CHAPTER 4 RESULTS AND DISCUSSION

In this study, we will be applying the FDM 13-point stencil, FDM 25-point stencil and IRBF 5-point stencil to verify the biharmonic problem as following.

4.1 Example1: Consider the uniformly sinusoidal load on a simply supported rectangular plate.

$$u(x,y)=\frac{q_0}{\pi^4 D\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$

where $q(x,y)=q_0\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$,

q_0 is the intensity of the load at center of the plate,

D is the flexural rigidity,

a,b are the side lengths of a rectangular plate.

Boundary conditions can be specified as

$$u\big|_{\Gamma u}=\bar{u}\quad,\quad \frac{\partial u}{\partial \boldsymbol{n}}\bigg|_{\Gamma q}=\bar{q}.$$

In this problem we used $a,b=1, D=\frac{q_0}{\pi^4}, h=0.05, \tau=0.001.$

The exact solution of this problem is $u(x,y)=\sin\pi x\sin\pi y$.

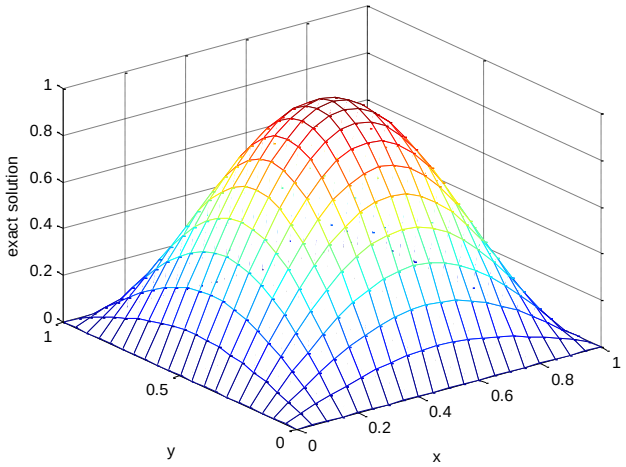

Figure 4.1 Surface of exact solution of example 1.

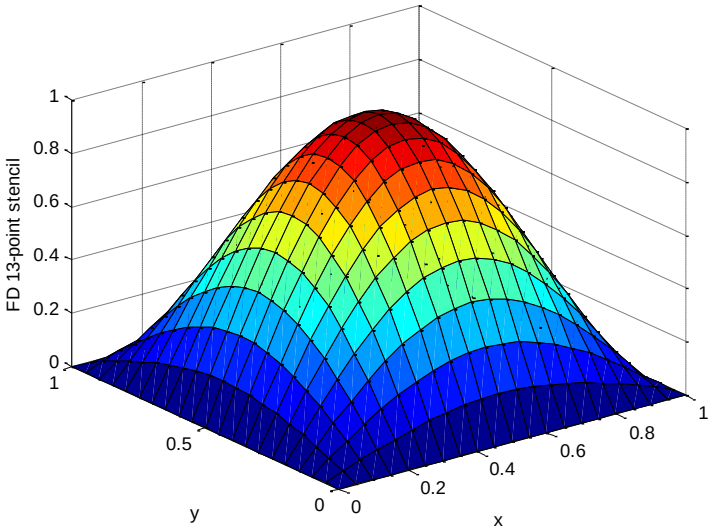

Figure 4.2 Surface of compact Finite Difference Method 13-point stencil of example 1.

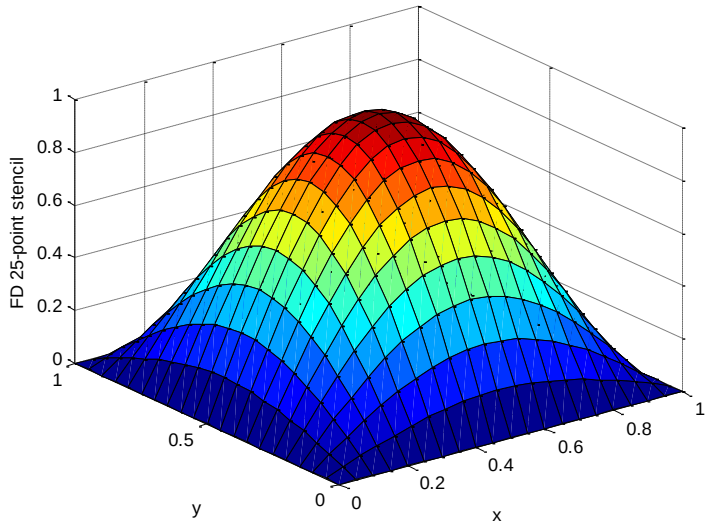

Figure 4.3 Surface of compact Finite Difference Method 25-point stencil of example 1.

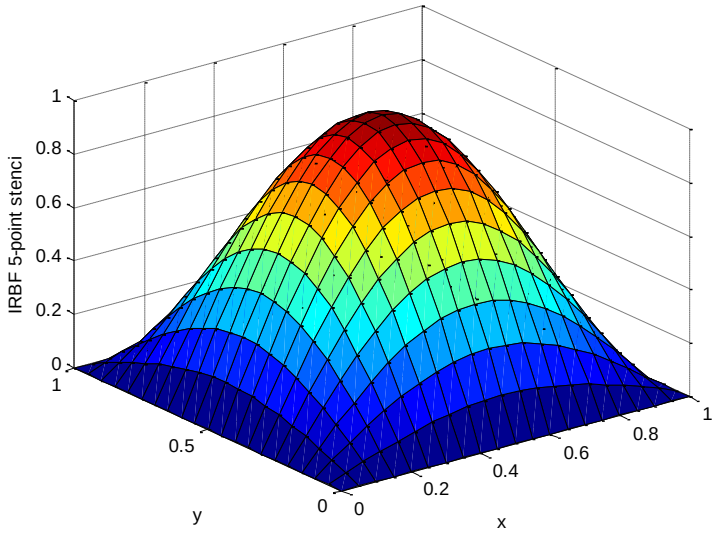

Figure 4.4 Surface of Integrated Radial Basis Functions 5-point stencil of example 1.

Table 4.1 The result of example 1 by using FDM 13-point and FDM 25-point stencils.

	Time (seconds)	The Number of iterations	Total of operation count	Ratio (FDM / IRBFs)
FDM 13-point	142.633	686996	13739920	21.78
FDM 25-point	18.698	14339	630916	

Experiments show that

1. The operation count of compact 25-point stencil less than compact 13-point stencil equal to 21.78 times.
2. The number of iterations compact 25-point stencil less than compact 13-point stencil equal to 47.91 times.
3. Elapsed time of compact 25-point stencil less than compact 13-point stencil equal to 123.935 times.

Table 4.2 The result of example 1 by using FDM 25-point stencil and IRBFs 5-point stencil.

	Time (seconds)	The Number of iterations	Total of operation count	Ratio (FDM / IRBFs)
FDM 25-point	18.698	14339	630916	16.709
IRBFs 5-point	0.172	609	37758	

Experiments show that

1. The operation count for all iteration of IRBFs 5-point stencil less than FDM 25-point stencil equal to 16.709 times.
2. The number of iterations of IRBFs 5-point stencil less than FDM 25-point stencil equal to 23.545 times.
3. Elapsed time of compact IRBFs 5-point stencil less than FDM 25-point stencil equal to 108.709 times.

4.2 Example2: Consider polynomial function as a exact solution

$$u(x,y) = x^2 (x-1)^2 \, y^2 (y-1)^2 .$$

Boundary conditions can be specified as

$$u|_{\Gamma u} = \bar{u} \quad , \quad \left.\frac{\partial u}{\partial \boldsymbol{n}}\right|_{\Gamma q} = \bar{q} .$$

In this problem we used $h = 0.05, \tau = 0.001$.

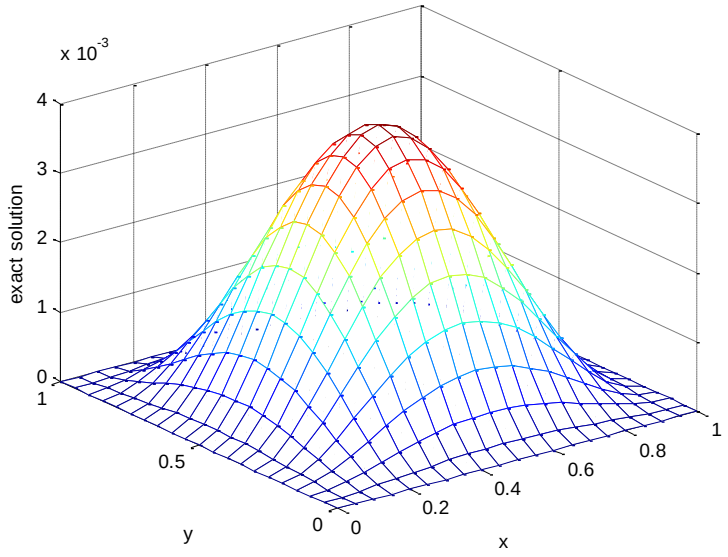

Figure 4.5 Surface of exact solution of example 2.

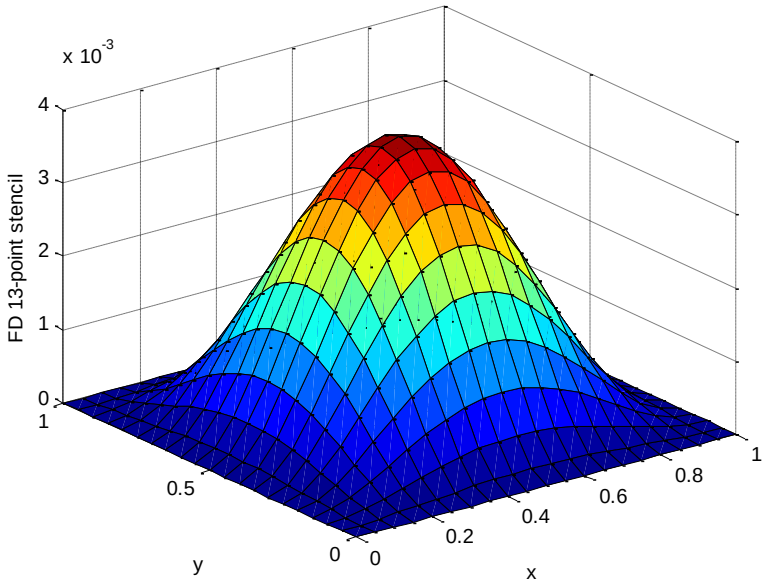

Figure 4.6 Surface of compact Finite Difference Method 13-point stencil of example 2.

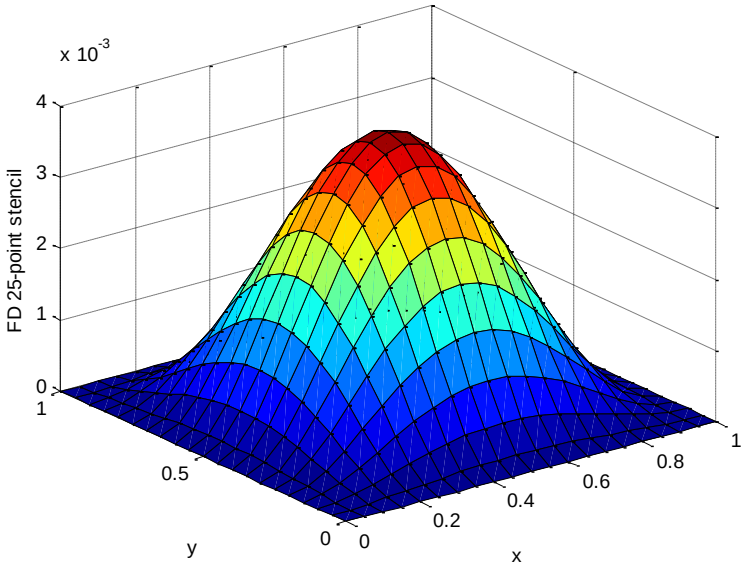

Figure 4.7 Surface of compact Finite Difference Method 25-point stencil of example 2.

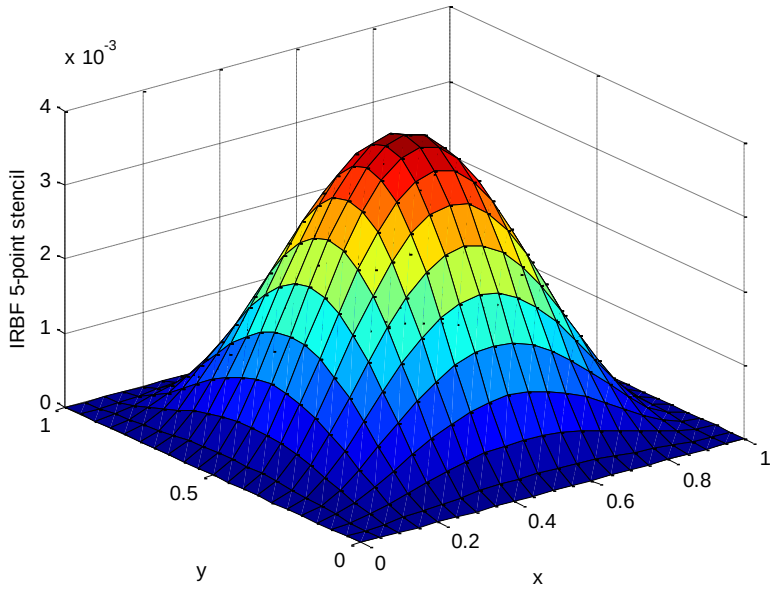

Figure 4.8 Surface of Integrated Radial Basis Functions 5-point stencil of example 2.

Table 4.3 The result of example 2 by using FDM 13-point stencils and FDM 25-point stencils.

	Time (seconds)	The Number of iterations	Total of operation count	Ratio (FDM/IRBF)
FDM 13-point	94.20	416697	8333940	30.73
FDM 25-point	7.364	6162	271128	

Experiments show that

1. The operation count for all iteration of compact 25-point stencil less than compact 13-point stencil equal to 28.776 times.
2. The number of iterations compact 25-point stencil less than compact 13-point stencil equal to 67.62 times.
3. Elapsed time of compact 25-point stencil less than compact 13-point stencil equal to 12.79 times.

Table 4.4 The result of example 2 by using FDM 25-point stencil and IRBFs 5-point stencil.

	Time (seconds)	The Number of iterations	Total of operation count	Ratio (FDM/IRBF)
FDM 25-point	7.364	6162	271128	11.661
IRBF 5-point	0.124	375	23250	

Experiments show that

1. The operation count for all iteration of IRBFs 5-point stencil less than FDM 25-point stencil equal to 11.661 times.
2. The number of iterations of IRBFs 5-point stencil less than FDM 25-point stencil equal to 16.432 times.
3. Elapsed time of IRBFs 5-point stencil less than FDM 25-point stencil equal to 59.387 times.