

Homogeneous of degree one ในราคาปัจจัยการผลิต

จากฟังก์ชันต้นทุนการผลิต (Cost Function)

$$c = c(w, y)$$

$$\ln c = f(\ln w, \ln y)$$

คุณสมบัติ Homogeneous of degree one in w ของฟังก์ชันต้นทุนการผลิต คือ

$$c(tw, y) = tc(w, y) \quad \text{for } t > 0$$

$$f(\ln tw, \ln y) = \ln tc$$

จาก Translog cost function สมการที่ (2) ในบทที่ 4

$$\begin{aligned} \ln C = & \alpha_0 + [\alpha_1 \ln w_1 + \alpha_2 \ln w_2] + [\beta_1 \ln y_1 + \beta_2 \ln y_2] \\ & + \frac{1}{2} [\alpha_{11} \ln w_1 \ln w_1 + \alpha_{12} \ln w_1 \ln w_2 + \alpha_{21} \ln w_2 \ln w_1 + \alpha_{22} \ln w_2 \ln w_2] \\ & + \frac{1}{2} [\beta_{11} \ln y_1 \ln y_1 + \beta_{12} \ln y_1 \ln y_2 + \beta_{21} \ln y_2 \ln y_1 + \beta_{22} \ln y_2 \ln y_2] \\ & + [\gamma_{11} \ln w_1 \ln y_1 + \gamma_{12} \ln w_1 \ln y_2 + \gamma_{21} \ln w_2 \ln y_1 + \gamma_{22} \ln w_2 \ln y_2] + \varepsilon \end{aligned}$$

พิสูจน์ คุณสมบัติ Homogeneous of degree one in w ของ Translog cost function ดังนี้

$$\begin{aligned} f(\ln tw, \ln y) = & \alpha_0 + [\alpha_1 \ln tw_1 + \alpha_2 \ln tw_2] + [\beta_1 \ln y_1 + \beta_2 \ln y_2] \\ & + \frac{1}{2} [\alpha_{11} \ln tw_1 \ln tw_1 + \alpha_{12} \ln tw_1 \ln tw_2 + \alpha_{21} \ln tw_2 \ln tw_1 + \alpha_{22} \ln tw_2 \ln tw_2] \\ & + \frac{1}{2} [\beta_{11} \ln y_1 \ln y_1 + \beta_{12} \ln y_1 \ln y_2 + \beta_{21} \ln y_2 \ln y_1 + \beta_{22} \ln y_2 \ln y_2] \\ & + [\gamma_{11} \ln tw_1 \ln y_1 + \gamma_{12} \ln tw_1 \ln y_2 + \gamma_{21} \ln tw_2 \ln y_1 + \gamma_{22} \ln tw_2 \ln y_2] + \varepsilon \\ f(\ln tw, \ln y) = & \alpha_0 + \alpha_1 \ln t + \alpha_1 \ln w_1 + \alpha_2 \ln t + \alpha_2 \ln w_2 + [\beta_1 \ln y_1 + \beta_2 \ln y_2] \\ & + \frac{\alpha_{11}}{2} (\ln t + \ln w_1)^2 + \frac{\alpha_{12}}{2} (\ln t + \ln w_1)(\ln t + \ln w_2) \\ & + \frac{\alpha_{21}}{2} (\ln t + \ln w_2)(\ln t + \ln w_1) + \frac{\alpha_{22}}{2} (\ln t + \ln w_2)^2 \\ & + \frac{1}{2} [\beta_{11} \ln y_1 \ln y_1 + \beta_{12} \ln y_1 \ln y_2 + \beta_{21} \ln y_2 \ln y_1 + \beta_{22} \ln y_2 \ln y_2] \\ & + \gamma_{11} (\ln t + \ln w_1) \ln y_1 + \gamma_{12} (\ln t + \ln w_1) \ln y_2 \\ & + \gamma_{21} (\ln t + \ln w_2) \ln y_1 + \gamma_{22} (\ln t + \ln w_2) \ln y_2 + \varepsilon \end{aligned}$$

$$\begin{aligned}
f(\ln tw, \ln y) = & \alpha_0 + \alpha_1 \ln w_1 + \alpha_2 \ln w_2 + [\beta_1 \ln y_1 + \beta_2 \ln y_2] \\
& + \frac{\alpha_{11}}{2} (\ln w_1)^2 + \frac{\alpha_{12}}{2} \ln w_1 \ln w_2 + \frac{\alpha_{21}}{2} \ln w_2 \ln w_1 + \frac{\alpha_{22}}{2} (\ln w_2)^2 \\
& + \frac{1}{2} [\beta_{11} \ln y_1 \ln y_1 + \beta_{12} \ln y_1 \ln y_2 + \beta_{21} \ln y_2 \ln y_1 + \beta_{22} \ln y_2 \ln y_2] \\
& + \gamma_{11} \ln w_1 \ln y_1 + \gamma_{12} \ln w_1 \ln y_2 + \gamma_{21} \ln w_2 \ln y_1 + \gamma_{22} \ln w_2 \ln y_2 \\
& + \alpha_1 \ln t + \alpha_2 \ln t + \frac{\alpha_{11}}{2} (\ln t)^2 + \alpha_{11} \ln t \ln w_1 + \frac{\alpha_{12}}{2} (\ln t)^2 \\
& + \frac{\alpha_{12}}{2} \ln t \ln w_2 + \frac{\alpha_{12}}{2} \ln w_1 \ln t + \frac{\alpha_{21}}{2} (\ln t)^2 + \frac{\alpha_{21}}{2} \ln t \ln w_1 \\
& + \frac{\alpha_{21}}{2} \ln t \ln w_2 + \alpha_{22} \ln t \ln w_2 + \frac{\alpha_{22}}{2} (\ln t)^2 + \gamma_{11} \ln t \ln y_1 + \gamma_{12} \ln t \ln y_2 \\
& + \gamma_{21} \ln t \ln y_1 + \gamma_{22} \ln t \ln y_2 + \varepsilon
\end{aligned}$$

$$\begin{aligned}
f(\ln tw, \ln y) = & \ln C + (\alpha_1 + \alpha_2) \ln t + \frac{1}{2} (\alpha_{11} + \alpha_{12}) (\ln t)^2 + \frac{1}{2} (\alpha_{21} + \alpha_{22}) (\ln t)^2 \\
& + \left(\alpha_{11} + \frac{\alpha_{12}}{2} + \frac{\alpha_{21}}{2} \right) \ln t \ln w_1 + \left(\alpha_{22} + \frac{\alpha_{12}}{2} + \frac{\alpha_{21}}{2} \right) \ln t \ln w_2 \\
& + (\gamma_{11} + \gamma_{21}) \ln t \ln y_1 + (\gamma_{12} + \gamma_{22}) \ln t \ln y_2 + \varepsilon
\end{aligned}$$

จะได้ว่า $f(\ln tw, \ln y) \neq \ln tc$ นั่นคือ Translog cost function ไม่มีคุณสมบัติ Homogeneous of degree one ในราคาปัจจัยการผลิต

ถ้ากำหนดเงื่อนไขต่างๆตามสมการที่ (4) ในบทที่ 4 ซึ่งประกอบด้วย

$$\begin{aligned}
\alpha_1 + \alpha_2 &= 1 & \alpha_{12} &= \alpha_{21} \\
\alpha_{11} + \alpha_{12} &= 0 & \alpha_{21} + \alpha_{22} &= 0 \\
\gamma_{11} + \gamma_{21} &= 0 & \gamma_{12} + \gamma_{22} &= 0
\end{aligned}$$

จะได้ว่า

$$\begin{aligned}
f(\ln tw, \ln y) &= \ln C + \ln t \\
&= \ln tC
\end{aligned}$$