{exp}\left\{-Q_q^2q^2-Q_k^2k^2\right\}. \tag{B.14}$$

The corresponding geometrical constant coupled to the spin-angular momentum:

$$F_{0,1}^{A_3}f_{0,1}^{A_3}(\nu,\lambda,\mu,m_f) = \Omega_{1,A_3}f_{1,A_3}+\Omega_{2,A_3}f_{2,A_3}+\Omega_{3,A_3}f_{3,A_3}, \tag{B.15}$$

Transition	$\langle f\vert O_{A_2}\vert i\rangle$
${}^{11}S_0 \to \omega\phi$	$\frac{5\left(-\sqrt{2}b_1+2b_2+\sqrt{6}b_3\right)}{54\sqrt{6}}$
${}^{33}S_1 \to \pi^0\phi$	$\frac{5}{486}\left(b_1+\sqrt{2}b_2+3\sqrt{3}b_3\right)$
${}^{31}S_0 \to \rho^0\phi$	$\frac{5\left(-\sqrt{2}b_1-2b_2+\sqrt{6}b_3\right)}{54\sqrt{6}}$
${}^{13}S_0 \to \eta\phi$	$\frac{5}{486}\left(b_1-\sqrt{2}b_2+3\sqrt{3}b_3\right)$

Table B.1: The spin-color-flavor wight $\langle f|O_{A_2}|i\rangle$ corresponding to the transition $p\bar{p}(L=0) \to \phi X(\ell_f=1)$ with the $5q$ component from chiral quark model.

where

$$\Omega_{1,A_3} = 6g_8N\pi^4\left(a_4+1\right)\sqrt{\frac{1}{Q_{p_1}^{10}}}\left(\frac{1}{Q_{p_3}^2}\right)^{3/2}\left(\frac{1}{Q_{p_4}^2}\right)^{3/2}\left(\beta_1-\beta_3+1\right)\delta(\vec{0}), \quad \text{(B.16)}$$

$$f_{1,A_3} = (-1)^{\lambda}\delta_{\lambda,-\nu}\delta_{\mu,m_f}, \quad \text{(B.17)}$$

$$\Omega_{2,A_3} = -6g_8N\pi^4\left(a_3-1\right)\left(a_4+1\right)\sqrt{\frac{1}{Q_{p_1}^{10}}}\left(\frac{1}{Q_{p_3}^2}\right)^{3/2}\left(\frac{1}{Q_{p_4}^2}\right)^{3/2}\delta(\vec{0}) \quad \text{(B.18)}$$

$$f_{2,A_3} = (-1)^{\mu}\delta_{\lambda,m_f}\delta_{\mu,-\nu}, \quad \text{(B.19)}$$

$$\Omega_{3,A_3} = -6g_8N\pi^4\left(a_3-1\right)\sqrt{\frac{1}{Q_{p_1}^{10}}}\left(\frac{1}{Q_{p_3}^2}\right)^{3/2}\left(\frac{1}{Q_{p_4}^2}\right)^{3/2}\left(\beta_1+\beta_4+1\right)\delta(\vec{0}), \quad \text{(B.20)}$$

and

$$f_{3,A_3} = (-1)^{\mu}\delta_{\mu,-\lambda}\delta_{\nu,m_f}. \quad \text{(B.21)}$$

Using the partial wave amplitude eq.(B.14) the transition amplitude $T_{if}^{ChQM}(\mathcal{O}_{A_3})$ can be written as

$$T_{if}^{ChQM}(\mathcal{O}_{A_3}) = \sum_{r=1}^{3}\lambda_{A_3}q\Omega_{r,A_3}\exp\left\{-Q_q^2q^2-Q_k^2k^2\right\}\langle f|O_{A_3}|i\rangle_r, \quad \text{(B.22)}$$

Transition	$\langle f\|O_{A_3}\|i\rangle_1$	$\langle f\|O_{A_3}\|i\rangle_2$	$\langle f\|O_{A_3}\|i\rangle_3$
${}^{11}S_0\rightarrow \omega\phi$	$-\frac{3\sqrt{2}b_1-6b_2+3\sqrt{6}b_3}{162\sqrt{6}}$	$-\frac{7\sqrt{2}b_1-14b_2-3\sqrt{6}b_3}{162\sqrt{6}}$	0
${}^{33}S_1\rightarrow \pi^0\phi$	$-\frac{3\sqrt{6}b_1-4\sqrt{3}b_2+9\sqrt{2}b_3}{486\sqrt{6}}$	$-\frac{3\sqrt{6}b_1+4\sqrt{3}b_2+9\sqrt{2}b_3}{486\sqrt{6}}$	$-\frac{9\sqrt{6}b_1-3\sqrt{2}b_3}{486\sqrt{6}}$
${}^{31}S_0\rightarrow \rho^0\phi$	$-\frac{2\sqrt{6}b_3-6b_2}{162\sqrt{6}}$	$-\frac{-6\sqrt{2}b_1-14b_2+2\sqrt{6}b_3}{162\sqrt{6}}$	$-\frac{\sqrt{6}b_3-9\sqrt{2}b_1}{162\sqrt{6}}$
${}^{13}S_1\rightarrow \eta_{ud}\phi$	$-\frac{\sqrt{2}b_1-2b_2}{243\sqrt{2}}$	$-\frac{\sqrt{2}b_1-2b_2}{243\sqrt{2}}$	$\frac{\sqrt{2}\left(\sqrt{2}b_1-2b_2\right)}{243}$

Table B.2: The spin-color-flavor wight $\langle f|O_{A_3}|i\rangle_r$ corresponding to the transition $p\bar{p}(L=0)\rightarrow \phi X(\ell_f=1)$ with the $5q$ component from chiral quark model.

with the spin-color-flavor wight

$$\langle f|O_{A_3}|i\rangle_r = \langle f|\sum_{\nu,\lambda}(-1)^{\nu+\lambda}\sigma^{56}_{-\nu}\sigma^{17}_{-\lambda}1^{56}_F 1^{17}_F 1^{56}_C 1^{17}_C f_{r,A_3}|i\rangle. \tag{B.23}$$

As the procedure we have done in the first two diagrams, $\langle f|O_{A_3}|i\rangle$ can be decoupled to be

$$\begin{aligned}\langle f|O_{A_3}|i\rangle_r = &\sum_{m_\epsilon,m_f}\sum_{m_\alpha,m_{4,8}}\sum_{m_\epsilon,m_f}\sum_{m_{KY},m_{\bar{p}}}\sum_{m_i,m_Y}\sum_{m_s,\mu}\sum_{\nu,\lambda}(-1)^{\nu+\lambda} \\ &\langle j,1,m_\epsilon,m_f|J',J'_z\rangle\langle 1,j_m,m_\alpha,m_{4,8}|j,m_\epsilon\rangle \\ &\langle\frac{1}{2},\frac{1}{2},m_{KY},m_{\bar{p}}|J,J_z\rangle\langle j_i,\frac{1}{2},m_i,m_Y|\frac{1}{2},m_{KY}\rangle \\ &\langle j_s,1,m_s,\mu|j_i,m_i\rangle f_{r,A_3}\langle SCF\rangle^{ChQM}_{A_3}\end{aligned} \tag{B.24}$$

with $\langle SCF\rangle^{ChQM}_{A_3}$ is given by

$$\begin{aligned}\langle SCF\rangle^{ChQM}_{A_2} = &\sum_{i=1}^{3} b_i\langle\chi_{1,m_\alpha}(\phi)\chi_{j_m,m_{4,8}}(X)| \\ &\sigma^{56}_{-\nu}\sigma^{17}_{-\lambda}1^{56}_F 1^{17}_F 1^{56}_C 1^{17}_C|\chi^i_{j_s,m_s}(q\bar{s})\chi^i_{\frac{1}{2},m_Y}(qqs)\chi_{\frac{1}{2},m_{\bar{p}}}(\bar{u}\bar{u}\bar{d})\rangle.\end{aligned} \tag{B.25}$$

Similarly, with the quark labeling as used in the spatial wave functions, we have $\langle f|O_{A_3}|i\rangle_r$ as shown in Table B.2

In order to combine the two transition amplitudes, we choose the radial parameters for the baryons and mesons: $R_B = 3.1\ GeV^{-1}$, $R_M = 4.1\ GeV^{-1}$ [17] and the size parameter between quark cluster $R = 4.1\ GeV^{-1}$. According to the vertex

Transition	$\langle f\vert O_{ChQM}\vert i\rangle \times 10^{-2}$
$^{11}S_0 \rightarrow \omega\phi$	-9.7
$^{33}S_1 \rightarrow \pi^0\phi$	3.1
$^{31}S_0 \rightarrow \rho^0\phi$	4.0
$^{13}S_0 \rightarrow \eta\phi$	1.3

Table B.3: The total spin-color-flavor wight $\langle f|O_{ChQM}|i\rangle$ corresponding to the transition $p\bar{p}(L = 0) \rightarrow \phi X(\ell_f = 1)$ with the $5q$ component from chiral quark model.

in the two quark line diagram, the effective strength can take same value, that is $\lambda_{A_2} = \lambda_{A_3} = \lambda_{ChQM}$. We have the total transition amplitude eq.(B.1) become

$$T_{if}^{ChQM} = \lambda_{ChQM} F_{0,1}^{ChQM} q \text{ exp} \left\{-Z_q^2 q^2 - Z_k^2 k^2\right\} \langle f|O_{ChQM}|i\rangle, \tag{B.26}$$

where $F_{0,1}^{ChQM} = \Omega^{A_2} = 4.9 \times 10^{-4}\ GeV^{-11}$, $Z_q \simeq 2.3\ GeV^{-1}$ and $Z_k \simeq 1.3\ GeV^{-1}$. The total spin-color-flavor wight is given by

$$\langle f|O_{ChQM}|i\rangle \simeq \langle f|O_{A_2}|i\rangle + 2(\langle f|O_{A_3}|i\rangle_1 - \langle f|O_{A_3}|i\rangle_2 + \langle f|O_{A_3}|i\rangle_3), \tag{B.27}$$

as listed in Table B.3.

Appendix C

Transition amplitude of the proton wave function with the figure of penta quark

The transition amplitude with $5q$ component of penta quark has the form as eq.(A.1) but the 5-quark component $|uuds\bar{s}\rangle$ is defined as

$$\chi_{\frac{1}{2},m_{ps\bar{s}}}(uuds\bar{s}) = |\{\chi_{1/2,m_{\bar{s}}}(\bar{s}) \otimes (\ell = 1, \mu)\}_{j_i,m_i} \otimes \chi_{s,s_z}(uuds)\rangle_{\frac{1}{2},m_{ps\bar{s}}}. \tag{C.1}$$

According to its configuration, which is constructed by coupling an $\bar{s}$ quark momentum $\vec{q}_1$ to the 4-quark $uuds$ momentum $\vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5$ with orbital angular momentum $(1, \mu)$, the 5-quark component spatial wave function is given by

$$\begin{aligned}\varphi_{uuds\bar{s}}(\vec{q}_1, .., \vec{q}_5) = N_{uuds\bar{s}}\exp\{-\frac{R_B^2}{2}[\frac{1}{2}(\vec{q}_2 - \vec{q}_3)^2 + \frac{1}{6}(\vec{q}_2 + \vec{q}_3 - 2\vec{q}_4)^2 \\ + \frac{1}{12}(\vec{q}_2 + \vec{q}_3 + \vec{q}_4 - 3\vec{q}_5)^2]\}Y_{1\mu}(\frac{\vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5 - 4\vec{q}_1}{\sqrt{20}}) \\ \exp\left\{-\frac{R_B^2}{2}(\frac{\vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5 - 4\vec{q}_1}{\sqrt{20}})^2\right\},\end{aligned} \tag{C.2}$$

With the spatial operator $\mathcal{O}_{A_1}^{spatail}$ and the spatial wave function of the antiproton and the mesons as defined in appendix A, and integrating with the delta functions as done in the case of $s\bar{s}$-sea quark , $I_{spatial}^{uuds}$ can be reduced to the spatial wave amplitude for S to P transition in the low-momentum approximation is given by

$$I^{uuds}_{spatial,0,1} = qF_{0,1}f_{0,1}(\nu,\lambda,\mu,m_f)\mathrm{exp}\left\{-Q_q^2q^2 - Q_k^2k^2\right\}, \tag{C.3}$$

with the geometrical constant and the spin-angular momentum function are given by

$$F^{uuds}_{0,1} = \frac{3}{8}\sqrt{5}N\pi^4\left(\frac{1}{Q^2_{p_2}}\right)^{3/2}\left(\frac{\left(\frac{1}{Q^2_{p_4}}\right)^{3/2}}{\left(Q^2_{p_3}\right)^{5/2}} - \frac{4\left(\frac{1}{Q^2_{p_3}}\right)^{3/2}}{\left(Q^2_{p_4}\right)^{5/2}}\right)(\beta_2 - 1)\,\delta^{(3)}(\vec{0}),$$
$$f^{uuds}_{0,1}(\nu,\lambda,\mu,m_f) = (-1)^{\nu}\delta_{\nu,-\lambda}\delta_{\mu,m_f}. \tag{C.4}$$

Decoupling the spin-color-flavor weight $\langle f|O_{A_1}|i\rangle^{uuds}$:

$$\begin{aligned}\langle f|O_{A_1}|i\rangle^{uuds} = &\sum_{m_\epsilon,m_f}\sum_{m_\alpha,m_{3,8}}\sum_{m_{ps\bar{s}},m_{\bar{p}}}\sum_{m_i,s_z}\sum_{m_{\bar{s}},\mu}\sum_{\nu,\lambda}\langle j,1,m_\epsilon,m_f|J',J'_z\rangle\\ &\langle J_s,j_m,m_\alpha,m_{3,8}|j,m_\epsilon\rangle\langle\frac{1}{2},\frac{1}{2},m_{ps\bar{s}},m_{\bar{p}}|J,J_z\rangle\\ &\langle j_i,s,m_i,s_z|\frac{1}{2},m_{ps\bar{s}}\rangle\langle\frac{1}{2},1,m_{\bar{s}},\mu|j_i,m_i\rangle\\ &(-1)^{\nu+\lambda}(-1)^{\nu}\delta_{\nu,-\lambda}\delta_{\mu,m_f}\langle SCF\rangle^{uuds},\end{aligned} \tag{C.5}$$

so the corresponding matrix element is given by

$$\begin{aligned}\langle SCF\rangle^{uuds} = &\langle\chi_{1,m_\alpha}(\phi)\chi_{j_m,m_{3,8}}(X)|\\ &\sigma^{56}_{-\nu}\sigma^{47}_{-\lambda}1^{56}_F1^{47}_F1^{56}_C1^{47}_C|\chi_{\frac{1}{2},m_{\bar{s}}}(\bar{s})\chi_{s,s_z}(uuds)\chi_{\frac{1}{2},m_{\bar{p}}}(\bar{u}\bar{u}\bar{d})\rangle.\end{aligned} \tag{C.6}$$

The corresponding spin-flavor weights can be obtained as shown in Table C.1.

Transition	$[31][31][22]_{A_1}$	$[31][211][22]_{A_1}$
$^{11}S_0 \rightarrow \omega\phi$	$\frac{5}{36\sqrt{6}}$	$\frac{5}{36\sqrt{6}}$
$^{33}S_1 \rightarrow \pi^0\phi$	$\frac{5}{108\sqrt{2}}$	$\frac{5}{108\sqrt{2}}$
$^{31}S_0 \rightarrow \rho^0\phi$	$\frac{13}{108\sqrt{6}}$	$\frac{13}{108\sqrt{6}}$
$^{13}S_1 \rightarrow \eta_{ud}\phi$	$\frac{1}{36\sqrt{2}}$	$\frac{1}{36\sqrt{2}}$

Table C.1: Spin-flavor matrix elements $\langle f|O_{A_I}|i\rangle$ for the transition $p\bar{p}(L=0) \rightarrow \phi X(\ell_f = 1)$ which are described by quark line diagram A_I with the 5-quark component of pentaquark.

Appendix D

The construction of the pentaquark wave functions

The spin, color and flavor wave functions of the q^4 configuration of the pentaquark can be worked out in the the Yamanochi technique [32]. Here we present some detail description to illustrate how to construct the presentation matrices of the irreducible representations of S_4. For instant, there are three Young tableaux for [3,1] with three corresponding basis function:

$$\begin{aligned}
\phi_{\eta}^{[3,1]} &= \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array} = |[3,1](2111)\rangle, \\
\phi_{\lambda}^{[3,1]} &= \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1}\end{array} = |[3,1](1211)\rangle, \\
\phi_{\rho}^{[3,1]} &= \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array} = |[3,1](1121)\rangle,
\end{aligned} \tag{D.1}$$

where $\phi_r^{[\bar{\lambda}]} = |[\bar{\lambda}](r_n, r_{n-1}, .., r_2, r_1)\rangle$ represents the Yamanochi basis or standard basis with r_i stands for the row from which a box is removed. Thus, there are three projection operators for irreducible representations [3,1]:

$$\begin{aligned}
W_{(2111)}^{[3,1]} &= \frac{3}{4!} \sum_i \langle[3,1](2111)|P_i|[3,1](2111)\rangle P_i, \\
W_{(1211)}^{[3,1]} &= \frac{3}{4!} \sum_i \langle[3,1](1211)|P_i|[3,1](1211)\rangle P_i, \\
W_{(1121)}^{[3,1]} &= \frac{3}{4!} \sum_i \langle[3,1](1121)|P_i|[3,1](1121)\rangle P_i,
\end{aligned} \tag{D.2}$$

where P_i stand for all the permutations of S_4 and the factor 3 is the dimension of the representation [3,1]. In order to evaluate the representation matrices for the permutation P_i, in case of permutation between object $n-1$ and n , we apply the operation of the element $(n-1,n)$ on the standard basis satisfies the following:

$$\begin{aligned}
(n-1,n)|[\bar{\lambda}](r,r,r_{n-2},..,r_2,1)\rangle &= |[\bar{\lambda}](r,r,r_{n-2},..,r_2,1)\rangle, \\
(n-1,n)|[\bar{\lambda}](r,r-1,r_{n-2},..,r_2,1)\rangle &= -|[\bar{\lambda}](r,r-1,r_{n-2},..,r_2,1)\rangle,
\end{aligned} \tag{D.3}$$

when $|[\bar{\lambda}](r,r-1,r_{n-2},..,r_2,1)\rangle$ not exists, and

$$\begin{aligned}
(n-1,n)|[\bar{\lambda}](r,s,r_{n-2},..,r_2,r_1)\rangle &= \sigma_{rs}|[\bar{\lambda}](r,s,r_{n-2},..,r_2,r_1)\rangle \\
&\quad +\sqrt{1-\sigma_{rs}^2}|[\bar{\lambda}](s,r,r_{n-2},..,r_2,r_1)\rangle,
\end{aligned} \tag{D.4}$$

where

$$\sigma_{rs} = \frac{1}{(\lambda_r - r) - (\lambda_s - s)}, \tag{D.5}$$

for $|[\bar{\lambda}](r,s,r_{n-2},..,r_2,r_1)\rangle$ and $|[\bar{\lambda}](s,r,r_{n-2},..,r_2,r_1)\rangle$ all exist and $r \neq s$. While the matrices of the elements (i,n) can be obtained by using relation

$$(i,n) = (n-1,n)(i,n-1)(n-1,n). \tag{D.6}$$

For example, the matrix of the element $(3,4)$:

$$\begin{aligned}
D^{[3,1]}(34) &= \begin{pmatrix}
\langle\phi_\eta^{[3,1]}|(34)|\phi_\eta^{[3,1]}\rangle & \langle\phi_\lambda^{[3,1]}|(34)|\phi_\eta^{[3,1]}\rangle & \langle\phi_\rho^{[3,1]}|(34)|\phi_\eta^{[3,1]}\rangle \\
\langle\phi_\eta^{[3,1]}|(34)|\phi_\lambda^{[3,1]}\rangle & \langle\phi_\lambda^{[3,1]}|(34)|\phi_\lambda^{[3,1]}\rangle & \langle\phi_\rho^{[3,1]}|(34)|\phi_\lambda^{[3,1]}\rangle \\
\langle\phi_\eta^{[3,1]}|(34)|\phi_\rho^{[3,1]}\rangle & \langle\phi_\lambda^{[3,1]}|(34)|\phi_\rho^{[3,1]}\rangle & \langle\phi_\rho^{[3,1]}|(34)|\phi_\rho^{[3,1]}\rangle
\end{pmatrix} \\
&= \begin{pmatrix}
-1/3 & 2\sqrt{2}/3 & 0 \\
2\sqrt{2}/3 & 1/3 & 0 \\
0 & 0 & 1
\end{pmatrix}.
\end{aligned} \tag{D.7}$$

A cycle permutation can be resolved into a product of transpositions, we have

$$\begin{aligned} D(ijk) &= D(ik)D(ij), \\ D(ijkl) &= D(il)D(ik)D(ij). \end{aligned} \tag{D.8}$$

By applying eq.(D.6) and eq.(D.8) with the matrix of the element $(n-1,n)$, all of the permutation matrices $D^{[3,1]}$ can be obtained such as

$$\begin{aligned} D(123) = D(13)D(12) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \\ &= \begin{pmatrix} -\frac{1}{3} & -\frac{\sqrt{2}}{3} & \sqrt{\frac{2}{3}} \\ -\frac{\sqrt{2}}{3} & \frac{5}{6} & \frac{1}{2\sqrt{3}} \\ \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2} \end{pmatrix}, \end{aligned} \tag{D.9}$$

$$\begin{aligned} D(1234) &= D(14)D(13)D(12) \\ &= \begin{pmatrix} -\frac{1}{3} & -\frac{\sqrt{2}}{3} & -\sqrt{\frac{2}{3}} \\ -\frac{\sqrt{2}}{3} & \frac{5}{6} & -\frac{1}{2\sqrt{3}} \\ -\sqrt{\frac{2}{3}} & -\frac{1}{2\sqrt{3}} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \\ &= \begin{pmatrix} -\frac{1}{3} & \frac{2\sqrt{2}}{3} & 0 \\ -\frac{\sqrt{2}}{3} & -\frac{1}{6} & \frac{\sqrt{3}}{2} \\ -\sqrt{\frac{2}{3}} & \frac{1}{4\sqrt{3}} - \frac{\sqrt{3}}{4} & -\frac{1}{2} \end{pmatrix}. \end{aligned} \tag{D.10}$$

Substituting $\langle\phi_i^{[3,1]}|(34)|\phi_i^{[3,1]}\rangle$ for all permutations into eq.(D.2) we obtain the projection opertor

$$\begin{aligned} W^{[3,1]}_{(2111)} = \frac{1}{8}(P_1 &- \frac{1}{3}P_{1,4}P_{2,3} - \frac{1}{3}P_{1,3}P_{2,4} - \frac{1}{3}P_{1,2}P_{3,4} + P_{1,2} + P_{1,3} \\ &- \frac{P_{1,4}}{3} + P_{2,3} - \frac{P_{2,4}}{3} - \frac{P_{3,4}}{3} + P_{1,2,3} - \frac{1}{3}P_{1,2,4} + P_{1,3,2} \\ &- \frac{1}{3}P_{1,3,4} - \frac{1}{3}P_{1,4,2} - \frac{1}{3}P_{1,4,3} - \frac{1}{3}P_{2,3,4} - \frac{1}{3}P_{2,4,3} \\ &- \frac{1}{3}P_{1,2,3,4} - \frac{1}{3}P_{1,2,4,3} - \frac{1}{3}P_{1,3,2,4} - \frac{1}{3}P_{1,3,4,2} \\ &- \frac{1}{3}P_{1,4,2,3} - \frac{1}{3}P_{1,4,3,2}), \end{aligned} \tag{D.11}$$

$$
\begin{aligned}
W^{[3,1]}_{(1211)} = \frac{1}{8}(P_1 &- \frac{2}{3}P_{1,4}P_{2,3} - \frac{1}{6}P_{1,3}P_{2,4} + \frac{1}{3}P_{1,2}P_{3,4} + P_{1,2} - \frac{P_{1,3}}{2} \\
&+ \frac{5P_{1,4}}{6} - \frac{P_{2,3}}{2} + \frac{5P_{2,4}}{6} + \frac{P_{3,4}}{3} - \frac{1}{2}P_{1,2,3} + \frac{5}{6}P_{1,2,4} \\
&- \frac{1}{2}P_{1,3,2} - \frac{1}{6}P_{1,3,4} + \frac{5}{6}P_{1,4,2} - \frac{1}{6}P_{1,4,3} - \frac{2}{3}P_{2,3,4} \\
&- \frac{2}{3}P_{2,4,3} - \frac{1}{6}P_{1,2,3,4} - \frac{1}{6}P_{1,2,4,3} - \frac{2}{3}P_{1,3,2,4} \\
&- \frac{1}{6}P_{1,3,4,2} - \frac{2}{3}P_{1,4,2,3} - \frac{1}{6}P_{1,4,3,2}),
\end{aligned}
\tag{D.12}
$$

and

$$
\begin{aligned}
W^{[3,1]}_{(1121)} = \frac{1}{8}(P_1 &+ \frac{1}{2}P_{1,3}P_{2,4} - P_{1,2}P_{3,4} - P_{1,2} + \frac{P_{1,3}}{2} + \frac{P_{1,4}}{2} + \frac{P_{2,3}}{2} \\
&+ \frac{P_{2,4}}{2} + P_{3,4} - \frac{1}{2}P_{1,2,3} - \frac{1}{2}P_{1,2,4} - \frac{1}{2}P_{1,3,2} + \frac{1}{2}P_{1,3,4} \\
&- \frac{1}{2}P_{1,4,2} + \frac{1}{2}P_{1,4,3} - \frac{1}{2}P_{1,2,3,4} - \frac{1}{2}P_{1,2,4,3} \\
&- \frac{1}{2}P_{1,3,4,2} - \frac{1}{2}P_{1,4,3,2}),
\end{aligned}
\tag{D.13}
$$

where P denotes the permutation on the objects that labeled by the subscript. Similarly, for the irreducible representation [2,2] and [2,1,1] that corresponding to Young tableaux

$$
\begin{aligned}
\phi^{[2,2]}_{\lambda} &= \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array} = |[2,2](2211)\rangle, \\
\phi^{[2,2]}_{\rho} &= \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array} = |[2,2](2121)\rangle,
\end{aligned}
\tag{D.14}
$$

$$
\begin{aligned}
\phi^{[2,1,1]}_{\eta} &= \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1}\end{array} = |[2,1,1](2111)\rangle, \\
\phi^{[2,1,1]}_{\lambda} &= \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array} = |[2,1,1](1211)\rangle, \\
\phi^{[2,1,1]}_{\rho} &= \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array} = |[2,1,1](1121)\rangle,
\end{aligned}
\tag{D.15}
$$

we have

$$\begin{aligned} W^{[2,2]}_{(2211)} = \frac{1}{24}(&2P_1 + 2P_{1,4}P_{2,3} + 2P_{1,3}P_{2,4} + 2P_{1,2}P_{3,4} + 2P_{1,2} - P_{1,3} \\ &-P_{1,4} - P_{2,3} - P_{2,4} + 2P_{3,4} - P_{1,2,3} - P_{1,2,4} - P_{1,3,2} \\ &-P_{1,3,4} - P_{1,4,2} - P_{1,4,3} - P_{2,3,4} - P_{2,4,3} - P_{1,2,3,4} \\ &-P_{1,2,4,3} + 2P_{1,3,2,4} - P_{1,3,4,2} + 2P_{1,4,2,3} - P_{1,4,3,2}), \end{aligned} \tag{D.16}$$

$$\begin{aligned} W^{[2,2]}_{(2121)} = \frac{1}{24}(&2P_1 + 2P_{1,4}P_{2,3} + 2P_{1,3}P_{2,4} + 2P_{1,2}P_{3,4} - 2P_{1,2} + P_{1,3} \\ &+P_{1,4} + P_{2,3} + P_{2,4} - 2P_{3,4} - P_{1,2,3} - P_{1,2,4} - P_{1,3,2} \\ &-P_{1,3,4} - P_{1,4,2} - P_{1,4,3} - P_{2,3,4} - P_{2,4,3} + P_{1,2,3,4} \\ &+P_{1,2,4,3} - 2P_{1,3,2,4} + P_{1,3,4,2} - 2P_{1,4,2,3} + P_{1,4,3,2}), \end{aligned} \tag{D.17}$$

$$\begin{aligned} W^{[2,1,1]}_{(2111)} = \frac{1}{8}(&P_1 - \frac{1}{3}P_{1,4}P_{2,3} - \frac{1}{3}P_{1,3}P_{2,4} - \frac{1}{3}P_{1,2}P_{3,4} - P_{1,2} - P_{1,3} \\ &+\frac{P_{1,4}}{3} - P_{2,3} + \frac{P_{2,4}}{3} + \frac{P_{3,4}}{3} + P_{1,2,3} - \frac{1}{3}P_{1,2,4} + P_{1,3,2} \\ &-\frac{1}{3}P_{1,3,4} - \frac{1}{3}P_{1,4,2} - \frac{1}{3}P_{1,4,3} - \frac{1}{3}P_{2,3,4} - \frac{1}{3}P_{2,4,3} \\ &+\frac{1}{3}P_{1,2,3,4} + \frac{1}{3}P_{1,2,4,3} + \frac{1}{3}P_{1,3,2,4} \\ &+\frac{1}{3}P_{1,3,4,2} + \frac{1}{3}P_{1,4,2,3} + \frac{1}{3}P_{1,4,3,2}), \end{aligned} \tag{D.18}$$

$$\begin{aligned} W^{[2,1,1]}_{(1211)} = \frac{1}{8}(&P_1 + \frac{1}{2}P_{1,3}P_{2,4} - P_{1,2}P_{3,4} + P_{1,2} - \frac{P_{1,3}}{2} - \frac{P_{1,4}}{2} - \frac{P_{2,3}}{2} \\ &-\frac{P_{2,4}}{2} - P_{3,4} - \frac{1}{2}P_{1,2,3} - \frac{1}{2}P_{1,2,4} - \frac{1}{2}P_{1,3,2} + \frac{1}{2}P_{1,3,4} \\ &-\frac{1}{2}P_{1,4,2} + \frac{1}{2}P_{1,4,3} + \frac{1}{2}P_{1,2,3,4} + \frac{1}{2}P_{1,2,4,3} \\ &+\frac{1}{2}P_{1,3,4,2} + \frac{1}{2}P_{1,4,3,2}), \end{aligned} \tag{D.19}$$

$$
\begin{aligned}
W^{[2,1,1]}_{(1121)} = \frac{1}{8}(P_1 - \frac{2}{3}P_{1,4}P_{2,3} - \frac{1}{6}P_{1,3}P_{2,4} + \frac{1}{3}P_{1,2}P_{3,4} - P_{1,2} + \frac{P_{1,3}}{2} \\
-\frac{5P_{1,4}}{6} + \frac{P_{2,3}}{2} - \frac{5P_{2,4}}{6} - \frac{P_{3,4}}{3} - \frac{1}{2}P_{1,2,3} + \frac{5}{6}P_{1,2,4} \\
-\frac{1}{2}P_{1,3,2} - \frac{1}{6}P_{1,3,4} + \frac{5}{6}P_{1,4,2} - \frac{1}{6}P_{1,4,3} \\
+\frac{1}{6}P_{1,2,4,3} + \frac{2}{3}P_{1,3,2,4} + \frac{1}{6}P_{1,3,4,2} \\
-\frac{2}{3}P_{2,3,4} - \frac{2}{3}P_{2,4,3} + \frac{1}{6}P_{1,2,3,4} \\
+\frac{2}{3}P_{1,4,2,3} + \frac{1}{6}P_{1,4,3,2}).
\end{aligned} \tag{D.20}
$$

By acting the obtained projection operators onto the four quark state $uuds$, the flavor wave function are obtained. The two corresponding spin wave functions $\chi_{[22]_{S_\lambda}}$ and $\chi_{[22]_{S_\rho}}$ can be constructed by the substitutions $u \leftrightarrow\uparrow$ and d, $s \leftrightarrow\downarrow$ in the flavor wave function, $\chi_{[22]_{F_\lambda}}$ and $\chi_{[22]_{F_\rho}}$, with an additional $1/\sqrt{2}$ in the normalization factor. Analogy, the color symmetry [211] is constructed by replacing $u \leftrightarrow R$, $d \leftrightarrow G$ and $s \leftrightarrow B$ in the flavor wave function $\chi_{[211]_F}$.

Finally in order to complete the q^4 wave function we have to couple the [211] color with [31] the spin-flavor wave function to be [31] spin-color-flavor wave function for $uuds$ configuration. We write down the all possible multiplication of $\chi_{[211]_{C_i}}$ with $\chi_{[31]_{FS_j}}$ in form of linear combination

$$
\chi_{[31]_{CSF}} = \sum_{i,j=\lambda,\rho,\eta} a_{i,j}\chi_{[211]_{C_i}}\chi_{[31]_{FS_j}}, \tag{D.21}
$$

where $a_{i,j}$ is the coefficient for making the combination correspond to [31] symmetry of spin-color-flavor wave function. There are only three components that give the four quark configuration is antisymmetric,

$$
\chi_{[31]_{CSF}} = a_{\lambda,\rho}\chi_{[211]_{C_\lambda}}\chi_{[31]_{FS_\rho}} + a_{\rho,\lambda}\chi_{[211]_{C_\rho}}\chi_{[31]_{FS_\lambda}} + a_{\eta,\eta}\chi_{[211]_{C_\eta}}\chi_{[31]_{FS_\eta}}. \tag{D.22}
$$

Transform $\chi_{[31]_{CSF}}$ with corresponding permutation $D(34)$, we have

$$
\begin{aligned}
(34)\chi_{[31]_{CSF}} = -a_{\lambda,\rho}\chi_{[211]_{C_\lambda}}\chi_{[31]_{FS_\rho}} \\
+a_{\rho,\lambda}\big(-\frac{1}{3}\chi_{[211]_{C_\rho}} + \frac{2\sqrt{2}}{3}\chi_{[211]_{C_\eta}}\big)\big(\frac{1}{3}\chi_{[31]_{FS_\lambda}} + \frac{2\sqrt{2}}{3}\chi_{[31]_{FS_\eta}}\big) \\
+a_{\eta,\eta}\big(\frac{2\sqrt{2}}{3}\chi_{[211]_{C_\rho}} + \frac{1}{3}\chi_{[211]_{C_\eta}}\big)\big(\frac{2\sqrt{2}}{3}\chi_{[31]_{FS_\lambda}} - \frac{1}{3}\chi_{[31]_{FS_\eta}}\big).
\end{aligned} \tag{D.23}
$$

Antisymmetric wave function require that

$$(34)\chi_{[31]_{CSF}} = -\chi_{[31]_{CSF}}, \tag{D.24}$$

this give a condition $a_{\rho,\lambda} = -a_{\eta,\eta}$ while $a_{\lambda,\rho}$ is arbitrary number. By choosing $a_{\lambda,\rho} = 1$ and normalizing $\chi_{[31]_{CSF}}$ with the obtained condition we obtain

$$\chi_{[31]_{CSF}} = \frac{1}{\sqrt{3}}\big(\chi_{[211]_{C_\lambda}}\chi_{[31]_{FS_\rho}} - \chi_{[211]_{C_\rho}}\chi_{[31]_{FS_\lambda}} + \chi_{[211]_{C_\eta}}\chi_{[31]_{FS_\eta}}\big). \tag{D.25}$$

For the spin-flavor $\chi_{[31]_{FS}}$ such as $|[31]_{FS_\eta}[31]_F[22]_S\rangle$ corresponding to Young tableau

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array} \tag{D.26}$$

There are only two product of the spin $\chi_{[22]_S}$ and flavor $\chi_{[31]_F}$ wave function, that are consistent with characteristic permutation of $\chi_{[31]_{FS}}$

$$\begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array} \otimes \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array}, \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array} \otimes \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array} \tag{D.27}$$

Hence, the spin-flavor $|[31]_{FS_\eta}[31]_F[22]_S\rangle$ can be assumed as

$$|[31]_{FS_\eta}[31]_F[22]_S\rangle = a_{\rho,\rho}\chi_{[31]_{F_\rho}}\chi_{[22]_{S_\rho}} + a_{\lambda,\lambda}\chi_{[31]_{F_\lambda}}\chi_{[22]_{S_\lambda}} \tag{D.28}$$

The coefficient $a_{\rho,\rho}$ and $a_{\lambda,\lambda}$ one can be evaluated by acting the matrix $D^{[3,1]}(123)$ (eq.(D.9)) and

$$D^{[22]}(123) = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \tag{D.29}$$

onto the corresponding wave function of eq.(D.28)

$$\begin{aligned} D^{[31]}(123)|[31]_{FS_\eta}[31]_F[22]_S\rangle = a_{\rho,\rho}D^{[31]}(123)\chi_{[31]_{F_\rho}}D^{[22]}(123)\chi_{[22]_{S_\rho}} \\ +a_{\lambda,\lambda}D^{[31]}(123)\chi_{[31]_{F_\lambda}}D^{[22]}(123)\chi_{[22]_{S_\lambda}}. \end{aligned} \tag{D.30}$$

We obtain

$$
\begin{aligned}
|[31]_{FS_\eta}[31]_F[22]_S\rangle = {} & a_{\rho,\rho}(-\frac{1}{2}\chi_{[31]_{F_\rho}} - \frac{\sqrt{3}}{2}\chi_{[31]_{F_\lambda}})(-\frac{1}{2}\chi_{[22]_{S_\rho}} - \frac{\sqrt{3}}{2}\chi_{[22]_{S_\lambda}}) \\
& + a_{\lambda,\lambda}(-\frac{1}{2}\chi_{[31]_{F_\lambda}} + \frac{\sqrt{3}}{2}\chi_{[31]_{F_\rho}})(\frac{\sqrt{3}}{2}\chi_{[22]_{S_\rho}} - \frac{1}{2}\chi_{[22]_{S_\lambda}}). \quad \text{(D.31)}
\end{aligned}
$$

Taking eq.(D.28) equal to eq.(D.31) and normalizing, give $a_{\rho,\rho} = a_{\lambda,\lambda} = 1/\sqrt{2}$ so we have the explicit form of the $|[31]_{FS_\eta}[31]_F[22]_S\rangle$.

Appendix E

Curriculum Vitae

E.1 Prof. Dr. Yupeng Yan

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Nakhon Ratchasima 30000, Thailand

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PROFESSIONAL EXPERIENCE

Professor (February 2007 - present)

School of Physics, Suranaree University of Technology, Thailand

Associate Professor (September 2002 - February 2007)

School of Physics, Suranaree University of Technology, Thailand

Assistant Professor (June 1999 - September 2002)

School of Physics, Suranaree University of Technology, Thailand

Lecturer (June 1997 - June 1999)

School of Physics, Suranaree University of Technology, Thailand

Research Officer (January 1996 - June 1997)

Department of Physics, University of the Witwatersrand, South Africa

FRD Post Doctoral Fellow (Foundation for Research Development of South Africa) (January 1995 - December 1995) Department of Physics, University of the Witwatersrand, South Africa

Lecturer (September 1987 - April 1990)

Department of Physics, Nankai University, P. R. China

DAAD Senior Visiting Researcher (Deutscher Akademischer Austauschdients) (November 2001 - Dcember 2001);

DAAD Senior Visiting Researcher (October 2000 - November 2000);

DAAD Visiting Researcher (April 1998 to June 1998,)

Institute for Theoretical Physics, Tuebingen University, Germany.

FRD Senior Research Officer (December 1999)

Department of Physics, University of the Witwatersrand, South Africa

Research Associate (July 1994 - December 1994),

Research Assistant (April 1990 - June 1994)

Institute for Theoretical Physics, Tuebingen University, Germany.

EDUCATION

Ph.D. in Physics (awarded on August 2, 1994)

Institute for Theoretical Physics, Tuebingen University, Germany.

Field of Study: Nuclear and Particle Theory

Title of Thesis: Nucleon-Antinucleon Bound States in Nonrelativistic Quark Models

Supervisor: Amand Faessler

Master of Science (awarded July 1987)

Department of Physics, Nankai University, P. R. China

Field of Study: Nuclear and Particle Theory

Title of Thesis: Vacuum Contribution to Nucleon-Nucleon Interaction

Supervisor: Guozu He

Bachelor of Science (awarded in July 1984)

Department of Physics, Nankai University, P. R. China

Field of Study: Electro-Optics

Title of Thesis: Design of Color-TV Bending Coil.

Supervisor: Shouqian Ding

AWARDS and RESEARCH GRANTS

SUT-National Research University project (2011-present) Research Center for Theoretical Physics

ThEP Fund (2009-present) Project: Study of strong interactions through exotic atoms, chiral perturbation theory and heavy ion collisions

CHE Fund (2007-2009) Project: Theoretical Physics

NRCT Fund (2006) Project: Dynamical Studies of Intermediate and High Energy Heavy Ion

SUT Fund (Oct. 2005 - Sep. 2007) Project: Study of Protonium Atoms in Sturmian Function Approach

NRCT Fund (NRCT: National Reaearch Council of Thailand) (Oct. 2001 - Sep. 2003) Project: Heavy Ion Reactions at Ultra-Relativistic Energies

RGJ Grant (RGJ: Royal Golden-Jubilee Ph.D. Project of Thailand, for more information see http://rgj.trf.or.th/eng.htm) (Oct. 2001 - Sep. 2006) Project: Low-Energy Pion-Proton Processes in Chiral Quark Models

RGJ Grant (October 2001 - September 2006) Project: Two-Pion and Two-Kaon Bound States in Chiral Quark Models

RGJ Grant (October 1998 - September 2003) Project: Baryon Weak and Electromagnetic Decays in Chiral Quark Models

RGJ Grant (October 1998 - September 2003) Project: Nucleon-Nucleon and Nucleon-Antinucleon Interactions

TRF Research Fund (Thailand Research Foundation) (Oct. 1997 - Sept. 1999) Suranaree University of Technology, Thailand

FRD Post Doctoral Fellowship (January 1995 - December 1995) (Foundation for Research Development of South Africa) University of the Witwatersrand, South Africa

Graduate Fellowship of Baden-Wuerttemberg of Germany (January 1992 - August 1994) Tuebingen University, Germany

INVITED LECTURES and TALKS:

The 9th conference on frontier topics of the interdisciplinary sciences of particle physics, nuclear physics and cosmology (July 20 - 24, 2010)

Talk: Decay widths of $X(1835)$ as $N\overline{N}$ bound state

Autumn School on Medium Energy Nuclear and Hadron Physics (October 8 - November 5, 2009)

Lectures: Antinucleon-nucleon interactions

International Workshop on the Physics of Excited Nucleon: NSTAR 2009 (April 19-22, 2009)

Talk: Electron-positron annihilation to nucleon-antinucleon pairs at low energies

The Fourth Asia-Pacific Conference on Few Body Problems in Physics (August 19-23, 2008)

Talk: Accurate evaluation of wave functions of pionium, kaonium and kaonic atom

The Third Asia-Pacific Conference on Few Body Problems in Physics (July 26-30, 2005)

Talk: Accurate Evaluation of Antiproton-Deuteron Atoms

CCAST World Laboratory Workshop (April 2-6, 2001)

China Center of the Advanced Science and Technology (CCAST), Beijing, P. R. China

Lectures: Proton-antiproton annihilation into two and three mesons (10 hours)

Nucleon-antinucleon atomic states (4 hours)

Quantum object is merely particle (4 hours)

(CCAST directed by T.D. Lee invites every year one outstanding young Chinese scholar working in each field abroad to give a series of lectures in Beijing)

Germany-East Asia Symposium of Nuclear and Particle Physics (May, 1998)

Talk: Proton-antiproton annihilation to two pions and two kaons

SELECTED PUBLICATIONS

1. X.Y. Liu, K. Khosonthongkee, A. Limphirat, P. Suebka, and Y. Yan, "Meson cloud contributions to baryon axial form factors", Phys. Rev. D 91, 034022 (2015).

2. W. Sreethawong, K. Xu and Y. Yan, "Exclusion of $c\bar{c}$ interpretation for X(3940)", J. Phys. G: Nucl. Part. Phys. 42 025001 (2015).

3. C. Herold, M. Nahrgang, Y. Yan, C. Kobdaj, "Multiplicity fluctuations at the quark-hadron phase transition from a fluid dynamical model", Journal of Physics: Conference Series 599, 12012 (2015).

4. S. J. Zheng, F. R. Xu, S. F. Shen, H. L. Liu, R. Wyss, and Y. P. Yan, "Shape coexistence and triaxiality in nuclei near ^{80}Zr", Phys. Rev. C 90, 064309 (2014).

5. Christoph Herold, Marlene Nahrgang, Yupeng Yan and Chinorat Kobdaj, "Net-baryon number variance and kurtosis within nonequilibrium chiral fluid dynamics", J. Phys. G: Nucl. Part. Phys. 41, 115106 (2014).

6. M. F. M. Lutz, D. Samart and Yupeng Yan, "Combined large-N_c and heavy-quark operator analysis for the chiral Lagrangian with charmed baryons", Physical Review D 90, 056006 (2014).

7. Yu-Liang Yan, Dai-Mei Zhou, Ayut Limphirat, Bao-Guo Dong, Yu-Peng Yan and Ben-hao Sa, "Simultaneously study for particle transverse sphericity and ellipticity in pp collisions at LHC energies", Nuclear Physics A 930, 187 (2014).

8. A. Limphirat, W. Sreethawong, K. Khosonthongkee, and Y. Yan, "Reaction $e^+e^- \to \bar{D}D$ and ψ' mesons", Physical Review D 89, 054030 (2014).

9. Dai-Mei Zhou, Zeng-Zeng Luo, Yun Cheng, Ayut Limphirat, Yu-Liang Yan, Yu-Peng Yan, Xu Cai and Ben-hao Sa, "Comparative study for non-statistical fluctuation of net- Proton, baryon, and charge multiplicities", J. Phys. G: Nucl. Part. Phys. 41, 065103 (2014).

10. X. Y. Liu, K. Khosonthongkee, A. Limphirat and Y. Yan, "Study of baryon octet electromagnetic form factors in perturbative chiral quark model", J. Phys. G: Nucl. Part. Phys. 41, 055008 (2014).

11. Khanchai Khosonthongkee and Yupeng Yan, "Low-Lying Baryons in Hybrid Quark Model", Few-Body Systems 55, 1037 (2014).

12. Wanchaloem Poonsawat, Ayut Limphirat, Dai-Mei Zhou, Yu-Liang Yan, Pornrad Srisawad, Chinorat Kobdaj, Yu-Peng Yan and Ben-hao Sa, "Net-Proton Nonstatistical Moments in High-Energy pp Collisions in PACIAE Model", Few-Body Systems 55, 1041 (2014).

13. X. Y. Liu, K. Khosonthongkee, A. Limphirat and Y. Yan, "Study of baryon octet charge form factors in perturbative chiral quark model", International Journal of Modern Physics: Conference Series 29, 1460252 (2014).

14. K. Xu, N. Ritjoho, S. Srisuphaphon and Y. Yan, "Estimation of ground state pentaquark masses", International Journal of Modern Physics: Conference Series 29, 1460251 (2014).

15. P. Srisawad, A. Harfield, S. Sombun, T. Katukum, O. Ketsungnoen, Y. M. Zheng, A. Limphirat and Y. Yan, "Influence of the in-medium kaon potential on kaon production in heavy ion collisions", Journal of Physics: Conference Series, 509, 012034 (2014).

16. Shuifa Shen, Guangbing Han, Shuxian Wen, Xuzhong Kang, Yupeng Yan, Zhijun Bai,Yican Wu, Xiaoguang Wu, Lihua Zhu, Guangsheng Li and Chuangye He, "High-Spin States and Level Structure in Stable Nucleus Strontium-84", Scientific Reports 3, 2740 (2013).

17. Shuifa Shen, Guangbing Han, Shuxian Wen, Yupeng Yan, Xiaoguang Wu, Lihua Zhu, Chuangye He, Guangsheng Li, "Magnetic rotation in Rubidium-84", Nuclear Science and Techniques 24, 030503 (2013).

18. P. Srisawad, Y. M. Zheng, A. Suksri, A. Harfield, A. Limphirat, Y. Yan, "In-Medium Kaon Potential and Nuclear Equation of State Measured in NucleusNucleus Collisions", Few-Body Systems, 54, 1449 (2013).

19. P. Srisawad, A. Suksri, S. Pholwiang, A. Harfield, Y. M. Zheng, Y. Yan, A. Limphirat, "Transverse mass spectra and rapidity distributions of K^+ in NI-NI collisions at 1.93 A GeV", Modern Physics Letters A, 28, 1350070 (2013).

20. Thomas Lang, Hendrik van Hees, Jan Steinheimer, Yu-Peng Yan and Marcus Bleicher, "Heavy quark transport at RHIC and LHC", J. Phys.: Conf. Ser. 426, 012032 (2013).

21. Christoph Herold, Yu-Peng Yan and Marcus Bleicher, "Signals for the QCD phase transition and critical point in a Langevin dynamical model", J. Phys.: Conf. Ser. 426, 012008 (2013).

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24. P. Srisawad, Y. M. Zheng, O. Katsungnoen, A. Limphirat and Y. Yan, "Azimuthal Distributions of K^+ Mesons in Heavy-Ion Collisions", Few-Body Systems, 54, 303 (2013).

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32. Dai-Mei Zhou, Ayut Limphirat, Yu-Liang Yan, Xiao-Mei Li, Yu-Peng Yan, Ben-Hao Sa, "Impact of parton rescattering on analysis of p+p collision data at LHC energies", Phys. Lett. B 694, 435 (2011).

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35. Amand Faessler, K. Khosonthongkee, C. Kobdaj, A. Limphirat, P. Suebka and Y. Yan, "Low-lying baryon decays in 3P0 quark model", accepted for publication in J. Phys. G: Nucl. Part. Phys. 37, 115002 (2010).

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46. Pornrad Srisawad, Yu-Ming Zheng, Yupeng Yan, Chinorat Kobdaj and Yong-Zhong Xing, "Collective flow in heavy-ion collisions for E_b = 0.25 - 1.15 GeV/nucleon", Modern Physics Letters A 24, 1063 (2009).

47. K. Kittimanapun, K. Khosonthongkee, C. Kobdaj, P. Suebka and Y. Yan, " $e^+e^- \to \omega\pi$ reaction and $\rho(1450)$ and $\rho(1700)$ mesons in a quark model", Phys. Rev. C79 025201 (2009).

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51. P. Srisawad, Y. M. Zheng, C. Fuchs, A. Faessler, Y. Yan, C. Kobdaj and Y.Z. Xing, "Sigma meson production in heavy ion collisions at intermediate energies", International Journal of Modern Physics A 22, 6219 (2007).

52. Y. Yan, P. Suebka, C. Kobdaj and K. Khosonthogkee, "Strong interaction in pionium", Nuclear Physics A 790, 402c (2007).

53. K. Khosonthogkee, N. Supanam, Y. Yan, Th. Gutsche and Amand Faessler, "N*(1440) decays in a hybrid baryon model", Nuclear Physics A 790, 518c (2007).

54. K. Pumsa-ard, W. Uchai and Y. Yan, "Meson exchange theory for high energy proton-proton scattering", International Journal of Modern Physics E, Vol. 15, No. 1 pp. 109-119 (2006).

55. P. Suebka, C. Kobdaj and Y. Yan, "$\pi\pi$ Reaction in non-relativistic quark model", International Journal of Modern Physics E, Vol. 14, No. 7 pp. 987-994 (2005).

56. Y. Yan, C. Kobdaj, P. Suebka, Y.M. Zheng, Amand Faessler, Th. Gutsche and V.E. Lyubovitskij, "Electron-positron annihilation into hadron-antihadron pairs", Phys. Rev. C71 025204 (2005).

57. P. Suebka and Y. Yan, "Accurate evaluation of pionium wave functions", Phys. Rev. C 70, 034006 (2004).

58. Yu-Ming Zheng, C. Fuchs, Amand Faessler, K. Shekhter, Yu-Peng Yan and Chinorat Kobdaj, "Covariant kaon dynamics and kaon flow in heavy ion collisions" Phys. Rev. C 69, 034907 (2004).

59. Ben-Hao Sa, Zhong-Qi Wang, Xu Cai, Dai-Mei Zhou, C. Kobdaj and Yu-Peng Yan, "Energy dependence of string fragmentation function and ϕ meson production" Commun. Theor. Phys. 41, 291 (2004).

60. Y. M. Zheng, C. Fuchs, Amand Faessler, K. Shekhter, P. Srisawad, Y. Yan and C. Kobdaj, "Influence of Chiral Mean Field on Kaon In-plane Flow in Heavy Ion Collisions" Commun. Theor. Phys. 41, 746 (2004).

61. K. Khosonthongkee, V.E. Lyubovitskij, Th. Gutsche, Amand Faessler, K. Pumsa-ard, S. Cheedket and Y. Yan, "Axial form factor of the nucleon in the perturbative chiral quark model", J. Phys. G: Nucl. Part. Phys. 30, 793 (2004).

62. S. Cheedket, V.E. Lyubovitskij, Th. Gutsche, Amand Faessler, K. Pumsa-ard and Y. Yan, "Electromagnetic form factors of the baryon octet in the purterbative chiral quark model", Eur. Phys. J. A. 20, 317 (2004).

63. Y. Yan, K. Pumsa-ard, R. Tegen, Th. Gutsche, V.E. Lyubovitskij and Amand Faessler, "Nucleon-Nucleon High-Energy Scattering", Int. J. Mod. Phys. E 12, 367 (2003).

64. Y. Yan, C. Kobdaj, W. Uchai, A. Faessler, T. Gutsche and Y.M. Zheng, "Electron-Position Annihilation into Nucleon-Antinucleon Pairs", Mod. Phys. Lett. A18, 370 (2003).

65. Y.M. Zheng, Z.L. Chu, C. Fuchs, A. Faessler, W. Xiao, D.P. Hua, Y. Yan, "Thansverse Flow of Kaons in Heavy-Ion Collisions", Chin. Phys. Lett. 19, 926 (2002).

66. Y. Yan, "Baryon Structure and Baryon Interaction", CCAST (World Laboratory) Workshop Series: Volume 129 (2001).

67. Y. Yan and R. Tegen, "On the Quark Substructure of the Hydrogen Nucleus", Suranaree Journal of Science and Technology Vol. 7, 42 (2000).

68. Y. Yan and R. Tegen, "Proton-Antiproton to two Pions and two Kaons in Baryon Exchange and Meson Pole Diagrams", Nucl. Phys. A648, 89 (1999).

69. Y. Yan and R. Tegen, "Scale Invariance of g_A/g_V in Dirac-scalar and Dirac-vector Quark Confining Potentials", ISMPE 13 (1998).

70. Y. Yan, T. Gutsche, R. Thierauf, A. Muhn and A. Faessler, "Quasinuclear Nucleon-Antinucleon Bound States in the Quark Annihilation Model", J. Phys. G23, 605 (1997).

71. Y. Yan, R. Tegen, T. Gutsche and A. Faessler, "Nucleon-Antinucleon Bound States and Sturmian Function Method", Phys. Rev. C56, 1596 (1997).

72. E. Bauer, T. Gutsche, A. Muhm, R. Thierauf, Y. Yan, A. Faessler and R.V. Mau, "The Rho Parameter of Low-Energy Proton-Antiproton Scattering in the 3P0 Quark Model", Phys. Lett. B386, 50 (1996).

73. Y. Yan and R. Tegen, "Role of Tensor Meson Pole and Delta Exchange Diagrams in Proton-Antiproton to two Pions", Phys. Rev. C54, 1441 (1996).

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75. R. Thierauf, T. Gutsche, Y. Yan, A. Muhm and A. Faessler, "The Non-Relativistic Quark Model and Nucleon-Antinucleon Interaction", Nucl. Phys. A588, 783 (1995).

76. Y. Yan, S.W. Huang and A. Faessler, "A Microscopic Quark Model of Pion Nucleon to Kaon Sigma Reactions for Heavy Ion Collisions", Phys. Lett. B354, 24 (1995).

E.2 Prof. Dr. Prasart Suebka

School of Physics, Suranaree University of Technology

Nakhon Ratchasima 30000, Thailand

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EDUCATION

Ph.D.(Physics), 1984

Arizona State University, U.S.A.

M. S. (Physics), 1979

Indiana University, U.S.A.

M. Sc. (Physics), 1974

Chulalongkorn Universitty, Thailand

B. Ed. (Hons.), 1972

Sri Nakharinwirote University, Thailand

SELECTED PUBLICATIONS

1. X.Y. Liu, K. Khosonthongkee, A. Limphirat, P. Suebka, and Y. Yan, "Meson cloud contributions to baryon axial form factors", Phys. Rev. D 91, 034022 (2015).

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3. Amand Faessler, K. Khosonthongkee, C. Kobdaj, A. Limphirat, P. Suebka and Y. Yan, "Low-lying baryon decays in 3P0 quark model", J. Phys. G: Nucl. Part. Phys. 37, 115002 (2010).

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5. Y. Yan, K. Khosonthongkee, C. Kobdaj, P. Suebka, " $e^+e^- \to N\bar{N}$ at Threshold and Proton Form Factor", J. Phys. G: Nucl. Part. Phys. 37, 075007 (2010).

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7. Y. Yan, C. Nualchimplee, P. Suebka, C. Kobdaj and K. Khosonthongkee, "Accurate evaluation of wave functions of pionium and kaonium", Modern Physics Letters A 24, 901 (2009).

8. K. Kittimanapun, K. Khosonthongkee, C. Kobdaj, P. Suebka and Y. Yan, " $e^+e^- \to \omega\pi$ reaction and $\rho(1450)$ and $\rho(1700)$ mesons in a quark model", Phys. Rev. C79 025201 (2009).

9. Y. Yan, K. Khosonthongkee, C. Kobdaj, P. Suebka, Th. Gutsche, Amand Faessler and V.E. Lyubovitskij, "$\bar{p}D$ atoms in realistic potentials", Physics Letter B 659, 555 (2008).

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12. J. E. Lowther, P. Manyum, and P. Suebka, "Electronic and structural properties of orthorhombic KTiOPo4 and related isomorphic materials" Physica Status Solidi (B) Basic Research 242 (7), pp. 1392-1398 (2005).

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Appendix F

Publications

1. S. Srisuphaphon, Y. Yan, Th. Gutsche and V. E. Lyubovitskij, "ϕ meson production in $p\bar{p}$ annihilation at rest," Phys. Rev. D **84**, 074035 (2011).
2. Y. Yan and S. Srisuphaphon, "Construction of multiquark states in group theory," Prog. Part. Nucl. Phys. **67**, 496 (2012).

PHYSICAL REVIEW D **84,** 074035 (2011)

ϕ meson production in $p\bar{p}$ annihilation at rest

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(Received 9 August 2011; published 27 October 2011)

Apparent channel-dependent violations of the Okubo-Zwieg-Iizuka (OZI) rule in nucleon-antinucleon annihilation reactions in the presence of an intrinsic strangeness component in the nucleon are discussed. Admixture of $s\bar{s}$ quark pairs in the nucleon wave function enables the direct coupling to the ϕ-meson in the annihilation channel without violating the OZI rule. Three forms are considered in this work for the strangeness content of the proton wave function, namely, the uud cluster with a $s\bar{s}$ sea-quark component, kaon-hyperon clusters based on a simple chiral quark model, and the pentaquark picture $uuds\bar{s}$. Nonrelativistic quark model calculations reveal that the strangeness magnetic moment μ_s and the strangeness contribution to the proton spin σ_s from the first two models are consistent with recent experimental data, where μ_s and σ_s are negative. For the third model, the $uuds$ subsystem with the configurations $[31]_{FS}[211]_F[22]_S$ and $[31]_{FS}[31]_F[22]_S$ leads to negative values of μ_s and σ_s. With effective quark line diagrams incorporating the $^3P_0$ model, we give estimates for the branching ratios of the annihilation reactions at rest $p\bar{p} \to \phi X$ ($X = \pi^0$, η, ρ^0, ω). Results for the branching ratios of ϕX production from atomic $p\bar{p}$ s-wave states are for the first and third model found to be strongly channel dependent, in good agreement with measured rates.

DOI: 10.1103/PhysRevD.84.074035 PACS numbers: 13.25.Ft, 13.25.Hw, 14.40.Lb, 14.40.Nd

I. INTRODUCTION

In the simple constituent quark model, where the proton is made of two constituent u quarks and one d quark, a good explanation of static properties, e.g. magnetic moment, can be achieved. However, experimental results of the pion-nucleon sigma term value, strange magnetic moment μ_s, strangeness contribution to nucleon form factor [1], as well as the apparent violations in nucleon-antinucleon annihilation reactions involving the ϕ meson [2] indicate that the proton might contain a substantial strange quark-antiquark ($s\bar{s}$) component. The strangeness sigma term appears to lie somewhere in the range of 2–7% of the nucleon mass [3]. The substantial Okubo-Zwieg-Iizuka (OZI) rule violations in the $N\bar{N}$ annihilation reactions involving the ϕ meson may suggest the presence of an intrinsic $s\bar{s}$ in the nucleon wave function [4], for instance, the presence of a $q^3s\bar{s}(\bar{q}^3s\bar{s})$ piece in the $N(\bar{N})$ wave function. With such an assumption, the ϕ meson could be produced in $N\bar{N}$ annihilation reactions via a shakeout or rearrangement of the strange quarks already stored in the nucleon without the violation of the OZI rule. There are other explanations of the OZI rule violation without introducing a strange component in the nucleon such as the resonance interpretation, instanton induced interaction [5], and rescattering [6].

The European Muon Collaboration spin experiment [7] on deep inelastic scattering of longitudinally polarized muons by longitudinally polarized protons revealed for the first time that the polarization of the strange quark sea may contribute a significant negative value to the proton spin σ_s. This experimental result was confirmed by the subsequent deep inelastic double polarization experiments. Reference [8] analyzed all the available data in a systematic way and found $\sigma_s = -0.10 \pm 0.03$. Among a large number of theoretical works, the chiral quark model (CQM) has been successfully applied to explain the spin and flavor structure of the proton [9,10]. With the fluctuation of the proton into a kaon and a hyperon, the negative polarization of the strange quark sea is explained and other theoretical results are derived consistent with the deep inelastic scattering experimental results [9]. The flavor and spin structures of the nucleon as well as other observables are studied in Ref. [10], with both pseudoscalar and vector mesons as well as octet and decuplet baryons included.

However, the configuration of strange quarks in the nucleon is still an open question. The strangeness magnetic moment μ_s can be extrapolated from the strange magnetic form factor $G_M^s(Q^2)$ at the momentum transfer $Q^2 = 0$ measured in the parity violation experiments of electron scattering from a nucleon [11]. Most experimental measurements suggest a positive value for μ_s, in contrast to the recent experiment data [12] and most theoretical calculations which have obtained negative values for this observable [13,14]. A recent work [15] has proposed a different form for the strangeness content of the proton. Instead of

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the 5-quark component, which consists of a uud cluster and a $s\bar{s}$ pair proposed for solving the puzzle of violation of the OZI rule, Ref. [15] treats the strange quark piece in terms of pentaquark configurations. Different pentaquark configurations that may be contained in the proton may yield both positive and negative values for the strangeness spin and magnetic moment of the proton.

The experimental results on μ_s, which is extracted from experimental data on $G_M^s(Q^2)$, are rather uncertain due to the large uncertainties in $G_M^s(Q^2)$ and the extrapolation approach. So it is believed that the proton-antiproton reactions involving ϕ production may be another platform to be applied to tackle the possible configuration of strange quarks in the proton. In the present work we consider the strange content in the proton wave function in three models, namely, the uud cluster with an $s\bar{s}$ sea-quark component, kaon-hyperon clusters based on the chiral quark model, and the pentaquark picture $uuds\bar{s}$. The theoretical σ_s, μ_s, and branching ratios of the reactions $p\bar{p} \to \phi X$ ($X = \pi^0, \eta, \rho^0, \omega$) will be compared to experimental data. We resort to the $^3P_0$ quark model [16] and the nearest threshold dominance model [17] to obtain quantitative predictions for the branching ratios of the annihilation reactions from atomic $p\bar{p}$ states with the relative orbital angular momentum $L = 0$ [18]. The paper is organized as follows. The proton wave functions are briefly described in Sec. II while σ_s and μ_s are calculated and discussed in Sec. III for various strangeness quark configurations. In Sec. IV we evaluate the branching ratios for the reactions $p\bar{p} \to \phi X$ for the three forms of proton wave functions by using the $^3P_0$ quark model. Finally a summary and conclusion are given in Sec. V.

II. PROTON WAVE FUNCTIONS

The proton wave function in the presence of strange quarks may include a 5-quark component $qqqs\bar{s}$ in addition to the uud quark component, taking generically the form

$$|p\rangle = A|uud\rangle + B|uuds\bar{s}\rangle, \tag{1}$$

where A and B are the amplitudes for the 3- and 5-quark components in the proton, respectively [19]. The possible spin-flavor structures of the 5-quark components discussed in the $N\bar{N}$ annihilation process are considered in the next three subsections.

A. Proton wave function with an explicit $s\bar{s}$ sea-quark component

We consider the idea that strange quarks are present in the form of an $s\bar{s}$ sea-quark component in the proton state. This idea was proposed for describing the apparent violation of the OZI rule in the ϕNN production process [20] and in more general form used to discuss the ϕ meson production in $N\bar{N}$ annihilation reactions [4]. The corresponding 5-quark component for this model can be written in Fock space as

$$\begin{aligned}|uuds\bar{s}\rangle^{s\bar{s}} = a_0&|(uud)_{(1/2)}(s\bar{s})_0\rangle_{(1/2)} \\ &+ a_1|(uud)_{1/2}(s\bar{s})_1\rangle_{(1/2)},\end{aligned} \tag{2}$$

where the subscripts denote the spin coupling of the quark clusters, and a_0 and a_1 represent the amplitudes for the spin 0 and spin 1 components of the admixed $s\bar{s}$ pairs.

B. Proton wave function based on a chiral quark model

In the chiral quark model, the dominant process is the fluctuation of a valence quark q into a quark q' plus a Goldstone boson (GB) which in turn forms a $(q\bar{q}')$ system [21]. After the fluctuation of the u and d quarks in the proton, one of these quarks turns into a quark plus a quark-antiquark pair involving a strange quark. This idea was considered, for example, for calculating the flavor and spin content of the proton [9,10]. To obtain the proton wave function we consider the SU(3) invariant interaction Lagrangian of baryon octet with nonet of pseudoscalar mesons,

$$\begin{aligned}\mathcal{L}_I = &-g_8\sqrt{2}(\alpha[\bar{B}BP]_F + (1-\alpha)[\bar{B}BP]_D) \\ &- g_1\frac{1}{\sqrt{3}}[\bar{B}BP]_S,\end{aligned} \tag{3}$$

where $g_8 = 3.8$ and $g_1 = 2.0$ are coupling constants [22] and α is known as the $F/(F+D)$ ratio with $F \simeq 0.51$, $D \simeq 0.76$ [23]. The square brackets denote the SU(3) invariant combinations,

$$[\bar{B}BP]_F = \mathrm{Tr}(\bar{B}PB) - \mathrm{Tr}(\bar{B}BP), \tag{4}$$

$$[\bar{B}BM]_D = \mathrm{Tr}(\bar{B}PB) + \mathrm{Tr}(\bar{B}BP) - \frac{2}{3}\mathrm{Tr}(\bar{B}B)\mathrm{Tr}(P), \tag{5}$$

$$[\bar{B}BP]_S = \mathrm{Tr}(\bar{B}B)\mathrm{Tr}(P), \tag{6}$$

where B and P are the baryon octet and pseudoscalar meson nonet matrices, respectively, given by

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}, \tag{7}$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}. \tag{8}$$

The part of the interaction Lagrangian which allows for a fluctuation of the proton into kaons and hyperons is contained in

$$\mathcal{L}_I = -g_1 \bar{p}\eta_1 p + g_8\bigg[\bar{p}\pi^0 + \frac{1-4\alpha}{\sqrt{3}}\bar{p}\eta_8 + \frac{1+2\alpha}{\sqrt{3}}\bar{\Lambda}K^- + (2\alpha-1)\bar{\Sigma}^0 K^- - \sqrt{2}\bar{n}\pi^- + \sqrt{2}(2\alpha-1)\bar{\Sigma}^- K^0\bigg]p + \cdots. \quad (9)$$

The final states resulting from pseudoscalar meson emission by the proton are summarized as

$$|\Psi\rangle \sim -g_1|p\eta_1\rangle + g_8\bigg[\frac{1-4\alpha}{\sqrt{3}}|p\eta_8\rangle + |p\pi^0\rangle + \frac{1+2\alpha}{\sqrt{3}}|\Lambda K^+\rangle + (2\alpha-1)|\Sigma^0 K^+\rangle - \sqrt{2}|n\pi^+\rangle + \sqrt{2}(2\alpha-1)|\Sigma^+ K^0\rangle\bigg]. \quad (10)$$

In the absence of the fluctuation, the proton is made up of the conventional two u quarks and one d quark. Thus $\Psi(p)$ may be interpreted as the 5-quark component of the proton wave function which is given by

$$|uuds\bar{s}\rangle^{\text{CQM}} = G_1|\Sigma^0 K^+\rangle + G_2|\Sigma^+ K^0\rangle + G_3|\Lambda^0 K^+\rangle + G_4|p\eta_1\rangle + G_5|p\eta_8\rangle, \quad (11)$$

where the G_i are the coefficients corresponding to the respective factor in Eq. (10). Each component in the last equation can be represented in terms of quark cluster configurations as

$$\begin{aligned} |p\eta_{1,8}\rangle &= |(uud)_{(1/2)}(s\bar{s})_0\rangle_{(1/2)},\\ |\Sigma^0 K^+\rangle &= |(uds)_{(1/2)}(u\bar{s})_0\rangle_{(1/2)},\\ |\Sigma^+ K^0\rangle &= |(uus)_{(1/2)}(d\bar{s})_0\rangle_{(1/2)},\\ |\Lambda^0 K^+\rangle &= |(usd)_{(1/2)}(u\bar{s})_0\rangle_{(1/2)}. \end{aligned} \quad (12)$$

C. Proton wave function including general configurations of the *uuds* subsystem

Another, more general form of the 5-quark component was proposed and analyzed in Ref. [15]. Instead of first generating a meson coupling to a baryon cluster, they considered the genuine 5-quark or $q^4\bar{q}$ pentaquark component in the proton. In this model the 5-quark component may be expressed in terms of the $uuds$ and the $\bar{s}$ wave functions as

$$|uuds\bar{s}\rangle^{uuds} = |(uuds)\bar{s}\rangle_{(1/2)}. \quad (13)$$

The flavor wave functions for the $uuds\bar{s}$ components are usually constructed by coupling the $uuds$ to the $\bar{s}$ flavor wave function. The configurations studied in [15] include at most one unit of orbital angular momentum. The favored configurations are connected to a positive sign for the strangeness magnetic moment and a negative one for the strangeness contribution to the proton spin.

III. STRANGENESS MAGNETIC MOMENT AND SPIN OF THE PROTON

In the nonrelativistic quark model the strangeness magnetic moment operator $\vec{\mu}_s$ and the strangeness contribution to the proton spin operator $\vec{\sigma}_s$ are defined as

$$\vec{\mu}_s = \frac{e}{2m_s}\sum_i \hat{S}_i(\hat{\ell}_s + \hat{\sigma}_s), \quad (14)$$

$$\vec{\sigma}_s = \hat{\sigma}_s + \hat{\sigma}_{\bar{s}}. \quad (15)$$

$\hat{S}_i$ is the strangeness counting operator with eigenvalue $+1$ for an s and -1 for an $\bar{s}$ quark and m_s is the constituent mass of the strange quark. To calculate the matrix elements of these operators explicit forms of the spin-flavor wave functions of the proton including orbital angular momentum are needed.

For the first model the spin-flavor wave function can be constructed by coupling the $|s\bar{s}\rangle_{j_s=0,1}$ configuration to the $|uud\rangle_{1/2}$ cluster. Since the admixed $s\bar{s}$ carries negative intrinsic parity, an orbital P-wave ($\ell = 1$) has to be introduced into the nucleon quark cluster wave function. The simplest configuration (see also Ref. [20]) corresponds to a $1S$-state of the $s\bar{s}$ pair moving in a P-wave relative to the (uud) valence quark cluster of the nucleon. Then the 5-quark component with total angular momentum $1/2$ can be written in the general form,

$$|uuds\bar{s}\rangle^{s\bar{s}}_{(1/2),m_{ps\bar{s}}=(1/2)} = \sum_{j_s,j_i=0,1}\alpha_{j_s j_i}|[(s\bar{s})_{j_s}\otimes\ell=1]_{j_i}\otimes(uud)_{(1/2)}\rangle_{(1/2),m_{ps\bar{s}}=(1/2)}, \quad (16)$$

with the normalization $\sum_{j_s,j_i=0,1}|\alpha_{j_s j_i}|^2 = 1$.

Similarly, for the proton wave function in the CQM, where the sea-quark contributions are embedded in the pseudoscalar mesons, a relative P-wave between the pseudoscalars and the uud or hyperon clusters has to be included. The spin-flavor wave function with spin $+1/2$ for each coupled meson-baryon state of Eq. (12) may be expressed as

$$|uuds\bar{s}\rangle^{\text{CQM}}_{(1/2),(1/2)} = |[(q\bar{s})_{j_s=0}\otimes\ell=1]_{j_i}\otimes(qqs)_s\rangle_{(1/2),m_{ps\bar{s}}=(1/2)}. \quad (17)$$

Wave functions of the pentaquark $uuds\bar{s}$ states employed in the third model are more complicated because no restrictions are set concerning the subclusters. One has to carefully consider the coupling of the color, spin, flavor, and spatial parts to construct the total wave functions [15]. The color part of the antiquark in the pentaquark states is a [11] antitriplet, denoted by the Weyl tableau of the SU(3) group. Hence the color symmetry of all the $uuds$ configurations is limited to a [211] triplet in order to form a pentaquark color singlet labeled by the Weyl tableau [222]. Three flavor symmetry patterns exist for the $uuds$ system corresponding to the octet representation for the proton: $[31]_F$, $[22]_F$, and $[211]_F$ characterized by the S_4 Young tableau. However, the pentaquark should be antisymmetric under any permutation of the 4-quark configuration. If the spatial wave function is symmetric, the spin-flavor part of the $uuds$ component must be a [31] state in order to form the antisymmetric color-spin-flavor $uuds$ part of the pentaquark wave function. For instance, the flavor symmetry representations $[31]_F$ and $[211]_F$ may combine with the spin symmetry state $[22]_S$ to form the mixed symmetry spin-flavor states $[31]_{FS}$ (the explicit forms may be found in [15,24,25]). In this work we consider only the case that the $uuds$ component is in the ground state with the spin symmetry $[22]_S$ corresponding to spin 0, and the relative orbital angular momentum between the $uuds$ component and the $\bar{s}$ is of one unit to obtain the positive parity for the proton wave function.

Theoretical results for the strangeness magnetic moment μ_s of the proton and the strangeness contribution to the proton spin σ_s are listed in Table I. In the first model we have fixed the configuration parameters as $\alpha_{1,0} = \alpha_{1,1} = \bar{\alpha}$. The strangeness magnetic moment μ_s depends explicitly on $\alpha_{0,1}$, which is related to the amplitude for the $s\bar{s}$ quark cluster with spin 0. Setting $\alpha_{0,1} = 0$ is equivalent to excluding the quantum number $J^{PC} = 0^{-+}$ for the $s\bar{s}$ admixture in the nucleon wave function connected to the production of η and η' in $N\bar{N}$ annihilation as discussed in [19,26]. The chiral quark model always gives results for μ_s and σ_s, which are negative, and the size of the strangeness contribution depends on the coupling g_8^2. For the third model, we show only the results for the cases where the $uuds$ component is in the ground state with the spin-flavor configurations $[31]_{FS}[211]_F[22]_S$ and $[31]_{FS}[31]_F[22]_S$ and the relative motion between the $uuds$ component and the $\bar{s}$ is a P-wave.

All the three models yield negative values for the strangeness contribution to the proton spin, which is consistent with present experimental results [7,8]. Negative values for the strangeness magnetic moment also result from all three models. Note that we restricted the considerations of Ref. [15] to the pentaquark components with the $uuds$ configurations $[31]_{FS}[211]_F[22]_S$ and $[31]_{FS}[31]_F[22]_S$, respectively.

TABLE I. Strangeness magnetic moment and spin of the proton for the three models of the 5-quark component.

$\lvert uuds\bar{s}\rangle$	$\mu_s(\frac{eB^2}{2m_s})$	$\sigma_s(B^2)$
$s\bar{s}$	$-0.55\bar{\alpha}\alpha_{0,1}$	$-1.22\bar{\alpha}^2$[18]
CQM	$-1.1g_8^2$	$-0.31g_8^2$
$[31]_{FS}[211]_F[22]_S$	$-\frac{1}{3}$[15]	$-\frac{1}{3}$[15]
$[31]_{FS}[31]_F[22]_S$	$-\frac{1}{3}$[15]	$-\frac{1}{3}$[15]

IV. $N\bar{N}$ TRANSITION AMPLITUDE AND BRANCHING RATIOS

To describe the annihilation reactions $N\bar{N} \to X\phi$ ($X = \pi^0, \eta, \rho^0, \omega$) we use an effective transition dynamics, which is evaluated in the context of a simple constituent quark model. In this specific process the ϕ meson couples to the intrinsic $s\bar{s}$ component of the nucleon, which is the leading order OZI allowed contribution. The process $p\bar{p}$ annihilation into ϕX involving the 5-quark components in the proton wave function can be described by the quark line diagrams of Fig. 1. In the hadronic transition the effective quark annihilation operator is taken with the quantum numbers of the vacuum ($^3P_0$, isospin $I = 0$ and color singlet). Meson decays and $N\bar{N}$ annihilation into two mesons are well described phenomenologically using such an effective quark-antiquark vertex. At least for meson decay, this approximation has been given a rigorous basis in strong-coupling QCD. The nonperturbative $q\bar{q}\ {}^3P_0$ vertex is defined according to [27]

$$V^{ij} = \sum_{\mu} \sigma_{-\mu}^{ij} Y_{1\mu}(\vec{q}_i - \vec{q}_j)\delta^{(3)}(\vec{q}_i + \vec{q}_j)(-1)^{1+\mu} 1_F^{ij} 1_C^{ij}, \tag{18}$$

where $Y_{1\mu}(\vec{q}) = |\vec{q}|\mathcal{Y}_{1\mu}(\hat{q})$ with $\mathcal{Y}_{1\mu}(\hat{q})$ being the spherical harmonics in momentum space, and 1_F^{ij} and 1_C^{ij} are unit

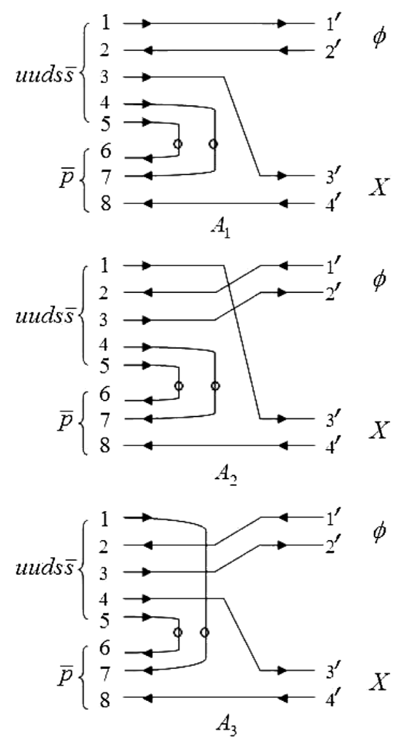

FIG. 1. Quark line diagrams for the production of two-meson final states in $p\bar{p}$ annihilation. Small circles refer to the effective vertex of the $^3P_0$ quark dynamics for $q\bar{q}$ annihilation. The first diagram corresponds to the shakeout of the intrinsic $s\bar{s}$ component of the proton wave function [4,18].

operators in flavor and color spaces, respectively. The spin operator $\sigma^{ij}_{-\mu}$ is part of the $^3P_0$ vertex, destroying or creating quark-antiquark pairs with spin 1.

In the momentum space representation the transition amplitudes for the quark diagrams of Fig. 1 are given by

$$T_{A_l} = \int d^3q_1 \ldots d^3q_8 d^3q_{1'} \ldots d^3q_{4'} \langle \phi X | \vec{q}_{1'} \ldots \vec{q}_{4'} \rangle \times \langle \vec{q}_{1'} \ldots \vec{q}_{4'} | \mathcal{O}_{A_l} | \vec{q}_1 \ldots \vec{q}_8 \rangle \langle \vec{q}_1 \ldots \vec{q}_8 | (uuds\bar{s}) \otimes (\bar{u}\,\bar{u}\,\bar{d}) \rangle \tag{19}$$

where $(\bar{u}\,\bar{u}\,\bar{d})$ stands for the antiproton wave function and $(uuds\bar{s})$ for the 5-quark component of the proton wave function. The effective operators $\mathcal{O}_{A_l}$ take the form

$$\mathcal{O}_{A_1} = \lambda_{A_1} \delta^{(3)}(\vec{q}_1 - \vec{q}_{1'}) \delta^{(3)}(\vec{q}_2 - \vec{q}_{2'}) \times \delta^{(3)}(\vec{q}_3 - \vec{q}_{3'}) \delta^{(3)}(\vec{q}_8 - \vec{q}_{4'}) V^{56} V^{47}, \tag{20}$$

$$\mathcal{O}_{A_2} = \lambda_{A_2} \delta^{(3)}(\vec{q}_2 - \vec{q}_{1'}) \delta^{(3)}(\vec{q}_3 - \vec{q}_{2'}) \times \delta^{(3)}(\vec{q}_1 - \vec{q}_{3'}) \delta^{(3)}(\vec{q}_8 - \vec{q}_{4'}) V^{56} V^{47}, \tag{21}$$

$$\mathcal{O}_{A_3} = \lambda_{A_3} \delta^{(3)}(\vec{q}_2 - \vec{q}_{1'}) \delta^{(3)}(\vec{q}_3 - \vec{q}_{2'}) \times \delta^{(3)}(\vec{q}_4 - \vec{q}_{3'}) \delta^{(3)}(\vec{q}_8 - \vec{q}_{4'}) V^{56} V^{17}. \tag{22}$$

The δ-functions represent the noninteracting and continuous quark-antiquark lines in the diagrams. The constants λ_{A_l} describe the effective strength of the transition topology and are considered to be overall fitting parameters in the phenomenological description of experimental data. Since the 5-quark component is treated as a small perturbative admixture in the proton ($B^2 \ll 1$), we ignore the transition amplitude with the term $\langle \vec{q}_1 \ldots \vec{q}_8 | (uuds\bar{s}) \otimes (\bar{u}\,\bar{u}\,\bar{d}\,\bar{s}\,s) \rangle$ or the rearrangement process [4].

In this work the internal spatial wave functions are taken in the harmonic oscillator approximation. For the mesons M (ϕ and X), the wave function can be expressed in terms of the quark momenta as

$$\langle M | \vec{q}_{i'} \vec{q}_{j'} \rangle \equiv \varphi_M(\vec{q}_{i'}, \vec{q}_{j'}) \chi_M = N_M \exp\left\{-\frac{R_M^2}{8}(\vec{q}_{i'} - \vec{q}_{j'})^2\right\} \chi_M, \tag{23}$$

with $N_M = (R_M^2/\pi)^{3/4}$ and R_M is the meson radial parameter. The spin-color-flavor wave function is denoted by χ_M. The baryon wave functions are given by

$$\langle B | \vec{q}_i \vec{q}_j \vec{q}_k \rangle \equiv \varphi_B \chi_B = N_B \exp\left\{-\frac{R_B^2}{4}\left[(\vec{q}_j - \vec{q}_k)^2 + \frac{(\vec{q}_j + \vec{q}_k - 2\vec{q}_i)^2}{3}\right]\right\} \chi_B, \tag{24}$$

where $N_B = (3R_B^2/\pi)^{3/2}$ and R_B is the baryon radial parameter. For the first and the second model the full 5-quark component wave function, resulting from the coupling of a meson to a baryon, is given by

$$\langle \vec{q}_1 \ldots \vec{q}_5 | uuds\bar{s} \rangle = \varphi_{uuds\bar{s}}(\vec{q}_1, \ldots, \vec{q}_5) \chi_{uuds\bar{s}} = N_{uuds\bar{s}} \exp\left\{-\frac{R_B^2}{4}\left[(\vec{q}_4 - \vec{q}_5)^2 + \frac{(\vec{q}_4 + \vec{q}_5 - 2\vec{q}_3)^2}{3}\right]\right\} \exp\left\{-\frac{R^2}{8}(\vec{q}_3 + \vec{q}_4 + \vec{q}_5 - \vec{q}_1 - \vec{q}_2)^2\right\} \times Y_{1\mu}(\vec{q}_3 + \vec{q}_4 + \vec{q}_5 - \vec{q}_1 - \vec{q}_2) \exp\left\{-\frac{R_M^2}{8}(\vec{q}_1 - \vec{q}_2)^2\right\} (\chi_B \otimes \chi_M). \tag{25}$$

The exponential form with the radial parameter R and the spherical harmonics $Y_{1\mu}$ together represent the internal relative P-wave between the 3-quark and 2-quark clusters.

For the third model the proton wave function includes a pentaquark component $uuds\bar{s}$ with the $uuds$ part in the ground state and the P-wave internal relative orbital angular momentum between $uuds$ and the $\bar{s}$. One may write the spatial wave function of the pentaquark component $uuds\bar{s}$ as

$$\varphi_{uuds\bar{s}}(\vec{q}_1, \ldots, \vec{q}_5) = N_{uuds\bar{s}} \exp\left\{-\frac{R_B^2}{4}\left[(\vec{q}_2 - \vec{q}_3)^2 + \frac{(\vec{q}_2 + \vec{q}_3 - 2\vec{q}_4)^2}{3} + \frac{(\vec{q}_2 + \vec{q}_3 + \vec{q}_4 - 3\vec{q}_5)^2}{6} + \frac{(\vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5 - 4\vec{q}_1)^2}{10}\right]\right\} Y_{1\mu}\left(\frac{\vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5 - 4\vec{q}_1}{\sqrt{20}}\right). \tag{26}$$

By choosing the plane wave basis for the relative motion of the proton and antiproton, the initial-state wave functions in the center of momentum system ($\vec{k} = \vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5$) are obtained as

$$\langle \vec{q}_1 \ldots \vec{q}_8 | (uuds\bar{s}) \otimes (\bar{u}\,\bar{u}\,\bar{d}) \rangle = \varphi_{uuds\bar{s},\bar{p}} [\chi_{uuds\bar{s}} \otimes \chi_{\bar{p}}]_{S,S_z}, \tag{27}$$

with

$$\varphi_{uuds\bar{s},\bar{p}} = \varphi_{uuds\bar{s}}\varphi_{\bar{p}}\delta^{(3)}(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5 - \vec{k}) \times \delta^{(3)}(\vec{q}_6 + \vec{q}_7 + \vec{q}_8 + \vec{k}). \quad (28)$$

The spins of the $p\bar{p}$ system are coupled to the total spin S with projection S_z. Similarly, the final state ϕX wave functions in the center of momentum system are given by ($\vec{q} = \vec{q}_{1'} + \vec{q}_{2'}$)

$$\langle \phi X|\vec{q}_{1'} \ldots \vec{q}_{4'}\rangle = \varphi_{\phi,X}[\chi_\phi \otimes \chi_X]_{j_i,m_\epsilon}, \quad (29)$$

with

$$\varphi_{\phi,X} = \varphi_\phi \varphi_X \delta^{(3)}(\vec{q} - \vec{q}_{1'} - \vec{q}_{2'})\delta^{(3)}(\vec{q} + \vec{q}_{3'} + \vec{q}_{4'}). \quad (30)$$

The spins of the two-meson states are coupled to j_i with projection m_ϵ.

In the low-momentum approximation, the transition amplitude T_{fi} of the annihilation reaction of the S-wave $\bar{p}p$ initial state i to the P-wave two-meson final state f with the quark line diagrams A_I as shown in Fig. 1 is derived as

$$T_{fi}(\vec{q}, \vec{k}) = \lambda_{A_I} F_{L=0,\ell_f=1}\, q \exp\{-Q_q^2 q^2 - Q_k^2 k^2\}\langle f|O_{A_I}|i\rangle. \quad (31)$$

The index i represents the initial state $^{2I+1,2S+1}L_J$, where L is the orbital angular momentum, S is the total spin, J is the total angular momentum, and I is the total isospin. The final state f is represented by the set of quantum numbers $f = \{\ell_f j J'\}$, where ℓ_f is the relative orbital angular momentum. The constants $F_{0,1}$, Q_q^2, and Q_k^2 are geometrical constants depending on the radial parameters. The matrix element $\langle f|O_{A_I}|i\rangle$ is the spin-flavor weight for a quark line diagram A_I. The detailed evaluation of the expression in Eq. (31) is given in the Appendix.

As we consider $p\bar{p}$ annihilations at rest where the strong interaction between the proton and antiproton may largely distort the $\bar{p}p$ hydrogenlike wave function at small distances [28], the effect of the initial-state interaction is in general not negligible. The inclusion of the initial-state interaction for the atomic state of the $p\bar{p}$ system results in the transition amplitude [29],

$$T_{f,LSJ}(\vec{q}) = \int d^3k T_{fi}(\vec{q}, \vec{k})\phi^I_{LSJ}(\vec{k}), \quad (32)$$

where $\phi^I_{LSJ}(\vec{k})$ is the protonium wave function in momentum space for fixed isospin I. The partial decay width for the transition of the $p\bar{p}$ state to the two-meson state ϕX is given by

$$\Gamma_{p\bar{p}\to\phi X} = \frac{1}{2E}\int \frac{d^3p_\phi}{2E_\phi}\frac{d^3p_X}{2E_X}\delta^{(3)}(\vec{p}_\phi + \vec{p}_X) \times \delta(E - E_\phi - E_X)|T_{f,LSJ}(\vec{q})|^2, \quad (33)$$

where E is the total energy ($E = 1.876$ GeV) and $E_{\phi,X} = \sqrt{m^2_{\phi,X} + \vec{p}^2_{\phi,X}}$ is the energy of outgoing meson ϕ and X with mass $m_{\phi,X}$ and momentum $\vec{p}_{\phi,X}$. With the explicit form of the transition amplitude given by Eq. (31), the partial decay width for the S to P transition ($L = 0$, $\ell_f = 1$) is written as

$$\Gamma_{p\bar{p}\to\phi X} = \lambda^2_{A_I} f(\phi, X)\langle f|O_{A_I}|i\rangle^2 \gamma(I, J), \quad (34)$$

with

$$\gamma(I, J) = |F_{0,1}\int d^3k\phi^I_{LSJ}(\vec{k})\exp\{-Q_k^2 k^2\}|^2 \quad (35)$$

and the kinematical phase-space factor defined by

$$f(\phi, X) = \frac{q^3}{8E^2}\exp\{-2Q_q^2 q^2\}. \quad (36)$$

The spin-flavor weights $\langle f|O_{A_I}|i\rangle$ for the transitions $N\bar{N} \to \phi X$ involving the different 5-quark components of the proton wave functions are listed in Table II. For the initial values of the total angular momentum J the statistical weights $1/4$ and $3/4$ have to be added for $J = 0$ and $J = 1$, respectively. Finally the branching ratio of S-wave $p\bar{p}$ annihilation to the final state ϕX is then given by

$$\mathrm{BR}(\phi, X) = \frac{(2J + 1)\Gamma_{p\bar{p}\to\phi X}}{4\Gamma_{\mathrm{tot}}(J)}, \quad (37)$$

where $\Gamma_{\mathrm{tot}}(J)$ is the total annihilation width of the $p\bar{p}$ atomic state with fixed principal quantum number [30].

The model dependence in Eq. (34) may be reduced by choosing a simplified phenomenological approach that has been applied in studies of two-meson branching ratios in nucleon-antinucleon [29] and radiative protonium annihilation [31]. Namely, the phase space factor of Eq. (36) is obtained in the harmonic oscillator approximation for the hadron wave function, depending on the relative momentum q and the masses of ϕX system. Instead we use a kinematical phase-space factor of the phenomenological form

TABLE II. Spin-flavor matrix elements $\langle f|O_{A_I}|i\rangle$ for the transitions $p\bar{p}(L = 0) \to \phi X(\ell_f = 1)$ which are described by the quark line diagram A_I. Here η_{ud} refers to the nonstrange flavor combination $\eta_{ud} = (u\bar{u} + d\bar{d})/\sqrt{2}$.

Transition	$s\bar{s}_{A_1}$	CQM	$[31][31][22]_{A_1}$	$[31][211][22]_{A_1}$
$^{11}S_0 \to \omega\phi$	$\frac{5}{9\sqrt{6}}$	-0.097	$\frac{5}{36\sqrt{6}}$	$\frac{5}{36\sqrt{6}}$
$^{33}S_1 \to \pi^0\phi$	$\frac{5}{27\sqrt{2}}$	0.031	$\frac{5}{108\sqrt{2}}$	$\frac{5}{108\sqrt{2}}$
$^{31}S_0 \to \rho^0\phi$	$\frac{13}{27\sqrt{6}}$	0.040	$\frac{13}{108\sqrt{6}}$	$\frac{13}{108\sqrt{6}}$
$^{13}S_1 \to \eta_{ud}\phi$	$\frac{1}{9\sqrt{2}}$	0.013	$\frac{1}{36\sqrt{2}}$	$\frac{1}{36\sqrt{2}}$

TABLE III. Branching ratio BR($\times 10^4$) for the transition $p\bar{p} \to \phi X$ ($X = \pi^0, \eta, \rho^0, \omega$) in $p\bar{p}$ annihilation at rest. The results indicated by $\star$ are normalized to the experimental values.

Transition	$\mathrm{BR}^{\mathrm{exp}}$	$\mathrm{BR}^{s\bar{s}}$	$\mathrm{BR}^{\mathrm{CQM}}$	$\mathrm{BR}^{[31][31][22]}$	$\mathrm{BR}^{[31][211][22]}$
${}^{11}S_0 \to \omega\phi$	6.3 ± 2.3	$6.3\star$	$6.3\star$	$6.3\star$	$6.3\star$
${}^{33}S_1 \to \pi^0\phi$	5.5 ± 0.7	5.4	1.6	5.4	5.4
${}^{31}S_0 \to \rho^0\phi$	3.4 ± 1.0	3.8	0.87	3.8	3.8
${}^{13}S_1 \to \eta\phi$	0.9 ± 0.3	1.4–1.8	0.20–0.27	1.4–1.8	1.4–1.8

$$f(\phi, X) = q \cdot \exp\{-a_s(s - s_{\phi X})^{1/2}\}, \qquad (38)$$

where $s_{\phi X} = (m_\phi + m_X)^{1/2}$ and $\sqrt{s} = (m_\phi^2 + q^2)^{1/2} + (m_X^2 + q^2)^{1/2}$. The constant $a_s = 1.2\ \mathrm{GeV}^{-1}$ is obtained from a phenomenological fit to the momentum dependence of various multipion final states in $p\bar{p}$ annihilation [17]. In addition, the functions $\gamma(I, J)$, depending on the initial-state interaction, are related to the probability for a protonium state to have isospin I and spin J with the normalization condition $\gamma(0, J) + \gamma(1, J) = 1$. Here we adopt for a protonium state the probability $\gamma(I, J)$ and the total decay width $\Gamma_{\mathrm{tot}}(J)$ obtained in an optical potential calculation [32], where explicit values are listed in [30].

In Table III we give the theoretical results for the branching ratios of Eq. (37) compared with experimental data. The branching ratios $\mathrm{BR}^{s\bar{s}}$, resulting from the first model where the proton wave function has an explicit $s\bar{s}$ admixture, have already been derived and studied in Ref. [18] by using the same approach. Annihilation processes in the first and third model are described by the quark line diagram A_1. Since the effective strength parameter λ_{A_1} is *a priori* unknown, it has to be adjusted to data. For this purpose one entry in the table (as indicated by $\star$) is normalized to the observed value.

For the second chiral model where the proton wave function contains a kaon-hyperon or eta-proton cluster component, all three quark line diagrams may have contributions to the $\bar{p}p$ annihilation process. However, the process proceeding by the diagram A_1 with the $|p\eta\rangle$ component in the proton wave function has no contribution to the transition because of orthogonality to the ϕ state. Therefore, the annihilation process in the second model can only be described by the quark line diagrams A_2 and A_3. Considering the same annihilation pattern in these two diagrams, for simplicity the two unknown strength parameters are of the same order with $\lambda_{A_2} = \lambda_{A_3}$. Model predictions are also normalized to experimental data (as indicated by $\star$). For final states with $X = \eta$, the physical η meson is produced by its nonstrange component η_{ud} with $\eta = \eta_{ud}(\sqrt{1/3}\cos\theta - \sqrt{2/3}\sin\theta)$ corresponding to a variation of the pseudoscalar mixing angle θ from $\theta = -10.7^o$ to $\theta = -20^o$.

As shown in Table III, the theoretical results of the first and third models, where the proton wave function possesses, respectively, a small kaon-hyperon component and a pentaquark, are in good agreement with the experimental data. Note that for these two cases the annihilation processes $p\bar{p} \to \phi X$ are described with the quark line diagram A_1.

Please note that in the present model we cannot give a reliable estimate for the genuine transition strength of the $p\bar{p}$ annihilation processes. Therefore, the measured production rates cannot be used to get an estimate for the $s\bar{s}$ content in the nucleon or for the coefficient B in Eq. (1).

V. SUMMARY

Three models have been studied for the proton involving intrinsic strangeness in the form of a 5-quark component $qqqs\bar{s}$ in the wave function. In particular, the proton wave function is made up of a uud configuration and a uud cluster with an $s\bar{s}$ sea-quark component, kaon-hyperon clusters based on the simple chiral quark model, or a pentaquark component $uuds\bar{s}$. We have calculated the strangeness magnetic moment μ_s and spin σ_s for the first and second models and generated negative values in line with recent experimental indication. Similarly, for the third model we pick these configurations, where negative values for μ_s and σ_s result [15].

We further applied quark line diagrams supplemented by the ${}^3P_0$ vertex to study the annihilation reactions $p\bar{p} \to \phi X$ ($X = \pi^0, \eta, \rho^0, \omega$) with the three types of proton wave functions. Excellent agreements of the model predictions in the first and third models with the experimental data are found for the branching ratios of the reactions of the $L = 0$ atomic $p\bar{p}$ state to ϕX ($X = \pi^0, \eta, \rho^0, \omega$).

ACKNOWLEDGMENTS

This work was supported by the DFG under Contract No. FA67/31-2. This research is also part of the European Community-Research Infrastructure Integrating Activity "Study of Strongly Interacting Matter" (HadronPhysics2), Grant Agreement No. 227431) and part of the Federal Targeted Program "Scientific and Scientific-Pedagogical Personnel of Innovative Russia" Contract No. 02.740.11.0238. We also acknowledge the generous help of Chun-Sheng An for providing us with the proton wave function with the 5-quark component in *uuds* subsystem used in this paper. The stay in Tubingen of Sorakrai Srisuphaphon was supported by the DAAD under

PKZ:A/07/98879, and the study at SUT was supported by Burapha University.

APPENDIX A: TRANSITION AMPLITUDES OF THE ANNIHILATION PROCESSES $p\bar{p} \to \phi X$

To describe the annihilation process $p\bar{p} \to \phi X$, where $X = \pi^0, \eta, \rho^0, \omega$ with the proton wave function with $s\bar{s}$ sea quark, we consider the shakeout of the intrinsic $s\bar{s}$ component of the proton wave function as indicated in the diagram A_1. With the operator $\mathcal{O}_{A_1}$ and the full account of the spin-flavor-color-orbital structure of the initial and final states, the transition amplitude can be written as

$$T_{if}^{s\bar{s}} = \lambda_{A_1}\left\langle f \left| \sum_{\nu,\lambda} (-1)^{\nu+\lambda} \sigma_{-\nu}^{56} \sigma_{-\lambda}^{47} 1_F^{56} 1_F^{47} 1_C^{56} 1_C^{47} I_{\text{spatial}}^{s\bar{s}} \right| i \right\rangle, \tag{A1}$$

where

$$|i\rangle = |\{\chi_{(1/2),m_{ps\bar{s}}}(uuds\bar{s}) \otimes \chi_{(1/2),m_{\bar{p}}}(\bar{u}\bar{u}\bar{d})\}_{S,S_z} \otimes (L,M)\rangle_{J,J_z}, \tag{A2}$$

$$|f\rangle = |\{\chi_{1,m_\alpha}(\phi) \otimes \chi_{j_m,m_{3',4'}}(X)\}_{j,m_\epsilon} \otimes (\ell_f, m_f)\rangle_{J,J_z}. \tag{A3}$$

The spin-flavor-color content of the clusters is denoted by $\chi \equiv \chi_\sigma \otimes \chi_F \otimes \chi_C$. The 5-quark component $\chi_{(1/2),m_{ps\bar{s}}}(uuds\bar{s})$ is defined as

$$\chi_{(1/2),m_{ps\bar{s}}}(uuds\bar{s}) = |\{\chi_{j_s,m_s}(s\bar{s}) \otimes (\ell = 1, \mu)\}_{j_i,m_i} \otimes \chi_{(1/2),m_p}(uud)\rangle_{(1/2),m_{ps\bar{s}}}. \tag{A4}$$

The spatial amplitude $I_{\text{spatial}}^{s\bar{s}}$ is explicitly given by

$$I_{\text{spatial}}^{s\bar{s}} = \int d^3q_1 \ldots d^3q_8 d^3q_{1'} \ldots d^3q_{4'} \varphi_{\phi,X} \mathcal{O}_{A_1}^{\text{spatial}} \varphi_{uuds\bar{s},\bar{p}}, \tag{A5}$$

where

$$\begin{aligned}\mathcal{O}_{A_1}^{\text{spatial}} &= Y_{1\lambda}(\vec{q}_4 - \vec{q}_7)\delta^{(3)}(\vec{q}_4 + \vec{q}_7) Y_{1\nu}(\vec{q}_5 - \vec{q}_6) \\ &\times \delta^{(3)}(\vec{q}_5 + \vec{q}_6)\delta^{(3)}(\vec{q}_1 - \vec{q}_{1'})\delta^{(3)}(\vec{q}_2 - \vec{q}_{2'}) \\ &\times \delta^{(3)}(\vec{q}_3 - \vec{q}_{3'})\delta^{(3)}(\vec{q}_8 - \vec{q}_{4'}).\end{aligned} \tag{A6}$$

Partial wave amplitudes can be obtained by projecting the transition amplitude onto the partial waves, where $L = 0$ and $l_f = 1$ corresponds to $\bar{p}p$ annihilation at rest. In the low-momentum approximation the integrals can be done analytically, and the partial wave amplitude in the leading order of the external momenta q is given by

$$I_{\text{spatial},L=0,l_f=1}^{s\bar{s}} = qF_{0,1}^{s\bar{s}} f_{0,1}^{s\bar{s}}(\nu,\lambda,\mu,m_f)\exp\{-Q_q^2q^2 - Q_k^2k^2\}. \tag{A7}$$

The geometrical constant $F_{0,1}^{s\bar{s}}$ and the spin-angular momentum function $f_{0,1}^{s\bar{s}}(\nu, \lambda, \mu, m_f)$ are given by

$$F_{0,1}^{s\bar{s}} = 2N\pi^2\left(\frac{1}{Q_{p_2}^2}\right)^{3/2}\left(\frac{3\sqrt{\pi}}{(Q_{p_4}^2)^{5/2}} - \frac{3\sqrt{\pi}}{4(Q_{p_3}^2)^{5/2}}\right),$$

$$f_{0,1}^{s\bar{s}}(\nu, \lambda, \mu, m_f) = (-1)^\nu \delta_{\nu,-\lambda}\delta_{\mu,m_f}, \tag{A8}$$

where $N = N_\phi N_X N_{uuds\bar{s}} N_{\bar{p}}$, and the coefficients in the exponential expression depend on the meson and baryon size parameters,

$$\begin{aligned} Q_k^2 &= \frac{4R_M^2R_B^2 + 9R^2R_B^2 + 3R_M^2R^2}{24(R_M^2 + 3R_B^2)}, \\ Q_q^2 &= \frac{12R_B^4 + 5R_M^2R_B^2 + 36R^2R_B^2 + 12R_M^2R^2}{24(R_M^2 + 3R_B^2)}, \\ Q_{p_2}^2 &= R_M^2, \qquad Q_{p_3}^2 = \frac{1}{2}(R_M^2 + 3R_B^2), \qquad Q_{p_4}^2 = 2R_B^2. \end{aligned} \tag{A9}$$

By using the spatial wave amplitude $I_{\text{spatial}}^{s\bar{s}}$ we obtain the transition amplitude $T_{if}^{s\bar{s}}$ taking the form as in Eq. (31) with the spin-color-flavor weight,

$$\langle f|O_{A_1}|i\rangle = \left\langle f \left| \sum_{\nu,\lambda}(-1)^{\nu+\lambda}\sigma_{-\nu}^{56}\sigma_{-\lambda}^{47} 1_F^{56} 1_F^{47} 1_C^{56} 1_C^{47}(-1)^\nu \times \delta_{\nu,-\lambda}\delta_{\mu,m_f} \right| i \right\rangle. \tag{A10}$$

According to the ${}^3P_0$ quark model the matrix element $\langle f|O_{A_1}|i\rangle$ can be evaluated by using the two-body matrix elements for spin, flavor, and color given by

$$\langle 0|\sigma_v^{ij}|\chi_{m_{ij}}^{J_{ij}}(ij)\rangle = \delta_{J_{ij},1}\delta_{m_{ij},-v}(-1)^v\sqrt{2}, \tag{A11}$$

$$\langle 0|1_F^{ij}|\chi_{t_{ij}}^{T_{ij}}(ij)\rangle = \delta_{T_{ij},0}\delta_{t_{ij},0}\sqrt{2}, \tag{A12}$$

and

$$\langle 0|1_C^{ij}|q_\alpha^i\bar{q}_\beta^j\rangle = \delta_{\alpha\beta}, \tag{A13}$$

where α and β are the color indices. The spin-color-flavor weights $\langle f|O_{A_1}|i\rangle$ are evaluated for various transitions, as listed in Table II.

In case of the simple chiral quark model, the annihilation processes are described by the quark line diagrams A_2 and A_3. Then the transition amplitude is set up as

$$T_{if}^{\text{CQM}} = T_{if}^{\text{CQM}}(\mathcal{O}_{A_2}) + T_{if}^{\text{CQM}}(\mathcal{O}_{A_3}), \tag{A14}$$

where the corresponding transition amplitudes for the two quark line diagrams are given by

$$T_{if}^{\rm CQM}(\mathcal{O}_{A_2}) = \lambda_{A_2}\left\langle f \left| \sum_{\nu,\lambda}(-1)^{\nu+\lambda}\sigma_{-\nu}^{56}\sigma_{-\lambda}^{47}1_F^{56}1_F^{47}1_C^{56}1_C^{47} \times I_{{\rm spatial},A_2}^{\rm CQM}\right| i\right\rangle \qquad (A15)$$

and

$$T_{if}^{\rm CQM}(\mathcal{O}_{A_3}) = \lambda_{A_3}\left\langle f \left| \sum_{\nu,\lambda}(-1)^{\nu+\lambda}\sigma_{-\nu}^{56}\sigma_{-\lambda}^{17}1_F^{56}1_F^{17}1_C^{56}1_C^{17} \times I_{{\rm spatial},A_3}^{\rm CQM}\right| i\right\rangle. \qquad (A16)$$

The initial state $|i\rangle$ and the final state $|f\rangle$ take the same form as defined in Eq. (A2) and (A3), but the 5-quark component in this case is given by

$$\chi_{(1/2),m_{KY}}(uuds\bar{s}) = \sum_{i=1}^{3} G_i|\{\chi^i_{j_s,m_s}(q\bar{s}) \otimes (\ell = 1, \mu)\}_{j_i,m_i} \otimes \chi^i_{(1/2),m_Y}(qqs)\rangle_{(1/2),m_{KY}}, \qquad (A17)$$

where $i = 1, 2, 3$ represent the kaon-hyperon clusters $K^+\Sigma^0$, $K^0\Sigma^+$ and $K^+\Lambda^0$, respectively, and the coefficients G_i are as defined in Eq. (11).

In the low-momentum approximation the partial wave amplitude from each of the quark line diagrams $A1$ and $A2$ in leading order of the external momentum q takes the general form as in Eq. (A7) but with different coefficients. In order to combine the two transition amplitudes, we choose the radial parameters for the baryons and mesons as $R_B = 3.1\ {\rm GeV}^{-1}$, $R_M = 4.1\ {\rm GeV}^{-1}$ [18] and the size parameter between the two quark clusters as $R = 4.1\ {\rm GeV}^{-1}$. Then the total transition amplitude Eq. (A14) becomes

$$T_{if}^{\rm CQM} = \lambda_{\rm CQM}F_{0,1}^{\rm CQM}q\exp\{-Z_q^2q^2 - Z_k^2k^2\}\langle f|O_{\rm CQM}|i\rangle, \qquad (A18)$$

where $F_{0,1}^{\rm CQM} = 4.9\times 10^{-4}\ {\rm GeV}^{-11}$, $Z_q \simeq 2.3\ {\rm GeV}^{-1}$ and $Z_k \simeq 1.3\ {\rm GeV}^{-1}$, and $\lambda_{A_2} = \lambda_{A_3} = \lambda_{\rm CQM}$. The total spin-color-flavor weight $\langle f|O_{\rm CQM}|i\rangle$ is calculated with the spin-angular momentum wave functions in Eq. (A17) and its elements are derived as

$$f_{0,1}^{\rm CQM} = -(-1)^{\nu}\delta_{\nu,-\lambda}\delta_{\mu,m_f} + 2(-1)^{\mu}\delta_{\mu,-\nu}\delta_{\lambda,m_f} + 2(-1)^{\lambda}\delta_{\mu,-\lambda}\delta_{\nu,m_f}. \qquad (A19)$$

Finally, we discuss the third model where the proton wave function includes a 5-quark component in the form of a pentaquark configuration. The ϕ production is described by only the quark line diagram A_1, and the transition amplitude takes the same form as Eq. (A1) but the 5-quark component $|uuds\bar{s}\rangle$ is given by

$$\chi_{(1/2),m_{ps\bar{s}}}(uuds\bar{s}) = |\{\chi_{(1/2),m_{\bar{s}}}(\bar{s}) \otimes (\ell = 1, \mu)\}_{j_i,m_i} \otimes \chi_{s,s_z}(uuds)\rangle_{(1/2),m_{ps\bar{s}}}. \qquad (A20)$$

In the low-momentum approximation, the partial wave amplitude and for the transition of the S-wave $\bar{p}p$ state to the P-wave two-meson final states takes the same form as Eq. (A7). The spin-angular momentum function $f_{0,1}^{s\bar{s}}(\nu, \lambda, \mu, m_f)$ is also the same as the one in Eq. (A8) but the corresponding geometrical constant is given by

$$F_{0,1} = -\frac{3}{16}\sqrt{5}N\pi^4\left(\frac{1}{Q_{p_2}^2}\right)^{3/2}\left(\frac{(\frac{1}{Q_{p_4}^2})^{3/2}}{(Q_{p_3}^2)^{5/2}} - \frac{4(\frac{1}{Q_{p_3}^2})^{3/2}}{(Q_{p_4}^2)^{5/2}}\right), \qquad (A21)$$

with the constants depending on the baryon and meson size parameters,

$$Q_k^2 = \frac{7R_B^2}{30} - \frac{R_B^4}{2(3R_B^2 + R_M^2)}, \quad Q_q^2 = \frac{1}{8}R_B^2\left(5 - \frac{R_B^2}{3R_B^2 + R_M^2}\right),$$
$$Q_{p_2}^2 = R_B^2 + \frac{R_M^2}{2}, \quad Q_{p_3}^2 = \frac{1}{2}(3R_B^2 + R_M^2), \quad Q_{p_4}^2 = 2R_B^2. \qquad (A22)$$

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Progress in Particle and Nuclear Physics 67 (2012) 496–501

Contents lists available at SciVerse ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

Construction of multiquark states in group theory

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ARTICLE INFO

Keywords:
Permutation group
Young tableau
Yamanouchi basis
Projection operator
Pentaquark

ABSTRACT

Permutation groups are applied to analyze the symmetries of multiquark systems and, as examples, wave functions of three-quark and pentaquark states are constructed systematically in the language of Yamanouchi basis.

1. Introduction

Reports of the $\Theta^+(1540)$ resonance with positive strangeness $S = +1$ by various experimental collaborations have sparked an enormous amount of experimental and theoretical studies of exotic baryons. The existence of the $\Theta^+(1540)$ is arguable, but the pentaquark configuration has been considered a possible component of the nucleon [1,2]. The simplest configuration of a baryon with $S = +1$ is $q^4\bar{q}$, a state made of four quarks and an antiquark. It is not straightforward to construct the wave functions, which play a crucial role for any theoretical calculation, of exotic baryons such as pentaquark states, q^3g states and q^6 states since the symmetries of multiquark systems are rather complicated. Here in this work we use the q^3 and pentaquark states as examples to analyze the symmetries of multiquark systems in the language of permutation groups, and construct their wave functions in the framework of Yamanouchi basis.

2. Permutation group and Yamanouchi basis

Yamanouchi basis is also called the standard basis in group theory. For the Young tabloid $\lambda \equiv [\lambda_1\lambda_2, \ldots,]$ in the permutation group S_n, the basis functions are written as

$$|[\lambda_1\lambda_2\cdots](r_nr_{n-1}\cdots r_2r_1)\rangle \tag{1}$$

where λ_i are the number of boxes in the ith row of the Young tabloid $[\lambda]$ and r_i stands for the row where a box is filled with the number i in a Young tableau of the Young tabloid $[\lambda]$. It is indeed that each Young tableau leads to one Yamanouchi basis function and all such defined functions for a Young tabloid together form a complete basis.

The permutation group S_{n-1} is a subgroup of the permutation group S_n, and the elements of S_n which are also in S_{n-1} keep the nth object unchanged. It can be proven according to the representation theory of finite group that

$$D(R) = \sum_r D_r(R), \quad \lambda_r - 1 \geq \lambda_{r+1} \tag{2}$$

where $D(R)$ is the representation $[\lambda]$ of $S_n, D_r(R)$ are the representation matrices of irreducible representations $[\lambda_1, \ldots, \lambda_{r-1}, \lambda_r - 1, \lambda_{r+1}, \ldots, \lambda_m]$ of S_{n-1}. From the above expression one can evaluate the representation matrix $D(R)$ $(R \in S_{n-1})$ of S_n if we know all the representation matrices of the permutation group S_{n-1}.

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0146-6410/$ – see front matter
doi:10.1016/j.ppnp.2012.01.017

Since any element of S_n can be resolved into a product of transpositions, the evaluation of representation matrices $D(R)$ ($R \ni S_{n-1}$) becomes evaluating the representation matrices of the transpositions (i, n). But due to

$$(i, n) = (n-1, n)(i, n-1)(n-1, n) \tag{3}$$

we need to evaluate only the matrix $D(n-1, n)$. The matrix $D(n-1, n)$ plus the known matrices $D(R \in S_{n-1})$ give the $[\lambda]$ representation matrices for all the elements in S_n.

The operation of the transposition $(n-1, n)$ of the permutation group S_n on the standard basis obeys the followings:

$$\begin{aligned}
&(n-1, n)|[\lambda](r, r, r_{n-2}, \dots,)\rangle = +|[\lambda](r, r, r_{n-2}, \dots,)\rangle, \\
&(n-1, n)|[\lambda](r, r-1, r_{n-2}, \dots,)\rangle = -|[\lambda](r, r-1, r_{n-2}, \dots,)\rangle
\end{aligned} \tag{4}$$

when $|[\lambda](r-1, r, r_{n-2}, \dots,)\rangle$ does not exist, and

$$(n-1, n)|[\lambda](r, s, r_{n-2}, \dots,)\rangle = \sigma_{rs}|[\lambda](r, s, r_{n-2}, \dots,)\rangle + \sqrt{1-\sigma_{rs}^2}|[\lambda](s, r, r_{n-2}, \dots,)\rangle \tag{5}$$

with

$$\sigma_{rs} = \frac{1}{(\lambda_r - r) - (\lambda_s - s)} \tag{6}$$

when $|[\lambda](r, s, r_{n-2}, \dots, r_2, r_1)\rangle$ and $|[\lambda](s, r, r_{n-2}, \dots, r_2, r_1)\rangle$ all exist and $r \neq s$.

As an example, we now construct the matrices of the representation [211] of S_4. There are three Young tableaux and hence three Yamanouchi basis functions for [211],

$$\phi_1 = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & \multicolumn{1}{c}{} \\ \cline{1-1} 4 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array} = |[211](3211)\rangle, \qquad \phi_2 = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1} 4 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array} = |[211](3121)\rangle, \qquad \phi_3 = \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1} 3 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array} = |[211](1321)\rangle. \tag{7}$$

For the elements (12), (13) and (23), we can directly write down the representation matrices since these elements are also the elements of S_3

$$\begin{aligned}
D^{[211]}(13) &= D^{[21]}(13) \oplus D^{[111]}(13) = \begin{pmatrix} -1/2 & -\sqrt{3}/2 & 0 \\ -\sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & -1 \end{pmatrix} \\
D^{[211]}(12) &= D^{[21]}(12) \oplus D^{[111]}(12) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \\
D^{[211]}(23) &= D^{[21]}(23) \oplus D^{[111]}(23) = \begin{pmatrix} -1/2 & \sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & -1 \end{pmatrix}.
\end{aligned} \tag{8}$$

Here we have used that $\phi_1 = |[211](3211)\rangle$ and $\phi_2 = |[211](3121)\rangle$ are respectively the [21] basis functions $|[21](211)\rangle$ and $|[21](121)\rangle$ of S_3 while ϕ_3 is the [111] basis function $|[21](321)\rangle$ of S_3, and

$$\begin{aligned}
&D^{[21]}(12) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad D^{[21]}(13) = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix}, \\
&D^{[21]}(23) = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}.
\end{aligned} \tag{9}$$

For the element (34) of S_4, we get from Eqs. (4) and (5),

$$\begin{aligned}
&(34)|[211](3211)\rangle = -|[211](3211)\rangle \\
&(34)|[211](3121)\rangle = \sigma_{31}|[211](3121)\rangle + \sqrt{1-\sigma_{31}^2}|[211](1321)\rangle \\
&(34)|[211](1321)\rangle = \sigma_{13}|[211](1321)\rangle + \sqrt{1-\sigma_{13}^2}|[211](3121)\rangle
\end{aligned} \tag{10}$$

with

$$\sigma_{31} = \frac{1}{(\lambda_3 - 3) - (\lambda_1 - 1)} = -\frac{1}{3} = -\sigma_{13}. \tag{11}$$

Thus in the Yamanouchi basis of ϕ_1, ϕ_2 and ϕ_3, the [211] representation matrix of the element (34) is

$$D(34) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1/3 & 2\sqrt{2}/3 \\ 0 & 2\sqrt{2}/3 & 1/3 \end{pmatrix}. \tag{12}$$

The representation matrices for other elements can be worked out from the matrices derived above.

3. q^3 baryon states

Baryon wave functions contain contributions of the spatial degrees of freedom and the internal degrees of freedom of color, flavor and spin. The internal degrees of freedom are taken to be the three light flavors u, d, s with spin 1/2 and three possible colors r, g, b. The quark transforms under the fundamental representation of $SU(m)$, whereas the antiquark transforms under the conjugate representation of $SU(m)$, with $m = 2, 3, 3, 6$ for the spin, flavor, color and spin–flavor degrees of freedom, respectively. The group representation theory states that n-quark states $|q_1\rangle|q_2\rangle\cdots|q_n\rangle$ form a m^n-dimensional direct product basis of $SU(m)$, and the direct product representation can be decomposed according to the irreducible representations of the permutation group S_n.

The explicit form of baryon spin and flavor functions can be easily derived in the framework of Yamanouchi basis by following the process: (1) Work out first the matrices of the irreducible representations of S_3 in Yamanouchi basis; (2) Construct the projection operator for each representation; (3) Let the projection operators act on arbitrary three-quark states to obtain the spin and flavor wave functions with the corresponding symmetries.

The projection operator corresponding to the Yamanouchi basis function $|[\lambda](r)\rangle$ of the representation $[\lambda]$ of S_n takes the form

$$W_{(r)}^{[\lambda]} = \sum_i \langle[\lambda](r)|R_i|[\lambda](r)\rangle R_i \tag{13}$$

where R_i stand for all the permutations of S_n. By letting $W_{(r)}^{[\lambda]}$ act on any function $f_1 f_2 \cdots f_n$, one could derive the corresponding standard basis function. Using the representation matrices of the permutation group S_3, we can directly write down the projection operators [3], for example,

$$\begin{aligned} P^\lambda &= 1 + (12) - \frac{1}{2}(13) - \frac{1}{2}(23) - \frac{1}{2}(123) - \frac{1}{2}(132) \\ P^\rho &= 1 - (12) + \frac{1}{2}(13) + \frac{1}{2}(23) - \frac{1}{2}(123) - \frac{1}{2}(132). \end{aligned} \tag{14}$$

Let the operators P^λ and P^ρ act on, for instance, the state udu (with $u \equiv \phi_u$ and $d \equiv \phi_d$). We have

$$\begin{aligned} P^\lambda udu &\Longrightarrow \phi_\lambda = \frac{1}{\sqrt{6}}\left[udu + duu - 2uud\right], \\ P^\rho udu &\Longrightarrow \phi_\rho = \frac{1}{\sqrt{2}}\left[udu - duu\right]. \end{aligned} \tag{15}$$

Spin–flavor wave functions of various permutation symmetries may be written in the general form,

$$\phi_{S,A,\lambda,\rho} = \sum_{i=S,A,\lambda,\rho}\ \sum_{j=S,A,\lambda,\rho} a_{ij}\phi_i\chi_j \tag{16}$$

where ϕ_i and χ_i are respectively the q^3 spin and flavor wave functions of symmetry (S), antisymmetry (A), λ-type and ρ-type. The coefficient a_{ij} in Eq. (16) can be determined by applying the permutation operators of S_3 to the general form. Check the simplest case,

$$(23)\left(a\phi_\lambda\chi_\lambda + b\phi_\rho\chi_\rho\right) = \left(\frac{1}{4}a + \frac{3}{4}b\right)\phi_\lambda\chi_\lambda + \left(\frac{3}{4}a + \frac{1}{4}b\right)\phi_\rho\chi_\rho - \frac{\sqrt{3}}{4}(a-b)\left(\phi_\lambda\chi_\rho + \phi_\rho\chi_\lambda\right). \tag{17}$$

$a = b$ leads to a fully symmetric spin–flavor wave function. Here we have used $D^{[21]}(23)$ in Eq. (9). The spin–flavor wave functions of other permutation symmetries can be worked out the same way.

4. Pentaquark states

It is not straightforward to construct the wave function of pentaquark states since the permutation symmetries of the five-particle system are rather complicated. Here we will first analyze the symmetries of pentaquark states in the language of permutation groups in a shortened manner. For the details we may refer to Ref. [4]. Then we construct the wave functions of pentaquark antidecuplet states in the framework of the Yamanouchi basis.

The algebraic structure of pentaquark states consists of the usual spin–flavor and color algebras

$$SU_{\mathrm{sf}}(6) \otimes SU_{\mathrm{c}}(3) \tag{18}$$

with

$$SU_{\mathrm{sf}}(6) = SU_{\mathrm{f}}(3) \otimes SU_{\mathrm{s}}(2). \tag{19}$$

In the language of group theory, the permutation symmetry of the four-quark configuration of pentaquark states is characterized by the S_4 Young tabloids [4], [31], [22], [211] and [1111]. That the pentaquark should be a color singlet demands that the color part of the pentaquark wave function must be a $[222]_1$ singlet. Since the color part of the antiquark in pentaquark states is a $[11]_3$ antitriplet

$$\psi^c(\bar{q}) = \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} \tag{20}$$

the color wave function of the four-quark configuration must be a $[211]_3$ triplet

$$\psi^c(q^4) = \begin{array}{|c|c|}\hline & \\ \hline \\ \cline{1-1} \\ \cline{1-1} \end{array} \tag{21}$$

That the total wave function of the four quark configuration is antisymmetric implies that its orbital-spin–flavor part must be a [31] state

$$\psi^{\text{osf}}(q^4) = \begin{array}{|c|c|c|}\hline & & \\ \hline \\ \cline{1-1} \end{array}\,. \tag{22}$$

The total wave function of the q^4 configuration may be written in the general form

$$\psi = \sum_{i,j=\lambda,\rho,\eta} a_{ij}\psi^c_{[211]_i}\psi^{\text{osf}}_{[31]_j}. \tag{23}$$

Considering that the [31] and [211] partitions are conjugate each other, an antisymmetric ψ may be formed by only three components, that is

$$\psi = a_{\lambda\rho}\phi^c_{[211]_\lambda}\psi^{\text{osf}}_{[31]_\rho} + a_{\rho\lambda}\psi^c_{[211]_\rho}\psi^{\text{osf}}_{[31]_\lambda} + a_{\eta\eta}\psi^c_{[211]_\eta}\psi^{\text{osf}}_{[31]_\eta}. \tag{24}$$

Applying the permutation (34) of S_4 to Eq. (24), we have

$$\begin{aligned}(34)\psi &= -a_{\lambda\rho}\psi^c_{[211]_\lambda}\psi^{\text{osf}}_{[31]_\rho} + a_{\rho\lambda}\left(-\frac{1}{3}\psi^c_{[211]_\rho} + \frac{2\sqrt{2}}{3}\psi^c_{[211]_\eta}\right)\left(\frac{1}{3}\psi^{\text{osf}}_{[31]_\lambda} + \frac{2\sqrt{2}}{3}\psi^{\text{osf}}_{[31]_\eta}\right) \\ &\quad + a_{\eta\eta}\left(\frac{2\sqrt{2}}{3}\psi^c_{[211]_\rho} + \frac{1}{3}\psi^c_{[211]_\eta}\right)\left(\frac{2\sqrt{2}}{3}\psi^{\text{osf}}_{[31]_\lambda} - \frac{1}{3}\psi^{\text{osf}}_{[31]_\eta}\right).\end{aligned} \tag{25}$$

An antisymmetric ψ demands $a_{\rho\lambda} = -a_{\eta\eta}$. Here we have used the [31] and [211] representation matrices for the permutation (34) of the S_4,

$$D^{[31]}(34) = \begin{pmatrix} -1/3 & 2\sqrt{2}/3 & 0 \\ 2\sqrt{2}/3 & 1/3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad D^{[211]}(34) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1/3 & 2\sqrt{2}/3 \\ 0 & 2\sqrt{2}/3 & 1/3 \end{pmatrix}. \tag{26}$$

By applying the permutation (12) or (23) of the S_4 to Eq. (24) the same way, we get $a_{\rho\lambda} = -a_{\lambda\rho}$. Finally, we derive a fully antisymmetric wave function for the q^4 configuration

$$\psi = \frac{1}{\sqrt{3}}\left(\psi^c_{[211]_\lambda}\psi^{\text{osf}}_{[31]_\rho} - \psi^c_{[211]_\rho}\psi^{\text{osf}}_{[31]_\lambda} + \psi^c_{[211]_\eta}\psi^{\text{osf}}_{[31]_\eta}\right). \tag{27}$$

The total wave function of ground state pentaquarks may take the form

$$\Psi = \frac{1}{\sqrt{3}}\left[\Psi^o_{[5]}\left(\Psi^c_{[211]_\lambda}\Psi^{\text{sf}}_{[31]_\rho} - \Psi^c_{[211]_\rho}\Psi^{\text{sf}}_{[31]_\lambda} + \Psi^c_{[211]_\eta}\Psi^{\text{sf}}_{[31]_\eta}\right)\right] \tag{28}$$

where Ψ^o, Ψ^c and Ψ^{sf} are respectively the orbital, color and spin–flavor parts of the pentaquark states. The subscripts [211] and [31] of the Ψ^c and Ψ^{sf} stand for the symmetries of the q^4 configuration in pentaquark states. For the pentaquark states with isospin $I = 0$ and strangeness $S = 1$, the spin–flavor wave function of the q^4 configuration must have the symmetry,

$$\begin{array}{ccccc} [31] & = & [22] & \otimes & [31] \\ SU_{\text{sf}}(6) & & SU_{\text{f}}(3) & & SU_{\text{s}}(2). \end{array} \tag{29}$$

Again, the spin–flavor wave functions of various permutation symmetries take the general form,

$$\psi^{\text{sf}}_{[31]} = \sum_{i=\lambda,\rho}\sum_{j=\lambda,\rho,\eta} a_{ij}\phi_{[22]_i}\chi_{[31]_j} \tag{30}$$

a_{ij} can be determined by acting the permutations of S_4 on the general form. The spin–flavor wave functions for the q^4 cluster are finally derived as,

$$\begin{aligned}
\psi^{sf}_{[31]_\rho} &= -\frac{1}{2}\phi_{[22]_\rho}\chi_{[31]_\lambda} - \frac{1}{2}\phi_{[22]_\lambda}\chi_{[31]_\rho} + \frac{1}{\sqrt{2}}\phi_{[22]_\rho}\chi_{[31]_\eta}\\
\psi^{sf}_{[31]_\lambda} &= -\frac{1}{2}\phi_{[22]_\rho}\chi_{[31]_\rho} + \frac{1}{2}\phi_{[22]_\lambda}\chi_{[31]_\lambda} + \frac{1}{\sqrt{2}}\phi_{[22]_\lambda}\chi_{[31]_\eta}\\
\psi^{sf}_{[31]_\eta} &= \frac{1}{\sqrt{2}}\phi_{[22]_\rho}\chi_{[31]_\rho} + \frac{1}{\sqrt{2}}\phi_{[22]_\lambda}\chi_{[31]_\lambda}.
\end{aligned} \tag{31}$$

The explicit form of the spin and flavor wave functions of the q^4 configuration of pentaquark states can be easily worked out in the Yamanouchi technique, following the three-step process as mentioned in the previous section. The λ-type and ρ-type projection operators for the representation [22] as well as the λ-type, ρ-type and η-type projection operators for the representation [31] are derived as follows:

$$\begin{aligned}
P_{[22]_\lambda} = {} & 2 + 2(12) - (13) - (14) - (23) - (24) + 2(34) + 2(12)(34) + 2(14)(23)\\
& + 2(13)(24) - (123) - (124) - (132) - (134) - (142) - (143) - (234) - (243)\\
& - (1234) - (1243) + 2(1324) - (1342) + 2(1423) - (1432),\\
P_{[22]_\rho} = {} & 2 - 2(12) + (13) + (14) + (23) + (24) - 2(34) + 2(12)(34) + 2(14)(23)\\
& + 2(13)(24) - (123) - (124) - (132) - (134) - (142) - (143) - (234) - (243)\\
& + (1234) + (1243) - 2(1324) + (1342) - 2(1423) + (1432),\\
P_{[31]_\lambda} = {} & 6 + 6(12) - 3(13) + 5(14) - 3(23) + 5(24) + 2(34) + 2(12)(34) - 4(14)(23)\\
& - 4(13)(24) - 3(123) + 5(124) - 3(132) - (134) + 5(142) - (143) - (234)\\
& - (243) - (1234) - (1243) - 4(1324) - (1342) - 4(1423) - (1432),\\
P_{[31]_\rho} = {} & 2 - 2(12) + (13) + (14) + (23) + (24) + 2(34) - 2(12)(34) - (123) - (124) - (132) + (134)\\
& - (142) + (143) + (234) + (243) - (1234) - (1243) - (1342) - (1432),\\
P_{[31]_\eta} = {} & 3 + 3(12) + 3(13) - (14) + 3(23) - (24) - (34) - (12)(34) - (14)(23) - (13)(24)\\
& + 3(123) - (124) + 3(132) - (134) - (142) - (143) - (234) - (243)\\
& - (1234) - (1243) - (1324) - (1342) - (1423) - (1432).
\end{aligned} \tag{32}$$

The flavor wave functions of the four-quark subsystem with the [22] symmetry are derived by operating $P_{[22]_{\lambda,\rho}}$ on any q^4 state. For instance,

$$\begin{aligned}
P_{[22]_\rho}(uudd) &\Longrightarrow \phi_{[22]_\rho} = \frac{1}{2}(dudu - duud + udud - uddu)\\
P_{[22]_\lambda}(uudd) &\Longrightarrow \phi_{[22]_\lambda} = \frac{1}{2\sqrt{3}}(duud + udud - 2uudd + uddu + dudu - 2dduu).
\end{aligned} \tag{33}$$

The flavor wave functions for the antidecuplet Θ^+ pentaquark with $I = I_3 = 0$ are given by

$$\Phi_{[22]_\rho} = \phi_{[22]_\rho}\bar{s}, \qquad \Phi_{[22]_\lambda} = \phi_{[22]_\lambda}\bar{s}. \tag{34}$$

The flavor states with other values of isospin (I, I_3) and strangeness S can be derived the same way.

The spin wave functions of the four-quark subsystem with the [31] symmetry can be derived by operating $P_{[31]_{\lambda,\rho,\eta}}$ on any q^4 spin state, for example, the state $\uparrow\uparrow\uparrow\downarrow$,

$$\begin{aligned}
P_{[31]_\eta}(\uparrow\uparrow\uparrow\downarrow) &\Longrightarrow \chi_{[31]_\eta}(s_{q^4} = 1, m_{q^4} = 1) = \frac{1}{2\sqrt{3}}\,|\downarrow\uparrow\uparrow\uparrow + \uparrow\downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow\uparrow - 3\uparrow\uparrow\uparrow\downarrow\rangle\\
P_{[31]_\rho}(\uparrow\uparrow\uparrow\downarrow) &\Longrightarrow \chi_{[31]_\rho}(s_{q^4} = 1, m_{q^4} = 1) = \frac{1}{\sqrt{2}}\,|\downarrow\uparrow\uparrow\uparrow - \uparrow\downarrow\uparrow\uparrow\rangle\\
P_{[31]_\lambda}(\uparrow\uparrow\uparrow\downarrow) &\Longrightarrow \chi_{[31]_\lambda}(s_{q^4} = 1, m_{q^4} = 1) = \frac{1}{\sqrt{6}}\,|\downarrow\uparrow\uparrow\uparrow + \uparrow\downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow\uparrow\rangle.
\end{aligned} \tag{35}$$

The total spin wave function of the pentaquark states is formed by combining the spin wave functions in Eq. (35) of the four-quark subsystem with the one of the antiquark, that is

$$\chi(q^4\bar{s})_{[31]_\alpha} = \sqrt{\frac{2}{3}}\chi_{[31]_\alpha}(s_{q^4} = 1, m_{q^4} = 1)\chi_{\bar{s}}(-1/2) - \sqrt{\frac{1}{3}}\chi_{[31]_\alpha}(s_{q^4} = 1, m_{q^4} = 0)\chi_{\bar{s}}(1/2) \tag{36}$$

with $\alpha = \rho, \lambda, \eta$. The states with other values of the projection m_s can be obtained the same way.

Combining the flavor wave functions in Eq. (34) and the spin wave functions, we derive the total spin–flavor wave function of the pentaquark state with isospin $I = 0$, strangeness $S = 1$ and spin $s = 1/2$,

$$\Psi^{sf}_{[31]_\rho} = -\frac{1}{2}\Phi_{[22]_\rho}\chi(q^4\bar{s})_{[31]_\lambda} - \frac{1}{2}\Phi_{[22]_\lambda}\chi(q^4\bar{s})_{[31]_\rho} + \frac{1}{\sqrt{2}}\Phi_{[22]_\rho}\chi(q^4\bar{s})_{[31]_\eta}$$
$$\Psi^{sf}_{[31]_\lambda} = -\frac{1}{2}\Phi_{[22]_\rho}\chi(q^4\bar{s})_{[31]_\rho} + \frac{1}{2}\Phi_{[22]_\lambda}\chi(q^4\bar{s})_{[31]_\lambda} + \frac{1}{\sqrt{2}}\Phi_{[22]_\lambda}\chi(q^4\bar{s})_{[31]_\eta}$$
$$\Psi^{sf}_{[31]_\eta} = \frac{1}{\sqrt{2}}\Phi_{[22]_\rho}\chi(q^4\bar{s})_{[31]_\rho} + \frac{1}{\sqrt{2}}\Phi_{[22]_\lambda}\chi(q^4\bar{s})_{[31]_\lambda}. \quad (37)$$

The color wave function of the q^4 configuration can be derived by applying the λ-type, ρ-type and η-type projection operators for the representation [211] of the S_4 to an arbitrary four-quark color state, for example, *RRGB*. We have

$$\begin{aligned}\psi^c_{[211]_\lambda}(R) &= \frac{1}{\sqrt{16}}(2|RRGB\rangle - 2|RRBG\rangle - |GRRB\rangle - |RGRB\rangle - |BRGR\rangle \\ &\quad - |RBGR\rangle + |BRRG\rangle + |GRBR\rangle + |RBRG\rangle + |RGBR\rangle),\\ \psi^c_{[211]_\rho}(R) &= \frac{1}{\sqrt{48}}(3|RGRB\rangle - 3|GRRB\rangle + 3|BRRG\rangle - 3|RBRG\rangle + 2|GBRR\rangle \\ &\quad - 2|BGRR\rangle - |BRGR\rangle + |RBGR\rangle + |GRBR\rangle - |RGBR\rangle),\\ \psi^c_{[211]_\eta}(R) &= \frac{1}{\sqrt{6}}(|BRGR\rangle + |RGBR\rangle + |GBRR\rangle - |RBGR\rangle - |GRBR\rangle - |BGRR\rangle).\end{aligned} \quad (38)$$

The singlet color wave function $\Psi^c_{[211]_j}(j = \lambda, \rho, \eta)$ in Eq. (28) is given by

$$\Psi^c_{[211]_j} = \frac{1}{\sqrt{3}}\left[\psi^c_{[211]_j}(R)\bar{R} + \psi^c_{[211]_j}(G)\bar{G} + \psi^c_{[211]_j}(B)\bar{B}\right] \quad (39)$$

where $\psi^c_{[211]_j}(G)$ and $\psi^c_{[211]_j}(B)$ are the color G and B wave functions of the q^4 cluster, derived by acting the projection operators in Eq. (32) on the states *GGRB* and *BBGR*, respectively.

So far we have worked out all the pieces of the pentaquark wave function in Eq. (28), which is for the ground state pentaquark with isospin $I = 0$, strangeness $S = 1$, spin $s = 1/2$ and $m_s = 1/2$.

5. Conclusion

The representation of permutation groups and the main features of the Yamanouchi basis are outlined. Three-quark and pentaquark states are analyzed in the language of permutation groups, and their wave functions are constructed in the framework of Yamanouchi basis. We show that the Yamanouchi basis approach is an easy, systematical way to work out multiquark wave functions.

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