Chapter 4

Summary and Open Problems

In this dissertation, we investigated four types of Rosa-type labelings of cubic graphs and multigraphs. We attempted to conduct a systemic study of which cubic graphs admit labelings that lead to cyclic decompositions of infinite classes of complete graphs (or multigraphs). We summarize our results in Section 4.1 and give some open problems in Section 4.2.

4.1   Summary

Graph labelings were introduced by Rosa [41] in 1967 as means of helping determine which graphs G with n edges will decompose the complete graph K_{2n+1}. Hundreds of research articles have been written about various labelings, especially graceful labelings (see the comprehensive dynamic survey maintained by Gallian [28]). Our interest is in investigations of labelings of cubic graphs. We chose cubic graphs because there has been much investigation of labelings of 2-regular graphs, but very little study of labelings of 3-regular graphs. We summarize our results below.

(1) We found free α-labelings of bipartite prisms, bipartite Möbius ladders, and of connected cubic bipartite graphs of order at most 14. These results will appear in [44].

(2) We found ρ-tripartite labelings of odd prisms, even Möbius ladders, and of connected cubic tripartite graphs of order at most 10. These results have been published in [46].

(3) We found graceful labelings of cubic graphs of order at most 12. These results will appear in [45].

(4) We introduced the concept of a 2-fold ρ^+-labeling of a bipartite multigraph and showed that $\tilde{C}_{2n}$, the cycle of length $2n$ with aletrnating double edges, admits a 2-fold ρ^+-labeling for all integers $n \geq 2$. These results will be submitted for publication in a forthcoming article.

4.2 Open Problems

We list several open problems that should be of interest to researchers in design theory.

The graph $2K_4$ is the only known cubic graph that does not admit a ρ-labeling. This prompts us to ask the following question.

Open Problem 1. Is there a cubic graph other than $2K_4$ that does not admit a ρ-labeling?

In Section 3.4, we showed that every cubic graph of order ≤ 12, other than $2K_4$ and $3K_4$, is graceful. Since K_4 is the only connected cubic graph that is not is not tripartite, this prompts us to ask the following question.

Open Problem 2. Is there every cubic tripartite graph graceful?

In Section 3.2, we showed that every cubic bipartite graph of order ≤ 14, admits a free α-labeling. We also showed that bipartite prisms and bipartite Möbius ladders admit free α-labelings. This prompts us to ask the following question.

Open Problem 3. Does every cubic bipartite graph admit a free α-labeling?

In Section 3.3, we showed that every connected cubic bipartite graph of order ≤ 10, admits a ρ-tripartite labeling. We also showed that tripartite prisms and tripartite Möbius ladders admit ρ-tripartite labelings. This prompts us to ask the following question.

Open Problem 4. Does every connected cubic tripartite graph admit a ρ-tripartite labeling?