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Some results on fuzzy hyperconnected spaces

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Abstract

This paper is a prolongation of the study of fuzzy hyperconnectedness. After studying the characteristic of fuzzy hyperconnectedness concerned with different concepts we ascertain an extension of one essential result of Park *et al.* (2003). In particular, we obtain one result based on fuzzy hyperconnectedness which points out a comparison between the fuzzy topological spaces and the general topological spaces. A necessary and sufficient condition for a fuzzy semicontinuous function to be a fuzzy almost continuous function is established while the said two continuities are independent to each other shown by Azad (1981).

Keywords: fuzzy hyperconnected space, fuzzy regular open set, fuzzy nowhere dense set, fuzzy semicontinuity, fuzzy almost continuity

1. Introduction

The concept of hyperconnectedness in topological spaces has been introduced by Steen and Seebach (1978). Then Ajmal *et al.* (1992) have studied some of the characterizations and basic properties of hyperconnected space. Thereafter several authors have devoted their work to investigate the various properties of hyperconnectedness in general topology. Later, Caldas *et al.* (2002) have only defined fuzzy hyperconnectedness in fuzzy topological space (in short, fts) for the exploration of the features of the so called fuzzy weakly semi-open function.

The main seek of this paper is to study the basic properties and preservations of fuzzy hyperconnectedness in a fts. Lastly, in this paper we define fuzzy feebly continuous function and constitute its characterizations based on fuzzy hyperconnectedness along with other existing functions namely fuzzy semicontinuous function and fuzzy almost continuous function.

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2. Preliminariesd

Throughout this paper, simply by X and Y we shall denote fts's (X,τ) and (Y,σ) respectively and $f: X \to Y$ will mean that f is a function from a fts (X,τ) to another fts (Y,σ). For a fuzzy set λ of X, $cl(\lambda)$, $int(\lambda)$ and $1_X - \lambda$ (or λ^c) will denote the closure of λ, the interior of λ and the complement of λ respectively, whereas the constant fuzzy sets taking on the values 0 and 1 on are denoted by 0_X and 1_X respectively.

2.1 Definition (Thangaraj & Balasubramanian, 2003) A fuzzy set λ in a fuzzy topological space (X,τ) is called a fuzzy dense if there exists no fuzzy closed set μ in (X,τ) such that $\lambda < \mu < 1$.

2.2 Definition (Thangaraj & Balasubramanian, 2003) A fuzzy set λ in a fuzzy topological space (X,τ) is called a fuzzy nowhere dense if there exists no non zero fuzzy open set μ in (X,τ) such that $\mu < cl(\lambda)$ i.e. $int\,cl = 0_X$.

2.3 Definition (Azad, 1981) A fuzzy set λ in a fuzzy topological space (X,τ) is called fuzzy semiopen (resp. fuzzy semiclosed) set on X if $\lambda\ clint\,\lambda$ (resp. $int\,cl\,\lambda\ \lambda$).

2.4 Definition (Azad, 1981) A fuzzy set λ in a fuzzy topological space (X,τ) is called fuzzy regular open (resp. fuzzy regular closed) set on X if $\lambda = int\, cl\, \lambda$ (resp. $\lambda = cl\, int\, \lambda$).

2.5 Definition (Bin Shahna, 1991) A fuzzy set λ in a fuzzy topological space (X,τ) is called fuzzy preopen (resp. fuzzy preclosed) set on X if $\lambda \; int\, cl\, \lambda$ (resp. $cl\, int\, \lambda \; \lambda$).

2.6 Definition (Yalvac, 1988) Let $\lambda \subset X$ be a fuzzy set and defined the following set

$sint(\lambda) = \vee\{\mu : \mu \subset \lambda, \mu$ is fuzzy semiopen set$\}$,

also $\lambda \supset sint(\lambda) \supset int(\lambda)$.

2.7 Definition (Balasubramanian *et al.*, 1997) A fuzzy set λ in a fuzzy topological space (X,τ) is called a generalized fuzzy closed (in short, gfc) $\Leftrightarrow$ $cl(\lambda) \; \mu$, whenever $\lambda \le \mu$ and λ is fuzzy open.

2.8 Definition (Park & Park, 2003) A fuzzy set λ in a fts X is called regular generalized fuzzy closed (in short, rgf-closed) if $cl(\lambda) \le \mu$, whenever $\lambda \le \mu$ and μ is fuzzy regular open in X.

2.9 Definition (Setupathy *et al.*, 1977) A fuzzy topological space (X,τ) is said to be fuzzy connected if X cannot be represented as the union of two non empty disjoint open fuzzy sets on .

2.10 Definition (Mukherjee & Ghosh, 1989) An fts (X,τ) is said to be fuzzy extremally disconnected (FED, for short) iff closure of every fuzzy open set is fuzzy open in X; equivalently, every fuzzy regular closed set is fuzzy open.

2.11 Definition (Thangaraj & Soundararajan, 2013). A fuzzy topological space (X,T) is called a fuzzy Volterra space if $cl\left(\wedge_{i=1}^{N} \lambda_i\right) = 1$, where λ_i's are fuzzy dense and fuzzy G_δ-sets in (X,T).

2.12 Definition (Thangaraj & Poongothai, 2013). Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy σ-Baire space if $int\left(\vee_{i=1}^{\infty} \lambda_i\right) = 0$, where λ_i's are fuzzy σ-nowhere dense sets in (X,T).

2.13 Definition (Thangaraj & Balasubramanian, 2001) A fuzzy topological space (X,T) is called a fuzzy P-space if countable intersection of fuzzy open sets in (X,T) is fuzzy open. That is, every non-zero fuzzy G_δ-set in (X,T) is fuzzy open in (X,T).

2.14 Definition (Arya & Deb, 1973) A function $f : X \to Y$ is said to be feebly continuous if, for every non empty open set V of Y, $f^{-1}(V) \neq \varnothing$ implies that $int\, f^{-1}(V) \neq \varnothing$.

2.15 Definition (Azad, 1981) A function $f : (X,\tau_X) \to (Y,\tau_Y)$ from a fts X into a fts Y is said to be fuzzy semi-continuous if $f^{-1}(\lambda)$ is a fuzzy semi-open set of X, for each fuzzy $\lambda \in \tau_Y$.

2.16 Definition (Ekici & Kerre, 2006) A function f: $(X,\tau) \to (Y,\sigma)$ is called fuzzy contra continuous if $f^{-1}(\mu) \in \tau$ for every fuzzy closed μ in Y.

2.17 Definition (Azad, 1981) A function $f : (X,\tau_X) \to (Y,\tau_Y)$ from a fts X into a fts Y is said to be fuzzy almost continuous mapping if $f^{-1}(\lambda) \in \tau X$ for each fuzzy regular open set λ of Y.

2.18 Definition (Park & Park, 2003) A function f: $X \to Y$ is called a regular generalized fuzzy continuous (in short, rgf-continuous) if the inverse image of every fuzzy closed set in Y is rgf-closed in X.

3. Characterizations of Fuzzy Hyperconnectedness

In this section, we establish some equivalent forms of fuzzy hyperconnectedness as a natural offshoot of the context. Here our endeavor is to extend one particular result by Azad (1981), which is imposed in his paper. Here we also find a relation in between fuzzy preclosed set and generalized fuzzy closed set underneath of fuzzy hyperconnectedness.

3.1 Definition (Caladas *et al.*, 2002) A fts (X,τ) is said to be fuzzy hyperconnected if every non-null fuzzy open subset of (X,τ) is fuzzy dense in (X,τ) .

3.2 Example

Let $X = \{a,b,c\}$, $\tau = \{0_X, 1_X, \{(a,0.5),(b,0.7),(c,0.7)\}, \{(a,0.4),(b,0.6),(c,0.5)\}\}$. Here every fuzzy open subset is fuzzy dense set. Therefore the fts (X,τ) is fuzzy hyperconnected space.

3.3 Remark Every fuzzy hyperconnected space is fuzzy connected. However the reverse of the above remark may not be true in general as shown in the following example.

3.4 Example

Let $X = \{a,b,c\}$, $\tau = \{0_X, 1_X, \{(a,0.5),(b,0.3),(c,0.2)\}, \{(a,0.4),(b,0.2),(c,0.1)\}\}$. Then the fts (X,τ) is fuzzy connected space but not fuzzy hyperconnected space since every nonempty fuzzy open set is not fuzzy dense set.

3.5 Theorem Every fuzzy hyperconnected space (X,τ) is fuzzy extremely disconnected.

Proof. Let us suppose that (X,τ) is fuzzy hyperconnected space. Then for any fuzzy open set μ, $cl(\mu) = 1_X$, which implies that $cl(\mu)$ is fuzzy open and as a consequence the space (X,τ) is fuzzy extremally disconnected.

3.6 Remark The converse of the above theorem may not be true in general.

3.7 Example

Let $X=\{a,b,c\}$, $\tau=\{0_X,1_X,\{(a,0.5),(b,0.3),(c,0.4)\},$ $\{(a,0.5),(b,0.7),(c,0.6)\},\{(a,0.3),(b,0.7),(c,0.2)\},$ $\{(a,0.5),(b,0.7),(c,0.4)\},\{(a,0.3),(b,0.3),(c,0.2)\}\}$. One can easily verify that the fts (X,τ) is fuzzy extremely disconnected but not fuzzy hyperconnected as the closure of every fuzzy open set is fuzzy open but not fuzzy dense set.

3.8 Theorem In any fts (X,τ), the following conditions are equivalent: (i) (X,τ) is fuzzy hyperconnected space. (ii) Every fuzzy subset of (X,τ) is either fuzzy dense or fuzzy nowhere dense set there in.

Proof. (i) $\Rightarrow$ (ii) Let (X,τ) be a fuzzy hyperconnected space and λ be any fuzzy subset such that $\lambda \le 1_X$. Suppose that λ is not fuzzy nowhere dense. Then we have $cl(1_X / cl(\lambda)) = 1_X / int(cl(\lambda)) \neq 1_X$ since $int(cl(\lambda)) \neq 0_X$ and this implies that $cl(int(cl(\lambda))) = 1_X$. But $l(int(cl(\lambda))) = 1_X \le cl(\lambda)$, which gives that $cl(\lambda) = 1_X$. Hence λ is a fuzzy dense set.

(ii) $\Rightarrow$ (i) Let be any nonempty fuzzy open set in *X*. Now for any nonempty fuzzy open set μ, we have $\mu < intcl\,\mu$ which implies that μ is not fuzzy nowhere dense set but using the given hypothesis, we have μ is fuzzy dense set; hence, the proof. Steen and Seebach (1970) have shown that any topological space (X,T) is hyperconnected iff every pair of nonempty open sets have non empty intersection. But the above equivalent condition is partially true in the context of a fts which is demonstrated as follows:

3.9 Proposition In any fuzzy hyperconnected space (X,τ), every pair of nonempty fuzzy open subset has a nonempty intersection.

Proof. Suppose that $\lambda \wedge \mu = 0_X$, for any two nonempty fuzzy open subsets λ and μ of (X,τ). This implies that $cl(\lambda) \wedge \mu = 0_X$ and λ is not a fuzzy dense set. Since λ is fuzzy open, then $0_X \neq \lambda \le int(cl(\lambda))$ and λ is not a fuzzy dense set. This is a contradiction. Thus, $\lambda \wedge \mu \neq 0_X$, for every nonempty fuzzy open subsets λ and μ of (X,τ). The result stated above is only the necessary condition to be a fuzzy hyperconnected space but not sufficient which is verified in the following example.

3.10 Example Let $X=\{a,b\}$, $\tau=\{0_X,1_X,\lambda_1,\lambda_2,\lambda_1 \vee \lambda_2, \lambda_1 \wedge \lambda_2\}$ where $\lambda_1=\{(a,0.5),(b,0.3)\}$, $\lambda_2=\{(a,0.4),(b,0.7)\}$. Here $\lambda_1 \wedge \lambda_2 = \{(a,0.4),(b,0.3)\}\} \neq 0_X$ but fts (X,τ) is not fuzzy hyperconnected. Normally in fuzzy topological space the following equality does not hold.

3.11 Remark In a fuzzy hyperconnected space (X,τ), if λ and μ are two fuzzy open sets then $cl(\lambda \wedge \mu) = cl(\lambda) \wedge cl(\mu)$.

3.12 Theorem Let (X,τ) be a fuzzy topological space. Then the following properties are equivalent: (i) (X,τ) is fuzzy hyperconnected, (ii) 1_X and 0_X are the only fuzzy regular open sets in *X*.

Proof. (i) $\Rightarrow$ (ii) Let (X,τ) be a fuzzy hyperconnected space. Suppose that μ is a nonempty fuzzy regular open set. We have $\mu = intcl(\mu)$. This implies that $(intcl(\mu))^c = cl(1_X - cl(\mu)) = \mu^c \neq 1_X$ since $\mu \neq 0_X$. But this is a contradiction to the assumption. Thus, 1_X and 0_X are the only fuzzy regular open sets in *X*. (ii) $\Rightarrow$ (i) Let 1_X and 0_X are the only fuzzy regular open sets in *X*. If possible suppose that *X* is not fuzzy hyperconnected. This implies that there exist a nonempty fuzzy open subset μ of *X* such that $cl(\mu) \neq 1_X$. We have $cl(int(\mu)) \neq 1_X$. This implies that the only possibility is, $cl(int(\mu)) = 0_X$. Therefore we have $cl(\mu) = 0_X$ where $\mu \neq 0_X$ which is absurd. And hence the fts (X,τ) is a fuzzy hyperconnected space.

3.13 Proposition In a fuzzy hyperconnected space (X,τ), any fuzzy subset λ of X is fuzzy semiopen set if $int(\lambda) \neq 0_X$.

Proof. Let *X* be a fuzzy hyperconnected space and λ be any fuzzy subset of *X* where $int(\lambda) \neq 0_X$. Therefore $cl(int(\lambda)) = 1_X$. Thus $\lambda \le cl(int(\lambda))$. Hence, every fuzzy subset λ is a fuzzy semiopen. Azad (1981) had shown that the collection of fuzzy semiopen sets falls short to form the structure of fuzzy topology due to lacking of finite intersection property of the same in a fts (X,τ). But we claim that the collection of fuzzy semiopen sets forms a fuzzy topology in fuzzy hyperconnected space, which is justified in the next proposition.

3.14 Proposition In a fuzzy hyperconnected space (X,τ), finite intersection of fuzzy semiopen sets is fuzzy semiopen set.

Proof. Let us consider that λ and μ are two nonempty fuzzy semiopen sets in a fuzzy hyperconnected space (X,τ). So $\lambda \le cl(int(\lambda))$ and $\mu \le cl(int(\mu))$. Therefore $cl(\lambda) = cl(int(\lambda)) = 1_X$ and $cl(\mu) = cl(int(\mu)) = 1_X$. Since λ and μ are two nonempty fuzzy semiopen sets in a fuzzy hyperconnected space and so $\lambda \wedge \mu \neq 0_X$. Consequently, $cl(int(\lambda \wedge \mu)) = cl(int(\lambda)) \wedge cl(int(\mu)) = 1_X$. Therefore $\lambda \wedge \mu \le cl(int(\lambda)) \wedge cl(int(\mu)) = cl(int(\lambda \wedge \mu))$. It implies $\lambda \wedge \mu$ is fuzzy semiopen set.

3.15 Proposition Every fuzzy nowhere dense set is generalized fuzzy closed set in a fts (X,τ).

Proof. Let λ be any fuzzy nowhere dense set in a fts (X,τ). Therefore $int\,cl(\lambda) = 0_X$ and there does not exist any fuzzy open set in between λ and $cl(\lambda)$. Also let us suppose that $\lambda \le \mu$ for any fuzzy open set μ and then we have always $cl(\lambda) \le \mu$. Therefore λ is generalized fuzzy closed set.

3.16 Theorem Let (X,τ) be a fts. Then the following conditions are equivalent: (i) (X,τ) is fuzzy hyperconnected, (ii) Every fuzzy preopen set is fuzzy dense set.

Proof. (i) $\Rightarrow$ (ii) Let us suppose that λ is any fuzzy preopen set. This implies that $\lambda \le int\,cl\,(\lambda)$. As a result from (i) we have $cl(\lambda) = cl(int(cl(\lambda))) = 1_X$. Therefore λ is fuzzy dense set. (ii) $\Rightarrow$ (i) Let λ be any fuzzy preopen set. So $\lambda \le int\, cl(\lambda)$. Consequently, (ii) we give that λ is fuzzy dense set. Therefore, $cl(\lambda) = cl(int(cl(\lambda))) = 1_X$. It means that (X,τ), is fuzzy hyperconnected.

3.17 Proposition In a fuzzy hyperconnected space every fuzzy preclosed set is a generalized fuzzy closed set which is defined by Balasubramanian *et al.* (1997).

Proof. Using proposition 3.15 and theorem 3.16 it can be straightforwardly established.

3.18 Proposition In a fuzzy hyperconnected space every fuzzy subset is regular generalized fuzzy closed set.

Proof. Let $\lambda \le \mu$ where μ is fuzzy regular open in *X*. Then $cl(\lambda) \le \mu$ since *X* is fuzzy hyperconnected space and in hyperconnected space only fuzzy regular open sets are 0_X and 1_X. Therefore every fuzzy subset is regular generalized fuzzy closed set. In 2014, Thangaraj *et al.* studied the relation between fuzzy topological P-space, fuzzy hyperconnected space and fuzzy Volterra space in a fts.

3.19 Proposition (Thangaraj & Soundararajan, 2014) If the fuzzy topological P-space (X,T) is a fuzzy hyperconnected space, then (X,T) is a fuzzy Volterra space.

4. Results on Fuzzy Functions in Fuzzy Hyperconnected Spaces

In this section, we consider certain mappings between fts's and study their behavior when either or both the domain and codomain spaces are replaced by fuzzy hyperconnected spaces from which we shall obtain significant information about those functions.

4.1 Definition A function $f:(X,\tau)\rightarrow(Y,\sigma)$ from a fts *X* into a fts *Y* is said to be fuzzy feebly continuous if for every fuzzy open set λ of *Y*, $f^{-1}(\lambda) \ne 0_X$ implies that $int\,(f^{-1}(\lambda)) \ne 0_X$.

4.2 Theorem Fuzzy feebly continuous function preserves fuzzy hyperconnectedness.

Proof. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a fuzzy feebly continuous surjection and let X is fuzzy hyperconnected space. Suppose that Y is not fuzzy hyperconnected. Then there exist two non empty fuzzy open sets λ_1 and λ_2 such that $\lambda_1 \wedge \lambda_2 = 0_Y$. It means that $f^{-1}(\lambda_1) \wedge f^{-1}(\lambda_2) = 0_X$. By the hypothesis if $f^{-1}(\lambda_1) \ne 0_X$, $f^{-1}(\lambda_2) \ne 0_X$, then $int(f^{-1}(\lambda_1)) \ne 0_X$ and $int(f^{-1}(\lambda_2)) \ne 0_X$. Here $f^{-1}(\lambda_1) \wedge f^{-1}(\lambda_2) = 0_X$, which implies that $int(f^{-1}(\lambda_1)) \wedge int(f^{-1}(\lambda_2)) = 0_X$. Then *X* is not fuzzy hyperconnected, which is a contradiction. Thus *Y* is fuzzy hyperconnected.

4.3 Theorem Every fuzzy semi continuous function is fuzzy feebly continuous in a fts (X,τ).

Proof. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a fuzzy semi continuous function. Suppose that λ is a fuzzy open subset of (Y,σ) such that $f^{-1}(\lambda) \ne 0_X$. This implies that $f^{-1}(\lambda)$ is a nonempty fuzzy semiopen set in (X,τ). As a result, $0_X \ne f^{-1}(\lambda) = sint(f^{-1}(\lambda)) \le int(f^{-1}(\lambda))$. As a result, $f:(X,\tau)\rightarrow(Y,\sigma)$ is fuzzy feebly continuous.

4.4 Remark Converse of the above theorem may not be true always.

4.5 Example

Let $X = Y = \{a,b,c\}$, $\tau = \{0_X, 1_X, \{(a,0.5),(b,0.4),(c,0.3)\}\}$ and $\sigma = \{0_Y, 1_Y, \{(a,0.5),(b,0.7),(c,0.7)\}\}$. Here $f:(X,\tau)\rightarrow(Y,\sigma)$, defined by $f(a)=a$, $f(b)=b$, $f(c)=c$. Then *f* is fuzzy feebly continuous function but not fuzzy semi continuous function. Since $f^{-1}\{(a,0.5),(b,0.7),(c,0.7)\} = \{(a,0.5),(b,0.7),(c,0.7)\}$ is not fuzzy semiopen set in *X*.

4.6 Proposition Let (X,τ) be a fuzzy hyperconnected space. If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a fuzzy feebly continuous function, then *f* is fuzzy semicontinuous function.

Proof. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ is a fuzzy feebly continuous and (X,τ) be a fuzzy hyperconnected space. Suppose that λ is a fuzzy open subset of (Y,σ) such that $f^{-1}(\lambda) \ne 0_X$. This implies that $int(f^{-1}(\lambda)) \ne 0_X$. Since (X,τ) is fuzzy hyperconnected space, therefore $int(f^{-1}(\lambda))$ is fuzzy semiopen set in (X,τ). As a result, *f* is fuzzy semicontinuous function.

4.7 Proposition Every fuzzy almost continuous function is fuzzy contra continuous function.

Proof. Let *f* be any fuzzy almost continuous function from *X* to *Y*. From the definition of fuzzy almost continuity we have $f(\lambda) \le int\,\mathrm{cl}(\mu)$, where λ and μ are two fuzzy open sets respectively in X and *Y*. It implies that $f(\lambda) \le \mathrm{cl}(\mu)$. Therefore *f* is fuzzy contra continuous function.

4.8 Remark But fuzzy contra continuous function may not be a fuzzy almost continuous function.

4.9 Example Let $X = Y = \{a,b,c\}$, $\tau = \{0_X, 1_X, \{(a,0.5),(b,0.8),(c,0.7)\}\}$ and $\sigma = \{0_Y, 1_Y, \{(a,0.5),(b,0.3),(c,0.2)\}\}$. Here we define $f:(X,\tau)\rightarrow(Y,\sigma)$, $f(a)=b$, $f(b)=c$, $f(c)=b$. Then f is fuzzy contra continuous function but not fuzzy almost continuous function. Since $f^{-1}\{(a,0.5),(b,0.3),(c,0.2)\} = \{(a,0.5),(b,0.2),(c,0.3)\}$ is not fuzzy open set in *X*.

4.10 Proposition Let $f:X\rightarrow Y$ be a fuzzy contra continuous function from *X* to *Y*, where *Y* is a fuzzy hyperconnected space. Then f is a fuzzy almost continuous function.

Proof. Since f is a fuzzy contra continuous so we get $f(\lambda) \leq \mathrm{cl}(\mu)$, where λ and μ is are fuzzy open sets in X and Y respectively. Again by the hypothesis X is a fuzzy hyperconnected space we have $f(\lambda) \leq cl(\mu) = 1_Y \Rightarrow f(\lambda) \leq intcl(\mu) = 1_Y$. Hence the proof. Azad (1981) proved that the concept of fuzzy semi continuity and fuzzy almost continuity are independent concepts in fts's. But in the next proposition we establish a relation between these two said notions.

4.11 Proposition Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a fuzzy semi continuous function from X to Y, where Y is a fuzzy hyperconnected space. Then f is a fuzzy almost continuous function.

Proof. Since f is a fuzzy semi continuous function then an inverse image of fuzzy open set in Y is fuzzy semi open in X. Again Y is fuzzy hyperconnected space. It gives that 1_Y and 0_Y are the only fuzzy regular open sets in Y using theorem 3.12. But it implies that $f^{-1}(1_Y) = 1_X$ and $f^{-1}(0_Y) = 0_X$. Therefore inverse image of every fuzzy regular open set in Y is fuzzy open set in X.

4.12 Proposition Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a fuzzy almost continuous function from X to Y, where X is a fuzzy hyperconnected space and $int\, f^{-1}(\lambda) \neq 0_X$ for any non empty fuzzy open set $\lambda \in \sigma$. Then f is a fuzzy semi continuous.

Proof. Since f is a fuzzy almost continuous function, so inverse image of fuzzy regular open set in Y is fuzzy open in X. Again by the hypothesis, X is fuzzy hyperconnected space with $int\, f^{-1}(\lambda) \neq 0_X$ for any non empty fuzzy open set $\lambda \in \sigma$. It means that inverse image of every nonempty fuzzy open set is fuzzy semiopen set in X. Since $cl\left(int\, f^{-1}(\lambda)\right) = 1_X \geq int\, f^{-1}(\lambda)$. On the other hand if any fuzzy open set $\lambda = 0_Y$, then $f^{-1}(\lambda) = 0_X$. Which gives that $f^{-1}(\lambda)$ is fuzzy semiopen set in X. Thus f is a fuzzy semi continuous.

4.13 Remark Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be any fuzzy function. Then f is fuzzy semicontinuous function iff f is fuzzy almost continuous with $int\, f^{-1}(\lambda) \neq 0_X$.

4.14 Proposition If X is fuzzy hyperconnected space then any fuzzy continuous function $f : X \rightarrow Y$ is regular generalized fuzzy continuous function.

Proof. Let $f : X \rightarrow Y$ is a fuzzy continuous function and X be a fuzzy hyperconnected space. Suppose that λ is any fuzzy open subset of Y. Then $f^{-1}(\lambda)$ is regular generalized fuzzy closed set in X. It implies that f is regular generalized fuzzy continuous function.

5. Conclusions

In this paper we sorted out the characteristic of different kinds of sets namely fuzzy preopen set, fuzzy preclosed set, generalized fuzzy closed set, regular generalized fuzzy closed set etc. in the ambience of fuzzy hyperconnected space. In literature it is found that in a fts every fuzzy closed set is itself a generalized fuzzy closed set and also a fuzzy preclosed set. But using proposition 3.15 and theorem 3.16 we established a relation between generalized fuzzy closed set and fuzzy preclosed set despite the fact that there is no relation between generalized fuzzy closed set and fuzzy preclosed set. Here we also studied the behavior of numerous functions namely fuzzy febbly continuous function, fuzzy almost continuous function, fuzzy semi continuous in the light of fuzzy hyperconnected space. At the end of this paper we culminated one particular implication which shows that fuzzy semicontinuity imply fuzzy almost continuity when the co domain space is restricted to a fuzzy hyperconnected space.

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