Chapter 3

Methodology

We generate magnetic field, which is static and homogeneous by using 2D+slab model of magnetic field turbulence. To study the separation between magnetic field line and charged particle trajectories, we simulate magnetic field lines corresponding to the initial guiding centers of the charged particles by numerically solving field line equation while the trajectories of particles is traced by solving equation of motion. After that the data are collected and analyzed by using new statistical approach. For the part of generating turbulent magnetic field with Taylor microscale, we show the method how to generate the spectrum and transform to time series in real space which are analogous to spacecraft data.

3.1 Generation of magnetic field

3.1.1 Turbulence Magnetic Field

Since the magnetic field in interplanetary is turbulent, we simulate magnetic field by setting up magnetic field parameters and specify power spectrum. In our simulations, we generate the magnetic field in the simulation box. We need to consider the effects of the simulation box, representations of turbulent field, and suitable length scale for simulated field lines. For turbulence case, the magnetic field is generated in wave number space (*k*-space) before conversion to real space. We instead define the power spectrum as a function in *k*-space, which is the Fourier transform of the magnetic correlation function $R_{ij}(\vec{r}) = < b_i(0)b_j(\vec{r}) >$. The spectrum that we usually use for the magnetic turbulence is a Komolgorov spectrum over a wide range of wave numbers. The magnetic fluctuations in equation (2.1) are composed of slab and 2D turbulence. Because the slab turbulence depends only on z and the 2D turbulence depends on x and y positions, we separately generate them in k_z and (k_x, k_y) spaces, respectively. After that, the magnetic field in Fourier space is converted to position space by an inverse fast Fourier transform. For numerical computation, we cannot generate the magnetic fluctuations continuously in space due to the limitation of the computer. Thus the magnetic field is constructed only on the grid points in the simulation box. To avoid bias due to a periodicity effect, we have to generate the magnetic field in a large (but finite) box. Therefore, in this part, the parameters that we need to input are the sizes in *x*, *y*, and *z* directions of the simulation box (L_x, L_y, and L_z), the number of grid points (N_x, N_y, and N_z), the total root-mean-squared fluctuation (δb), the fraction of 2D and slab energy, the shapes of the 2D and slab power spectra, and coherence lengths (λ_z an $\lambda_\perp$).

Slab Turbulence

For slab turbulence, we set the power spectrum for simulations as

$$P_{xx}^{slab}(k_z) = P_{yy}^{slab}(k_z) = \frac{C^{slab}}{[1+(k_z\lambda)^2]^{5/6}}, \tag{3.1}$$

where C^{slab} is a normalization constant that depends on the slab energy and λ is the parallel coherence length. From the function of the slab spectrum, the slab magnetic fluctuations in k_z space are

$$b_x^{slab}(k_z) = \sqrt{P_{xx}^{slab}(k_z)}\ \exp[i\phi(k_z)] \tag{3.2}$$

$$b_y^{slab}(k_z) = \sqrt{P_{yy}^{slab}(k_z)}\ \exp[i\phi(k_z)], \tag{3.3}$$

where ϕ is a random phase number and k_z is a discrete number which is $k_z = j2\pi/L_z$, for $j = 1, 2, 3, \dots, N_z/2 - 1$.

2D Turbulence

For 2D turbulence, we instead specify the power spectrum $A(k_x, k_y)$ because the power spectra $P_{xx}^{slab}(k_x, k_y)$ and $P_{yy}^{slab}(k_x, k_y)$ can be written in terms of $A(k_x, k_y)$ as

$$A(k_\perp) = \frac{C^{2D}}{[1+(k_\perp\lambda_\perp)^2]^{7/3}}. \tag{3.4}$$

From the relationship between magnetic fluctuation and potential function in *k*-space, the 2D fluctuations in (k_x, k_y) are

$$b_x^{2D}(k_x, k_y) = -ik_y\sqrt{A(k_\perp)}\ \exp[i\phi(k_x, k_y)] \tag{3.5}$$

$$b_y^{2D}(k_x, k_y) = ik_x\sqrt{A(k_\perp)}\ \exp[i\phi(k_x, k_y)], \tag{3.6}$$

3.1.2 Simple Gaussian 2D Field

Since we can specify the function of simple Gaussian 2D Field directly in real space. From equation (2.3), we write potential function in Cartesian coordinate

$$a(x,y) = A_0 \exp\left[-\frac{(x-x_0)^2 + (y-y_0)^2}{2\sigma^2}\right], \tag{3.7}$$

where x_0 and y_0 are center of the Gaussian island. From the relationship $\vec{b}_{2D} = \vec{\nabla} a(x,y) \times \hat{z}$, we can write

$$b_x = \frac{\partial a(x,y)}{\partial y} = \frac{-(y-y_0)a(x,y)}{\sigma^2} \tag{3.8}$$

$$b_y = -\frac{\partial a(x,y)}{\partial x} = \frac{(x-x_0)a(x,y)}{\sigma^2}. \tag{3.9}$$

3.1.3 Turbulent Magnetic Field with Taylor Microscale

Once we have specified the shape of spectrum as equation (2.4), we can generate realizations of the signal in the frequency domain, $F(f)$, as

$$F(f) = \sqrt{P(f)} \exp[i\phi] \tag{3.10}$$

where ϕ is a random phase. Then a fast Fourier transform (FFT) is used to convert the function $F(f)$ into the real time domain. In the simulations reported here, we employ this approach to obtain 2^{22} data points for the time series.

We next compute the Taylor microscale from the data set we generated by employing the definition equation (2.4). In Table (3.1), we give the Taylor microscale values for a range of dissipation scale indices q corresponding to the generic power spectrum shown in Figure 3.1. (Note that the spectra are given here as Fourier amplitudes squared, which can easily be converted to power spectral density.) We will treat these expected values of the Taylor microscale as the true or exact Taylor microscale values for the synthetic time series data. To examine and test our extrapolation method, we use only one-eighth of the original data. The purpose of defining this subset is that any consistent method will provide good (and even convergent) values of τ_{TS} when the time resolution Δt of the estimates is very fine, i.e., the spectral cutoff is resolved and $\Delta t f_{max} < 1/2$. However, our motivation is to obtain reasonably accurate values of τ_{TS} when the effective resolution of the data sampling is adjusted so that we are not in this asymptotic regime – a circumstance that is more likely to be realized in practice when analyzing spacecraft data.

With the subset of our discrete time series, we compute the second order structure function. This can be used to obtain an estimate of the correlation function. We then determine the radius of curvature from correlation function and an estimate of the Taylor microscale. In the following section, we will demonstrate an extrapolation technique (Weygand et al. 2007, 2009, 2010, 2011) to estimate Taylor microscale from a series of parabolic fits of the correlation function near the origin. The details of the technique we use to analyze are given in Chapter 6.

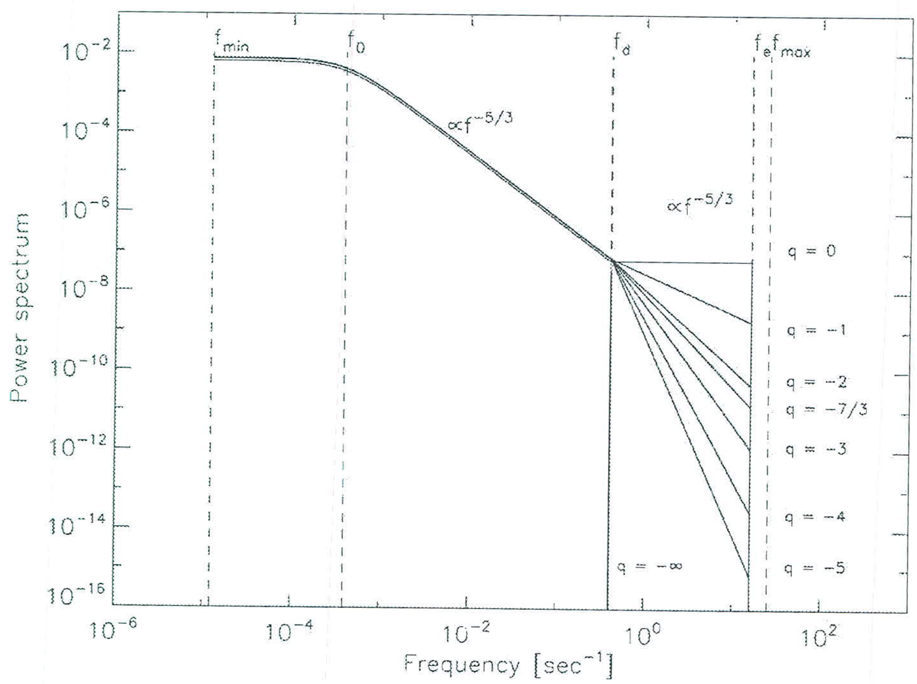

Figure 3.1 The power spectrum for a number of values of q in the dissipation range.

Table 3.1 Showing index q which we vary for each case and their Taylor scales when we fix dissipation scale (τ_d = 2.5 s).

Case	τ_{TS}^{expect} [s]	τ_{TS}^{expect} $[\tau_d]$
$q = -\infty$	6.569	2.63
$q = -5$	5.097	2.04
$q = -4$	4.368	1.75
$q = -3$	2.869	1.15
$q = -7/3$	1.607	0.64
$q = -2$	1.095	0.44
$q = -1$	0.095	0.028

3.2 Particle Simulations

For the charged particle q and mass m, moving with velocity ($\vec{v}$) a magnetic field ($\vec{B}$), without electric field ($\vec{E}$), we can write motion equation by Newton's Lorentz force ($\vec{F}_B$):

$$\vec{F}_B = m\frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B}). \tag{3.11}$$

For our work, we a bit adapt equation (3.11) for simulation (Tooprakai et al., 2007),

$$\frac{d\vec{v}'}{dt'} = \alpha(\vec{v}' \times \vec{B}'), \tag{3.12}$$

where $\alpha = (qB_0\tau_0)/(\gamma m_0)$ and the quantities $\vec{v}'$, $\vec{B}'$, and t' are normalized quantities which have units as scale to the speed of light (c), the mean magnetic field (B_0), the time scale $\tau_0 = \lambda/c$, respectively. Note that λ is the slab turbulence coherence length.

We can find trajectories of the charged particles, when we know the equation of motion of the charged particles. In this work, we use Newton's Lorentz force equation to find positions of the charged particles by using fourth-order Runge-Kutta method with adaptive time stepping regulated by a fifth-order error estimate step (Press, Teukolsky, Vetterling & Flannery, 1992; Dalena, Chuychai, Mace, Greco, Qin & Matthaeus, 2012).

3.3 Magnetic Field Line Simulations

When we know the value of the magnetic field at each grid point, we can trace the magnetic field line that is tangent everywhere to the magnetic field ($\vec{B}$). The differential equation of the magnetic field line is

$$d\vec{l} \times \vec{B} = 0. \tag{3.13}$$

In Cartesian coordinates, $d\vec{l}$ is (dx, dy, dz) and $\vec{B}$ is (B_x, B_y, B_z). From equation (3.13), it can be written as

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z}. \tag{3.14}$$

In our model, we use $\vec{B} = B_0\hat{z} + b_x\hat{x} + b_y\hat{y}$ so we obtain

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_0}. \tag{3.15}$$

Finally, we can write the differential equation for the magnetic field line as

$$\frac{dx}{dz} = \frac{b_x(x,y,z)}{b_0} = \frac{b_x^{slab}(z) + b_x^{2D}(x,y)}{B_0} \tag{3.16}$$

$$\frac{dy}{dz} = \frac{b_y(x,y,z)}{b_0} = \frac{b_x^{slab}(z) + b_x^{2D}(x,y)}{B_0} \tag{3.17}$$

After that the differential equation of the magnetic field line is solved by using fourth order Runge Kutta method with adaptive step size as same as we use in particle simulation to find positions of the magnetic field lines x_{FL}, y_{FL}, z_{FL}.

3.4 Simulation and Analysis Method for Separation between Charged Particles and Field Lines

We simulate 1,000 pairs of particle trajectories (protons) and their initial field lines with starting points located at the initial guiding center (GC) of the particles. As the trajectories of the particles are traced by equation (3.12), their GCs are also computed from the radius of curve of the particle orbits, $\vec{\rho}$:

$$\vec{\rho} = \frac{\vec{B} \times \vec{p}}{qB^2}, \tag{3.18}$$

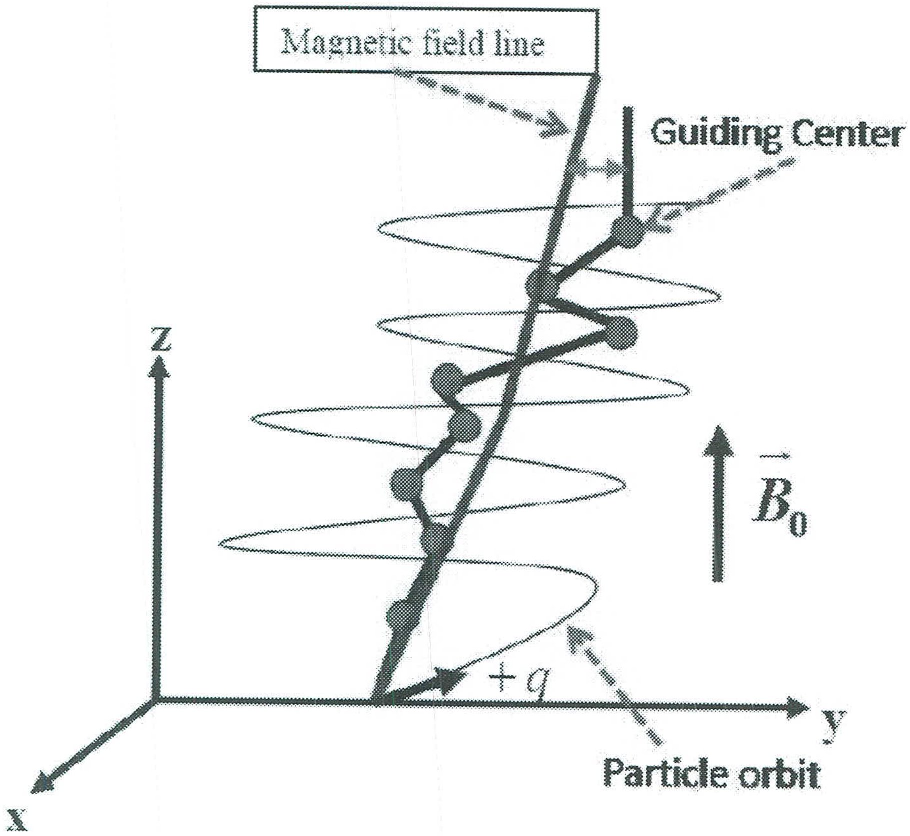

Figure 3.3 The diagram of separation between the guiding center and magnetic field line (Wikee 2013).