ผนวก ก

Translog Cost Function ที่มาจาก Taylor series expansion

จาก Taylor series expansion ตัวแปร n ตัว

$$f(x_1,...,x_n)=\sum_{j=0}^{\infty}\left\{\frac{1}{j!}\left[\sum_{k=1}^{n}(x_k-a_k)\frac{\partial}{\partial x_k'}\right]^j f(x_1',...,x_n')\right\}_{x_1'=a_1,...,x_n'=a_n}$$

ดังนั้น Second order Taylor series expansion ตัวแปร n ตัว จะได้

$$f(x_1,...,x_n)\approx\sum_{j=0}^{2}\left\{\frac{1}{j!}\left[\sum_{k=1}^{n}(x_k-a_k)\frac{\partial}{\partial x_k'}\right]^j f(x_1',...,x_n')\right\}_{x_1'=a_1,...,x_n'=a_n}$$

$$=\frac{1}{0!}f(a_1,...,a_n)+\frac{1}{1!}\left[\sum_{k=1}^{n}(x_k-a_k)\frac{\partial f}{\partial x_k'}\right]$$

$$+\left(\frac{1}{2!}\left[\sum_{k=1}^{n}(x_k-a_k)\frac{\partial}{\partial x_k'}\right]^2 f(x_1',...,x_n')\right)_{x_1'=a_1,...,x_n'=a_n}$$

ดังนั้นสามารถหา $f(x_1,...,x_m,x_{m+1},...,x_{m+n})$ ได้ดังนี้

$$f(x_1,...,x_m,x_{m+1},...,x_{m+n})$$

$$\approx\frac{1}{0!}f(a_1,...,a_m,a_{m+1},...,a_{m+n})+\frac{1}{1!}\left[\sum_{k=1}^{m}(x_k-a_k)\frac{\partial f}{\partial x_k'}\right]+\frac{1}{1!}\left[\sum_{k=m+1}^{m+n}(x_k-a_k)\frac{\partial f}{\partial x_k'}\right]$$

$$+\left(\frac{1}{2!}\left[\sum_{k=1}^{m}(x_k-a_k)\frac{\partial}{\partial x_k'}+\sum_{k=m+1}^{m+n}(x_k-a_k)\frac{\partial}{\partial x_k'}\right]^2 f(x_1',...,x_m',x_{m+1}',...,x_{m+n}')\right)_{x_1'=a_1,...,x_{m+n}'=a_{m+n}}$$

$$=f(a_1,...,a_m,a_{m+1},...,a_{m+n})+\left[\sum_{k=1}^{m}(x_k-a_k)\frac{\partial f}{\partial x_k'}\right]+\left[\sum_{k=m+1}^{m+n}(x_k-a_k)\frac{\partial f}{\partial x_k'}\right]$$

$$+\frac{1}{2}\left[\sum_{k=1}^{m}(x_k-a_k)\frac{\partial}{\partial x_k'}\right]^2 f+\frac{1}{2}\left[\sum_{k=m+1}^{m+n}(x_k-a_k)\frac{\partial}{\partial x_k'}\right]^2 f$$

$$+\frac{2}{2}\left[\sum_{k=1}^{m}(x_k-a_k)\frac{\partial}{\partial x_k'}\right]\left[\sum_{k=m+1}^{m+n}(x_k-a_k)\frac{\partial}{\partial x_k'}\right]f$$

ให้ $x_{m+1} = y_1, x_{m+2} = y_2, x_{m+3} = y_3, ..., x_{m+n} = y_n$ และ

$a_{m+1} = b_1, a_{m+2} = b_2, a_{m+3} = b_3, ..., a_{m+n} = b_n$ จะได้ว่า

$$f(x_1,...,x_m,y_1,...,y_n)$$

$$\approx f\left(a_1,...,a_m,b_1,...,b_n\right) + \left[\sum_{i=1}^{m}\left(x_i - a_i\right)\frac{\partial f}{\partial x_i}\right] + \left[\sum_{k=1}^{n}\left(y_k - b_k\right)\frac{\partial f}{\partial y_k}\right]$$
$$+\frac{1}{2}\left[\sum_{i=1}^{m}\left(x_i - a_i\right)\frac{\partial}{\partial x_i}\right]^2 f + \frac{1}{2}\left[\sum_{k=1}^{n}\left(y_k - b_k\right)\frac{\partial}{\partial y_k}\right]^2 f$$
$$+\left[\sum_{i=1}^{m}\left(x_i - a_i\right)\frac{\partial}{\partial x_i}\right]\left[\sum_{k=1}^{n}\left(y_k - b_k\right)\frac{\partial}{\partial y_k}\right] f$$

$$= f\left(a_1,...,a_m,b_1,...,b_n\right) + \sum_{i=1}^{m}\left(x_i - a_i\right)\frac{\partial f}{\partial x_i} + \sum_{k=1}^{n}\left(y_k - b_k\right)\frac{\partial f}{\partial y_k}$$
$$+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\left(x_i - a_i\right)\left(x_j - a_j\right)\frac{\partial^2 f}{\partial x_i \partial x_j} + \frac{1}{2}\sum_{k=1}^{n}\sum_{h=1}^{n}\left(y_k - b_k\right)\left(y_h - b_h\right)\frac{\partial^2 f}{\partial y_k \partial y_h}$$
$$+\sum_{i=1}^{m}\sum_{k=1}^{n}\left(x_i - a_i\right)\left(y_k - b_k\right)\frac{\partial^2 f}{\partial x_i \partial y_k}$$

จาก Cost Function

$$C = g(w_1,...,w_m,y_1,...,y_n)$$

$$\ln C = f(\ln w_1,...,\ln w_m, \ln y_1,...,\ln y_n)$$

จะได้ว่า

$$\ln C \approx f\left(a_1,...,a_m,b_1,...,b_n\right) + \sum_{i=1}^{m}\left(\ln w_i - a_i\right)\frac{\partial \ln C}{\partial \ln w_i} + \sum_{k=1}^{n}\left(\ln y_k - b_k\right)\frac{\partial \ln C}{\partial \ln y_k}$$
$$+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\left(\ln w_i - a_i\right)\left(\ln w_j - a_j\right)\frac{\partial^2 \ln C}{\partial \ln w_i \partial \ln w_j} + \frac{1}{2}\sum_{k=1}^{n}\sum_{h=1}^{n}\left(\ln y_k - b_k\right)\left(\ln y_h - b_h\right)\frac{\partial^2 \ln C}{\partial \ln y_k \partial \ln y_h}$$
$$+\sum_{i=1}^{m}\sum_{k=1}^{n}\left(\ln w_i - a_i\right)\left(\ln y_k - b_k\right)\frac{\partial^2 \ln C}{\partial \ln w_i \partial \ln y_k}$$

เมื่อ ทำ Taylor series รอบๆ จุด 0 นั่นคือ $a_1,...,a_m,b_1,...,b_n = 0$ จะได้ว่า

$$\ln C \approx f(0) + \sum_{i=1}^{m}(\ln w_i)\frac{\partial \ln C}{\partial \ln w_i}\bigg|_0 + \sum_{k=1}^{n}(\ln y_k)\frac{\partial \ln C}{\partial \ln y_k}\bigg|_0$$
$$+\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}(\ln w_i)(\ln w_j)\frac{\partial^2 \ln C}{\partial \ln w_i \partial \ln w_j}\bigg|_0 + \frac{1}{2}\sum_{k=1}^{n}\sum_{h=1}^{n}(\ln y_k)(\ln y_h)\frac{\partial^2 \ln C}{\partial \ln y_k \partial \ln y_h}\bigg|_0$$
$$+\sum_{i=1}^{m}\sum_{k=1}^{n}(\ln w_i)(\ln y_k)\frac{\partial^2 \ln C}{\partial \ln w_i \partial \ln y_k}\bigg|_0$$

แทน

$$\alpha_0 = f(0)$$
$$\alpha_i = \frac{\partial \ln C}{\partial \ln w_i}\bigg|_0$$
$$\beta_k = \frac{\partial \ln C}{\partial \ln y_k}\bigg|_0$$
$$\alpha_{ij} = \frac{\partial^2 \ln C}{\partial \ln w_i \partial \ln w_j}\bigg|_0$$
$$\beta_{kh} = \frac{\partial^2 \ln C}{\partial \ln y_k \partial \ln y_h}\bigg|_0$$
$$\gamma_{ik} = \frac{\partial^2 \ln C}{\partial \ln w_i \partial \ln y_k}\bigg|_0$$

จะได้ Translog Cost Function ดังนี้

$$\ln C \approx \alpha_0 + \sum_{i=1}^{m}\alpha_i \ln w_i + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\alpha_{ij}\ln w_i \ln w_j + \sum_{k=1}^{n}\beta_k \ln y_k$$
$$+\frac{1}{2}\sum_{k=1}^{n}\sum_{h=1}^{n}\beta_{kh}\ln y_k \ln y_h + \sum_{i=1}^{m}\sum_{k=1}^{n}\gamma_{ik}\ln w_i \ln y_k$$