ภาคผนวก ข

การคำนวณค่า $b(0)$

$$b(0)=\left\{\frac{\left[\int_{-\infty}^{\infty}u^2K(u)du\right]^2\int_{-1}^{0}K_0(u)^2\,du}{\left[\int_{-1}^{0}u^2K_0(u)du\right]^2\int_{-\infty}^{\infty}K(u)^2\,du}\right\}^{\frac{1}{5}}$$

ฟังก์ชันเคอร์เนลแบบเกาส์เซียน คือ $K(u)=\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}u^2)$

ฟังก์ชันเคอร์เนลของจุดสิ้นสุด คือ $K_{(0)}(u)=6+18u+12u^2$

พิจารณา $\int_{-\infty}^{\infty}K(u)^2\,du$

$$\int_{-\infty}^{\infty}K(u)^2\,du=\int_{-\infty}^{\infty}\left[\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}u^2)\right]^2du=\int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\exp(-u^2)\right]du$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp(-u^2)du=\frac{1}{2\pi}\left(\frac{\sqrt{\pi}}{2}\times\frac{2}{\sqrt{\pi}}\right)\int_{-\infty}^{\infty}\exp(-u^2)du$$

$$=\frac{1}{4\sqrt{\pi}}\left(\frac{2}{\sqrt{\pi}}\right)\int_{-\infty}^{\infty}\exp(-u^2)du$$

$$=\frac{1}{4\sqrt{\pi}}\left(\frac{2}{\sqrt{\pi}}\right)\left(\int_{-\infty}^{0}\exp(-u^2)du+\int_{0}^{\infty}\exp(-u^2)du\right)$$

$$=\frac{1}{4\sqrt{\pi}}\left(\frac{2}{\sqrt{\pi}}\int_{-\infty}^{0}\exp(-u^2)du+\frac{2}{\sqrt{\pi}}\int_{0}^{\infty}\exp(-u^2)du\right)$$

$$=\frac{1}{4\sqrt{\pi}}\left(\frac{2}{\sqrt{\pi}}\int_{-\infty}^{0}\exp(-u^2)du+\frac{2}{\sqrt{\pi}}\int_{0}^{\infty}\exp(-u^2)du\right)$$

$$=\frac{1}{4\sqrt{\pi}}\left(-\frac{2}{\sqrt{\pi}}\int_{0}^{-\infty}\exp(-u^2)du+\frac{2}{\sqrt{\pi}}\int_{0}^{\infty}\exp(-u^2)du\right)$$

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Error function : $erf(x)=\frac{2}{\sqrt{\pi}}\int_{0}^{x}\exp(-u^2)du$

ซึ่ง $erf(\infty)=1$ และ $erf(-\infty)=-1$ (แสดงรายละเอียดในภาคผนวก ค)

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$$=\frac{1}{4\sqrt{\pi}}\left(-erf(-\infty)+erf(\infty)\right)=\frac{1}{4\sqrt{\pi}}\left(-(-1)+1\right)=\frac{2}{4\sqrt{\pi}}$$
$$=\frac{1}{2\sqrt{\pi}}$$

ดังนั้น $\int_{-\infty}^{\infty}K(u)^2du=\frac{1}{2\sqrt{\pi}}$

พิจารณา $\int_{-\infty}^{\infty}u^2K(u)du$

$$\int_{-\infty}^{\infty}u^2K(u)du=\int_{-\infty}^{\infty}u^2\left[\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}u^2)\right]du=\int_{-\infty}^{\infty}\left[\frac{u^2}{\sqrt{2\pi}}\exp(-\frac{u^2}{2})\right]du$$
$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[u^2\exp(-\frac{u^2}{2})\right]du=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[(u)(u)\exp(-\frac{u^2}{2})\right]du$$
$$=\frac{1}{\sqrt{2\pi}}\left[\left(-u\exp(-\frac{u^2}{2})\Big|_{-\infty}^{\infty}\right)+\int_{-\infty}^{\infty}\exp(-\frac{u^2}{2})du\right]$$
$$=-\frac{1}{\sqrt{2\pi}}\left(u\exp(-\frac{u^2}{2})\Big|_{-\infty}^{\infty}\right)+\frac{1}{\sqrt{2\pi}}\left(\int_{-\infty}^{\infty}\exp(-\frac{u^2}{2})du\right)$$
$$=-\frac{1}{\sqrt{2\pi}}\left(\lim_{u\to\infty}\frac{u}{\exp(\frac{u^2}{2})}-\lim_{u\to-\infty}\frac{u}{\exp(\frac{u^2}{2})}\right)+\frac{1}{\sqrt{2\pi}}(\sqrt{2})\int_{-\infty}^{\infty}\exp-(\frac{u}{\sqrt{2}})^2d(\frac{u}{\sqrt{2}})$$

กำหนดให้ $t=\frac{u}{\sqrt{2}}$

$$=-\frac{1}{\sqrt{2\pi}}\left(0-0\right)+\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\exp(-t^2)dt=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\exp(-t^2)dt$$
$$=\left(\frac{1}{2}\right)(2)\left(\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\exp(-t^2)dt\right)=\left(\frac{1}{2}\right)\left(\frac{2}{\sqrt{\pi}}\int_{-\infty}^{\infty}\exp(-t^2)dt\right)$$
$$=\frac{1}{2}\left(\frac{2}{\sqrt{\pi}}\int_{-\infty}^{0}\exp(-t^2)dt+\frac{2}{\sqrt{\pi}}\int_{0}^{\infty}\exp(-t^2)dt\right)$$

$$= \frac{1}{2}\left(-\frac{2}{\sqrt{\pi}} \int_{0}^{-\infty} \exp(-t^2)dt + \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} \exp(-t^2)dt \right)$$

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Error function : $erf(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} \exp(-u^2)du$

ซึ่ง $erf(\infty) = 1$ และ $erf(-\infty) = -1$ (แสดงรายละเอียดในภาคผนวก ค)

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$$= \frac{1}{2}\left(-erf(-\infty) + erf(\infty)\right) = \frac{1}{2}\left(-(-1)+1\right) = \frac{2}{2}$$

$$= 1$$

ดังนั้น $\int_{-\infty}^{\infty} u^2 K(u)du = 1$

พิจารณา $\int_{-1}^{0} K_0(u)^2 du$

$$\int_{-1}^{0} K_0(u)^2 du = \int_{-1}^{0} (6+18u+12u^2)^2 du = \int_{-1}^{0} (144u^4 + 432u^3 + 468u^2 + 216u + 36)du$$

$$= \frac{144u^5}{5} + \frac{432u^4}{4} + \frac{468u^3}{3} + \frac{216u^2}{2} + 36u + c \Big|_{-1}^{0}$$ เมื่อ c คือค่าคงที่

$$= \frac{144u^5}{5} + 108u^4 + 156u^3 + 108u^2 + 36u + c \Big|_{-1}^{0}$$

$$= \left[\frac{144(0)^5}{5} + 108(0)^4 + 156(0)^3 + 108(0)^2 + 36(0) + c\right] - \left[\frac{144(-1)^5}{5} + 108(-1)^4 + 156(-1)^3 + 108(-1)^2 + 36(-1) + c\right]$$

$$= c - \left[-\frac{144}{5} + 108 - 156 + 108 - 36 + c\right] = \frac{144}{5} - 108 + 156 - 108 + 36$$

$$= \frac{24}{5}$$

ดังนั้น $\int_{-1}^{0} K_0(u)^2 du = \frac{24}{5}$

พิจารณา $\int_{-1}^{0} u^2 K_0(u)du$

$$\int_{-1}^{0} u^2 K_0(u)du = \int_{-1}^{0} u^2(6+18u+12u^2)du = \int_{-1}^{0}(6u^2+18u^3+12u^4)du$$

$$= \frac{6u^3}{3}+\frac{18u^4}{4}+\frac{12u^5}{5}+c\Big|_{-1}^{0} \quad \text{เมื่อ } c \text{ คือค่าคงที่}$$

$$= 2u^3+\frac{9u^4}{2}+\frac{12u^5}{5}+c\Big|_{-1}^{0}$$

$$= \left[2(0)^3+\frac{9(0)^4}{2}+\frac{12(0)^5}{5}+c\right]-\left[2(-1)^3+\frac{9(-1)^4}{2}+\frac{12(-1)^5}{5}+c\right]$$

$$= c-\left[-2+\frac{9}{2}-\frac{12}{5}+c\right] = 2-\frac{9}{2}+\frac{12}{5}$$

$$= -\frac{1}{10}$$

ดังนั้น $\int_{-1}^{0} u^2 K_0(u)du = -\frac{1}{10}$

ทำการคำนวณค่า $b(0)$ ได้ดังนี้

$$b(0) = \left\{\frac{\left[\int_{-\infty}^{\infty} u^2 K(u)du\right]^2 \int_{-1}^{0} K_0(u)^2 du}{\left[\int_{-1}^{0} u^2 K_0(u)du\right]^2 \int_{-\infty}^{\infty} K(u)^2 du}\right\}^{\frac{1}{5}}$$

$$= \left\{\frac{(1)^2\left(\frac{24}{5}\right)}{\left(-\frac{1}{10}\right)^2\left(\frac{1}{2\sqrt{\pi}}\right)}\right\}^{\frac{1}{5}} = \left\{\frac{(1)\left(\frac{24}{5}\right)}{\left(\frac{1}{100}\right)\left(\frac{1}{2\sqrt{\pi}}\right)}\right\}^{\frac{1}{5}}$$

$$= \left\{\frac{(24)(100)(2\sqrt{\pi})}{5}\right\}^{\frac{1}{5}} = (960\sqrt{\pi})^{\frac{1}{5}}$$

$$\approx 4.427786896$$

ดังนั้น จะได้ว่า $b(0) \approx 4.427786896$