

ภาคผนวก ๔.

ระเบียบวิธีเชิงตัวเลขในการวิเคราะห์กระบวนการถ่ายเทคุณสมบัติ มวลสารและความดันในวัสดุพื้นที่ไม่อิ่มตัวภายในได้พัฒนาไมโครเวฟ

สมการถ่ายเทมวลสาร (Mass Transport Equation)

สมการถ่ายเทมวลสาร เป็นสมการที่อธิบายถึงการกระจายของความชื้นในระบบและอัตราการเปลี่ยนแปลงของความชื้นเมื่อระยะเวลาการอบแห้งเปลี่ยนไป สำหรับกระบวนการอบแห้งด้วยพัฒนาไมโครเวฟสมการถ่ายเทมวลสารเกิดจากสมมุติฐานการเคลื่อนที่ของของไนโตรสูตรูปแบบ เขียนได้ดังนี้

$$\varphi \frac{\partial}{\partial t} \{s + Y_v(1-s)\} + \frac{\partial}{\partial z} \left[\frac{KK_{rl}}{\mu_l} \left(\frac{\partial P_c}{\partial z} - \frac{\partial P_g}{\partial z} + g_z \right) + Y_v \frac{KK_{rg}}{\mu_g} \left(-\frac{\partial P_g}{\partial z} + \rho_g g_z \right) - Y_g D_m \frac{\partial}{\partial z} (W_v) \right] = 0 \quad (\text{๔.1})$$

$$\text{โดยที่ } \frac{\rho_v}{\rho_l} = Y_v, \quad \frac{\rho_g}{\rho_l} = Y_g, \quad \frac{Y_v}{Y_g} = W_v$$

โดยเมื่อวิเคราะห์ด้วยระเบียบวิธีเชิงตัวเลขโดยวิธีผลต่างสี่บเนื่อง (finite difference) บนพื้นฐานวิธีปริมาตรควบคุม (control volume) จะได้

$$\begin{aligned} \text{เมื่อ } \frac{KK_{rl}}{\mu_l} \Bigg|_{k \pm \frac{l}{2}} &= \frac{\frac{KK_{rl}}{\mu_l} \Bigg|_k + \frac{KK_{rl}}{\mu_l} \Bigg|_{k \pm l}}{2}, & \frac{KK_{rg}}{\mu_g} \Bigg|_{k \pm \frac{l}{2}} &= \frac{\frac{KK_{rg}}{\mu_g} \Bigg|_k + \frac{KK_{rg}}{\mu_g} \Bigg|_{k \pm l}}{2} \\ D_{k \pm \frac{l}{2}}^{n+1} &= \frac{D_k^{n+1} + D_{k \pm l}^{n+1}}{2} & & \quad (\text{๔.2}) \end{aligned}$$

$$\text{และ } s^{n+1} = s^n - \Delta s, \quad P_g^{n+1} = P_g^n - \Delta P_g$$

$$T^{n+1} = T^n - \Delta T \quad (\text{๔.3})$$

ดังนั้น

$$\begin{aligned}
 & \frac{\phi}{\Delta t} \left\{ (1 - s_{ir}) (s_{ek}^{n+1} - s_{ek}^n) + (1 - s_{ir}) (Y_{vk}^{n+1} (1 - s_{ek}^{n+1}) - Y_{vk}^n (1 - s_{ek}^n)) \right\} \\
 & \left[\left. \frac{KK_{rl}}{\mu_l} \right|_{k+\frac{1}{2}} \left(\frac{P_{ck+1}^{n+1} - P_{ck}^{n+1}}{\Delta z} - \frac{P_{gk+1}^{n+1} - P_{gk}^{n+1}}{\Delta z} + g_z \right) - \left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \left(\frac{P_{ck}^{n+1} - P_{ck-1}^{n+1}}{\Delta z} - \frac{P_{gk}^{n+1} - P_{gk-1}^{n+1}}{\Delta z} + g_z \right) \right] \\
 & + \frac{I}{\Delta z} \left\{ + Y_{vk}^{n+1} \left[\left. \frac{KK_{rg}}{\mu_g} \right|_{k+\frac{1}{2}} \left(\frac{-P_{gk+1}^{n+1} + P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) - \left. \frac{KK_{rg}}{\mu_g} \right|_{k-\frac{1}{2}} \left(\frac{-P_{gk}^{n+1} + P_{gk-1}^{n+1}}{\Delta z} + \rho_g g_z \right) \right] \right. \\
 & \left. - Y_{vk}^{n+1} \left[D_{mk+\frac{1}{2}}^{n+1} \left(\frac{w_{vk+1}^{n+1} - w_{vk}^{n+1}}{\Delta z} \right) - D_{mk-\frac{1}{2}}^{n+1} \left(\frac{w_{vk}^{n+1} - w_{vk-1}^{n+1}}{\Delta z} \right) \right] \right\} \\
 & = 0 = F_k^{n+1}(s, P_g, T)
 \end{aligned} \tag{¶.4}$$

กรณีพิจารณาโนนดภายใน (Internal node)

เมื่อพิจารณาสมการถ่ายเทมวัลสารเที่ยบกับความชื้น (เที่ยบ s)

ณ ตำแหน่ง k

$$\begin{aligned}
 & \frac{\partial F_k^{n+1}}{\partial s_{ek}^{n+1}} = \frac{\phi}{\Delta t} (1 - s_{ir}) \left\{ (1 - Y_{vk}^{n+1}) \right. \\
 & \left. \left[\left. \frac{KK_{rl}}{\mu_l} \right|_{k+\frac{1}{2}} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{-P_{ck}^{n+1}}{\Delta z} \right) + \left(\frac{P_{ck+1}^{n+1} - P_{ck}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \right)_k \right) \right. \\
 & \left. - \left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{P_{ck}^{n+1}}{\Delta z} \right) - \left(\frac{P_{ck}^{n+1} - P_{ck-1}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \right)_k \right) \right\} \\
 & + \frac{I}{\Delta z} \left\{ - \left[\left. \frac{KK_{rl}}{\mu_l} \right|_{k+\frac{1}{2}} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{-P_{gk}^{n+1}}{\Delta z} \right) + \left(\frac{P_{gk+1}^{n+1} - P_{gk}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \right)_k \right) \right. \\
 & \left. - \left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{P_{gk}^{n+1}}{\Delta z} \right) - \left(\frac{P_{gk}^{n+1} - P_{gk-1}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \right)_k \right] \right\} \\
 & + \left[\left(Y_{vk}^{n+1} \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \right)_k \right) \left(\frac{P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \right] - \left(Y_{vk-1}^{n+1} \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \right)_k \right) \left(\frac{-P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \\
 & - Y_{vk}^{n+1} \left[\frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} D_{mk}^{n+1} \left(\frac{w_{vk+1}^{n+1} - w_{vk}^{n+1}}{\Delta z} \right) - \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} D_{mk}^{n+1} \left(\frac{w_{vk}^{n+1} - w_{vk-1}^{n+1}}{\Delta z} \right) \right] \\
 & = 0 = Fa
 \end{aligned} \tag{¶.5}$$

ณ ตำแหน่ง $k+1$

$$\frac{\partial F_{k+1}^{n+1}}{\partial s_{ek}^{n+1}} = \frac{I}{\Delta z} \left\{ \begin{aligned} & \left[\left(\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{-P_{ck}^{n+1}}{\Delta z} \right) + \left(\frac{P_{ck+1}^{n+1} - P_{ck}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \right] \\ & - \left(\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{-P_{gk}^{n+1}}{\Delta z} \right) + \left(\frac{P_{gk+1}^{n+1} - P_{gk}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \\ & + \left[\left(Y_{vk}^{n+1} \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \Big|_k \right) \right) \left(\frac{P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \right] \\ & - Y_{vk}^{n+1} \left[\frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} D_{mk+1}^{n+1} \left(\frac{w_{vk+1}^{n+1} - w_{vk}^{n+1}}{\Delta z} \right) \right] \end{aligned} \right\} \\ & = Fb \end{math>$$

(¶.6)

ณ ตำแหน่ง $k-1$

$$\frac{\partial F_{k-1}^{n+1}}{\partial s_{ek}^{n+1}} = \frac{I}{\Delta z} \left\{ \begin{aligned} & \left[\left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{P_{ck}^{n+1}}{\Delta z} \right) - \left(\frac{P_{ck}^{n+1} - P_{ck-1}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \right] \\ & - \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{P_{gk}^{n+1}}{\Delta z} \right) - \left(\frac{P_{gk}^{n+1} - P_{gk-1}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \\ & + \left[\left(Y_{vk-1}^{n+1} \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \Big|_k \right) \right) \left(\frac{-P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \right] - Y_{vk}^{n+1} \left[-\frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} D_{mk-1}^{n+1} \left(\frac{w_{vk}^{n+1} - w_{vk-1}^{n+1}}{\Delta z} \right) \right] \end{aligned} \right\}$$

$= 0 = Fc$

(¶.7)

ເນື້ອພິຈາລະນາສົມກາຮ່າຍເທມວລສາຣເຖີຍບກັບຄວາມດັນ (ເຖີຍບ P)

ນີ້ ຕຳແໜ່ງ k

$$\begin{aligned}
 \frac{\partial F_k^{n+1}}{\partial P_k^{n+1}} &= \frac{\phi}{\Delta t} (1 - s_{ir}) \left\{ (1 - s_{ek}^{n+1}) \frac{\partial}{\partial P_k^{n+1}} Y_{vk}^{n+1} \right\} \\
 &\quad \left[\left(\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{-P_{ck}^{n+1}}{\Delta z} \right) + \left(\frac{P_{ck+1}^{n+1} - P_{ck}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \right. \\
 &\quad \left. \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{P_{ck}^{n+1}}{\Delta z} \right) - \left(\frac{P_{ck}^{n+1} - P_{ck-1}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \right] \\
 &+ \frac{I}{\Delta z} \left\{ - \left[\left(\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{-P_{gk}^{n+1}}{\Delta z} \right) + \left(\frac{P_{gk+1}^{n+1} - P_{gk}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \right. \right. \\
 &\quad \left. \left. - \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{P_{gk}^{n+1}}{\Delta z} \right) - \left(\frac{P_{gk}^{n+1} - P_{gk-1}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \right] \right. \\
 &\quad \left. + \left[\left(Y_{vk}^{n+1} \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \Big|_k \right) \right) \left(\frac{P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \right] - \left(Y_{vk-1}^{n+1} \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \Big|_k \right) \right) \left(\frac{-P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \right] \right] \\
 &\quad \left. - Y_{vk}^{n+1} \left[\frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk}^{n+1} \left(\frac{w_{vk+1}^{n+1} - w_{vk}^{n+1}}{\Delta z} \right) - \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk}^{n+1} \left(\frac{w_{vk}^{n+1} - w_{vk-1}^{n+1}}{\Delta z} \right) \right] \right\} \\
 &= 0 = Fd
 \end{aligned}$$

(¶.8)

ณ ตำแหน่ง $k+1$

$$\frac{\partial F_{k+1}^{n+1}}{\partial P_k^{n+1}} = \frac{I}{\Delta z} \left[\begin{aligned} & \left[\left(\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{-P_{ck}^{n+1}}{\Delta z} \right) + \left(\frac{P_{ck+1}^{n+1} - P_{ck}^{n+1}}{\Delta z} + g_z \right) \frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \right] \\ & - \left[\left(\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{-P_{gk}^{n+1}}{\Delta z} \right) + \left(\frac{P_{gk+1}^{n+1} - P_{gk}^{n+1}}{\Delta z} + g_z \right) \frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \right] \\ & + \left[\left(Y_{vk}^{n+1} \frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \Big|_k \right) \right) \left(\frac{P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \right] \\ & - Y_{vk}^{n+1} \left[\frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk}^{n+1} \left(\frac{w_{vk+1}^{n+1} - w_{vk}^{n+1}}{\Delta z} \right) \right] \end{aligned} \right] \\ & = 0 = Fe \end{aligned} \tag{¶.9}$$

ณ ตำแหน่ง $k-1$

$$\frac{\partial F_{k-1}^{n+1}}{\partial P_k^{n+1}} = \frac{I}{\Delta z} \left[\begin{aligned} & \left[\left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{P_{ck}^{n+1}}{\Delta z} \right) - \left(\frac{P_{ck}^{n+1} - P_{ck-1}^{n+1}}{\Delta z} + g_z \right) \frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \right] \\ & - \left[\left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{P_{gk}^{n+1}}{\Delta z} \right) - \left(\frac{P_{gk}^{n+1} - P_{gk-1}^{n+1}}{\Delta z} + g_z \right) \frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \right] \\ & + \left[\left(Y_{vk-1}^{n+1} \frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \Big|_k \right) \right) \left(\frac{-P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \right] \\ & - Y_{vk}^{n+1} \left[-\frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk}^{n+1} \left(\frac{w_{vk}^{n+1} - w_{vk-1}^{n+1}}{\Delta z} \right) \right] \end{aligned} \right] \\ & = 0 = Ff \end{aligned} \tag{¶.10}$$

ເນື້ອ ພິຈາລະນາສມກາຮັດຢ່າຍເທມວລສາງເຖິງປັບອຸນຫກົມ (ເຖິງປ ຖ)

ብນ ຕຳແໜ່ງ k

$$\begin{aligned}
 \frac{\partial F_k^{n+1}}{\partial T_k^{n+1}} &= \frac{\phi}{\Delta t} (1 - s_{ir}) \left\{ (1 - s_{ek}^{n+1}) \frac{\partial}{\partial T_k^{n+1}} Y_{vk}^{n+1} \right\} \\
 &\quad \left\{ \left[\begin{array}{l} \left(\frac{KK_{rl}}{\mu_l} \right)_{k+\frac{1}{2}} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{-P_{ck}^{n+1}}{\Delta z} \right) + \left(\frac{P_{ck+1}^{n+1} - P_{ck}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \right)_k \\ - \left(\frac{KK_{rl}}{\mu_l} \right)_{k-\frac{1}{2}} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{P_{ck}^{n+1}}{\Delta z} \right) - \left(\frac{P_{ck}^{n+1} - P_{ck-1}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \right)_k \\ \left(\frac{KK_{rl}}{\mu_l} \right)_{k+\frac{1}{2}} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{-P_{gk}^{n+1}}{\Delta z} \right) + \left(\frac{P_{gk+1}^{n+1} - P_{gk}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \right)_k \\ - \left(\frac{KK_{rl}}{\mu_l} \right)_{k-\frac{1}{2}} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{P_{gk}^{n+1}}{\Delta z} \right) - \left(\frac{P_{gk}^{n+1} - P_{gk-1}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \right)_k \\ + \frac{I}{\Delta z} \left[\begin{array}{l} \left(Y_{vk}^{n+1} \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \right)_k \right) \left(\frac{P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) - \left(Y_{vk-1}^{n+1} \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \right)_k \right) \left(-\frac{P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \\ - Y_{vk}^{n+1} \left[\frac{I}{2} \frac{\partial}{\partial T_k^{n+1}} D_{mk}^{n+1} \left(\frac{w_{vk+1}^{n+1} - w_{vk}^{n+1}}{\Delta z} \right) - \frac{I}{2} \frac{\partial}{\partial T_k^{n+1}} D_{mk}^{n+1} \left(\frac{w_{vk}^{n+1} - w_{vk-1}^{n+1}}{\Delta z} \right) \right] \end{array} \right] \end{array} \right] \right\} \\
 &= 0 = Fg
 \end{aligned}$$

(¶.11)

ብນ ຕຳແໜ່ງ k+1

$$\begin{aligned}
 \frac{\partial F_k^{n+1}}{\partial T_k^{n+1}} &= \frac{I}{\Delta z} \left\{ \begin{array}{l} \left(\frac{KK_{rl}}{\mu_l} \right)_{k+\frac{1}{2}} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{-P_{ck}^{n+1}}{\Delta z} \right) + \left(\frac{P_{ck+1}^{n+1} - P_{ck}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \right)_k \\ - \left(\frac{KK_{rl}}{\mu_l} \right)_{k+\frac{1}{2}} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{-P_{gk}^{n+1}}{\Delta z} \right) + \left(\frac{P_{gk+1}^{n+1} - P_{gk}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \right)_k \\ + \left[\left(Y_{vk}^{n+1} \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \right)_k \right) \left(\frac{P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \right] - Y_{vk}^{n+1} \left[\frac{I}{2} \frac{\partial}{\partial T_k^{n+1}} D_{mk}^{n+1} \left(\frac{w_{vk+1}^{n+1} - w_{vk}^{n+1}}{\Delta z} \right) \right] \end{array} \right\} \\
 &= 0 = Fh
 \end{aligned}$$

(¶.12)

ณ ตำแหน่ง k-1

$$\begin{aligned}
 \frac{\partial F_{k-l}^{n+1}}{\partial T_k^{n+1}} = \frac{I}{\Delta z} & \left\{ \left[\left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{P_{ck}^{n+1}}{\Delta z} \right) - \left(\frac{P_{ck}^{n+1} - P_{ck-l}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \right] \right. \\
 & \left. - \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{P_{gk}^{n+1}}{\Delta z} \right) - \left(\frac{P_{gk}^{n+1} - P_{gk-l}^{n+1}}{\Delta z} + g_z \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \right] \\
 & + \left[- \left(Y_{vk-l}^{n+1} \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \Big|_k \right) \left(\frac{-P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \right) \right] \\
 & \left. - Y_{vk}^{n+1} \left[- \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} D_{mk}^{n+1} \left(\frac{w_{vk}^{n+1} - w_{vk-l}^{n+1}}{\Delta z} \right) \right] \right\} \\
 & = 0 = Fi
 \end{aligned} \tag{๔.13}$$

จากนั้นทำการประยุกต์สำหรับพิจารณากรณีโนนดที่เงื่อนไขขอบเขต (Boundary Conditions) โดยทำการประยุกต์ เช่นเดียวกับกรณีโนนดภายใน (Internal node) ดังเช่นในภาคผนวก ก

สมการความดัน (Total Pressure Equation)

สมการความดัน เป็นสมการที่อธิบายถึงการกระจายของสนามความดันในระบบและอัตราการเปลี่ยนแปลงของความดันในวัสดุพื้นเมื่อระยะเวลาการออบแห้งเปลี่ยนไป โดยการกระจายความดันเกิดขึ้นจากสมมุติฐานการเคลื่อนตัวและการเปลี่ยนแปลงสถานะของไอล์ในวัสดุพื้น สามารถนำมาเขียนได้ดังนี้

$$\phi \frac{\partial}{\partial t} \{Y_a (I - s)\} + \frac{\partial}{\partial z} \left[Y_a \frac{KK_{rg}}{\mu_g} \left(-\frac{\partial p_g}{\partial z} + \rho_g g_z \right) - Y_g D_m \frac{\partial}{\partial z} \left(\frac{\rho_a}{\rho_g} \right) \right] = 0 \tag{๔.14}$$

$$\text{โดยที่ } \frac{\rho_a}{\rho_l} = Y_a$$

และสามารถวิเคราะห์ด้วยระเบียบวิธีเชิงตัวเลขโดยวิธีผลต่างสี่บเนื่อง (finite difference) บนพื้นฐานวิธีปริมาตรควบคุม (control volume) ดังนี้

$$\begin{aligned}
& \frac{\phi}{\Delta t} \left\{ (1 - s_{ir}) \left[Y_{vk}^{n+1} (1 - s_{ek}^{n+1}) - Y_{vk}^n (1 - s_{ek}^n) \right] \right\} \\
& + \frac{I}{\Delta z} \left\{ -Y_{gk}^{n+1} \left[D_{mk+\frac{1}{2}}^{n+1} \left(\frac{\left(\frac{\rho_a}{\rho_g} \right)_{k+1}^{n+1} - \left(\frac{\rho_a}{\rho_g} \right)_k^{n+1}}{\Delta z} \right) - D_{mk-\frac{1}{2}}^{n+1} \left(\frac{\left(\frac{\rho_a}{\rho_g} \right)_k^{n+1} - \left(\frac{\rho_a}{\rho_g} \right)_{k-1}^{n+1}}{\Delta z} \right) \right] \right\} \\
& = 0 = G_k^{n+1}(s, P_g, T)
\end{aligned} \tag{¶.15}$$

กรณีพิจารณาโนนดภายใน (Internal node)

เมื่อพิจารณาสมการถ่ายเทมวัลสารเที่ยบกับความชื้น (เที่ยบ s)

ณ ตำแหน่ง k

$$\begin{aligned}
& \frac{\partial G_k^{n+1}}{\partial s_{ek}^{n+1}} = \frac{\phi}{\Delta t} (1 - s_{ir}) Y_{vk}^{n+1} \\
& + \frac{I}{\Delta z} \left\{ Y_{ak}^{n+1} \left[\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{P_{gk}^{n+1}}{\Delta z} \right) \left(\frac{-P_{gk+1}^{n+1} + P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \right. \\
& \left. - \frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{1}{2}} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{-P_{gk}^{n+1}}{\Delta z} \right) \left(\frac{-P_{gk}^{n+1} + P_{gk-1}^{n+1}}{\Delta z} + \rho_g g_z \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \\
& - Y_{gk}^{n+1} \left[\frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} D_{mk+\frac{1}{2}}^{n+1} \left(\frac{\left(\frac{\rho_a}{\rho_g} \right)_{k+1}^{n+1} - \left(\frac{\rho_a}{\rho_g} \right)_k^{n+1}}{\Delta z} \right) - \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} D_{mk-\frac{1}{2}}^{n+1} \left(\frac{\left(\frac{\rho_a}{\rho_g} \right)_k^{n+1} - \left(\frac{\rho_a}{\rho_g} \right)_{k-1}^{n+1}}{\Delta z} \right) \right] \\
& = 0 = Ga
\end{aligned} \tag{¶.16}$$

ณ ตำแหน่ง k+1

$$\begin{aligned}
 \frac{\partial G_{k+l}^{n+I}}{\partial s_{ek}^{n+I}} &= \frac{I}{\Delta z} \left\{ Y_{ak}^{n+I} \left[\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{l}{2}} \frac{\partial}{\partial s_{ek}^{n+I}} \left(\frac{P_{gk}^{n+I}}{\Delta z} \right) \left(\frac{-P_{gk+l}^{n+I} + P_{gk}^{n+I}}{\Delta z} + \rho_g g_z \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+I}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \right. \\
 &\quad \left. - Y_{gk}^{n+I} \left[\frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+I}} D_{mk+\frac{l}{2}}^{n+I} \left(\frac{\left(\frac{\rho_a}{\rho_g} \right)_{k+l}^{n+I} - \left(\frac{\rho_a}{\rho_g} \right)_k^{n+I}}{\Delta z} \right) \right] \right\} \\
 &= 0 = Gb
 \end{aligned}$$

(¶.17)

ณ ตำแหน่ง k-1

$$\begin{aligned}
 \frac{\partial G_{k-l}^{n+I}}{\partial s_{ek}^{n+I}} &= \frac{I}{\Delta z} \left\{ Y_{ak}^{n+I} \left[-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{l}{2}} \frac{\partial}{\partial s_{ek}^{n+I}} \left(\frac{-P_{gk}^{n+I}}{\Delta z} \right) \left(\frac{-P_{gk-l}^{n+I} + P_{gk}^{n+I}}{\Delta z} + \rho_g g_z \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+I}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \right. \\
 &\quad \left. - Y_{gk}^{n+I} \left[-\frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+I}} D_{mk-\frac{l}{2}}^{n+I} \left(\frac{\left(\frac{\rho_a}{\rho_g} \right)_k^{n+I} - \left(\frac{\rho_a}{\rho_g} \right)_{k-l}^{n+I}}{\Delta z} \right) \right] \right\} \\
 &= 0 = Gc
 \end{aligned}$$

(¶.18)

ເນື້ອ ພິຈາລະນາສມກາຮັດຢ່າຍເທົ່ວລສາງເຖິງບັນດາຄວາມດັນ (ເຖິງປ P)

ณ ຕຳແໜ່ງ k

$$\begin{aligned} \frac{\partial G_k^{n+1}}{\partial P_k^{n+1}} &= \frac{\phi}{\Delta t} (1 - s_{ir}) \frac{\partial}{\partial P_k^{n+1}} Y_{vk}^{n+1} \\ &+ \frac{1}{\Delta z} \left\{ Y_{ak}^{n+1} \left[\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{P_{gk}^{n+1}}{\Delta z} \right) \left(\frac{-P_{gk+1}^{n+1} + P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \right. \\ &\quad \left. - \frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{-P_{gk}^{n+1}}{\Delta z} \right) \left(\frac{-P_{gk-1}^{n+1} + P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \\ &- Y_{gk}^{n+1} \left[\frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk+\frac{1}{2}}^{n+1} \left(\frac{\left(\frac{\rho_a}{\rho_g} \right)_{k+1}^{n+1} - \left(\frac{\rho_a}{\rho_g} \right)_k^{n+1}}{\Delta z} \right) - \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk-\frac{1}{2}}^{n+1} \left(\frac{\left(\frac{\rho_a}{\rho_g} \right)_k^{n+1} - \left(\frac{\rho_a}{\rho_g} \right)_{k-1}^{n+1}}{\Delta z} \right) \right] \right\} \\ &= 0 = Gd \end{aligned}$$

(¶.19)

ณ ຕຳແໜ່ງ k+1

$$\begin{aligned} \frac{\partial G_{k+1}^{n+1}}{\partial P_k^{n+1}} &= \frac{1}{\Delta z} \left\{ Y_{ak}^{n+1} \left[\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{P_{gk}^{n+1}}{\Delta z} \right) \left(\frac{-P_{gk+1}^{n+1} + P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \right. \\ &\quad \left. - Y_{gk}^{n+1} \left[\frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk+\frac{1}{2}}^{n+1} \left(\frac{\left(\frac{\rho_a}{\rho_g} \right)_{k+1}^{n+1} - \left(\frac{\rho_a}{\rho_g} \right)_k^{n+1}}{\Delta z} \right) \right] \right\} \\ &= 0 = Ge \end{aligned}$$

(¶.20)

ณ ตำแหน่ง k-1

$$\begin{aligned}
 \frac{\partial G_{k-1}^{n+1}}{\partial P_k^{n+1}} &= \frac{I}{\Delta z} \left\{ Y_{ak}^{n+1} \left[-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{l}{2}} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{-P_{gk}^{n+1}}{\Delta z} \right) \left(\frac{-P_{gk+1}^{n+1} + P_{gk-l}^{n+1}}{\Delta z} + \rho_g g_z \right) \frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \right. \\
 &\quad \left. - Y_{gk}^{n+1} \left[-\frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk-\frac{l}{2}}^{n+1} \left(\frac{\left(\frac{\rho_a}{\rho_g} \right)_k^{n+1} - \left(\frac{\rho_a}{\rho_g} \right)_{k-l}^{n+1}}{\Delta z} \right) \right] \right\} \\
 &= 0 = Gf
 \end{aligned}
 \tag{¶.21}$$

เมื่อ พิจารณาสมการถ่ายเทمواลสารเที่ยบกับคุณภาพ (เที่ยบ T)

ณ ตำแหน่ง k

$$\begin{aligned}
 \frac{\partial G_k^{n+1}}{\partial T_k^{n+1}} &= \frac{\phi}{\Delta t} (1 - s_{ir}) \frac{\partial}{\partial T_k^{n+1}} Y_{vk}^{n+1} \\
 &+ \frac{I}{\Delta z} \left\{ Y_{ak}^{n+1} \left[\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{l}{2}} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{P_{gk}^{n+1}}{\Delta z} \right) \left(\frac{-P_{gk+1}^{n+1} + P_{gk}^{n+1}}{\Delta z} + \rho_g g_z \right) \frac{I}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \right. \\
 &\quad \left. - Y_{gk}^{n+1} \left[-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{l}{2}} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{-P_{gk}^{n+1}}{\Delta z} \right) \left(\frac{-P_{gk}^{n+1} + P_{gk-l}^{n+1}}{\Delta z} + \rho_g g_z \right) \frac{I}{2} \frac{\partial}{\partial T_k^{n+1}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \right\} \\
 &= 0 = Gg
 \end{aligned}
 \tag{¶.22}$$

ณ ตำแหน่ง k+1

$$\begin{aligned}
 & \frac{\partial G_{k+l}^{n+I}}{\partial T_k^{n+I}} = \frac{I}{\Delta z} \left\{ Y_{ak}^{n+I} \left[\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{l}{2}} \frac{\partial}{\partial T_k^{n+I}} \left(\frac{P_{gk}^{n+I}}{\Delta z} \right) \left(\frac{-P_{gk+l}^{n+I} + P_{gk}^{n+I}}{\Delta z} + \rho_g g_z \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+I}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \right. \\
 & \quad \left. - Y_{gk}^{n+I} \left[\frac{1}{2} \frac{\partial}{\partial T_k^{n+I}} D_{mk+\frac{l}{2}}^{n+I} \left(\frac{\left(\frac{\rho_a}{\rho_g} \right)_{k+l}^{n+I} - \left(\frac{\rho_a}{\rho_g} \right)_k^{n+I}}{\Delta z} \right) \right] \right\} \\
 & = 0 = Gh
 \end{aligned} \tag{¶.23}$$

ณ ตำแหน่ง k-1

$$\begin{aligned}
 & \frac{\partial G_{k-l}^{n+I}}{\partial T_k^{n+I}} = \frac{I}{\Delta z} \left\{ Y_{ak}^{n+I} \left[-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{l}{2}} \frac{\partial}{\partial T_k^{n+I}} \left(\frac{-P_{gk}^{n+I}}{\Delta z} \right) \left(\frac{-P_{gk}^{n+I} + P_{gk-l}^{n+I}}{\Delta z} + \rho_g g_z \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+I}} \left(\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \right. \\
 & \quad \left. - Y_{gk}^{n+I} \left[-\frac{1}{2} \frac{\partial}{\partial T_k^{n+I}} D_{mk-\frac{l}{2}}^{n+I} \left(\frac{\left(\frac{\rho_a}{\rho_g} \right)_k^{n+I} - \left(\frac{\rho_a}{\rho_g} \right)_{k-l}^{n+I}}{\Delta z} \right) \right] \right\} \\
 & = 0 = Gi
 \end{aligned} \tag{¶.24}$$

จากนั้นทำการประยุกต์สำหรับพิจารณากรณีโนนดที่เงื่อนไขขอบเขต (Boundary Conditions) โดยทำการประยุกต์ เช่นเดียวกับกรณีโนนดภายใน (Internal node) ดังเช่นในภาคผนวก ก

สมการถ่ายเทความร้อน (Heat Transport Equation)

สมการถ่ายเทความร้อน เป็นสมการที่อธิบายถึงการกระจายของสนามอุณหภูมิในระบบและอัตราการเปลี่ยนแปลงของอุณหภูมิในวัสดุพื้นเมือง กระบวนการอุบัติเหตุและการตอบแหน่งเปลี่ยนไป การกระจายอุณหภูมิเกิดขึ้นจากสมมุติฐานสมการพาสำหรับของเหลวที่ไหลในวัสดุพื้นเมือง สมการนำของโครงสร้างของแข็งในวัสดุพื้นเมือง และสมการปริมาณความร้อนที่วัสดุผลิตได้ซึ่งเป็นปริมาณความร้อนที่วัสดุผลิตขึ้นจากการดูดซับพลังงานไมโครเวฟ และสามารถนำมาเขียนได้ดังนี้

$$\begin{aligned} & \frac{\partial}{\partial t} \left[(\rho_l c_{pl}) \phi + \left((\rho c_p)_a + (\rho c_p)_v \right) \phi (1-s) + \rho_p c_{pp} (1-\phi) T \right] \\ & + \frac{\partial}{\partial z} \left[(\rho_l c_{pl} w_l + (\rho_a c_{pa} + \rho_v c_{pv}) w_g) T \right] \\ & = \frac{\partial}{\partial z} \left[\lambda \frac{\partial T}{\partial z} \right] - H_v \left\{ \frac{\partial}{\partial t} \left\{ \rho_v \phi (1-s) \right\} + \frac{\partial}{\partial z} \left[\rho_v \frac{KK_{rg}}{\mu_g} \left(-\frac{\partial P_g}{\partial z} + \rho_g g_z \right) - \rho_g D_m \frac{\partial}{\partial z} \left(\frac{\rho_v}{\rho_g} \right) \right] \right\} + Q \end{aligned} \quad (\text{ก.25})$$

เมื่อ

$$(\rho c_p)_T = \rho C_p \phi + ((\rho c_p)_a + (\rho c_p)_v) \phi (1-s) + \rho_p C_{pp} (1-\phi) \quad (\text{ก.26})$$

ดังนั้น

$$\begin{aligned} & \frac{\partial}{\partial t} \left\{ (\rho c_p)_T T \right\} + \frac{\partial}{\partial z} \left[(\rho_l c_{pl} w_l + (\rho_a c_{pa} + \rho_v c_{pv}) w_g) T \right] = \frac{\partial}{\partial z} \left[\lambda \frac{\partial T}{\partial z} \right] \\ & - H_v \left\{ \frac{\partial}{\partial t} \left\{ \rho_v \phi (1-s) \right\} + \frac{\partial}{\partial z} \left[\rho_v \frac{KK_{rg}}{\mu_g} \left(-\frac{\partial P_g}{\partial z} + \rho_g g_z \right) - \rho_g D_m \frac{\partial}{\partial z} \left(\frac{\rho_v}{\rho_g} \right) \right] \right\} + Q \end{aligned} \quad (\text{ก.27})$$

และสามารถวิเคราะห์ด้วยระเบียบวิธีเชิงตัวเลขโดยวิธีผลต่างสี่บเนื่อง (finite difference) บนพื้นฐานวิธีปริมาตรควบคุม (control volume) ดังนี้

$$\begin{aligned} \text{เมื่อ} \quad & \left. \frac{KK_{rl}}{\mu_l} \right|_{k \pm \frac{l}{2}} = \frac{\left. \frac{KK_{rl}}{\mu_l} \right|_k + \left. \frac{KK_{rl}}{\mu_l} \right|_{k \pm l}}{2} \quad \left. \frac{KK_{rg}}{\mu_g} \right|_{k \pm \frac{l}{2}} = \frac{\left. \frac{KK_{rg}}{\mu_g} \right|_k + \left. \frac{KK_{rg}}{\mu_g} \right|_{k \pm l}}{2} \\ & D_{k \pm \frac{l}{2}}^{n+1} = \frac{D_k^{n+1} + D_{k \pm l}^{n+1}}{2} \quad \lambda_{k \pm \frac{l}{2}}^{n+1} = \frac{\lambda_k^{n+1} + \lambda_{k \pm l}^{n+1}}{2} \end{aligned} \quad (\text{ก.28})$$

และ

$$s^{n+1} = s^n - \Delta s$$

$$P_g^{n+1} = P_g^n - \Delta P_g$$

$$T^{n+1} = T^n - \Delta T$$

(ก.29)

$$\begin{aligned}
& \frac{I}{\Delta t} \left\{ \rho_l c_{pl} \phi (1 - s_{ir}) (s_{ek}^{n+1} - s_{ek}^n) + \rho_g c_{pg} \phi (1 - s_{ir}) \left[(1 - s_{ek}^{n+1}) - (1 - s_{ek}^n) \right] + \rho_p c_{pp} (1 - \phi) (T_k^{n+1} - T_k^n) \right. \\
& \left. \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) \left(\frac{P_{gk+l}^{n+1} - P_{gk}^{n+1}}{\Delta z} \right) - \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) \left(\frac{P_{gk}^{n+1} - P_{gk-l}^{n+1}}{\Delta z} \right) \right] \right\} \\
& + \frac{I}{\Delta z} \left\{ \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) \left(\frac{P_{ck+l}^{n+1} - P_{ck}^{n+1}}{\Delta z} \right) - \rho_l c_{pl} \rho_l g \right. \right. \\
& \left. \left. + \left[-\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) \left(\frac{P_{ck}^{n+1} - P_{ck-l}^{n+1}}{\Delta z} \right) \right] \right] \right\} (T_k^{n+1} - T_{k-l}^{n+1}) \\
& + \left[\left(\rho c_p \right)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} \right) \left(\frac{P_{gk+l}^{n+1} - P_{gk}^{n+1}}{\Delta z} \right) - \left(\rho c_p \right)_{av} \rho_g g \right. \\
& \left. + \left[-\left(\rho c_p \right)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{1}{2}} \right) \left(\frac{P_{gk}^{n+1} - P_{gk-l}^{n+1}}{\Delta z} \right) \right] \right] \\
& + \frac{I}{\Delta z} \left[\lambda_{k+\frac{1}{2}}^{n+1} \left(\frac{T_{k+l}^{n+1} - T_k^{n+1}}{\Delta z} \right) - \lambda_{k-\frac{1}{2}}^{n+1} \left(\frac{T_k^{n+1} - T_{k-l}^{n+1}}{\Delta z} \right) \right] + \frac{H_v \rho_v \phi}{\Delta t} \left\{ (1 - s_{ir}) (s_{ek}^{n+1} - s_{ek}^n) \right\} \\
& \left. \left[\rho_{vk}^{n+1} \frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} \left(\rho_g g_z \right) - \rho_{vk-l}^{n+1} \frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{1}{2}} \left(\rho_g g_z \right) \right] \right\} \\
& + \frac{I}{\Delta z} \left\{ \left[\rho_{gk}^{n+1} D_{mk+\frac{1}{2}}^{n+1} \left(\left(\frac{\rho_v}{\rho_g} \right)_{k+l}^{n+1} - \left(\frac{\rho_v}{\rho_g} \right)_k^{n+1} \right) - \rho_{gk-l}^{n+1} D_{mk-\frac{1}{2}}^{n+1} \left(\left(\frac{\rho_v}{\rho_g} \right)_k^{n+1} - \left(\frac{\rho_v}{\rho_g} \right)_{k-l}^{n+1} \right) \right] \right\} \\
& - Q = 0
\end{aligned}$$

(¶.30)

นำ Δz^2 คูณตลอดสมการ

$$\begin{aligned}
 & \frac{\Delta z^2}{\Delta t} \left\{ \rho_l c_{pl} \phi (1 - s_{ir}) (s_{ek}^{n+1} - s_{ek}^n) \right. \\
 & + \rho_g c_{pg} \phi (1 - s_{ir}) [(1 - s_{ek}^{n+1}) - (1 - s_{ek}^n)] + \rho_p c_{pp} (1 - \phi) \left. (T_k^{n+1} - T_k^n) \right\} \\
 & + \left\{ \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (P_{gk+1}^{n+1} - P_{gk}^{n+1}) - \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (P_{gk}^{n+1} - P_{gk-1}^{n+1}) \right] \right. \\
 & + \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (P_{ck+1}^{n+1} - P_{ck}^{n+1}) - \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (P_{ck}^{n+1} - P_{ck-1}^{n+1}) \right] \\
 & + \left. \left[-\rho_l c_{pl} \rho_l g \Delta z \right. \right. \\
 & + \left[\left(\boldsymbol{\alpha}_p \right)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} \right) (P_{gk+1}^{n+1} - P_{gk}^{n+1}) - \left(\boldsymbol{\alpha}_p \right)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{1}{2}} \right) (P_{gk}^{n+1} - P_{gk-1}^{n+1}) \right] \\
 & \left. \left. - \left(\boldsymbol{\alpha}_p \right)_{av} \rho_g g \Delta z \right] \right\} (T_k^{n+1} - T_{k-1}^{n+1}) \\
 & + \left[\lambda_{k+\frac{1}{2}}^{n+1} (T_{k+1}^{n+1} - T_k^{n+1}) - \lambda_{k-\frac{1}{2}}^{n+1} (T_k^{n+1} - T_{k-1}^{n+1}) \right] + \frac{H_v \rho_v \phi \Delta z^2}{\Delta t} \{ (1 - s_{ir}) (s_{ek}^{n+1} - s_{ek}^n) \} \\
 & + \left\{ \rho_{vk}^{n+1} \frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} (\rho_g g_z \Delta z) - \rho_{vk-1}^{n+1} \frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{1}{2}} (\rho_g g_z \Delta z) \right\} \\
 & + \left\{ - \left(\rho_{gk}^{n+1} D_{mk+\frac{1}{2}}^{n+1} \left(\left(\frac{\rho_v}{\rho_g} \right)^{n+1}_{k+1} - \left(\frac{\rho_v}{\rho_g} \right)^{n+1}_k \right) - \rho_{gk-1}^{n+1} D_{mk-\frac{1}{2}}^{n+1} \left(\left(\frac{\rho_v}{\rho_g} \right)^{n+1}_k - \left(\frac{\rho_v}{\rho_g} \right)^{n+1}_{k-1} \right) \right) \right\} \\
 & - Q = 0 = E_k^{n+1}(s, P_g, T)
 \end{aligned}$$

(¶.31)

กรณีพิจารณาโหนดภายใน (Internal node)

เมื่อพิจารณาสมการถ่ายเทมวลสารเที่ยบกับความชื้น (เที่ยบ s)

ณ ตำแหน่ง k

$$\frac{\partial E_k^{n+1}}{\partial s_{ek}^{n+1}} = \frac{\Delta x^2}{\Delta t} \left\{ \begin{array}{l} \left[\rho_l c_{pl} \phi(I - s_{ir}) + \rho_g c_{pg} \phi(I - s_{ir})(-I) \right] [T_k^{n+1} - T_k^n] \\ + \left[\rho_l c_{pl} \phi(I - s_{ir}) (s_{ek}^{n+1} - s_{ek}^n) \right. \\ \left. + \rho_g c_{pg} \phi(I - s_{ir}) [(I - s_{ek}^{n+1}) - (I - s_{ek}^n)] \right] \frac{\partial}{\partial s_{ek}^{n+1}} (T_k^{n+1}) \\ + \rho_p c_{pp} (I - \phi) \end{array} \right\}$$

$$+ \left[\begin{array}{l} \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (P_{gk+1}^{n+1} - P_{gk}^{n+1}) \frac{\partial}{\partial s_{ek}^{n+1}} (T_k^{n+1}) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{\partial}{\partial s_{ek}^{n+1}} (-P_{gk}^{n+1}) \\ + \rho_l c_{pl} (P_{gk+1}^{n+1} - P_{gk}^{n+1}) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \\ - \left(\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (P_{gk}^{n+1} - P_{gk-1}^{n+1}) \frac{\partial}{\partial s_{ek}^{n+1}} (T_k^{n+1}) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{\partial}{\partial s_{ek}^{n+1}} (P_{gk}^{n+1}) \right) \\ + \rho_l c_{pl} (P_{gk}^{n+1} - P_{gk-1}^{n+1}) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \end{array} \right]$$

$$+ \left[\begin{array}{l} \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (P_{ck+1}^{n+1} - P_{ck}^{n+1}) \frac{\partial}{\partial s_{ek}^{n+1}} (T_k^{n+1}) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{\partial}{\partial s_{ek}^{n+1}} (-P_{ck}^{n+1}) \\ + \rho_l c_{pl} (P_{ck+1}^{n+1} - P_{ck}^{n+1}) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \end{array} \right]$$

$$+ \left[\begin{array}{l} \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (P_{ck}^{n+1} - P_{ck-1}^{n+1}) \frac{\partial}{\partial s_{ek}^{n+1}} (T_k^{n+1}) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{\partial}{\partial s_{ek}^{n+1}} (P_{ck}^{n+1}) \\ + \rho_l c_{pl} (P_{ck}^{n+1} - P_{ck-1}^{n+1}) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \end{array} \right]$$

$$- \rho_l c_{pl} \rho_l g \Delta x \frac{\partial}{\partial s_{ek}^{n+1}} (T_k^{n+1})$$

$$\begin{aligned}
& + \left(\left(\boldsymbol{\alpha}_p \right)_{av} \left| -\frac{KK_{rg}}{\mu_g} \right|_{k+\frac{I}{2}} \left(P_{gk+I}^{n+I} - P_{gk}^{n+I} \right) \frac{\partial}{\partial s_{ek}^{n+I}} \left(T_k^{n+I} \right) + \left(\boldsymbol{\alpha}_p \right)_{av} \left| -\frac{KK_{rg}}{\mu_g} \right|_{k+\frac{I}{2}} \left(T_k^{n+I} - T_{k-I}^{n+I} \right) \frac{\partial}{\partial s_{ek}^{n+I}} \left(-P_{gk}^{n+I} \right) \right. \right. \\
& \quad \left. \left. + \left(\boldsymbol{\alpha}_p \right)_{av} \left(P_{gk+I}^{n+I} - P_{gk}^{n+I} \right) \left(T_k^{n+I} - T_{k-I}^{n+I} \right) \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+I}} \left(-\frac{KK_{rl}}{\mu_l} \right)_k \right) \right. \\
& \quad \left. - \left(\left(\boldsymbol{\alpha}_p \right)_{av} \left| -\frac{KK_{rg}}{\mu_g} \right|_{k-\frac{I}{2}} \left(P_{gk}^{n+I} - P_{gk-I}^{n+I} \right) \frac{\partial}{\partial s_{ek}^{n+I}} \left(T_k^{n+I} \right) + \left(\boldsymbol{\alpha}_p \right)_{av} \left| -\frac{KK_{rg}}{\mu_g} \right|_{k-\frac{I}{2}} \left(T_k^{n+I} - T_{k-I}^{n+I} \right) \frac{\partial}{\partial s_{ek}^{n+I}} \left(P_{gk}^{n+I} \right) \right. \right. \\
& \quad \left. \left. + \left(\boldsymbol{\alpha}_p \right)_{av} \left(P_{gk}^{n+I} - P_{gk-I}^{n+I} \right) \left(T_k^{n+I} - T_{k-I}^{n+I} \right) \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+I}} \left(-\frac{KK_{rg}}{\mu_g} \right)_k \right) \right) \\
& \quad - \left(\boldsymbol{\alpha}_p \right)_{av} \rho_g g \Delta t \frac{\partial}{\partial s_{ek}^{n+I}} \left(T_k^{n+I} \right) \\
& \quad + H_v \left[\left[\frac{\Delta t^2}{\Delta t} \rho_v \phi (1 - s_{ir}) (-I) + \left[\rho_v \rho_g g \Delta t \frac{\partial}{\partial s_{ek}^{n+I}} \left(\frac{KK_{rg}}{\mu_g} \right)_k \right] \right] \right. \\
& \quad \left. - \left[D_{mk+\frac{I}{2}}^{n+I} \frac{\partial}{\partial s_{ek}^{n+I}} \left(-P_{vk}^{n+I} \right) + \left(P_{vk+I}^{n+I} - P_{vk}^{n+I} \right) \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+I}} D_{mk}^{n+I} \right] \right. \\
& \quad \left. - \left[D_{mk-\frac{I}{2}}^{n+I} \frac{\partial}{\partial s_{ek}^{n+I}} \left(P_{vk}^{n+I} \right) + \left(P_{vk}^{n+I} - P_{vk-I}^{n+I} \right) \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+I}} D_{mk}^{n+I} \right] \right] \\
& \quad + \left[\lambda_{k+\frac{I}{2}}^{n+I} \frac{\partial}{\partial s_{ek}^{n+I}} \left(-T_k^{n+I} \right) + \left(T_{k+I}^{n+I} - T_k^{n+I} \right) \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+I}} \lambda_k^{n+I} \right] - \left[\lambda_{k-\frac{I}{2}}^{n+I} \frac{\partial}{\partial s_{ek}^{n+I}} \left(T_k^{n+I} \right) + \left(T_k^{n+I} - T_{k-I}^{n+I} \right) \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+I}} \lambda_k^{n+I} \right] \\
& \quad - Q = 0 = Ea
\end{aligned}$$

(๘.32)

លេខ ទីរាយអន្តែង k+1

$$\begin{aligned}
 \frac{\partial E_{k+l}^{n+1}}{\partial s_{ek}^{n+1}} = & \left(\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{l}{2}} \right) \left(P_{gk+l}^{n+1} - P_{gk}^{n+1} \right) \frac{\partial}{\partial s_{ek}^{n+1}} \left(T_k^{n+1} \right) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{l}{2}} \right) \left(T_k^{n+1} - T_{k-l}^{n+1} \right) \frac{\partial}{\partial s_{ek}^{n+1}} \left(-P_{gk}^{n+1} \right) \right. \\
 & \left. + \rho_l c_{pl} \left(P_{gk+l}^{n+1} - P_{gk}^{n+1} \right) \left(T_k^{n+1} - T_{k-l}^{n+1} \right) \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \\
 & + \left(\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{l}{2}} \right) \left(P_{ck+l}^{n+1} - P_{ck}^{n+1} \right) \frac{\partial}{\partial s_{ek}^{n+1}} \left(T_k^{n+1} \right) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{l}{2}} \right) \left(T_k^{n+1} - T_{k-l}^{n+1} \right) \frac{\partial}{\partial s_{ek}^{n+1}} \left(-P_{ck}^{n+1} \right) \right. \\
 & \left. + \rho_l c_{pl} \left(P_{ck+l}^{n+1} - P_{ck}^{n+1} \right) \left(T_k^{n+1} - T_{k-l}^{n+1} \right) \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \\
 & + \left((\alpha_p)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{l}{2}} \right) \left(P_{gk+l}^{n+1} - P_{gk}^{n+1} \right) \frac{\partial}{\partial s_{ek}^{n+1}} \left(T_k^{n+1} \right) + (\alpha_p)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{l}{2}} \right) \left(T_k^{n+1} - T_{k-l}^{n+1} \right) \frac{\partial}{\partial s_{ek}^{n+1}} \left(-P_{gk}^{n+1} \right) \right. \\
 & \left. + (\alpha_p)_{av} \left(P_{gk+l}^{n+1} - P_{gk}^{n+1} \right) \left(T_k^{n+1} - T_{k-l}^{n+1} \right) \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(-\frac{KK_{rg}}{\mu_g} \Big|_k \right) \right) \\
 & + H_v \left(- \left[D_{mk+\frac{l}{2}}^{n+1} \frac{\partial}{\partial s_{ek}^{n+1}} \left(-P_{vk}^{n+1} \right) + \left(P_{vk+l}^{n+1} - P_{vk}^{n+1} \right) \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+1}} D_{mk}^{n+1} \right] \right. \\
 & \left. + \left[\lambda_{k+\frac{l}{2}}^{n+1} \frac{\partial}{\partial s_{ek}^{n+1}} \left(-T_k^{n+1} \right) + \left(T_{k+l}^{n+1} - T_k^{n+1} \right) \frac{I}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \lambda_k^{n+1} \right] \right) \\
 & - Q = 0 = Eb
 \end{aligned}$$

(¶.33)

លេខ ទីរាយអនុវត្ត k-1

$$\begin{aligned}
 \frac{\partial E_{k-l}^{n+I}}{\partial s_{ek}^{n+I}} = & - \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \right) \left(P_{gk}^{n+I} - P_{gk-l}^{n+I} \right) \frac{\partial}{\partial s_{ek}^{n+I}} \left(T_k^{n+I} \right) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \right) \left(T_k^{n+I} - T_{k-l}^{n+I} \right) \frac{\partial}{\partial s_{ek}^{n+I}} \left(P_{gk}^{n+I} \right) \right. \\
 & \left. + \rho_l c_{pl} \left(P_{gk}^{n+I} - P_{gk-l}^{n+I} \right) \left(T_k^{n+I} - T_{k-l}^{n+I} \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+I}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \\
 & - \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \right) \left(P_{ck}^{n+I} - P_{ck-l}^{n+I} \right) \frac{\partial}{\partial s_{ek}^{n+I}} \left(T_k^{n+I} \right) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \right) \left(T_k^{n+I} - T_{k-l}^{n+I} \right) \frac{\partial}{\partial s_{ek}^{n+I}} \left(P_{ck}^{n+I} \right) \right. \\
 & \left. + \rho_l c_{pl} \left(P_{ck}^{n+I} - P_{ck-l}^{n+I} \right) \left(T_k^{n+I} - T_{k-l}^{n+I} \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+I}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \\
 & - \left[(\rho c_p)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{l}{2}} \right) \left(P_{gk}^{n+I} - P_{gk-l}^{n+I} \right) \frac{\partial}{\partial s_{ek}^{n+I}} \left(T_k^{n+I} \right) + (\rho c_p)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{l}{2}} \right) \left(T_k^{n+I} - T_{k-l}^{n+I} \right) \frac{\partial}{\partial s_{ek}^{n+I}} \left(P_{gk}^{n+I} \right) \right. \\
 & \left. + (\rho c_p)_{av} \left(P_{gk}^{n+I} - P_{gk-l}^{n+I} \right) \left(T_k^{n+I} - T_{k-l}^{n+I} \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+I}} \left(-\frac{KK_{rg}}{\mu_g} \Big|_k \right) \right] \\
 & + H_v \left(D_{mk-\frac{l}{2}} \frac{\partial}{\partial s_{ek}^{n+I}} \left(P_{vk}^{n+I} \right) + \left(P_{vk}^{n+I} - P_{vk-l}^{n+I} \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+I}} D_{mk}^{n+I} \right) \\
 & - \left[\lambda_{k-\frac{l}{2}}^{n+I} \frac{\partial}{\partial s_{ek}^{n+I}} \left(T_k^{n+I} \right) + \left(T_k^{n+I} - T_{k-l}^{n+I} \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+I}} \lambda_k^{n+I} \right] \\
 & - Q = 0 = Ec
 \end{aligned}$$

(¶.34)

ເນື້ອ ພິຈາລະນາສົມກາຮຕ່າຍເທົມວລສາງເຖິງປັບຄວາມດັນ (ເຖິງປ ພ)

ນ ຕຳແໜ່ງ k

$$\begin{aligned}
 \frac{\partial E_k^{n+1}}{\partial P_k^{n+1}} = & \frac{\Delta z^2}{\Delta t} \left\{ \left[\rho_l c_{pl} \phi(I - s_{ir}) \frac{\partial}{\partial P_k^{n+1}} (s_{ek}^{n+1}) + \rho_g c_{pg} \phi(I - s_{ir}) \frac{\partial}{\partial P_k^{n+1}} (-s_{ek}^{n+1}) \right] (T_k^{n+1} - T_k^n) + \right. \\
 & \left. \left[\rho_l c_{pl} \phi(I - s_{ir}) (s_{ek}^{n+1} - s_{ek}^n) + \rho_g c_{pg} \phi(I - s_{ir}) [(I - s_{ek}^{n+1}) - (I - s_{ek}^n)] \right] \frac{\partial}{\partial P_k^{n+1}} (T_k^{n+1}) \right\} \\
 & + \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (P_{gk+1}^{n+1} - P_{gk}^{n+1}) \frac{\partial}{\partial P_k^{n+1}} (T_k^{n+1}) \right. \\
 & + \rho_l c_{pl} (P_{gk+1}^{n+1} - P_{gk}^{n+1}) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (T_k^{n+1} - T_{k-1}^{n+1}) (-I) \\
 & + \left. \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (P_{gk}^{n+1} - P_{gk-1}^{n+1}) \frac{\partial}{\partial P_k^{n+1}} (T_k^{n+1}) \right. \right. \\
 & \left. \left. + \rho_l c_{pl} (P_{gk}^{n+1} - P_{gk-1}^{n+1}) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (T_k^{n+1} - T_{k-1}^{n+1}) \right] \right. \\
 & \left. \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (P_{ck+1}^{n+1} - P_{ck}^{n+1}) \frac{\partial}{\partial P_k^{n+1}} (T_k^{n+1}) \right. \right. \\
 & + \rho_l c_{pl} (P_{ck+1}^{n+1} - P_{ck}^{n+1}) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \\
 & + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{\partial}{\partial P_k^{n+1}} (-P_{ck}^{n+1}) \\
 & + \left. \left. \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (P_{ck}^{n+1} - P_{ck-1}^{n+1}) \frac{\partial}{\partial P_k^{n+1}} (T_k^{n+1}) \right. \right. \right. \\
 & + \rho_l c_{pl} (P_{ck}^{n+1} - P_{ck-1}^{n+1}) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \\
 & + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{\partial}{\partial P_k^{n+1}} (P_{ck}^{n+1}) \\
 & \left. \left. \left. - \rho_l c_{pl} \rho_l g \Delta z \frac{\partial}{\partial P_k^{n+1}} (T_k^{n+1}) \right] \right]
 \end{aligned}$$

$$\begin{aligned}
& + \left(\rho c_p \right)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{l}{2}} \right) \left(P_{gk+l}^{n+1} - P_{gk}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) \\
& + \left(\rho c_p \right)_{av} \left(P_{gk+l}^{n+1} - P_{gk}^{n+1} \right) \left(T_k^{n+1} - T_{k-l}^{n+1} \right) \frac{l}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\frac{KK_g}{\mu_g} \Big|_k \right) + \left(\rho c_p \right)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{l}{2}} \right) \left(T_k^{n+1} - T_{k-l}^{n+1} \right) (-l) \\
& - \left(\rho c_p \right)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{l}{2}} \right) \left(P_{gk}^{n+1} - P_{gk-l}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) \\
& + \left(\rho c_p \right)_{av} \left(P_{gk}^{n+1} - P_{gk-l}^{n+1} \right) \left(T_k^{n+1} - T_{k-l}^{n+1} \right) \frac{l}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\frac{KK_g}{\mu_g} \Big|_k \right) + \left(\rho c_p \right)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{l}{2}} \right) \left(T_k^{n+1} - T_{k-l}^{n+1} \right) \\
& - \left(\rho c_p \right)_{av} \rho_g g \Delta z \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) \\
& + H_v \left[\left[\frac{\Delta z^2}{\Delta t} \rho_v \phi (1 - s_{ir}) \frac{\partial}{\partial P_k^{n+1}} \left(-s_{ek}^{n+1} \right) + \left[\rho_v \rho_g g \Delta z \frac{\partial}{\partial P_k^{n+1}} \left(\frac{KK_{rg}}{\mu_g} \Big|_k \right) \right] \right] \right] \\
& - \left[D_{mk+\frac{l}{2}}^{n+1} \frac{\partial}{\partial P_k^{n+1}} \left(-P_{vk}^{n+1} \right) + \left(P_{vk+l}^{n+1} - P_{vk}^{n+1} \right) \frac{l}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk}^{n+1} \right] \\
& - \left[D_{mk-\frac{l}{2}}^{n+1} \frac{\partial}{\partial P_k^{n+1}} \left(P_{vk}^{n+1} \right) + \left(P_{vk}^{n+1} - P_{vk-l}^{n+1} \right) \frac{l}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk}^{n+1} \right] \\
& + \left[\lambda_{k+\frac{l}{2}}^{n+1} \frac{\partial}{\partial P_k^{n+1}} \left(-T_k^{n+1} \right) + \left(T_{k+l}^{n+1} - T_k^{n+1} \right) \frac{l}{2} \frac{\partial}{\partial P_k^{n+1}} \lambda_k^{n+1} \right] - \left[\lambda_{k-\frac{l}{2}}^{n+1} \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) + \left(T_k^{n+1} - T_{k-l}^{n+1} \right) \frac{l}{2} \frac{\partial}{\partial P_k^{n+1}} \lambda_k^{n+1} \right] \\
& - Q = 0 = Ed
\end{aligned}$$

(¶.35)

ณ ตำแหน่ง $k+1$

$$\begin{aligned}
 \frac{\partial E_{k+1}^{n+1}}{\partial P_k^{n+1}} = & \left(\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) \left(P_{gk+1}^{n+1} - P_{gk}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) + \rho_l c_{pl} \left(P_{gk+1}^{n+1} - P_{gk}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \right) \\
 & + \left(\frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) - I \right) \\
 & + \left(\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) \left(P_{ck+1}^{n+1} - P_{ck}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) + \rho_l c_{pl} \left(P_{ck+1}^{n+1} - P_{ck}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \right) \\
 & + \left(\frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(-P_{ck}^{n+1} \right) \right) \\
 & + \left((\rho c_p)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} \right) \left(P_{gk+1}^{n+1} - P_{gk}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) + (\rho c_p)_{av} \left(P_{gk+1}^{n+1} - P_{gk}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \right) \\
 & + \left(\frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\frac{KK_g}{\mu_g} \Big|_k \right) + (\rho c_p)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) - I \right) \\
 & + H_v \left(- \left[D_{mk+\frac{1}{2}}^{n+1} \frac{\partial}{\partial P_k^{n+1}} \left(-P_{vk}^{n+1} \right) + \left(P_{vk+1}^{n+1} - P_{vk}^{n+1} \right) \frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk}^{n+1} \right] \right) \\
 & + \left[\lambda_{k+\frac{1}{2}}^{n+1} \frac{\partial}{\partial P_k^{n+1}} \left(-T_k^{n+1} \right) + \left(T_{k+1}^{n+1} - T_k^{n+1} \right) \frac{I}{2} \frac{\partial}{\partial P_k^{n+1}} \lambda_k^{n+1} \right] \\
 & - Q = 0 = Ee
 \end{aligned}$$

(¶.36)

លេខ ទីរាយអនុវត្ត k-1

$$\begin{aligned}
 \frac{\partial E_{k-l}^{n+I}}{\partial P_k^{n+I}} = & - \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \right) (P_{gk}^{n+I} - P_{gk-l}^{n+I}) \frac{\partial}{\partial P_k^{n+I}} (T_k^{n+I}) + \rho_l c_{pl} (P_{gk}^{n+I} - P_{gk-l}^{n+I}) (T_k^{n+I} - T_{k-l}^{n+I}) \right] \\
 & - \left[\frac{I}{2} \frac{\partial}{\partial P_k^{n+I}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \right) (T_k^{n+I} - T_{k-l}^{n+I}) \right. \\
 & - \left. \left. \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \right) (P_{ck}^{n+I} - P_{ck-l}^{n+I}) \frac{\partial}{\partial P_k^{n+I}} (T_k^{n+I}) + \rho_l c_{pl} (P_{ck}^{n+I} - P_{ck-l}^{n+I}) (T_k^{n+I} - T_{k-l}^{n+I}) \right. \right. \\
 & - \left. \left. \frac{I}{2} \frac{\partial}{\partial P_k^{n+I}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \right) (T_k^{n+I} - T_{k-l}^{n+I}) \frac{\partial}{\partial P_k^{n+I}} (P_{ck}^{n+I}) \right. \right. \\
 & - \left. \left. (\rho c_p)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{l}{2}} \right) (P_{gk}^{n+I} - P_{gk-l}^{n+I}) \frac{\partial}{\partial P_k^{n+I}} (T_k^{n+I}) + (\rho c_p)_{av} (P_{gk}^{n+I} - P_{gk-l}^{n+I}) (T_k^{n+I} - T_{k-l}^{n+I}) \right. \right. \\
 & - \left. \left. \frac{I}{2} \frac{\partial}{\partial P_k^{n+I}} \left(-\frac{KK_g}{\mu_g} \Big|_k \right) + (\rho c_p)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{l}{2}} \right) (T_k^{n+I} - T_{k-l}^{n+I}) \right. \right. \\
 & + H_v \left(- \left[D_{mk-\frac{l}{2}}^{n+I} \frac{\partial}{\partial P_k^{n+I}} (P_{vk}^{n+I}) + (P_{vk}^{n+I} - P_{vk-l}^{n+I}) \frac{I}{2} \frac{\partial}{\partial P_k^{n+I}} D_{mk}^{n+I} \right] \right. \\
 & - \left. \left[\lambda_{k-\frac{l}{2}}^{n+I} \frac{\partial}{\partial P_k^{n+I}} (T_k^{n+I}) + (T_k^{n+I} - T_{k-l}^{n+I}) \frac{I}{2} \frac{\partial}{\partial P_k^{n+I}} \lambda_k^{n+I} \right] \right. \\
 & - Q = 0 = Ef
 \end{aligned}$$

(Allison.37)

ເນື້ອ ພິຈາລະນາສົມກາຮຕ່າຍເທມວລສາງເຖິງປັບອຸນຫກົມ (ເຖິງປ ຖ)

ນ ຕຳແໜ່ງ k

$$\begin{aligned}
 \frac{\partial E_k^{n+I}}{\partial T_k^{n+I}} = & \frac{\Delta^2}{\Delta t} \left\{ \left[\rho_l c_{pl} \phi(I - s_{ir}) \frac{\partial}{\partial T_k^{n+I}} (s_{ek}^{n+I}) + \rho_g c_{pg} \phi(I - s_{ir}) \frac{\partial}{\partial T_k^{n+I}} (-s_{ek}^{n+I}) \right] (T_k^{n+I} - T_k^n) + \right. \\
 & \left. \left[\rho_l c_{pl} \phi(I - s_{ir}) (s_{ek}^{n+I} - s_{ek}^n) + \rho_g c_{pg} \phi(I - s_{ir}) [(I - s_{ek}^{n+I}) - (I - s_{ek}^n)] + \rho_p c_{pp} (I - \phi) \right] \right\} \\
 & + \left[\left. \begin{aligned}
 & \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (P_{gk+I}^{n+I} - P_{gk}^{n+I}) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (T_k^{n+I} - T_{k-I}^{n+I}) \frac{\partial}{\partial T_k^{n+I}} (-P_{gk}^{n+I}) \right] \\
 & + \rho_l c_{pl} (P_{gk+I}^{n+I} - P_{gk}^{n+I}) (T_k^{n+I} - T_{k-I}^{n+I}) \frac{1}{2} \frac{\partial}{\partial T_k^{n+I}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right)
 \end{aligned} \right] \\
 & + \left[\begin{aligned}
 & \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (P_{gk}^{n+I} - P_{gk-I}^{n+I}) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (T_k^{n+I} - T_{k-I}^{n+I}) \frac{\partial}{\partial T_k^{n+I}} (P_{gk}^{n+I}) \right] \\
 & + \rho_l c_{pl} (P_{gk}^{n+I} - P_{gk-I}^{n+I}) (T_k^{n+I} - T_{k-I}^{n+I}) \frac{1}{2} \frac{\partial}{\partial T_k^{n+I}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right)
 \end{aligned} \right] \\
 & + \left[\begin{aligned}
 & \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (P_{ck+I}^{n+I} - P_{ck}^{n+I}) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (T_k^{n+I} - T_{k-I}^{n+I}) \frac{\partial}{\partial T_k^{n+I}} (-P_{ck}^{n+I}) \right] \\
 & + \rho_l c_{pl} (P_{ck+I}^{n+I} - P_{ck}^{n+I}) (T_k^{n+I} - T_{k-I}^{n+I}) \frac{1}{2} \frac{\partial}{\partial T_k^{n+I}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right)
 \end{aligned} \right] \\
 & + \left[\begin{aligned}
 & \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (P_{ck}^{n+I} - P_{ck-I}^{n+I}) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{1}{2}} \right) (T_k^{n+I} - T_{k-I}^{n+I}) \frac{\partial}{\partial T_k^{n+I}} (P_{ck}^{n+I}) \right] \\
 & + \rho_l c_{pl} (P_{ck}^{n+I} - P_{ck-I}^{n+I}) (T_k^{n+I} - T_{k-I}^{n+I}) \frac{1}{2} \frac{\partial}{\partial T_k^{n+I}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right)
 \end{aligned} \right] \\
 & - \rho_l c_{pl} \rho_l g \Delta x
 \end{aligned}$$

$$\begin{aligned}
& + \left. \left(\alpha_p \right)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} \right) \left(P_{gk+I}^{n+I} - P_{gk}^{n+I} \right) + \left(\alpha_p \right)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} \right) \left(T_k^{n+I} - T_{k-I}^{n+I} \right) \frac{\partial}{\partial T_k^{n+I}} \left(-P_{gk}^{n+I} \right) \right. \\
& \left. + \left(\alpha_p \right)_{av} \left(P_{gk+I}^{n+I} - P_{gk}^{n+I} \right) \left(T_k^{n+I} - T_{k-I}^{n+I} \right) \frac{I}{2} \frac{\partial}{\partial T_k^{n+I}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \\
& - \left. \left(\alpha_p \right)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{1}{2}} \right) \left(P_{gk}^{n+I} - P_{gk-I}^{n+I} \right) + \left(\alpha_p \right)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{1}{2}} \right) \left(T_k^{n+I} - T_{k-I}^{n+I} \right) \frac{\partial}{\partial T_k^{n+I}} \left(P_{gk}^{n+I} \right) \right. \\
& \left. - \left(\alpha_p \right)_{av} \left(P_{gk}^{n+I} - P_{gk-I}^{n+I} \right) \left(T_k^{n+I} - T_{k-I}^{n+I} \right) \frac{I}{2} \frac{\partial}{\partial T_k^{n+I}} \left(-\frac{KK_{rg}}{\mu_g} \Big|_k \right) \right) \\
& \left[\frac{\Delta t^2}{\Delta t} \rho_v \phi(I-s_{ir}) \frac{\partial}{\partial T_k^{n+I}} \left(-s_{ek}^{n+I} \right) + \left[\rho_v \rho_g g \Delta t \frac{\partial}{\partial T_k^{n+I}} \left(\frac{KK_{rg}}{\mu_g} \Big|_k \right) \right] \right] \\
& + H_v \left[- \left[D_{mk+\frac{1}{2}}^{n+I} \frac{\partial}{\partial T_k^{n+I}} \left(-P_{vk}^{n+I} \right) + \left(P_{vk+I}^{n+I} - P_{vk}^{n+I} \right) \frac{I}{2} \frac{\partial}{\partial T_k^{n+I}} D_{mk}^{n+I} \right] \right. \\
& \left. - \left[D_{mk-\frac{1}{2}}^{n+I} \frac{\partial}{\partial T_k^{n+I}} \left(P_{vk}^{n+I} \right) + \left(P_{vk}^{n+I} - P_{vk-I}^{n+I} \right) \frac{I}{2} \frac{\partial}{\partial T_k^{n+I}} D_{mk}^{n+I} \right] \right] \\
& + \left[\lambda_{k+\frac{1}{2}}^{n+I} (-I) + \left(T_{k+I}^{n+I} - T_k^{n+I} \right) \frac{I}{2} \frac{\partial}{\partial T_k^{n+I}} \lambda_k^{n+I} \right] - \left[\lambda_{k-\frac{1}{2}}^{n+I} + \left(T_k^{n+I} - T_{k-I}^{n+I} \right) \frac{I}{2} \frac{\partial}{\partial T_k^{n+I}} \lambda_k^{n+I} \right] \\
& - Q = 0 = Eg
\end{aligned}$$

(¶.38)

លេខ ទំនាក់អង្គភាព k+1

$$\begin{aligned}
 \frac{\partial E_{k+1}^{n+1}}{\partial T_k^{n+1}} = & \left(\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (P_{gk+1}^{n+1} - P_{gk}^{n+1}) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{\partial}{\partial T_k^{n+1}} (-P_{gk}^{n+1}) \right. \\
 & \left. + \rho_l c_{pl} (P_{gk+1}^{n+1} - P_{gk}^{n+1}) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \\
 & + \left(\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (P_{ck+1}^{n+1} - P_{ck}^{n+1}) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k+\frac{1}{2}} \right) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{\partial}{\partial T_k^{n+1}} (-P_{ck}^{n+1}) \right. \\
 & \left. + \rho_l c_{pl} (P_{ck+1}^{n+1} - P_{ck}^{n+1}) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \\
 & + \left((\alpha_p)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} \right) (P_{gk+1}^{n+1} - P_{gk}^{n+1}) + (\alpha_p)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k+\frac{1}{2}} \right) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{\partial}{\partial T_k^{n+1}} (-P_{gk}^{n+1}) \right. \\
 & \left. + (\alpha_p)_{av} (P_{gk+1}^{n+1} - P_{gk}^{n+1}) (T_k^{n+1} - T_{k-1}^{n+1}) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right) \\
 & + H_v \left[- \left[D_{mk+\frac{1}{2}}^{n+1} \frac{\partial}{\partial T_k^{n+1}} (-P_{vk}^{n+1}) + (P_{vk+1}^{n+1} - P_{vk}^{n+1}) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} D_{mk}^{n+1} \right] \right] \\
 & + \left[\lambda_{k+\frac{1}{2}}^{n+1} (-I) + (T_{k+1}^{n+1} - T_k^{n+1}) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \lambda_k^{n+1} \right] \\
 & - Q = 0 = Eh
 \end{aligned}$$

(Allison 39)

ณ ตำแหน่ง k-1

$$\begin{aligned}
 \frac{\partial E_{k-l}^{n+1}}{\partial T_k^{n+1}} = & - \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \right) (P_{gk}^{n+1} - P_{gk-l}^{n+1}) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \right) (T_k^{n+1} - T_{k-l}^{n+1}) \frac{\partial}{\partial T_k^{n+1}} (P_{gk}^{n+1}) \right. \\
 & \left. + \rho_l c_{pl} (P_{gk}^{n+1} - P_{gk-l}^{n+1}) (T_k^{n+1} - T_{k-l}^{n+1}) \frac{I}{2} \frac{\partial}{\partial T_k^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \\
 & - \left[\rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \right) (P_{ck}^{n+1} - P_{ck-l}^{n+1}) + \rho_l c_{pl} \left(-\frac{KK_{rl}}{\mu_l} \Big|_{k-\frac{l}{2}} \right) (T_k^{n+1} - T_{k-l}^{n+1}) \frac{\partial}{\partial T_k^{n+1}} (P_{ck}^{n+1}) \right. \\
 & \left. + \rho_l c_{pl} (P_{ck}^{n+1} - P_{ck-l}^{n+1}) (T_k^{n+1} - T_{k-l}^{n+1}) \frac{I}{2} \frac{\partial}{\partial T_k^{n+1}} \left(-\frac{KK_{rl}}{\mu_l} \Big|_k \right) \right] \\
 & - \left[(\rho c_p)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{l}{2}} \right) (P_{gk}^{n+1} - P_{gk-l}^{n+1}) + (\rho c_p)_{av} \left(-\frac{KK_{rg}}{\mu_g} \Big|_{k-\frac{l}{2}} \right) (T_k^{n+1} - T_{k-l}^{n+1}) \frac{\partial}{\partial T_k^{n+1}} (P_{gk}^{n+1}) \right. \\
 & \left. + (\rho c_p)_{av} (P_{gk}^{n+1} - P_{gk-l}^{n+1}) (T_k^{n+1} - T_{k-l}^{n+1}) \frac{I}{2} \frac{\partial}{\partial T_k^{n+1}} \left(-\frac{KK_{rg}}{\mu_g} \Big|_k \right) \right] \\
 & + H_v \left[- \left[D_{mk-\frac{l}{2}}^{n+1} \frac{\partial}{\partial T_k^{n+1}} (P_{vk}^{n+1}) + (P_{vk}^{n+1} - P_{vk-l}^{n+1}) \frac{I}{2} \frac{\partial}{\partial T_k^{n+1}} D_{mk}^{n+1} \right] \right. \\
 & \left. - \left[\lambda_{k-\frac{l}{2}}^{n+1} + (T_k^{n+1} - T_{k-l}^{n+1}) \frac{I}{2} \frac{\partial}{\partial T_k^{n+1}} \lambda_k^{n+1} \right] \right] \\
 & - Q = 0 = Ei
 \end{aligned} \tag{4.40}$$

จากนั้นทำการประยุกต์สำหรับพิจารณากรณีโหนดที่เงื่อนไขขอบเขต (Boundary Conditions) โดยทำการประยุกต์ เช่นเดียวกับกรณีโหนดภายใน (Internal node) ดังเช่นในภาคผนวก ก.