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Thesis Title : Computational Evaluation of $f(m,n,x)=\int_0^\infty \frac{u^m e^{-u^2}du}{u^n+x^n}$ for $m,n \in \{0,1,2\}$ and $x \geq 0$

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Abstract

E.T. Goodwin and J. Staton (1948) evaluated the function $f(x)=\int_0^\infty \frac{e^{-u^2}du}{u+x}$ to four decimals accuracy for the range $0.02 \leq x \leq 10.00$ by using ascending series, asymptotic series and numerical solution of a differential equation. They showed that each method was valid over a restricted range of x values. However, the results of Goodwin and Staton were unnecessarily complicated because of the old – fashioned computational techniques available in 1948.

In this thesis, their methods are extended to approximate values of the function $f(m,n,x)=\int_0^\infty \frac{u^m e^{-u^2}du}{u^n+x^n}$ for $m,n \in \{0,1,2\}$ and $x \geq 0$. Modern computational methods are also used to extend and check the Goodwin and Staton results.

In the first section of the thesis, the function $f(0,1,x)=\int_0^\infty \frac{e^{-u^2}du}{u+x}$ considered by Goodwin and Staton is evaluated to six decimals accuracy and compared with the values in their paper. Further, the function $f(0,1,x)$ is evaluated in the limits as $x \to 0$ and $x \to \infty$.

In the second section of the thesis, the efficiency of the numerical integration routines in MATLAB is investigated by comparing values obtained from the integrals $\int_0^\infty \frac{e^{-u^2}du}{2}$, $\int_0^\infty \frac{ue^{-u^2}du}{2}$ and $\int_0^\infty \frac{u^2e^{-u^2}du}{2}$ with the exact analytical values.

In the third section of the thesis, the other 5 cases of the function $f(m,n,x)$ for $m,n \in \{0,1,2\}$ are evaluated by using ascending series, asymptotic series, numerical solution of differential equations and numerical integration, and the results from the different methods are compared.

The thesis concludes with two appendices. In appendix A, the behaviour of the function $g(x) = \int_0^\infty \frac{e^{-u^2}du}{u+x^2}$ is investigated and the results are applied to the physical problem of an electrical transmission line. In appendix B, a more detailed analysis is given of some of the numerical methods used throughout the thesis.

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