

# CONTENTS

	PAGE
ENGLISH ABSTRACT	ii
THAI ABSTRACT	iii
ACKNOWLEDGEMENT	iv
CONTENTS	v
LIST OF TABLES	viii
LIST OF FIGURES	ix
LIST OF TECHNICAL VOCABULARY AND ABBREVIATIONS	x
 <b>CHAPTER</b>	
<b>1. INTRODUCTION</b>	<b>1</b>
1.1 Rational	1
1.2 Literature Review	1
1.3 Objective of the Thesis	2
1.4 Scopes of the Thesis	2
 <b>2. THEOREM</b>	<b>3</b>
2.1 Radial Basis Functions (RBFs) Method	3
2.2 Integrated Radial Basis Functions (IRBFs)	6
2.3 Biharmonic Equation	6
 <b>3. METHODOLOGY</b>	<b>7</b>
3.1 Finite difference approximation of biharmonic equation with $2^{nd}$ order accuracy.	7
3.1.1 The finite difference approximation of $\frac{\partial^4 u}{\partial x^4} \Big _{i,j}$ with $2^{nd}$ order accuracy.	7
3.1.2 The finite difference approximation of $2 \frac{\partial^4 u}{\partial x^2 \partial y^2} \Big _{i,j}$ with $2^{nd}$ order accuracy.	8
3.1.3 The finite difference approximation of $\frac{\partial^4 u}{\partial y^4} \Big _{i,j}$ with $2^{nd}$ order accuracy.	9
3.2 Finite difference approximation of biharmonic equation with $4^{th}$ order accuracy.	9
3.2.1 The finite difference approximation of $\frac{\partial^4 u}{\partial x^4} \Big _{i,j}$ with $4^{th}$ order accuracy.	9
3.2.2 The finite difference approximation of $2 \frac{\partial^4 u}{\partial x^2 \partial y^2} \Big _{i,j}$ with $4^{th}$ order accuracy.	12

	<b>PAGE</b>
3.2.3 The finite difference approximation of $\frac{\partial^4 u}{\partial y^4} \Big _{i,j}$ with 4 <sup>th</sup> order accuracy.	12
3.3 Integrated Radial Basis Functions (IRBF <sub>s</sub> ).	15
3.4 Iteration Method	18
3.4.1 Iteration method of Finite Difference Method (FDM) with 2 <sup>nd</sup> order accuracy.	18
3.4.2 Iteration method of Finite Difference Method (FDM) with 4 <sup>th</sup> order accuracy.	19
3.4.3 Iteration method of Integrate Radial Basis Functions (IRBFs)	20
3.5 Boundary Condition	20
3.5.1 Backward difference approximation of $\frac{\partial u}{\partial x} \Big _{i=N,j}$ with 2 <sup>nd</sup> order accuracy.	20
3.5.2 Backward difference approximation of $\frac{\partial u}{\partial y} \Big _{i,j=N}$ with 2 <sup>nd</sup> order accuracy.	21
3.5.3 Forward difference approximation of $\frac{\partial u}{\partial x} \Big _{i=N,j}$ with 2 <sup>nd</sup> order accuracy.	21
3.5.4 Forward difference approximation of $\frac{\partial u}{\partial y} \Big _{i,j=N}$ with 2 <sup>nd</sup> order accuracy.	22
3.5.5 Backward difference approximation of $\frac{\partial u}{\partial x} \Big _{i=N,j}$ with 4 <sup>th</sup> order accuracy.	23
3.5.6 Backward difference approximation of $\frac{\partial u}{\partial y} \Big _{i,j=N}$ with 4 <sup>th</sup> order accuracy.	24
3.5.7 Forward difference approximation of $\frac{\partial u}{\partial x} \Big _{i=N,j}$ with 4 <sup>th</sup> order accuracy.	25
3.5.8 Forward difference approximation of $\frac{\partial u}{\partial y} \Big _{i,j=N}$ with 4 <sup>th</sup> order accuracy.	26
<b>4. RESULTS AND DISCUSSION</b>	<b>27</b>
4.1 Example1	27
4.2 Example2	31
<b>5. CONCLUSION AND RECOMMENDATION</b>	<b>35</b>
5.1 Conclusion	35
5.2 Recommendation	35

	<b>PAGE</b>
<b>REFERENCES</b>	<b>36</b>
<b>APPENDIX</b>	<b>38</b>
<b>BIOGRAPHY</b>	<b>52</b>

## LIST OF TABLES

TABLE	PAGE
1.1 Some commonly used radial basis functions.	4
4.1 The result of example 1 by using FDM 13-point stencil and FDM 25-point stencils.	29
4.2 The result of example 1 by using FDM 25-point stencil and IRBFs 5-point stencils.	30
4.3 The result of example 2 by using FDM 13-point stencil and FDM 25 point stencils.	33
4.4 The result of example 2 by using FDM 25-point stencil and IRBFs 5-point stencils.	34

## LIST OF FIGURES

FIGURE	PAGE
3.1 Diagram of biharmonic equation with $2^{nd}$ order accuracy.	9
3.2 Diagram of $\frac{\partial^4 u}{\partial x^4} \Big _{i,j}$ with $4^{th}$ order accuracy.	11
3.3 Diagram of $2 \frac{\partial^4 u}{\partial x^2 \partial y^2} \Big _{i,j}$ with $4^{th}$ order accuracy.	12
3.4 Diagram of $\frac{\partial^4 u}{\partial y^4} \Big _{i,j}$ with $4^{th}$ order accuracy.	14
3.5 Diagram of biharmonic equation with $4^{th}$ order accuracy.	14
3.6 Schematic outline of a 5-point stencil.	15
3.7 Diagram of iteration method biharmonic equation at $2^{nd}$ order accuracy.	19
3.8 Diagram of iteration method biharmonic equation with $4^{th}$ order accuracy.	19
4.1 Surface of exact solution of example 1	27
4.2 Surface of compact Finite Difference Method 13-point stencil of example 1.	28
4.3 Surface of compact Finite Difference Method 25-point stencil of example 1.	28
4.4 Surface of Integrated Radial Basis Functions 5-point stencil of example 1.	29
4.5 Surface of exact solution of example 2	31
4.6 Surface of compact Finite Difference Method 13-point stencil of example 2.	32
4.7 Surface of compact Finite Difference Method 25-point stencil of example 2.	32
4.8 Surface of Integrated Radial Basis Functions 5-point stencil of example 2.	33

## LIST OF TECHNICAL VOCABULARY AND ABBREVIATIONS

$D$	=	Flexural rigidity
$h$	=	Grid size
$n$	=	Distant data
$\mathbf{n}$	=	Outward unit normal vector to the boundary $\Gamma$
$C_k$	=	Constant of integration
$G_k(x), \phi(r)$	=	Radial Basis Functions
$H_k(x), \overline{H}_k(x)$	=	Integrated Radial Basis Functions
$\beta$	=	Factor
$\varepsilon$	=	Shape parameter
$\tau$	=	Time step
$q_0$	=	Intensity of the load at center of the plate
$a_k^2$	=	Multiquadric width
$w_k$	=	Weight of node $k$
$\overline{u}$	=	Prescribed potential on essential $\Gamma_u$
$\overline{q}$	=	Normal flux on essential $\Gamma_q$
$\nabla^2$	=	Two-dimensional Laplace operator
$\ \cdot\ $	=	Euclidean norm