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ANALYSIS OF EMPIRICAL COMPRESSION INDEX EQUATIONS USING THE VOID RATIO

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This paper proposes a new method to evaluate the reliability of published empirical formulas in terms of accuracy and applicability to different soil types. Different empirical models are proposed to properly approximate the compression index for a wide range of void ratio and soil types. They were developed using a unique technique and a substantial number of published regression equations and compression data. Familiar empirical equations were examined for their reliability in predicting the compression index of clay for any void ratio. A comparison was made between available and newly-proposed empirical formulas using combined regression data sets compiled independently by several authors. The newly proposed empirical compression index equations are applicable to wide ranges of clay soils and void ratios, validating other published relationships. The degree of scatter and variations in the computed compression index values are minimized for any void ratio.

Keywords: Consolidation, Regression equations, Reliability, Standard error, Clay, Settlement calculations.

1 INTRODUCTION

Several empirical equations have been developed to relate compression index (C_c) to soil index properties. Some equations are supposed to reflect C_c of all soils while others are limited to specific soil types and/or geography (Peck and Reed 1954). Most authors used the correlation coefficient (R^2) as a lone measure to justify their applicability to a wide range of soils. Little or no information was provided relative to the number of data points used and/or the standard error. Further, the lack of uniformity in data collection and data interpretation makes it difficult to verify the accuracy of derived empirical equations. However, a large number of publications are now available to warrant a closer look at the validity, accuracy, and usefulness of many available empirical formulas for C_c estimation of fine-grained soils to their in-situ void ratio (e_o). The more widely used equations to estimate C_c are those developed by Nishida (1956), Hough (1957), and Bowles (1989). Besides statistical measures, these equations seem to lack a logical and/or theoretical basis. The applicability of many of these equations to organic soils has not been established. 3-D models clearly show that consolidation pressures cannot be ignored in organic soils irrespective of the index property being used; C_c for clay sediments is actually related to consolidation pressure. Al-Khafaji and Andersland (1981) showed that the use of C_c in settlement calculations for organic soils is not justified. For a majority of practical problems, combining mineral and organic soils data is not suitable. This paper undertakes an exhaustive comparative study of available empirical equations, comparing their applicability to available published and independently-collected data. Also, more insight is provided into the nature of future development of empirical equations.

2 AVAILABLE EMPIRICAL COMPRESSION INDEX EQUATIONS

Empirical equations to estimate the C_c are valuable because they are generally viewed as substitutes for consolidation tests. Approximate C_c values are important in preliminary settlement studies and indicate the magnitude of C_c for conducting consolidation tests. The soil index property used to estimate C_c should be easily measured in the laboratory. Some empirical formulas linearly relate C_c to void ratio as shown in Table 1.

Table 1. Empirical equations for compression index approximation using void ratio.

Equation	**Applicability**	**Reference**
$C_c = 0.40(e_o – 0.25)$	All natural soils	Azzouz *et al.* 1976
$C_c = 1.15(e_o – 0.35)$	All clays	Nishida 1956
$C_c = 0.54(e_o – 0.35)$	All natural soils	Nishida 1956
$C_c = 0.75(e_o – 0.50)$	Soils of very low plasticity	Sowers 1970
$C_c = 0.156e_o + 0.0107$	All clays	Bowles 1989
$C_c = 0.43(e_o – 0.25)$	Brazilian clays	Cozzolino 1961
$C_c = 0.35(e_o – 0.5)$	Organic soils	Hough 1957

The empirical expressions in Table 1 share one commonality – all are based on regression analysis of laboratory test data. It is interesting to note that Nishida (1956) provided two equations relating to all clays and to all natural soils that are significantly different. Hough (1957) was the first to recognize that important differences exist between organic and mineral clay soils and suggested two different empirical equations to estimate C_c for the two types of soils. He also introduced several formulas to estimate C_c for cohesionless soils. Lambe and Whitman (1969) suggested that empirical expressions were not reliable, based in part on a graphical correlation between the ratio $C_c/(1 + e_0)$ versus natural water content for a number of soil samples.

3 VALIDITY OF EMPIRICAL COMPRESSION INDEX EQUATIONS

Examination of equations published by Azzouz *et al.* (1976), Serajuddin (1969), Cozzolino (1961), and Hough (1957) have similar slopes but different intercepts. This illustrates the need for an objective and rational method to validate empirical equations for compression index approximation. While nonlinear and multiple regression equations may be applicable in certain cases, these are not recommended due to inherently large fluctuations in approximated dependent parameters (C_c). Therefore, a new method is proposed to qualitatively and quantitatively determine the validity of linear regression equations used to estimate C_c. A number of regression equations were developed using one or more combinations of independently compiled data sets published by Rendon-Herrero (1980) and the linear empirical formulas listed in Table 1. A linear model relating the C_c to e_o was assumed:

$$C_c = \propto_e + \beta_e e_o \tag{1}$$

α_e and β_e are the regression coefficients relating C_c to e_o for a given data set. Regression analysis was then performed using the combined data set (76 data points). Objectivity and unbiased analysis require that one must not be selective in choosing data points used in regression analysis. For this reason, the range of C_c was arbitrarily limited in the ranges of 0-1 and 0-0.5, and the corresponding regression equations were developed. The e_o was limited to ranges of 0-3, 0-2, 0-1, and 0-0.75 and the corresponding regression equations determined. This process was applied to each of the two independent data sets reported by Rendon-Herrero (1980), using the same limits on C_c and e_o. The resulting regression coefficients α_e and β_e, correlation coefficients (R^2), standard errors σ_e, average void ratio e_{avg}, and average compression index C_{cavg} are shown in Table 2.

Table 2. Regression analysis results for compression index as a function of *in situ* void ratio (based on data reported by Rendon-Herrero 1980).

Eq. No.	C_{cavg}	e_{avg}	Limit	# Points	R^2 (%)	σ_e	α_e	β_e
R1	0.349	1.142	Full Range	76	95.6	0.0758	−0.145964	0.433883
e-1	0.259	0.951	$0 \le C_c \le 1.0$	69	92.6	0.0604	−0.107009	0.385004
e-2	0.203	0.809	$0 \le C_c \le 0.5$	63	77.3	0.0568	−0.145964	0.433883
e-3	0.293	1.023	$0 \le e \le 3.0$	72	94.4	0.0644	−0.126327	0.409942
e-4	0.208	0.821	$0 \le e \le 2.0$	64	79.3	0.0566	−0.094456	0.368087
e-5	0.153	0.682	$0 \le e \le 1.0$	48	47.0	0.0529	−0.047026	0.293533
e-6	0.119	0.576	$0 \le e \le 0.75$	30	24.3	0.0392	0.006019	0.19655

Careful examination of the regression equations reveals that reducing the data in Table 2 by only a few points has dramatic effects on the R^2 value and associated standard error. This is true irrespective of the total number of data points analyzed. This makes it difficult to decide which data points to include or exclude from the analysis. Note that reducing the number of data points form 76 in Eq. R1 to 30 data points in Eq. e-6 reduces the correction coefficient from 95.6% to 24.3%. In general, one should use empirical formulas with high correlation and low standard error. On that basis, one may select a number of empirical equations for a given range of C_c or e_o. The implication is that no regression equation can do the job of correctly predicting the C_c over the full range of e_o values expected for soil. Hence, Eq. R1 is likely the most reasonable empirical expression for soils with e_o of less than 1.14. This is because it is based on 76 data points and has the highest R^2 of 94.3% with a relatively small corresponding standard error. Other empirical expressions may be selected for different e_o ranges. The derived empirical expressions appear to be varied and dependent on the number of data points involved. At first glance, it seems impossible to derive any substantive conclusions. Fortunately, consideration of the regression coefficients α_e and β_e shows that they are related linearly irrespective of R^2. Although regression coefficients corresponding to small correlations indicate lack of trend, the relationship between the regression coefficients holds as shown in Figure 1.

Clearly, a linear relationship exists between the α_e and β_e regression coefficients. The equation of the line has a correlation coefficient of R^2=0.994 and is given as follows:

$$\propto_e = 0.13721 - 0.64312\beta_e \tag{2}$$

Eq. (2) is referred to as the compression void ratio line and is believed to be a property of soils being considered. The published linear empirical equations (see e1 through e-6 listed in Table 2) relating compression index to void ratio are shown graphically in Figure 2.

Clearly, the regression coefficients for equations e-1, e-2, e-3, e-4, and e-5 plot closely to the derived regression Line. Nishida's empirical formula for all clays deviates appreciably from the line. Also, both Bowles and Huffs equations appear to be the least accurate predictors of the compression index from all those shown in Table 1. Note that Nishida's equation proposed for all soils is accurate because it is very close to the derived regression line shown in Figure 2. Koppula (1981) found that in comparison with other well-known relationships, Nishida's equation performed very poorly in predicting compression index of cohesive soils from the province of Alberta, Canada. Also, even though, equation e-5 is close to the line, the correlation coefficients of equations with positive intercepts are not very high. Although, the various empirical formulas were based on entirely different data from that used in the derivation of the Line, it is obvious that there is a definite trend. In effect, these empirical formulas verify the legitimacy of the newly developed line.

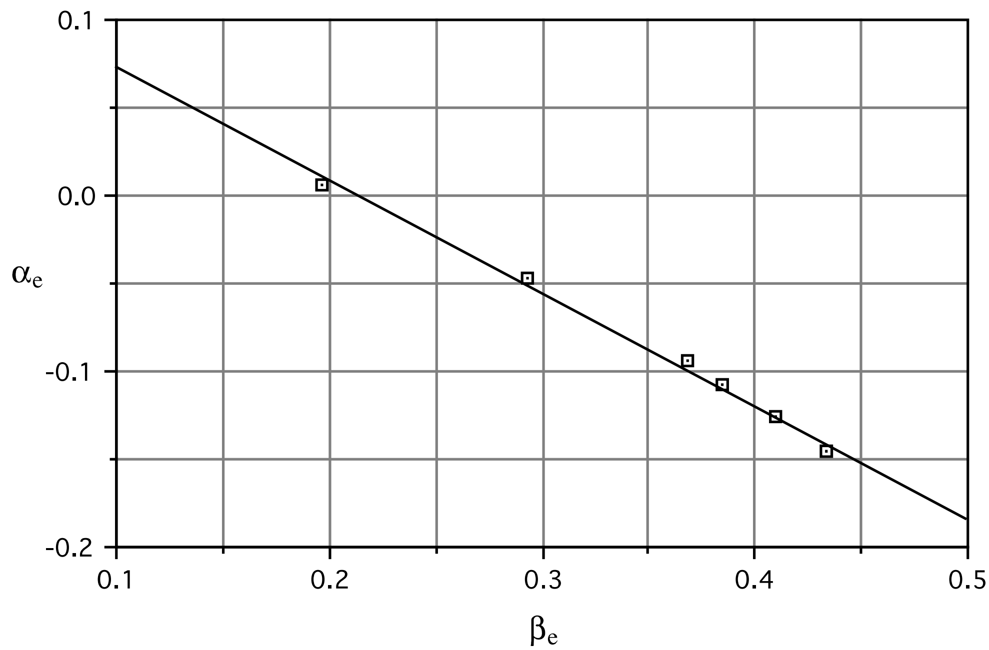

Figure 1. Relationship between regression coefficients for relationships between compression index and is situ void ratio.

Based on the observation that large β_e values indicate large correlation, it is possible to suggest an empirical relationship approximately within the range of the data used. Therefore, choosing equation e-3 which has a reasonably large slope, high correlation, and small standard error, a value of $\beta_e = 0.40$ is selected. Substituting into Eq. (2) gives an $\alpha_e = -0.1200$.

$$C_c = -0.12 + 0.40 = 0.40(e_o - 0.3) \tag{3}$$

It is important to note that Eq. (3) is based on a maximum in situ void ratio of less than 3.0. This excludes the application of equation to most organic soils. Also, the empirical evaluation of compression index for soils with void ratios of less than 0.30 is not possible.

It is evident that additional empirical equations can be suggested which correspond to those listed in Tables 1 and 2. Thus, using the predefined ranges of independent variables along with Eq. (2), it should be possible to refine these formulas. However, it is important to only know that such expressions will be associated with fewer data points than used in the derivation of Eq. (3). Consequently, caution must be exercised when relationships are based on experimental limited data.

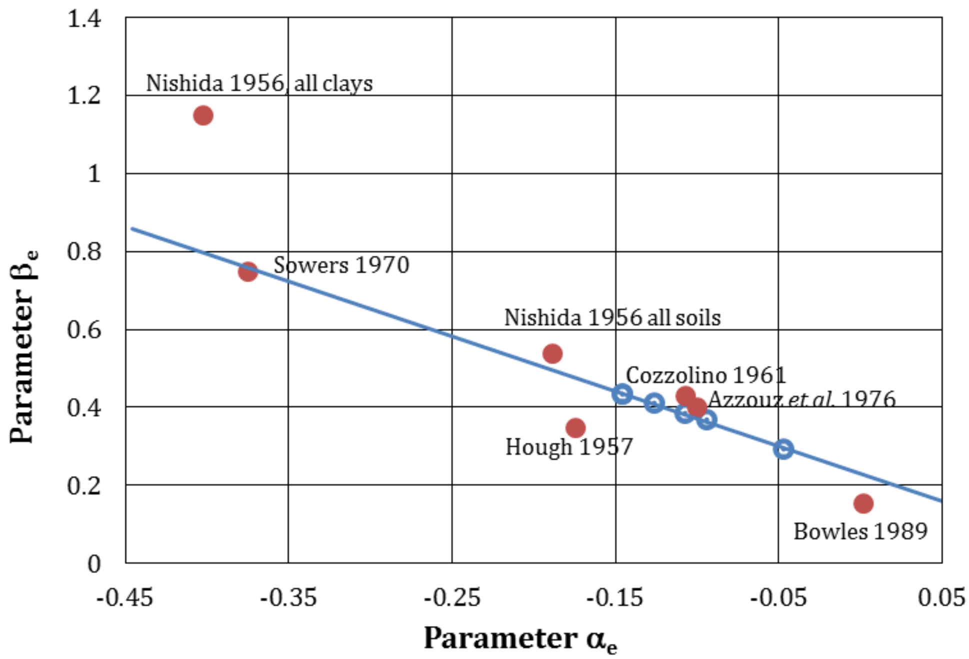

Figure 2. Relationship between regression coefficients developed between the compression index and the *in situ* void ratio.

4 SUMMARY AND CONCLUSIONS

Most empirical equations used to estimate compression index of soils in terms of soil index properties have been developed using data for disturbed as well as undisturbed soils. The variability of soil parameters, soil types, and machine and operator errors makes it impossible to suggest a unified approach to compression index estimation. Unlike mineral soils, organic soils are highly compressible and their properties change under constant effective consolidation pressure (Al-Khafaji and Andersland 1981). Consequently, prediction of C_c should be limited to mineral soils. Most empirical formulas to estimate C_c are based on liquid limit, water content, and void ratio assuming linear relationships with one independent variable. While some of these empirical equations are restricted to specific soils, others are supposedly applicable to all soils. Use of these formulas is often legitimized based on the R^2 value but no attempt has been made to examine the associated standard errors. Consideration of a number of widely-known empirical compression equations with data revealed interesting and useful possibilities. Examination of data scatter reveals that high values of e_o are generally associated with organic soils. The inclusion of such data points in derivations of empirical formulas could alter the applicability of many of these equations to mineral soils. The variability of C_c relating to organic soils is well documented. In fact, Al-Khafaji and Andersland (1981) have shown that the use of C_c in settlement calculations of organic soils is not justified. Based on work presented in this paper (Figure 1), it may become possible to define regions of applicability of published empirical equations to a variety of soils.

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