Chapter 4

Effect of Reduced Dimensionality of the Magnetic Field Fluctuations on the Cross-Field Motion of Charged Particles

In interplanetary space, the transport of energetic charged particles is influenced by a turbulent magnetic field. Previous studies have shown that a two-component (2D+slab) magnetic model (Matthaeus, Goldstein, and Roberts 1990; Bieber, Wanner, and Matthaeus 1996) of turbulence is a useful model for the magnetic field in the heliosphere. Normally, the diffusive behavior of charged particles in a turbulent magnetic field is observed when they approach the long time limit. The charged particles are often assumed to follow and diffuse according to the random walk of the field lines but some theories of perpendicular particle transport, such as nonlinear guiding center theory (NLGC) (Matthaeus et al. 2003; Ruffolo et al. 2008; Ruffolo et al. 2012), implicitly assume some true cross-field diffusion in which particles separate from the field line connected to their initial location. Furthermore, such cross-field diffusion is of specific interest because it is the only way that particles can diffuse across boundaries of magnetic field topology, such as the heliospheric current sheet and boundaries of interplanetary magnetic flux ropes. Jokipii et al. (1993) and Jones et al. (1998) derived a theorem showing that charged particles in an arbitrary electromagnetic field with at least one ignorable spatial coordinate remain forever tied to a given magnetic flux surface. In this work, we consider charged particles in one- and two-component magnetic fields to illustrate the effect of reduced dimensionality of magnetic fluctuations on the cross-field motion. A simple 2D flux tube and slab turbulence are used for fluctuations with we can turn the dimensionality on and off.

4.1 Simulation Setup

In the simulation, we set the box length in the z direction as $10{,}000\lambda$ and the number of grid points is $N_z = 4{,}194{,}304$. The magnetic parameters are $\delta B_{slab}/B_0 = 0.5$, $\sigma = 0.5\lambda$, and $A_0 = 0.8436 B_0 \lambda$.

The 100 MeV protons were released in random direction distribution. There are three cases that we have explored in our work: particle motion in a pure 2D field, in pure slab turbulence and in 2D+slab fields. These three cases can demonstrate the effect of reduced dimensionality of the magnetic field fluctuations. That is a reduction to one dimensionality can be studied in the case of pure 2D field with azimuthal symmetry $\left[B_0\hat{z} + \vec{b}_{2D}(r)\right]$ and in the case of pure slab turbulence $\left[B_0\hat{z} + \vec{b}_{slab}(z)\right]$. The 2D+slab case represents fully three dimensional fluctuations. Note that in our results all length scale is in unit of λ and time is in unit of λ/c.

4.2 Results

Case I: Particles in the pure 2D field

We inject all 1,000 protons of 100 MeV at a random angles along certain initial radius (r_0) of the 2D flux tube. The results are shown in Table 1 and Figure 1 for $r_0 = 0.5\lambda$. From the results, we found that $\langle(x_{GC}(t) - x_{FL}[z(t)])^2\rangle \sim r_0^2$. It can be explained by the drift motion in the azimuthal direction (z direction) while the particles still stay near the flux surface. Figure 4.2 shows similar results for 10 MeV particles started at $r_0 = 0.1\lambda$).
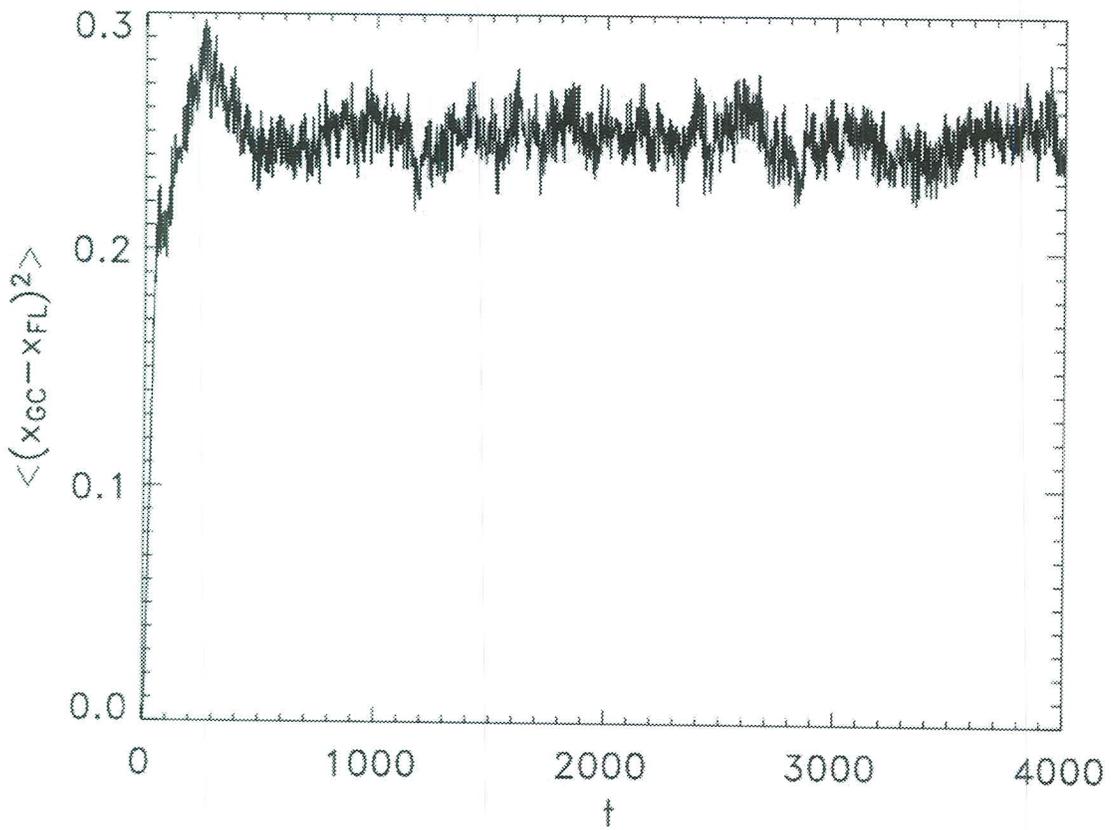
Figure 4.1 The mean squared separation between guiding centers of 100 MeV particles starting at $r_0 = 0.5\lambda$ with their field lines in the pure 2D field. The value close to r_0^2 implies confinement to flux surface.
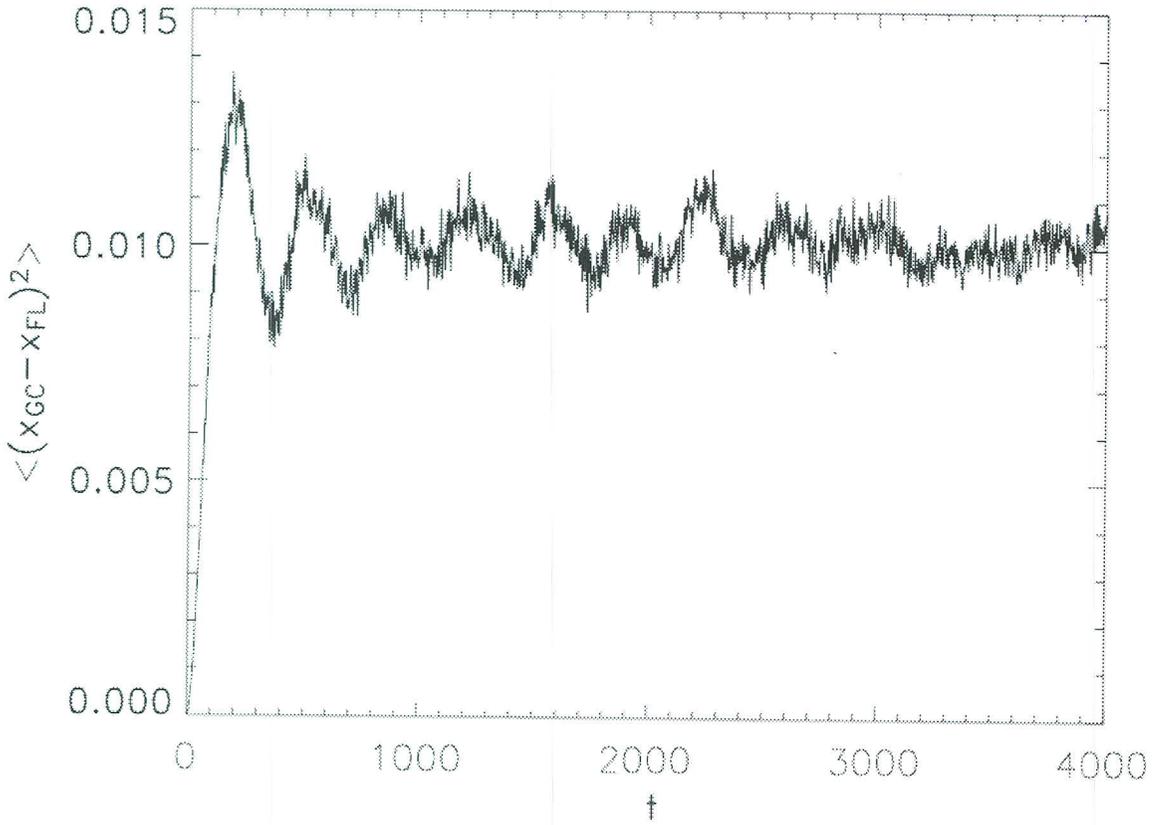
Figure 4.2 The mean squared separation between guiding centers of 10 MeV particles starting at $r_0 = 0.1\lambda$ with their field lines in the pure 2D field.

Table 4.1 Saturation values for 100 MeV particles in the pure 2D field when we vary the location of the initial positions.

r_0	$\langle(x_{GC} - x_{FL})^2\rangle$	$\langle(y_{GC} - y_{FL})^2\rangle$
0.1	0.0149	0.0149
0.3	0.0900	0.0902
0.5	0.249	0.250
0.7	0.492	0.493
0.9	0.825	0.826

Case II: Particles in slab turbulence

In this case, the particles are started at random positions in the simulation box. The results in Figure 4.3 show that the particles in slab turbulence have very little separation for their field lines. The particles stick with their initial field lines.
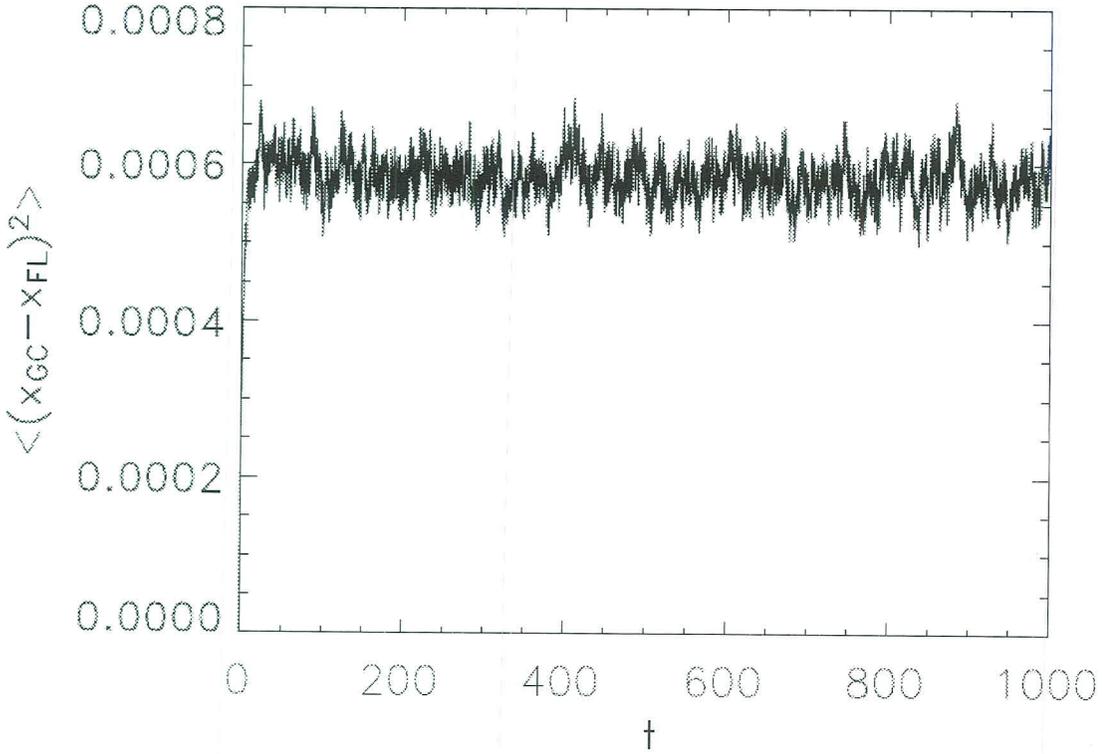

Figure 4.3 The mean squared separation between guiding centers of 100 MeV particles and their field lines in pure slab turbulence. The low value indicates confinement of the particle to remain near the same field line.

Case III: Particles in 2D+slab fields

In the case of three dimensional fluctuations, we can see that the particles have unconstrained cross-field motion illustrated in Figure 4.4.
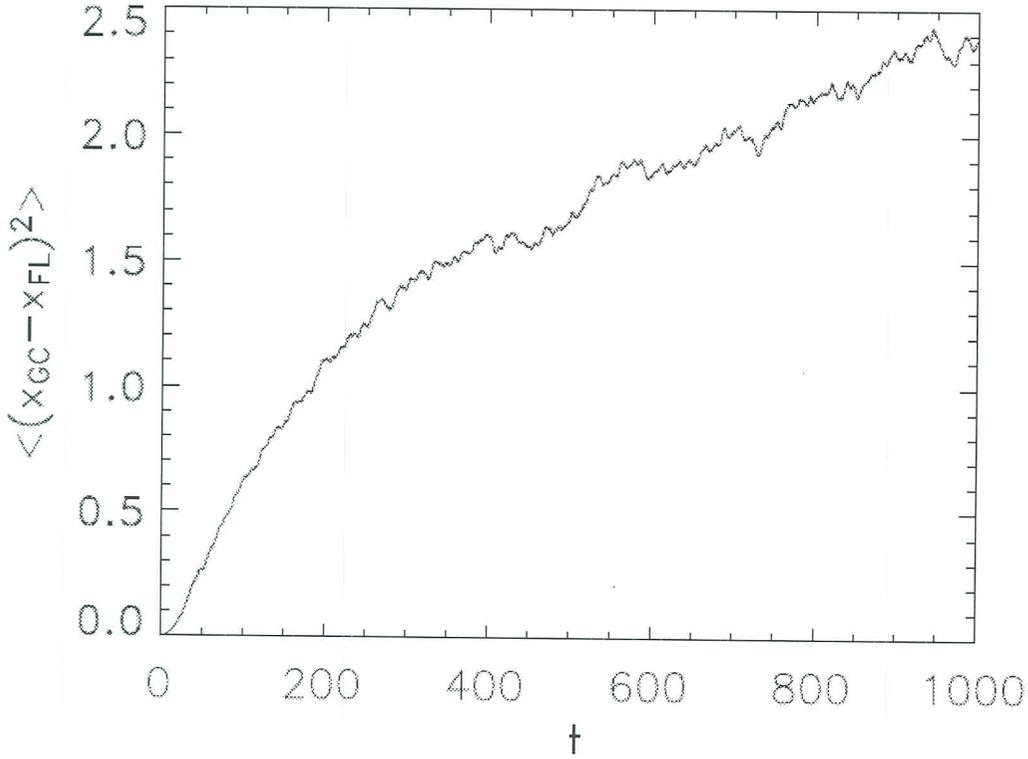
Figure 4.4 The mean squared separation spreading between guiding centers of 100 MeV particles and their field lines in Gaussian 2D field + slab turbulence. The continual increase indicates unconstrained cross-field motion in this fully three-dimensional case.

4.3 Conclusions and Discussion

We have examined the cross-field motion of particles in the one- and two-component magnetic field models. Our results show the effect of reduced of dimensionalities of the magnetic fluctuations on the cross-field motion. When the fluctuations have two ignorable coordinates in one case (pure slab), no cross-field motion can be observed, with particles constrained to remain near one field line, and in another case (pure 2D with azimuthal symmetry), the particles are constrained to remain near on-flux surface. Unconstrained cross-field motion occurs only when the magnetic field has fully three dimensions. These results are consistent with the theorem of Jokipii et al. (1993) and Jones et al. (1998) which states that in an

EM field with an ignorable coordinate, the particle motion remains near a single magnetic flux surface. The particles in the pure slab case stick to their initial field lines because the theorem applied to the ignorable x coordinate constrains the particle to one magnetic flux surface, application to the ignorable y coordinate constrains the same particle to a different flux surface, and the intersection of the two surfaces in simply the original field line. For the pure 2D case, application of the theorem to the ignorable coordinate z coordinate implies a constraint to remain near a flux surface of constant r, which is the same surface. Thus there is no further constraint from the second application of the theorem, the particles are confined on the flux surface but have the drift motions in the z (or ϕ) direction that cause the saturate values in our statistics approximately r_0^2. This study will help us to understand the cross-field motion of energetic particles in interplanetary space and will play an important role in developing the transport theory of charged particles in magnetic turbulence.