ภาคผนวก ข.

ระเบียบวิธีเชิงตัวเลขในการวิเคราะห์กระบวนการถ่ายเทความร้อน มวลสารและความดันในวัสดุพรุนที่ไม่อิ่มตัวภายใต้พลังงานไมโครเวฟ

สมการถ่ายเทมวลสาร (Mass Transport Equation)

สมการถ่ายเทมวลสาร เป็นสมการที่อธิบายถึงการกระจายของความชื้นในระนาบและอัตราการเปลี่ยนแปลงของความชื้นเมื่อระยะเวลาการอบแห้งเปลี่ยนไป สำหรับกระบวนการอบแห้งด้วยพลังงานไมโครเวฟสมการถ่ายเทมวลสารเกิดจากสมมุติฐานการเคลื่อนที่ของของไหลในวัสดุพรุน เขียนได้ดังนี้

$$\varphi \frac{\partial}{\partial t}\{s + Y_v(1-s)\} + \frac{\partial}{\partial z}\left[\frac{KK_{rl}}{\mu_l}\left(\frac{\partial P_c}{\partial z} - \frac{\partial P_g}{\partial z} + g_z\right) + Y_v \frac{KK_{rg}}{\mu_g}\left(-\frac{\partial P_g}{\partial z} + \rho_g g_z\right) - Y_g D_m \frac{\partial}{\partial z}(W_v)\right] = 0 \quad \text{(ข.1)}$$

โดยที่ $\frac{\rho_v}{\rho_l} = Y_v$, $\frac{\rho_g}{\rho_l} = Y_g$, $\frac{Y_v}{Y_g} = W_v$

โดยเมื่อวิเคราะห์ด้วยระเบียบวิธีเชิงตัวเลขโดยวิธีผลต่างสืบเนื่อง (finite difference) บนพื้นฐานวิธีปริมาตรควบคุม (control volume) จะได้

เมื่อ $\left.\frac{KK_{rl}}{\mu_l}\right|_{k\pm\frac{1}{2}} = \frac{\left.\frac{KK_{rl}}{\mu_l}\right|_k + \left.\frac{KK_{rl}}{\mu_l}\right|_{k\pm1}}{2}$, $\left.\frac{KK_{rg}}{\mu_g}\right|_{k\pm\frac{1}{2}} = \frac{\left.\frac{KK_{rg}}{\mu_g}\right|_k + \left.\frac{KK_{rg}}{\mu_g}\right|_{k\pm1}}{2}$

$$D^{n+1}_{k\pm\frac{1}{2}} = \frac{{D_k}^{n+1} + {D_{k\pm1}}^{n+1}}{2} \quad \text{(ข.2)}$$

และ $s^{n+1} = s^n - \Delta s$, ${P_g}^{n+1} = {P_g}^n - \Delta P_g$

$$T^{n+1} = T^n - \Delta T \quad \text{(ข.3)}$$

ดังนั้น

$$\frac{\phi}{\Delta t}\left\{(1-s_{ir})(s_{ek}^{n+1}-s_{ek}^{n})+(1-s_{ir})(Y_{vk}^{n+1}(1-s_{ek}^{n+1})-Y_{vk}^{n}(1-s_{ek}^{n})\right\}$$

$$+\frac{1}{\Delta z}\left\{\begin{array}{l}\left[\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\left(\frac{P_{ck+1}^{n+1}-P_{ck}^{n+1}}{\Delta z}-\frac{P_{gk+1}^{n+1}-P_{gk}^{n+1}}{\Delta z}+g_z\right)-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\left(\frac{P_{ck}^{n+1}-P_{ck-1}^{n+1}}{\Delta z}-\frac{P_{gk}^{n+1}-P_{gk-1}^{n+1}}{\Delta z}+g_z\right)\right]\\ +Y_{vk}^{n+1}\left[\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\left(\frac{-P_{gk+1}^{n+1}+P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)-\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\left(\frac{-P_{gk}^{n+1}+P_{gk-1}^{n+1}}{\Delta z}+\rho_g g_z\right)\right]\\ -Y_{vk}^{n+1}\left[D_{mk+\frac{1}{2}}^{n+1}\left(\frac{w_{vk+1}^{n+1}-w_{vk}^{n+1}}{\Delta z}\right)-D_{mk-\frac{1}{2}}^{n+1}\left(\frac{w_{vk}^{n+1}-w_{vk-1}^{n+1}}{\Delta z}\right)\right]\end{array}\right\}$$

$$=0=F_k^{n+1}\left(s,P_g,T\right) \qquad \text{(ข.4)}$$

กรณีพิจารณาโหนดภายใน (Internal node)

เมื่อพิจารณาสมการถ่ายเทมวลสารเทียบกับความชื้น (เทียบ s)

ณ ตำแหน่ง k

$$\frac{\partial F_k^{n+1}}{\partial s_{ek}^{n+1}}=\frac{\phi}{\Delta t}(1-s_{ir})\left\{(1-Y_{vk}^{n+1})\right\}$$

$$+\frac{1}{\Delta z}\left\{\begin{array}{l}\left[\begin{array}{l}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\frac{-P_{ck}^{n+1}}{\Delta z}\right)+\left(\frac{P_{ck+1}^{n+1}-P_{ck}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\\ \left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\frac{P_{ck}^{n+1}}{\Delta z}\right)-\left(\frac{P_{ck}^{n+1}-P_{ck-1}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\end{array}\right]\\ -\left[\begin{array}{l}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\frac{-P_{gk}^{n+1}}{\Delta z}\right)+\left(\frac{P_{gk+1}^{n+1}-P_{gk}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\\ \left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\frac{P_{gk}^{n+1}}{\Delta z}\right)-\left(\frac{P_{gk}^{n+1}-P_{gk-1}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\end{array}\right]\\ +\left[\left(Y_{vk}^{n+1}\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)\left(\frac{P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\right)-\left(Y_{vk-1}^{n+1}\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)\left(\frac{-P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\right)\right]\\ -Y_{vk}^{n+1}\left[\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}D_{mk}^{n+1}\left(\frac{w_{vk+1}^{n+1}-w_{vk}^{n+1}}{\Delta z}\right)-\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}D_{mk}^{n+1}\left(\frac{w_{vk}^{n+1}-w_{vk-1}^{n+1}}{\Delta z}\right)\right]\end{array}\right\}$$

$$=0=Fa \qquad \text{(ข.5)}$$

ณ ตำแหน่ง k+1

$$\frac{\partial F_{k+1}^{n+1}}{\partial s_{ek}^{n+1}} = \frac{1}{\Delta z}\left\{\begin{array}{l}\left[\begin{array}{l}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\frac{-P_{ck}^{n+1}}{\Delta z}\right)+\left(\frac{P_{ck+1}^{n+1}-P_{ck}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\\ -\left(\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\frac{-P_{gk}^{n+1}}{\Delta z}\right)+\left(\frac{P_{gk+1}^{n+1}-P_{gk}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\end{array}\right]\\ +\left[\left(Y_{vk}^{n+1}\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)\left(\frac{P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\right)\right]\\ -Y_{vk}^{n+1}\left[\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}D_{mk+1}^{n+1}\left(\frac{w_{vk+1}^{n+1}-w_{vk}^{n+1}}{\Delta z}\right)\right]\end{array}\right\}$$

$$= Fb \qquad \text{(ข.6)}$$

ณ ตำแหน่ง k-1

$$\frac{\partial F_{k-1}^{n+1}}{\partial s_{ek}^{n+1}} = \frac{1}{\Delta z}\left\{\begin{array}{l}\left[\begin{array}{l}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\frac{P_{ck}^{n+1}}{\Delta z}\right)-\left(\frac{P_{ck}^{n+1}-P_{ck-1}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\\ -\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\frac{P_{gk}^{n+1}}{\Delta z}\right)-\left(\frac{P_{gk}^{n+1}-P_{gk-1}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\end{array}\right]\\ +\left[-\left(Y_{vk-1}^{n+1}\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)\left(\frac{-P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\right)\right]-Y_{vk}^{n+1}\left[-\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}D_{mk-1}^{n+1}\left(\frac{w_{vk}^{n+1}-w_{vk-1}^{n+1}}{\Delta z}\right)\right]\end{array}\right\}$$

$$= 0 = Fc \qquad \text{(ข.7)}$$

เมื่อพิจารณาสมการถ่ายเทมวลสารเทียบกับความดัน (เทียบ P)

ณ ตำแหน่ง k

$$\frac{\partial F_k^{n+1}}{\partial P_k^{n+1}} = \frac{\phi}{\Delta t}(1-s_{ir})\left\{(1-s_{ek}^{n+1})\frac{\partial}{\partial P_k^{n+1}}Y_{vk}^{n+1}\right\}$$

$$+\frac{1}{\Delta z}\left\{\begin{array}{l}
\left[\begin{array}{l}
\left(\frac{KK_{rl}}{\mu_l}\Big|_{k+\frac{1}{2}}\frac{\partial}{\partial P_k^{n+1}}\left(\frac{-P_{ck}^{n+1}}{\Delta z}\right)+\left(\frac{P_{ck+1}^{n+1}-P_{ck}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\Big|_k\right)\right) \\
\left(-\frac{KK_{rl}}{\mu_l}\Big|_{k-\frac{1}{2}}\frac{\partial}{\partial P_k^{n+1}}\left(\frac{P_{ck}^{n+1}}{\Delta z}\right)-\left(\frac{P_{ck}^{n+1}-P_{ck-1}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\Big|_k\right)\right)
\end{array}\right] \\
-\left[\begin{array}{l}
\left(\frac{KK_{rl}}{\mu_l}\Big|_{k+\frac{1}{2}}\frac{\partial}{\partial P_k^{n+1}}\left(\frac{-P_{gk}^{n+1}}{\Delta z}\right)+\left(\frac{P_{gk+1}^{n+1}-P_{gk}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\Big|_k\right)\right) \\
\left(-\frac{KK_{rl}}{\mu_l}\Big|_{k-\frac{1}{2}}\frac{\partial}{\partial P_k^{n+1}}\left(\frac{P_{gk}^{n+1}}{\Delta z}\right)-\left(\frac{P_{gk}^{n+1}-P_{gk-1}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\Big|_k\right)\right)
\end{array}\right] \\
+\left[\left(Y_{vk}^{n+1}\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\frac{KK_{rg}}{\mu_g}\Big|_k\right)\right)\left(\frac{P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\right)-\left(Y_{vk-1}^{n+1}\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\frac{KK_{rg}}{\mu_g}\Big|_k\right)\left(\frac{-P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\right)\right] \\
-Y_{vk}^{n+1}\left[\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}D_{mk}^{n+1}\left(\frac{w_{vk+1}^{n+1}-w_{vk}^{n+1}}{\Delta z}\right)-\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}D_{mk}^{n+1}\left(\frac{w_{vk}^{n+1}-w_{vk-1}^{n+1}}{\Delta z}\right)\right]
\end{array}\right\}$$

$$=0=Fd \qquad \text{(ข.8)}$$

ณ ตำแหน่ง k+1

$$\frac{\partial F_{k+1}^{n+1}}{\partial P_k^{n+1}} = \frac{1}{\Delta z}\left\{\begin{array}{l}\left[\left(\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}}\left(\frac{-P_{ck}^{n+1}}{\Delta z}\right)+\left(\frac{P_{ck+1}^{n+1}-P_{ck}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\right. \\ \left.-\left(\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}}\left(\frac{-P_{gk}^{n+1}}{\Delta z}\right)+\left(\frac{P_{gk+1}^{n+1}-P_{gk}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\right] \\ +\left[\left(Y_{vk}^{n+1}\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)\left(\frac{P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\right)\right] \\ -Y_{vk}^{n+1}\left[\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}D_{mk}^{n+1}\left(\frac{w_{vk+1}^{n+1}-w_{vk}^{n+1}}{\Delta z}\right)\right]\end{array}\right\}$$

$$= 0 = Fe \qquad \text{(ข.9)}$$

ณ ตำแหน่ง k-1

$$\frac{\partial F_{k-1}^{n+1}}{\partial P_k^{n+1}} = \frac{1}{\Delta z}\left\{\begin{array}{l}\left[\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}}\left(\frac{P_{ck}^{n+1}}{\Delta z}\right)-\left(\frac{P_{ck}^{n+1}-P_{ck-1}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\right. \\ \left.-\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}}\left(\frac{P_{gk}^{n+1}}{\Delta z}\right)-\left(\frac{P_{gk}^{n+1}-P_{gk-1}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\right] \\ +\left[-\left(Y_{vk-1}^{n+1}\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)\left(\frac{-P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\right)\right] \\ -Y_{vk}^{n+1}\left[-\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}D_{mk}^{n+1}\left(\frac{w_{vk}^{n+1}-w_{vk-1}^{n+1}}{\Delta z}\right)\right]\end{array}\right\}$$

$$= 0 = Ff \qquad \text{(ข.10)}$$

เมื่อ พิจารณาสมการถ่ายเทมวลสารเทียบกับอุณหภูมิ (เทียบ T)

ณ ตำแหน่ง k

$$\frac{\partial F_k^{n+1}}{\partial T_k^{n+1}} = \frac{\phi}{\Delta t}(1-s_{ir})\left\{(1-s_{ek}^{n+1})\frac{\partial}{\partial T_k^{n+1}}Y_{vk}^{n+1}\right\}$$
$$+\frac{1}{\Delta z}\left\{\begin{array}{l}\left[\begin{array}{l}\left(\frac{KK_{rl}}{\mu_l}\Big|_{k+\frac{1}{2}}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{-P_{ck}^{n+1}}{\Delta z}\right)+\left(\frac{P_{ck+1}^{n+1}-P_{ck}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\Big|_k\right)\right)\\ -\left(\frac{KK_{rl}}{\mu_l}\Big|_{k-\frac{1}{2}}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{P_{ck}^{n+1}}{\Delta z}\right)-\left(\frac{P_{ck}^{n+1}-P_{ck-1}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\Big|_k\right)\right)\end{array}\right]\\ -\left[\begin{array}{l}\left(\frac{KK_{rl}}{\mu_l}\Big|_{k+\frac{1}{2}}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{-P_{gk}^{n+1}}{\Delta z}\right)+\left(\frac{P_{gk+1}^{n+1}-P_{gk}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\Big|_k\right)\right)\\ -\left(\frac{KK_{rl}}{\mu_l}\Big|_{k-\frac{1}{2}}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{P_{gk}^{n+1}}{\Delta z}\right)-\left(\frac{P_{gk}^{n+1}-P_{gk-1}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\Big|_k\right)\right)\end{array}\right]\\ +\left[\left(Y_{vk}^{n+1}\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{KK_{rg}}{\mu_g}\Big|_k\right)\right)\left(\frac{P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\right)-\left(Y_{vk-1}^{n+1}\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{KK_{rg}}{\mu_g}\Big|_k\right)\left(\frac{-P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\right)\right]\\ -Y_{vk}^{n+1}\left[\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}D_{mk}^{n+1}\left(\frac{w_{vk+1}^{n+1}-w_{vk}^{n+1}}{\Delta z}\right)-\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}D_{mk}^{n+1}\left(\frac{w_{vk}^{n+1}-w_{vk-1}^{n+1}}{\Delta z}\right)\right]\end{array}\right\}$$
$$=0=Fg$$

(ข.11)

ณ ตำแหน่ง k+1

$$\frac{\partial F_{k+1}^{n+1}}{\partial T_k^{n+1}} = \frac{1}{\Delta z}\left\{\begin{array}{l}\left[\left(\frac{KK_{rl}}{\mu_l}\Big|_{k+\frac{1}{2}}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{-P_{ck}^{n+1}}{\Delta z}\right)+\left(\frac{P_{ck+1}^{n+1}-P_{ck}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\Big|_k\right)\right)\right.\\ \left.-\left(\frac{KK_{rl}}{\mu_l}\Big|_{k+\frac{1}{2}}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{-P_{gk}^{n+1}}{\Delta z}\right)+\left(\frac{P_{gk+1}^{n+1}-P_{gk}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\Big|_k\right)\right)\right]\\ +\left[\left(Y_{vk}^{n+1}\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{KK_{rg}}{\mu_g}\Big|_k\right)\right)\left(\frac{P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\right)\right]-Y_{vk}^{n+1}\left[\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}D_{mk}^{n+1}\left(\frac{w_{vk+1}^{n+1}-w_{vk}^{n+1}}{\Delta z}\right)\right]\end{array}\right\}$$
$$=0=Fh$$

(ข.12)

ณ ตำแหน่ง k-1

$$\frac{\partial F_{k-1}^{n+1}}{\partial T_k^{n+1}} = \frac{1}{\Delta z}\left\{\begin{array}{l}\left[\begin{array}{l}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{P_{ck}^{n+1}}{\Delta z}\right)-\left(\frac{P_{ck}^{n+1}-P_{ck-1}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\\ -\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{P_{gk}^{n+1}}{\Delta z}\right)-\left(\frac{P_{gk}^{n+1}-P_{gk-1}^{n+1}}{\Delta z}+g_z\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right)\end{array}\right]\\ +\left[-\left(Y_{vk-1}^{n+1}\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)\left(\frac{-P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\right)\right]\\ -Y_{vk}^{n+1}\left[-\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}D_{mk}^{n+1}\left(\frac{w_{vk}^{n+1}-w_{vk-1}^{n+1}}{\Delta z}\right)\right]\end{array}\right\}$$

$$= 0 = Fi \qquad \text{(ข.13)}$$

จากนั้นทำการประยุกต์สำหรับพิจารณากรณีโหนดที่เงื่อนไขขอบเขต (Boundary Conditions) **โดยทำ**การประยุกต์ **เช่นเดียวกับกรณี**โหนดภายใน (Internal node) ดังเช่นในภาคผนวก ก

สมการความดัน (Total Pressure Equation)

สมการความดัน เป็นสมการที่อธิบายถึงการกระจายของสนามความดันในระนาบและอัตราการเปลี่ยนแปลงของความดันในวัสดุพรุนเมื่อระยะเวลาการอบแห้งเปลี่ยนไป โดยการกระจายความดันเกิดขึ้นจากสมมุติฐานการเคลื่อนตัวและการเปลี่ยนแปลงสถานะของของไหลในวัสดุพรุน สามารถนำมาเขียนได้ดังนี้

$$\phi\frac{\partial}{\partial t}\left\{Y_a(1-s)\right\}+\frac{\partial}{\partial z}\left[Y_a\frac{KK_{rg}}{\mu_g}\left(-\frac{\partial p_g}{\partial z}+\rho_g g_z\right)-Y_g D_m\frac{\partial}{\partial z}\left(\frac{\rho_a}{\rho_g}\right)\right]=0 \qquad \text{(ข.14)}$$

โดยที่ $\frac{\rho_a}{\rho_l} = Y_a$

และสามารถวิเคราะห์ด้วยระเบียบวิธีเชิงตัวเลขโดยวิธีผลต่างสืบเนื่อง (finite difference) บนพื้นฐานวิธีปริมาตรควบคุม (control volume) ดังนี้

$$\frac{\phi}{\Delta t}\left\{(1-s_{ir})\left[\left(Y_{vk}^{n+1}(1-s_{ek}^{n+1})-Y_{vk}^{n}(1-s_{ek}^{n})\right]\right\}\right.$$

$$+\frac{1}{\Delta z}\left\{\begin{array}{l} Y_{ak}^{n+1}\left[\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\left(\frac{-P_{gk+1}^{n+1}+P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)-\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\left(\frac{-P_{gk}^{n+1}+P_{gk-1}^{n+1}}{\Delta z}+\rho_g g_z\right)\right] \\ -Y_{gk}^{n+1}\left[D_{mk+\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_{k+1}^{n+1}-\left(\frac{\rho_a}{\rho_g}\right)_{k}^{n+1}}{\Delta z}\right)-D_{mk-\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_{k}^{n+1}-\left(\frac{\rho_a}{\rho_g}\right)_{k-1}^{n+1}}{\Delta z}\right)\right] \end{array}\right\}$$

$$=0=G_k^{n+1}\left(s,P_g,T\right)$$

(ข.15)

กรณีพิจารณาโหนดภายใน (Internal node)

เมื่อพิจารณาสมการถ่ายเทมวลสารเทียบกับความชื้น (เทียบ s)

ณ ตำแหน่ง k

$$\frac{\partial G_k^{n+1}}{\partial s_{ek}^{n+1}}=\frac{\phi}{\Delta t}(1-s_{ir})Y_{vk}^{n+1}$$

$$+\frac{1}{\Delta z}\left\{\begin{array}{l} Y_{ak}^{n+1}\left[\begin{array}{l}\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\frac{P_{gk}^{n+1}}{\Delta z}\right)\left(\frac{-P_{gk+1}^{n+1}+P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right) \\ -\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\frac{-P_{gk}^{n+1}}{\Delta z}\right)\left(\frac{-P_{gk}^{n+1}+P_{gk-1}^{n+1}}{\Delta z}+\rho_g g_z\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right)\end{array}\right] \\ -Y_{gk}^{n+1}\left[\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}D_{mk+\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_{k+1}^{n+1}-\left(\frac{\rho_a}{\rho_g}\right)_{k}^{n+1}}{\Delta z}\right)-\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}D_{mk-\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_{k}^{n+1}-\left(\frac{\rho_a}{\rho_g}\right)_{k-1}^{n+1}}{\Delta z}\right)\right] \end{array}\right\}$$

$$=0=Ga$$

(ข.16)

ณ ตำแหน่ง k+1

$$\frac{\partial G_{k+1}^{n+1}}{\partial s_{ek}^{n+1}} = \frac{1}{\Delta z}\left\{\begin{array}{l} Y_{ak}^{n+1}\left[\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}} \frac{\partial}{\partial s_{ek}^{n+1}}\left(\frac{P_{gk}^{n+1}}{\Delta z}\right)\left(\frac{-P_{gk+1}^{n+1}+P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right] \\ -Y_{gk}^{n+1}\left[\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}} D_{mk+\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_{k+1}^{n+1}-\left(\frac{\rho_a}{\rho_g}\right)_k^{n+1}}{\Delta z}\right)\right] \end{array}\right\}$$

$$= 0 = Gb \qquad \text{(ข.17)}$$

ณ ตำแหน่ง k-1

$$\frac{\partial G_{k-1}^{n+1}}{\partial s_{ek}^{n+1}} = \frac{1}{\Delta z}\left\{\begin{array}{l} Y_{ak}^{n+1}\left[-\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}} \frac{\partial}{\partial s_{ek}^{n+1}}\left(\frac{-P_{gk}^{n+1}}{\Delta z}\right)\left(\frac{-P_{gk}^{n+1}+P_{gk-1}^{n+1}}{\Delta z}+\rho_g g_z\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right] \\ -Y_{gk}^{n+1}\left[-\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}} D_{mk-\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_k^{n+1}-\left(\frac{\rho_a}{\rho_g}\right)_{k-1}^{n+1}}{\Delta z}\right)\right] \end{array}\right\}$$

$$= 0 = Gc \qquad \text{(ข.18)}$$

เมื่อ พิจารณาสมการถ่ายเทมวลสารเทียบกับความดัน (เทียบ P)

ณ ตำแหน่ง k

$$\frac{\partial G_k^{n+1}}{\partial P_k^{n+1}} = \frac{\phi}{\Delta t}(1-s_{ir})\frac{\partial}{\partial P_k^{n+1}} Y_{vk}^{n+1}$$

$$+\frac{1}{\Delta z}\left\{ Y_{ak}^{n+1}\left[\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}}\left(\frac{P_{gk}^{n+1}}{\Delta z}\right)\left(\frac{-P_{gk+1}^{n+1}+P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right) \right.\right.$$

$$\left.\left. -\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}}\left(\frac{-P_{gk}^{n+1}}{\Delta z}\right)\left(\frac{-P_{gk}^{n+1}+P_{gk-1}^{n+1}}{\Delta z}+\rho_g g_z\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right) \right] \right.$$

$$\left. -Y_{gk}^{n+1}\left[\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}} D_{mk+\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_{k+1}^{n+1}-\left(\frac{\rho_a}{\rho_g}\right)_k^{n+1}}{\Delta z}\right) - \frac{1}{2}\frac{\partial}{\partial P_k^{n+1}} D_{mk-\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_k^{n+1}-\left(\frac{\rho_a}{\rho_g}\right)_{k-1}^{n+1}}{\Delta z}\right) \right] \right\}$$

$$= 0 = Gd \qquad \text{(ข.19)}$$

ณ ตำแหน่ง k+1

$$\frac{\partial G_{k+1}^{n+1}}{\partial P_k^{n+1}} = \frac{1}{\Delta z}\left\{ Y_{ak}^{n+1}\left[\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}}\left(\frac{P_{gk}^{n+1}}{\Delta z}\right)\left(\frac{-P_{gk+1}^{n+1}+P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right) \right] \right.$$

$$\left. -Y_{gk}^{n+1}\left[\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}} D_{mk+\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_{k+1}^{n+1}-\left(\frac{\rho_a}{\rho_g}\right)_k^{n+1}}{\Delta z}\right) \right] \right\}$$

$$= 0 = Ge \qquad \text{(ข.20)}$$

ณ ตำแหน่ง k-1

$$\frac{\partial G_{k-1}^{n+1}}{\partial P_k^{n+1}} = \frac{1}{\Delta z}\left\{ Y_{ak}^{n+1}\left[-\frac{KK_{rg}}{\mu_g}\bigg|_{k-\frac{1}{2}} \frac{\partial}{\partial P_k^{n+1}}\left(\frac{-P_{gk}^{n+1}}{\Delta z}\right)\left(\frac{-P_{gk}^{n+1}+P_{gk-1}^{n+1}}{\Delta z}+\rho_g g_z\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\bigg|_k\right)\right] \right.$$
$$\left. -Y_{gk}^{n+1}\left[-\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}} D_{mk-\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_k^{n+1} - \left(\frac{\rho_a}{\rho_g}\right)_{k-1}^{n+1}}{\Delta z} \right)\right] \right\}$$
$$= 0 = Gf$$

(ข.21)

เมื่อ พิจารณาสมการถ่ายเทมวลสารเทียบกับอุณหภูมิ (เทียบ T)

ณ ตำแหน่ง k

$$\frac{\partial G_k^{n+1}}{\partial T_k^{n+1}} = \frac{\phi}{\Delta t}(1-s_{ir})\frac{\partial}{\partial T_k^{n+1}} Y_{vk}^{n+1}$$
$$+\frac{1}{\Delta z}\left\{ Y_{ak}^{n+1}\left[\frac{KK_{rg}}{\mu_g}\bigg|_{k+\frac{1}{2}} \frac{\partial}{\partial T_k^{n+1}}\left(\frac{P_{gk}^{n+1}}{\Delta z}\right)\left(\frac{-P_{gk+1}^{n+1}+P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\bigg|_k\right) \right.\right.$$
$$\left. -\frac{KK_{rg}}{\mu_g}\bigg|_{k-\frac{1}{2}} \frac{\partial}{\partial T_k^{n+1}}\left(\frac{-P_{gk}^{n+1}}{\Delta z}\right)\left(\frac{-P_{gk}^{n+1}+P_{gk-1}^{n+1}}{\Delta z}+\rho_g g_z\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\frac{KK_{rl}}{\mu_l}\bigg|_k\right)\right]$$
$$\left. -Y_{gk}^{n+1}\left[\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}} D_{mk+\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_{k+1}^{n+1} - \left(\frac{\rho_a}{\rho_g}\right)_k^{n+1}}{\Delta z} \right) - \frac{1}{2}\frac{\partial}{\partial T_k^{n+1}} D_{mk-\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_k^{n+1} - \left(\frac{\rho_a}{\rho_g}\right)_{k-1}^{n+1}}{\Delta z} \right)\right] \right\}$$
$$= 0 = Gg$$

(ข.22)

ณ ตำแหน่ง k+1

$$\frac{\partial G_{k+1}^{n+1}}{\partial T_k^{n+1}} = \frac{1}{\Delta z}\left\{\begin{array}{l} Y_{ak}^{n+1}\left[\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}} \frac{\partial}{\partial T_k^{n+1}}\left(\frac{P_{gk}^{n+1}}{\Delta z}\right)\left(\frac{-P_{gk+1}^{n+1}+P_{gk}^{n+1}}{\Delta z}+\rho_g g_z\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right] \\ -Y_{gk}^{n+1}\left[\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}} D_{mk+\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_{k+1}^{n+1}-\left(\frac{\rho_a}{\rho_g}\right)_k^{n+1}}{\Delta z}\right)\right] \end{array}\right\}$$

$$= 0 = Gh \qquad \text{(ข.23)}$$

ณ ตำแหน่ง k-1

$$\frac{\partial G_{k-1}^{n+1}}{\partial T_k^{n+1}} = \frac{1}{\Delta z}\left\{\begin{array}{l} Y_{ak}^{n+1}\left[-\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}} \frac{\partial}{\partial T_k^{n+1}}\left(\frac{-P_{gk}^{n+1}}{\Delta z}\right)\left(\frac{-P_{gk}^{n+1}+P_{gk-1}^{n+1}}{\Delta z}+\rho_g g_z\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\right] \\ -Y_{gk}^{n+1}\left[-\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}} D_{mk-\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_a}{\rho_g}\right)_k^{n+1}-\left(\frac{\rho_a}{\rho_g}\right)_{k-1}^{n+1}}{\Delta z}\right)\right] \end{array}\right\}$$

$$= 0 = Gi \qquad \text{(ข.24)}$$

จากนั้นทำการประยุกต์สำหรับพิจารณากรณีโหนดที่เงื่อนไขขอบเขต (Boundary Conditions) **โดยทำ**การประยุกต์ **เช่นเดียวกับกรณี**โหนดภายใน (Internal node) ดังเช่นในภาคผนวก ก

สมการถ่ายเทความร้อน (Heat Transport Equation)

สมการถ่ายเทความร้อน เป็นสมการที่อธิบายถึงการกระจายของสนามอุณหภูมิในระนาบและอัตราการเปลี่ยนแปลงของอุณหภูมิในวัสดุพรุนเมื่อระยะเวลาการอบแห้งเปลี่ยนไป การกระจายอุณหภูมิเกิดขึ้นจากสมมุติฐานสมการพาสำหรับของไหลที่ไหลในวัสดุพรุน สมการนำของโครงสร้างของแข็งในวัสดุพรุน และสมการปริมาณความร้อนที่วัสดุผลิตได้ซึ่งเป็นปริมาณความร้อนที่วัสดุผลิตขึ้นจากการดูดซับพลังงานไมโครเวฟ และสามารถนำมาเขียนได้ดังนี้

$$\frac{\partial}{\partial t}\left\{\left(\rho_l c_{pl}\phi s+\left\{\left(\rho c_P\right)_a+\left(\rho c_p\right)_v\right\}\phi(1-s)+\rho_p c_{pp}(1-\phi)\right)T\right\}$$
$$+\frac{\partial}{\partial z}\left[\left\{\rho_l c_{pl} w_l+\left(\rho_a c_{pa}+\rho_v c_{pv}\right)w_g\right\}T\right]$$
$$=\frac{\partial}{\partial z}\left[\lambda\frac{\partial T}{\partial z}\right]-H_v\left\{\frac{\partial}{\partial t}\left\{\rho_v\phi(1-s)\right\}+\frac{\partial}{\partial z}\left[\rho_v\frac{KK_{rg}}{\mu_g}\left(-\frac{\partial P_g}{\partial z}+\rho_g g_z\right)-\rho_g D_m\frac{\partial}{\partial z}\left(\frac{\rho_v}{\rho_g}\right)\right]\right\}+Q \qquad \text{(ข.25)}$$

เมื่อ

$$\left(\rho C_p\right)_T=\rho_l C_{pl}\phi s+\left\{\left(\rho C_p\right)_a+\left(\rho C_p\right)_v\right\}\phi(1-s)+\rho_p C_{pp}(1-\phi) \qquad \text{(ข.26)}$$

ดังนั้น

$$\frac{\partial}{\partial t}\left\{\left(\rho c_p\right)_T T\right\}+\frac{\partial}{\partial z}\left[\left\{\rho_l c_{pl} w_l+\left(\rho_a c_{pa}+\rho_v c_{pv}\right)w_g\right\}T\right]=\frac{\partial}{\partial z}\left[\lambda\frac{\partial T}{\partial z}\right]$$
$$-H_v\left\{\frac{\partial}{\partial t}\left\{\rho_v\phi(1-s)\right\}+\frac{\partial}{\partial z}\left[\rho_v\frac{KK_{rg}}{\mu_g}\left(-\frac{\partial P_g}{\partial z}+\rho_g g_z\right)-\rho_g D_m\frac{\partial}{\partial z}\left(\frac{\rho_v}{\rho_g}\right)\right]\right\}+Q \qquad \text{(ข.27)}$$

และสามารถวิเคราะห์ด้วยระเบียบวิธีเชิงตัวเลขโดยวิธีผลต่างสืบเนื่อง (finite difference) บนพื้นฐานวิธีปริมาตรควบคุม (control volume) ดังนี้

เมื่อ $\left.\frac{KK_{rl}}{\mu_l}\right|_{k\pm\frac{1}{2}}=\frac{\left.\frac{KK_{rl}}{\mu_l}\right|_k+\left.\frac{KK_{rl}}{\mu_l}\right|_{k\pm1}}{2}$ $\left.\frac{KK_{rg}}{\mu_g}\right|_{k\pm\frac{1}{2}}=\frac{\left.\frac{KK_{rg}}{\mu_g}\right|_k+\left.\frac{KK_{rg}}{\mu_g}\right|_{k\pm1}}{2}$

$$D_{k\pm\frac{1}{2}}^{n+1}=\frac{D_k{}^{n+1}+D_{k\pm1}{}^{n+1}}{2} \qquad \lambda_{k\pm\frac{1}{2}}^{n+1}{}^{n+1}=\frac{\lambda_k{}^{n+1}+\lambda_{k\pm1}{}^{n+1}}{2} \qquad \text{(ข.28)}$$

และ

$$s^{n+1}=s^n-\Delta s$$
$$P_g{}^{n+1}=P_g{}^n-\Delta P_g$$
$$T^{n+1}=T^n-\Delta T \qquad \text{(ข.29)}$$

$$\frac{1}{\Delta t}\left\{\rho_l c_{pl}\phi(1-s_{ir})({s_{ek}}^{n+1}-{s_{ek}}^{n})+\rho_g c_{pg}\phi(1-s_{ir})\left[\left(1-{s_{ek}}^{n+1}\right)-\left(1-{s_{ek}}^{n}\right)\right]+\rho_p c_{pp}(1-\phi)\right\}\left(T_k^{n+1}-T_k^{n}\right)$$

$$+\frac{1}{\Delta z}\left\{\begin{array}{l}\left(\left[\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(\frac{P_{gk+1}^{n+1}-P_{gk}^{n+1}}{\Delta z}\right)-\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(\frac{P_{gk}^{n+1}-P_{gk-1}^{n+1}}{\Delta z}\right)\right]\right)\\ +\left[\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(\frac{P_{ck+1}^{n+1}-P_{ck}^{n+1}}{\Delta z}\right)\\ -\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(\frac{P_{ck}^{n+1}-P_{ck-1}^{n+1}}{\Delta z}\right)\end{array}\right]-\rho_l c_{pl}\rho_l g\\ +\left(\left[\begin{array}{l}\left(\rho c_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\right)\left(\frac{P_{gk+1}^{n+1}-P_{gk}^{n+1}}{\Delta z}\right)\\ -\left(\rho c_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\right)\left(\frac{P_{gk}^{n+1}-P_{gk-1}^{n+1}}{\Delta z}\right)\end{array}\right]-\left(\rho c_p\right)_{av}\rho_g g\right)\end{array}\right\}\left(T_k^{n+1}-T_{k-1}^{n+1}\right)$$

$$+\frac{1}{\Delta z}\left[\lambda_{k+\frac{1}{2}}^{n+1}\left(\frac{T_{k+1}^{n+1}-T_k^{n+1}}{\Delta z}\right)-\lambda_{k-\frac{1}{2}}^{n+1}\left(\frac{T_k^{n+1}-T_{k-1}^{n+1}}{\Delta z}\right)\right]+\frac{H_v\rho_v\phi}{\Delta t}\left\{(1-s_{ir})({s_{ek}}^{n+1}-{s_{ek}}^{n})\right\}$$

$$+\frac{1}{\Delta z}\left\{\begin{array}{l}\left(\rho_{vk}^{n+1}\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\left(\rho_g g_z\right)-\rho_{vk-1}^{n+1}\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\left(\rho_g g_z\right)\right)\\ -\left(\rho_{gk}^{n+1}D_{mk+\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_v}{\rho_g}\right)_{k+1}^{n+1}-\left(\frac{\rho_v}{\rho_g}\right)_{k}^{n+1}}{\Delta z}\right)-\rho_{gk-1}^{n+1}D_{mk-\frac{1}{2}}^{n+1}\left(\frac{\left(\frac{\rho_v}{\rho_g}\right)_{k}^{n+1}-\left(\frac{\rho_v}{\rho_g}\right)_{k-1}^{n+1}}{\Delta z}\right)\right)\end{array}\right\}$$

$$-Q=0$$

(ป.30)

นำ Δz^2 คูณตลอดสมการ

$$
\begin{aligned}
&\frac{\Delta z^2}{\Delta t}\left\{\begin{array}{l}\rho_l c_{pl}\phi(1-s_{ir})\left(s_{ek}{}^{n+1}-s_{ek}{}^{n}\right)\\ +\rho_g c_{pg}\phi(1-s_{ir})\left[\left(1-s_{ek}{}^{n+1}\right)-\left(1-s_{ek}{}^{n}\right)\right]+\rho_p c_{pp}(1-\phi)\end{array}\right\}\left(T_k^{n+1}-T_k^{n}\right)\\
&+\left\{\begin{array}{l}\left(\left[\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)-\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)\right]\right.\\ +\left[\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(P_{ck+1}^{n+1}-P_{ck}^{n+1}\right)-\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(P_{ck}^{n+1}-P_{ck-1}^{n+1}\right)\right]\\ \left.-\rho_l c_{pl}\rho_l g\Delta z\right)\end{array}\right\}\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\\
&+\left\{\left(\left[\left(\alpha_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\right)\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)-\left(\alpha_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\right)\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)\right]\right.\right.\\
&\left.\left.-\left(\alpha_p\right)_{av}\rho_g g\Delta z\right)\right\}\\
&+\left[\lambda_{k+\frac{1}{2}}^{n+1}\left(T_{k+1}^{n+1}-T_k^{n+1}\right)-\lambda_{k-\frac{1}{2}}^{n+1}\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\right]+\frac{H_v\rho_v\phi\Delta z^2}{\Delta t}\left\{(1-s_{ir})\left(s_{ek}{}^{n+1}-s_{ek}{}^{n}\right)\right\}\\
&+\left\{\begin{array}{l}\left(\rho_{vk}^{n+1}\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\left(\rho_g g_z\Delta z\right)-\rho_{vk-1}^{n+1}\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\left(\rho_g g_z\Delta z\right)\right)\\ -\left(\rho_{gk}^{n+1}D_{mk+\frac{1}{2}}^{n+1}\left(\left(\frac{\rho_v}{\rho_g}\right)_{k+1}^{n+1}-\left(\frac{\rho_v}{\rho_g}\right)_{k}^{n+1}\right)-\rho_{gk-1}^{n+1}D_{mk-\frac{1}{2}}^{n+1}\left(\left(\frac{\rho_v}{\rho_g}\right)_{k}^{n+1}-\left(\frac{\rho_v}{\rho_g}\right)_{k-1}^{n+1}\right)\right)\end{array}\right\}\\
&-Q=0=E_k^{n+1}\left(s,P_g,T\right)
\end{aligned}
\qquad \text{(ข.31)}
$$

กรณีพิจารณาโหนดภายใน (Internal node)

เมื่อพิจารณาสมการถ่ายเทมวลสารเทียบกับความชื้น (เทียบ s)

ณ ตำแหน่ง k

$$\frac{\partial E_k^{n+1}}{\partial s_{ek}^{n+1}}=\frac{\Delta z^2}{\Delta t}\left\{\begin{array}{l}\left[\rho_l c_{pl}\phi(1-s_{ir})+\rho_g c_{pg}\phi(1-s_{ir})(-1)\right]\left(T_k^{n+1}-T_k^{n}\right)\\ +\left\{\begin{array}{l}\rho_l c_{pl}\phi(1-s_{ir})\left(s_{ek}{}^{n+1}-s_{ek}{}^{n}\right)\\ +\rho_g c_{pg}\phi(1-s_{ir})\left[\left(1-s_{ek}{}^{n+1}\right)-\left(1-s_{ek}{}^{n}\right)\right]\\ +\rho_p c_{pp}(1-\phi)\end{array}\right\}\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)\end{array}\right\}$$

$$+\left[\begin{array}{l}\left(\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)+\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(-P_{gk}^{n+1}\right)\right.\\ \left.+\rho_l c_{pl}\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right)\right)\\ -\left(\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)+\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(P_{gk}^{n+1}\right)\\ +\rho_l c_{pl}\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right)\end{array}\right)\end{array}\right]$$

$$+\left[\begin{array}{l}\left(\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(P_{ck+1}^{n+1}-P_{ck}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)+\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(-P_{ck}^{n+1}\right)\right.\\ \left.+\rho_l c_{pl}\left(P_{ck+1}^{n+1}-P_{ck}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right)\right)\\ -\left(\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(P_{ck}^{n+1}-P_{ck-1}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)+\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(P_{ck}^{n+1}\right)\\ +\rho_l c_{pl}\left(P_{ck}^{n+1}-P_{ck-1}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right)\end{array}\right)\end{array}\right]$$

$$-\rho_l c_{pl}\rho_l g\Delta z\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)$$

$$
\begin{aligned}
&+\left(\begin{array}{l}
\left(\rho\alpha_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\right)\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)+\left(\rho\alpha_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(-P_{gk}^{n+1}\right)\\
+\left(\rho\alpha_p\right)_{av}\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)
\end{array}\right)\\
&-\left(\begin{array}{l}
\left(\rho\alpha_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\right)\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)+\left(\rho\alpha_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(P_{gk}^{n+1}\right)\\
+\left(\rho\alpha_p\right)_{av}\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)
\end{array}\right)\\
&-\left(\rho\alpha_p\right)_{av}\rho_g g\Delta z\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)\\
&+H_v\left(\begin{array}{l}
\left[\frac{\Delta z^2}{\Delta t}\rho_v\phi\left(1-s_{ir}\right)(-1)+\left[\rho_v\rho_g g\Delta z\frac{\partial}{\partial s_{ek}^{n+1}}\left(\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)\right]\right]\\
-\left[D_{mk+\frac{1}{2}}^{n+1}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-P_{vk}^{n+1}\right)+\left(P_{vk+1}^{n+1}-P_{vk}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}D_{mk}^{n+1}\right]\\
-\left[D_{mk-\frac{1}{2}}^{n+1}\frac{\partial}{\partial s_{ek}^{n+1}}\left(P_{vk}^{n+1}\right)+\left(P_{vk}^{n+1}-P_{vk-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}D_{mk}^{n+1}\right]
\end{array}\right)\\
&+\left[\lambda_{k+\frac{1}{2}}^{n+1}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-T_k^{n+1}\right)+\left(T_{k+1}^{n+1}-T_k^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\lambda_k^{n+1}\right]-\left[\lambda_{k-\frac{1}{2}}^{n+1}\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)+\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\lambda_k^{n+1}\right]\\
&-Q=0=Ea
\end{aligned}
$$

(ข.32)

ณ ตำแหน่ง k+1

$$\frac{\partial E_{k+1}^{n+1}}{\partial s_{ek}^{n+1}}=\left(\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)+\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(-P_{gk}^{n+1}\right)\\ +\rho_l c_{pl}\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\end{array}\right)$$

$$+\left(\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(P_{ck+1}^{n+1}-P_{ck}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)+\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(-P_{ck}^{n+1}\right)\\ +\rho_l c_{pl}\left(P_{ck+1}^{n+1}-P_{ck}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)\end{array}\right)$$

$$+\left(\begin{array}{l}\left(\rho_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\right)\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(T_k^{n+1}\right)+\left(\rho_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial s_{ek}^{n+1}}\left(-P_{gk}^{n+1}\right)\\ +\left(\rho_p\right)_{av}\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)\end{array}\right)$$

$$+H_v\left(-\left[D_{mk+\frac{1}{2}}^{n+1}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-P_{vk}^{n+1}\right)+\left(P_{vk+1}^{n+1}-P_{vk}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}D_{mk}^{n+1}\right]\right)$$

$$+\left[\lambda_{k+\frac{1}{2}}^{n+1}\frac{\partial}{\partial s_{ek}^{n+1}}\left(-T_k^{n+1}\right)+\left(T_{k+1}^{n+1}-T_k^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial s_{ek}^{n+1}}\lambda_k^{n+1}\right]$$

$$-Q=0=Eb$$

(ข.33)

ณ ตำแหน่ง k-1

$$\frac{\partial E_{k-1}^{n+1}}{\partial s_{ek}^{n+1}} = -\left(\begin{array}{l} \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \right) \left(P_{gk}^{n+1} - P_{gk-1}^{n+1} \right) \frac{\partial}{\partial s_{ek}^{n+1}} \left(T_k^{n+1} \right) + \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{\partial}{\partial s_{ek}^{n+1}} \left(P_{gk}^{n+1} \right) \\ + \rho_l c_{pl} \left(P_{gk}^{n+1} - P_{gk-1}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_k \right) \end{array} \right)$$

$$- \left(\begin{array}{l} \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \right) \left(P_{ck}^{n+1} - P_{ck-1}^{n+1} \right) \frac{\partial}{\partial s_{ek}^{n+1}} \left(T_k^{n+1} \right) + \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{\partial}{\partial s_{ek}^{n+1}} \left(P_{ck}^{n+1} \right) \\ + \rho_l c_{pl} \left(P_{ck}^{n+1} - P_{ck-1}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_k \right) \end{array} \right)$$

$$- \left(\begin{array}{l} \left(\rho c_p \right)_{av} \left(-\left. \frac{KK_{rg}}{\mu_g} \right|_{k-\frac{1}{2}} \right) \left(P_{gk}^{n+1} - P_{gk-1}^{n+1} \right) \frac{\partial}{\partial s_{ek}^{n+1}} \left(T_k^{n+1} \right) + \left(\rho c_p \right)_{av} \left(-\left. \frac{KK_{rg}}{\mu_g} \right|_{k-\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{\partial}{\partial s_{ek}^{n+1}} \left(P_{gk}^{n+1} \right) \\ + \left(\rho c_p \right)_{av} \left(P_{gk}^{n+1} - P_{gk-1}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \left(-\left. \frac{KK_{rg}}{\mu_g} \right|_k \right) \end{array} \right)$$

$$+ H_v \left(D_{mk-\frac{1}{2}}^{n+1} \frac{\partial}{\partial s_{ek}^{n+1}} \left(P_{vk}^{n+1} \right) + \left(P_{vk}^{n+1} - P_{vk-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} D_{mk}^{n+1} \right)$$

$$- \left[\lambda_{k-\frac{1}{2}}^{n+1} \frac{\partial}{\partial s_{ek}^{n+1}} \left(T_k^{n+1} \right) + \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial s_{ek}^{n+1}} \lambda_k^{n+1} \right]$$

$$- Q = 0 = Ec$$

(ข.34)

เมื่อ พิจารณาสมการถ่ายเทมวลสารเทียบกับความดัน (เทียบ P)

ณ ตำแหน่ง k

$$\frac{\partial E_k^{n+1}}{\partial P_k^{n+1}} = \frac{\Delta z^2}{\Delta t}\left\{\begin{array}{l}\left[\rho_l c_{pl}\phi(1-s_{ir})\frac{\partial}{\partial P_k^{n+1}}\left(s_{ek}^{n+1}\right)+\rho_g c_{pg}\phi(1-s_{ir})\frac{\partial}{\partial P_k^{n+1}}\left(-s_{ek}^{n+1}\right)\right]\left(T_k^{n+1}-T_k^{n}\right)+ \\ \left\{\begin{array}{l}\rho_l c_{pl}\phi(1-s_{ir})\left(s_{ek}{}^{n+1}-s_{ek}{}^{n}\right)+\rho_g c_{pg}\phi(1-s_{ir})\left[\left(1-s_{ek}{}^{n+1}\right)-\left(1-s_{ek}{}^{n}\right)\right] \\ +\rho_p c_{pp}(1-\phi)\end{array}\right\}\frac{\partial}{\partial P_k^{n+1}}\left(T_k^{n+1}\right)\end{array}\right\}$$

$$+\left[\begin{array}{l}\left(\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\frac{\partial}{\partial P_k^{n+1}}\left(T_k^{n+1}\right) \\ +\rho_l c_{pl}\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right)+\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)(-1)\end{array}\right) \\ -\left(\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)\frac{\partial}{\partial P_k^{n+1}}\left(T_k^{n+1}\right) \\ +\rho_l c_{pl}\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right)+\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\end{array}\right)\end{array}\right]$$

$$+\left[\begin{array}{l}\left(\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(P_{ck+1}^{n+1}-P_{ck}^{n+1}\right)\frac{\partial}{\partial P_k^{n+1}}\left(T_k^{n+1}\right) \\ +\rho_l c_{pl}\left(P_{ck+1}^{n+1}-P_{ck}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right) \\ +\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial P_k^{n+1}}\left(-P_{ck}^{n+1}\right)\end{array}\right) \\ -\left(\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(P_{ck}^{n+1}-P_{ck-1}^{n+1}\right)\frac{\partial}{\partial P_k^{n+1}}\left(T_k^{n+1}\right) \\ +\rho_l c_{pl}\left(P_{ck}^{n+1}-P_{ck-1}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right) \\ +\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial P_k^{n+1}}\left(P_{ck}^{n+1}\right)\end{array}\right)\end{array}\right]$$

$$-\rho_l c_{pl}\rho_l g\Delta z\frac{\partial}{\partial P_k^{n+1}}\left(T_k^{n+1}\right)$$

$$
\begin{aligned}
&+\left(\begin{array}{l}
(\rho c_p)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\right)\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\frac{\partial}{\partial P_k^{n+1}}\left(T_k^{n+1}\right)\\
+(\rho c_p)_{av}\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(-\left.\frac{KK_g}{\mu_g}\right|_k\right)+(\rho c_p)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)(-1)
\end{array}\right)\\
&-\left(\begin{array}{l}
(\rho c_p)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\right)\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)\frac{\partial}{\partial P_k^{n+1}}\left(T_k^{n+1}\right)\\
+(\rho c_p)_{av}\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\left(-\left.\frac{KK_g}{\mu_g}\right|_k\right)+(\rho c_p)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)
\end{array}\right)\\
&-(\rho c_p)_{av}\rho_g g\Delta z\frac{\partial}{\partial P_k^{n+1}}\left(T_k^{n+1}\right)\\
&+H_v\left(\begin{array}{l}
\left[\frac{\Delta z^2}{\Delta t}\rho_v\phi(1-s_{ir})\frac{\partial}{\partial P_k^{n+1}}\left(-s_{ek}^{n+1}\right)+\left[\rho_v\rho_g g\Delta z\frac{\partial}{\partial P_k^{n+1}}\left(\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)\right]\right]\\
-\left[D_{mk+\frac{1}{2}}^{n+1}\frac{\partial}{\partial P_k^{n+1}}\left(-P_{vk}^{n+1}\right)+\left(P_{vk+1}^{n+1}-P_{vk}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}D_{mk}^{n+1}\right]\\
-\left[D_{mk-\frac{1}{2}}^{n+1}\frac{\partial}{\partial P_k^{n+1}}\left(P_{vk}^{n+1}\right)+\left(P_{vk}^{n+1}-P_{vk-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}D_{mk}^{n+1}\right]
\end{array}\right)\\
&+\left[\lambda_{k+\frac{1}{2}}^{n+1}\frac{\partial}{\partial P_k^{n+1}}\left(-T_k^{n+1}\right)+\left(T_{k+1}^{n+1}-T_k^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\lambda_k^{n+1}\right]-\left[\lambda_{k-\frac{1}{2}}^{n+1}\frac{\partial}{\partial P_k^{n+1}}\left(T_k^{n+1}\right)+\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial P_k^{n+1}}\lambda_k^{n+1}\right]\\
&-Q=0=Ed
\end{aligned}
$$

(ข.35)

ณ ตำแหน่ง k+1

$$\frac{\partial E_{k+1}^{n+1}}{\partial P_k^{n+1}} = \left(\begin{array}{l} \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k+\frac{1}{2}} \right) \left(P_{gk+1}^{n+1} - P_{gk}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) + \rho_l c_{pl} \left(P_{gk+1}^{n+1} - P_{gk}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \\ \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k} \right) + \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k+\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) (-1) \end{array} \right)$$

$$+ \left(\begin{array}{l} \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k+\frac{1}{2}} \right) \left(P_{ck+1}^{n+1} - P_{ck}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) + \rho_l c_{pl} \left(P_{ck+1}^{n+1} - P_{ck}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \\ \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k} \right) + \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k+\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(-P_{ck}^{n+1} \right) \end{array} \right)$$

$$+ \left(\begin{array}{l} \left(\rho c_p \right)_{av} \left(-\left. \frac{KK_{rg}}{\mu_g} \right|_{k+\frac{1}{2}} \right) \left(P_{gk+1}^{n+1} - P_{gk}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) + \left(\rho c_p \right)_{av} \left(P_{gk+1}^{n+1} - P_{gk}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \\ \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\left. \frac{KK_g}{\mu_g} \right|_{k} \right) + \left(\rho c_p \right)_{av} \left(-\left. \frac{KK_{rg}}{\mu_g} \right|_{k+\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) (-1) \end{array} \right)$$

$$+ H_v \left(-\left[D_{mk+\frac{1}{2}}^{n+1} \frac{\partial}{\partial P_k^{n+1}} \left(-P_{vk}^{n+1} \right) + \left(P_{vk+1}^{n+1} - P_{vk}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk}^{n+1} \right] \right)$$

$$+ \left[\lambda_{k+\frac{1}{2}}^{n+1} \frac{\partial}{\partial P_k^{n+1}} \left(-T_k^{n+1} \right) + \left(T_{k+1}^{n+1} - T_k^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \lambda_k^{n+1} \right]$$

$$- Q = 0 = Ee \qquad \text{(ข.36)}$$

ณ ตำแหน่ง k-1

$$\frac{\partial E_{k-1}^{n+1}}{\partial P_k^{n+1}} = -\left(\begin{array}{l} \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \right) \left(P_{gk}^{n+1} - P_{gk-1}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) + \rho_l c_{pl} \left(P_{gk}^{n+1} - P_{gk-1}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \\ \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k} \right) + \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \end{array} \right)$$

$$- \left(\begin{array}{l} \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \right) \left(P_{ck}^{n+1} - P_{ck-1}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) + \rho_l c_{pl} \left(P_{ck}^{n+1} - P_{ck-1}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \\ \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k} \right) + \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(P_{ck}^{n+1} \right) \end{array} \right)$$

$$- \left(\begin{array}{l} \left(\rho c_p \right)_{av} \left(-\left. \frac{KK_{rg}}{\mu_g} \right|_{k-\frac{1}{2}} \right) \left(P_{gk}^{n+1} - P_{gk-1}^{n+1} \right) \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) + \left(\rho c_p \right)_{av} \left(P_{gk}^{n+1} - P_{gk-1}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \\ \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \left(-\left. \frac{KK_g}{\mu_g} \right|_{k} \right) + \left(\rho c_p \right)_{av} \left(-\left. \frac{KK_{rg}}{\mu_g} \right|_{k-\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \end{array} \right)$$

$$+ H_v \left(- \left[D_{mk-\frac{1}{2}}^{n+1} \frac{\partial}{\partial P_k^{n+1}} \left(P_{vk}^{n+1} \right) + \left(P_{vk}^{n+1} - P_{vk-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} D_{mk}^{n+1} \right] \right)$$

$$- \left[\lambda_{k-\frac{1}{2}}^{n+1} \frac{\partial}{\partial P_k^{n+1}} \left(T_k^{n+1} \right) + \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial P_k^{n+1}} \lambda_k^{n+1} \right]$$

$$- Q = 0 = Ef$$

(ข.37)

เมื่อ พิจารณาสมการถ่ายเทมวลสารเทียบกับอุณหภูมิ (เทียบ T)

ณ ตำแหน่ง k

$$\frac{\partial E_k^{n+1}}{\partial T_k^{n+1}}=\frac{\Delta z^2}{\Delta t}\left\{\begin{array}{l}\left[\rho_l c_{pl}\phi(1-s_{ir})\frac{\partial}{\partial T_k^{n+1}}({s_{ek}}^{n+1})+\rho_g c_{pg}\phi(1-s_{ir})\frac{\partial}{\partial T_k^{n+1}}(-{s_{ek}}^{n+1})\right]\left(T_k^{n+1}-T_k^{n}\right)+\\ \left\{\rho_l c_{pl}\phi(1-s_{ir})({s_{ek}}^{n+1}-{s_{ek}}^{n})+\rho_g c_{pg}\phi(1-s_{ir})\left[\left(1-{s_{ek}}^{n+1}\right)-\left(1-{s_{ek}}^{n}\right)\right]+\rho_p c_{pp}(1-\phi)\right\}\end{array}\right\}$$

$$+\left[\begin{array}{l}\left(\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)+\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial T_k^{n+1}}\left(-P_{gk}^{n+1}\right)\\ +\rho_l c_{pl}\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right)\end{array}\right)\\ -\left(\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)+\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial T_k^{n+1}}\left(P_{gk}^{n+1}\right)\\ +\rho_l c_{pl}\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right)\end{array}\right)\end{array}\right]$$

$$+\left[\begin{array}{l}\left(\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(P_{ck+1}^{n+1}-P_{ck}^{n+1}\right)+\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k+\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial T_k^{n+1}}\left(-P_{ck}^{n+1}\right)\\ +\rho_l c_{pl}\left(P_{ck+1}^{n+1}-P_{ck}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right)\end{array}\right)\\ -\left(\begin{array}{l}\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(P_{ck}^{n+1}-P_{ck-1}^{n+1}\right)+\rho_l c_{pl}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k-\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial T_k^{n+1}}\left(P_{ck}^{n+1}\right)\\ +\rho_l c_{pl}\left(P_{ck}^{n+1}-P_{ck-1}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_{k}\right)\end{array}\right)\end{array}\right]$$

$$-\rho_l c_{pl}\rho_l g\Delta z$$

$$
\begin{aligned}
&+\left(\begin{array}{l}
\left(\alpha_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\right)\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)+\left(\alpha_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k+\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial T_k^{n+1}}\left(-P_{gk}^{n+1}\right)\\
+\left(\alpha_p\right)_{av}\left(P_{gk+1}^{n+1}-P_{gk}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(-\left.\frac{KK_{rl}}{\mu_l}\right|_k\right)
\end{array}\right)\\
&-\left(\begin{array}{l}
\left(\alpha_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\right)\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)+\left(\alpha_p\right)_{av}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_{k-\frac{1}{2}}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{\partial}{\partial T_k^{n+1}}\left(P_{gk}^{n+1}\right)\\
+\left(\alpha_p\right)_{av}\left(P_{gk}^{n+1}-P_{gk-1}^{n+1}\right)\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\left(-\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)
\end{array}\right)-\left(\alpha_p\right)_{av}\rho_g g\Delta z\\
&+H_v\left[\begin{array}{l}
\frac{\Delta z^2}{\Delta t}\rho_v\phi(1-s_{ir})\frac{\partial}{\partial T_k^{n+1}}\left(-s_{ek}^{n+1}\right)+\left[\rho_v\rho_g g\Delta z\frac{\partial}{\partial T_k^{n+1}}\left(\left.\frac{KK_{rg}}{\mu_g}\right|_k\right)\right]\\
-\left[D_{mk+\frac{1}{2}}^{n+1}\frac{\partial}{\partial T_k^{n+1}}\left(-P_{vk}^{n+1}\right)+\left(P_{vk+1}^{n+1}-P_{vk}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}D_{mk}^{n+1}\right]\\
-\left[D_{mk-\frac{1}{2}}^{n+1}\frac{\partial}{\partial T_k^{n+1}}\left(P_{vk}^{n+1}\right)+\left(P_{vk}^{n+1}-P_{vk-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}D_{mk}^{n+1}\right]
\end{array}\right]\\
&+\left[\lambda_{k+\frac{1}{2}}^{n+1}(-1)+\left(T_{k+1}^{n+1}-T_k^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\lambda_k^{n+1}\right]-\left[\lambda_{k-\frac{1}{2}}^{n+1}+\left(T_k^{n+1}-T_{k-1}^{n+1}\right)\frac{1}{2}\frac{\partial}{\partial T_k^{n+1}}\lambda_k^{n+1}\right]\\
&-Q=0=Eg
\end{aligned}
\qquad \text{(ป.38)}
$$

ณ ตำแหน่ง k+1

$$\frac{\partial E_{k+1}^{n+1}}{\partial T_k^{n+1}} = \left(\begin{array}{l} \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k+\frac{1}{2}} \right) \left(P_{gk+1}^{n+1} - P_{gk}^{n+1} \right) + \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k+\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{\partial}{\partial T_k^{n+1}} \left(-P_{gk}^{n+1} \right) \\ + \rho_l c_{pl} \left(P_{gk+1}^{n+1} - P_{gk}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k} \right) \end{array} \right)$$

$$+ \left(\begin{array}{l} \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k+\frac{1}{2}} \right) \left(P_{ck+1}^{n+1} - P_{ck}^{n+1} \right) + \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k+\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{\partial}{\partial T_k^{n+1}} \left(-P_{ck}^{n+1} \right) \\ + \rho_l c_{pl} \left(P_{ck+1}^{n+1} - P_{ck}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k} \right) \end{array} \right)$$

$$+ \left(\begin{array}{l} \left(\alpha_p \right)_{av} \left(-\left. \frac{KK_{rg}}{\mu_g} \right|_{k+\frac{1}{2}} \right) \left(P_{gk+1}^{n+1} - P_{gk}^{n+1} \right) + \left(\alpha_p \right)_{av} \left(-\left. \frac{KK_{rg}}{\mu_g} \right|_{k+\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{\partial}{\partial T_k^{n+1}} \left(-P_{gk}^{n+1} \right) \\ + \left(\alpha_p \right)_{av} \left(P_{gk+1}^{n+1} - P_{gk}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k} \right) \end{array} \right)$$

$$+ H_v \left[- \left[D_{mk+\frac{1}{2}}^{n+1} \frac{\partial}{\partial T_k^{n+1}} \left(-P_{vk}^{n+1} \right) + \left(P_{vk+1}^{n+1} - P_{vk}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} D_{mk}^{n+1} \right] \right]$$

$$+ \left[\lambda_{k+\frac{1}{2}}^{n+1} \left(-1 \right) + \left(T_{k+1}^{n+1} - T_k^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \lambda_k^{n+1} \right]$$

$$- Q = 0 = Eh$$

(ข.39)

ณ ตำแหน่ง k-1

$$\frac{\partial E_{k-1}^{n+1}}{\partial T_k^{n+1}} = -\left(\begin{array}{l} \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \right) \left(P_{gk}^{n+1} - P_{gk-1}^{n+1} \right) + \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{\partial}{\partial T_k^{n+1}} \left(P_{gk}^{n+1} \right) \\ + \rho_l c_{pl} \left(P_{gk}^{n+1} - P_{gk-1}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k} \right) \end{array} \right)$$

$$- \left(\begin{array}{l} \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \right) \left(P_{ck}^{n+1} - P_{ck-1}^{n+1} \right) + \rho_l c_{pl} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k-\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{\partial}{\partial T_k^{n+1}} \left(P_{ck}^{n+1} \right) \\ + \rho_l c_{pl} \left(P_{ck}^{n+1} - P_{ck-1}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(-\left. \frac{KK_{rl}}{\mu_l} \right|_{k} \right) \end{array} \right)$$

$$- \left(\begin{array}{l} \left(\rho c_p \right)_{av} \left(-\left. \frac{KK_{rg}}{\mu_g} \right|_{k-\frac{1}{2}} \right) \left(P_{gk}^{n+1} - P_{gk-1}^{n+1} \right) + \left(\rho c_p \right)_{av} \left(-\left. \frac{KK_{rg}}{\mu_g} \right|_{k-\frac{1}{2}} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{\partial}{\partial T_k^{n+1}} \left(P_{gk}^{n+1} \right) \\ + \left(\rho c_p \right)_{av} \left(P_{gk}^{n+1} - P_{gk-1}^{n+1} \right) \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \left(-\left. \frac{KK_{rg}}{\mu_g} \right|_{k} \right) \end{array} \right)$$

$$+ H_v \left[- \left[D_{mk-\frac{1}{2}}^{n+1} \frac{\partial}{\partial T_k^{n+1}} \left(P_{vk}^{n+1} \right) + \left(P_{vk}^{n+1} - P_{vk-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} D_{mk}^{n+1} \right] \right]$$

$$- \left[\lambda_{k-\frac{1}{2}}^{n+1} + \left(T_k^{n+1} - T_{k-1}^{n+1} \right) \frac{1}{2} \frac{\partial}{\partial T_k^{n+1}} \lambda_k^{n+1} \right]$$

$$- Q = 0 = Ei$$

(ข.40)

จากนั้นทำการประยุกต์สำหรับพิจารณากรณีโหนดที่เงื่อนไขขอบเขต (Boundary Conditions) **โดยทำ**การประยุกต์ **เช่นเดียวกับกรณี**โหนดภายใน (Internal node) ดังเช่นในภาคผนวก ก.