

ON GENERALIZATION OF PRIMENESS IN MODULE CATEGORY

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ABSTRACT

Prime ideals play an important role in the study of ring structures, especially in commutative algebras.

Motivating the definition of prime submodules of Sanh et al. in 2008, we introduced and investigated the class of IFP and nearly prime submodules. Using our notions, we generalized the Anderson's Theorem, following that for a finitely generated, quasi-projective, fully IFP module M , which is a self-generator, if every minimal prime submodule over a proper fully invariant submodule U of M is finitely generated, then there are finitely many minimal prime submodules over U .

The main result in this thesis is that a finitely generated right R -module is Noetherian if and only if every nearly prime submodule is finitely generated. This can be considered as a generalization of Cohen's Theorem in commutative rings.

KEY WORDS: IFP MODULES / FULLY IFP MODULES
ANDERSON'S THEOREM / NEARLY PRIME SUBMODULES
COHEN'S THEOREM / NOETHERIAN MODULES

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