APPENDIX

Source code of example 1 by using Finite Difference Method 13-point stencil

```
clear all;
format longG;
%% define parameter
a=0;
b=1;
c=0;
d=1;
epsilon=0.5*10^(-4);
xystep=0.05;
delt=0.001;
x=a:xystep:b;
y=c:xystep:d;
[tmp nx]=size(x);
[tmp ny]=size(y);
%% define exact solution
cof=1;
uuE=@(xx,yy) cof*(sin(xx*pi)*sin(yy*pi));
uu4xE=@(xx,yy) cof*(sin(xx*pi)*(pi^4)*sin(yy*pi));
uu4yE=@(xx,yy) cof*(sin(xx*pi)*(pi^4)*sin(yy*pi));
uuxxyyE=@(xx,yy) cof*(sin(xx*pi)*(pi^4)*sin(yy*pi));
ffE =@(xx,yy) uu4xE(xx,yy)+2*uuxxyyE(xx,yy)+uu4yE(xx,yy);
uE=zeros(nx,ny);
f=zeros(nx,ny);
for i=1:nx
   for j=1:ny
      uE(i,j)=uuE(x(i),y(j));
      f(i,j)=ffE(x(i),y(j));
   end;
end;
%% compute IC
u0=zeros(nx,ny);
u0(1,:)=uE(1,:);
u0(2,:)=uE(2,:);
u0(nx,:)=uE(nx,:);
u0(nx-1,:)=uE(nx-1,:);
u0(:,1)=uE(:,1);
u0(:,2)=uE(:,2);
u0(:,ny)=uE(:,ny);
u0(:,ny-1)=uE(:,ny-1);
u1=u0;
it_max=10000000;
tic;
for it=1:it_max
   for i=3:nx-2;
```

```
    for j=3:ny-2
        u1(i,j)=u0(i,j)+delt*((xystep^4)*f(i,j) ...
          -(u0(i-2,j)+u0(i+2,j)+u0(i,j-2)+u0(i,j+2)) ...
          +8*(u0(i-1,j)+u0(i+1,j)+u0(i,j-1)+u0(i,j+1)) ...
          -2*(u0(i-1,j-1)+u0(i+1,j-1)+u0(i-1,j+1)+u0(i+1,j+1)) ...
          -20*u0(i,j));
      end;
   end;
   TOL=max(max(abs(u1-uE)));
   if TOL < epsilon
      break;
   end;
   u0=u1;
   disp([it TOL]);
end;
toc;
disp([it TOL]);
figure;
mesh(x,y,uE');
figure;
surf(x,y,u1');
```

Source code of example 1 by using Finite Difference Method 25-point stencil

```
clear all;
format longG;
%% define parameter
a=0;
b=1;
c=0;
d=1;
epsilon=0.5*10^(-4);
xystep=0.05;
delt=0.001;
x=a:xystep:b;
y=c:xystep:d;
[tmp nx]=size(x);
[tmp ny]=size(y);
%% define exact solution
cof=1;
uuE=@(xx,yy) cof*(sin(xx*pi)*sin(yy*pi));
uu4xE=@(xx,yy) cof*(sin(xx*pi)*(pi^4)*sin(yy*pi));
uu4yE=@(xx,yy) cof*(sin(xx*pi)*(pi^4)*sin(yy*pi));
uuxxyyE=@(xx,yy) cof*(sin(xx*pi)*(pi^4)*sin(yy*pi));
ffE =@(xx,yy) uu4xE(xx,yy)+2*uuxxyyE(xx,yy)+uu4yE(xx,yy);
uE=zeros(nx,ny);
f=zeros(nx,ny);
for i=1:nx
```

```
    for j=1:ny
        uE(i,j)=uuE(x(i),y(j));
        f(i,j)=ffE(x(i),y(j));
    end;
end;
%% compute IC
u0=zeros(nx,ny);
u0(1,:)=uE(1,:);
u0(2,:)=uE(2,:);
u0(nx,:)=uE(nx,:);
u0(nx-1,:)=uE(nx-1,:);
u0(:,1)=uE(:,1);
u0(:,2)=uE(:,2);
u0(:,ny)=uE(:,ny);
u0(:,ny-1)=uE(:,ny-1);
u1=u0;
it_max=10000000;
tic;
for it=1:it_max
    for i=3:nx-2;
        for j=3:ny-2
            u1(i,j)=u0(i,j)+delt*((xystep^4)*(12*f(i,j)-f(i-2,j)-f(i+2,j)-f(i,j-2)…
                -f(i,j+2)+16*(f(i-1,j)+f(i+1,j)+f(i,j-1)+f(i,j+1))) ...
                +5*(u0(i-2,j-2)+u0(i+2,j-2)+u0(i-2,j+2)+u0(i+2,j+2)) ...
                -44*(u0(i-1,j-2)+u0(i+1,j-2)+u0(i-1,j+2)+u0(i+1,j+2)+u0(i-2,j-1)…
                +u0(i+2,j-1)+u0(i-2,j+1)+u0(i+2,j+1)) ...
                +6*(u0(i,j-2)+u0(i,j+2)+u0(i-2,j)+u0(i+2,j)) ...
                +128*(u0(i-1,j-1)+u0(i+1,j-1)+u0(i-1,j+1)+u0(i+1,j+1)) ...
                +120*(u0(i,j-1)+u0(i,j+1)+u0(i-1,j)+u0(i+1,j)) ...
                -684*u0(i,j));
        end;
    end;
    TOL=max(max(abs(u1-uE)));
    if TOL < epsilon
        break;
    end;
    u0=u1;
    disp([it TOL]);
end;
toc;
disp([it TOL]);
figure;
mesh(x,y,uE');
figure;
surf(x,y,u1');
```

Source code of example 1 by using Integrated Radial Basis Functions 5-point stencil

```
clear all;
format longG;
%% define parameter
G=@(xx,xxk,aak) sqrt((xx-xxk)^2+aak^2);
H0b=@(xx,xxk,aak) (1/4*(2*xx-2*xxk))*sqrt(xx^2-
2*xx*xxk+xxk^2+aak^2)+(1/2)*aak^2*ln(xx-xxk+sqrt(xx^2-
2*xx*xxk+xxk^2+aak^2));
H1b=@(xx,xxk,aak) (1/6)*(xx^2-2*xx*xxk+xxk^2+aak^2)^(3/2)+(1/2)*aak^2
*log(xx-xxk+sqrt(xx^2-2*xx*xxk+xxk^2+aak^2))*xx-(1/2)*aak^2*sqrt(xx^2-
2*xx*xxk+xxk^2+aak^2)-(1/2)*aak^2*xxk*log(xx-xxk+sqrt(xx^2-
2*xx*xxk+xxk^2+aak^2));
% G=@(xx,xxk,aak) exp(-aak^2*(xx-xxk)^2);
% H0b=@(xx,xxk,aak) (1/2)*sqrt(pi)*erf(aak*xx-aak*xxk)/aak;
% H1b=@(xx,xxk,aak) (1/2)*sqrt(pi)*erf(aak*xx-aak*xxk)*xx/aak;
a=0;
b=1;
c=0;
d=1;
xystep=0.05;
x=a:xystep:b;
y=c:xystep:d;
[tmp nx]=size(x);
[tmp ny]=size(y);
ak=0.5;
epsilon=0.5*10^(-4);
eta=zeros(5,nx,ny);
theta=zeros(5,nx,ny);
for i=3:nx-2
   for j=3:ny-2
      x1=x(i-1);
      x3=x(i);
      x5=x(i+1);
      C=zeros(5,5);
      C(1,1)=H1b(x1,x1,ak);
      C(1,2)=H1b(x1,x3,ak);
      C(1,3)=H1b(x1,x5,ak);
      C(1,4)=x1;
      C(1,5)=1;
      C(2,1)=H1b(x3,x1,ak);
      C(2,2)=H1b(x3,x3,ak);
      C(2,3)=H1b(x3,x5,ak);
      C(2,4)=x3;
      C(2,5)=1;
      C(3,1)=H1b(x5,x1,ak);
      C(3,2)=H1b(x5,x3,ak);
      C(3,3)=H1b(x5,x5,ak);
```

```
      C(3,4)=x5;
      C(3,5)=1;
      C(4,1)=G(x1,x1,ak);
      C(4,2)=G(x1,x3,ak);
      C(4,3)=G(x1,x5,ak);
      C(4,4)=0;
      C(4,5)=0;
      C(5,1)=G(x5,x1,ak);
      C(5,2)=G(x5,x3,ak);
      C(5,3)=G(x5,x5,ak);
      C(5,4)=0;
      C(5,5)=0;
      iC=inv(C);
      eta(:,i,j)=[G(x3,x1,ak) G(x3,x3,ak) G(x3,x5,ak) 0 0]*iC;
      y1=y(i-1);
      y3=y(i);
      y5=y(i+1);
      C=zeros(5,5);
      C(1,1)=H1b(y1,y1,ak);
      C(1,2)=H1b(y1,y3,ak);
      C(1,3)=H1b(y1,y5,ak);
      C(1,4)=y1;
      C(1,5)=1;
      C(2,1)=H1b(y3,y1,ak);
      C(2,2)=H1b(y3,y3,ak);
      C(2,3)=H1b(y3,y5,ak);
      C(2,4)=y3;
      C(2,5)=1;
      C(3,1)=H1b(y5,y1,ak);
      C(3,2)=H1b(y5,y3,ak);
      C(3,3)=H1b(y5,y5,ak);
      C(3,4)=y5;
      C(3,5)=1;
      C(4,1)=G(y1,y1,ak);
      C(4,2)=G(y1,y3,ak);
      C(4,3)=G(y1,y5,ak);
      C(4,4)=0;
      C(4,5)=0;
      C(5,1)=G(y5,y1,ak);
      C(5,2)=G(y5,y3,ak);
      C(5,3)=G(y5,y5,ak);
      C(5,4)=0;
      C(5,5)=0;
      iC=inv(C);
      theta(:,i,j)=[G(y3,y1,ak) G(y3,y3,ak) G(y3,y5,ak) 0 0]*iC;
   end;
end;
%% define exact solution
cof=1;
```

```
uuE=@(xx,yy) cof*(sin(xx*pi)*sin(yy*pi));
uuxxE=@(xx,yy) cof*(sin(xx*pi)*(-pi^2)*sin(yy*pi));
uuyyE=@(xx,yy) cof*(sin(xx*pi)*(-pi^2)*sin(yy*pi));
uu4xE=@(xx,yy) cof*(sin(xx*pi)*(pi^4)*sin(yy*pi));
uu4yE=@(xx,yy) cof*(sin(xx*pi)*(pi^4)*sin(yy*pi));
uuxxyyE=@(xx,yy) cof*(sin(xx*pi)*(pi^4)*sin(yy*pi));
vvE =@(xx,yy) uuxxE(xx,yy)+uuyyE(xx,yy);
vvxxE =@(xx,yy) uu4xE(xx,yy)+uuxxyyE(xx,yy);
vvyyE =@(xx,yy) uuxxyyE(xx,yy)+uu4yE(xx,yy);
ffE =@(xx,yy) uu4xE(xx,yy)+2*uuxxyyE(xx,yy)+uu4yE(xx,yy);
uE=zeros(nx,ny);
vE=zeros(nx,ny);
f=zeros(nx,ny);
for i=1:nx
   for j=1:ny
      uE(i,j)=uuE(x(i),y(j));
      vE(i,j)=vvE(x(i),y(j));
      f(i,j)=ffE(x(i),y(j));
   end;
end;

%% compute IC
v0=zeros(nx,ny);
v0(1,:)=vE(1,:);
v0(2,:)=vE(2,:);
v0(nx,:)=vE(nx,:);
v0(nx-1,:)=vE(nx-1,:);
v0(:,1)=vE(:,1);
v0(:,2)=vE(:,2);
v0(:,ny)=vE(:,ny);
v0(:,ny-1)=vE(:,ny-1);
u0=zeros(nx,ny);
u0(1,:)=uE(1,:);
u0(2,:)=uE(2,:);
u0(nx,:)=uE(nx,:);
u0(nx-1,:)=uE(nx-1,:);
u0(:,1)=uE(:,1);
u0(:,2)=uE(:,2);
u0(:,ny)=uE(:,ny);
u0(:,ny-1)=uE(:,ny-1);
v1=v0;
u1=u0;
it_max=100000;
tic;
for it=1:it_max
   for i=3:nx-2;
      for j=3:ny-2
```

```
            v1(i,j)=(f(i,j) -(eta(4,i,j)*(v0(i-2,j)-2*v0(i-1,j)+v0(i,j))/(xystep^2) ...
            +eta(5,i,j)*(v0(i,j)-2*v0(i+1,j)+v0(i+2,j))/(xystep^2)) ...
            -(theta(4,i,j)*(v0(i,j-2)-2*v0(i,j-1)+v0(i,j))/(xystep^2) ...
            +theta(5,i,j)*(v0(i,j)-2*v0(i,j+1)+v0(i,j+2))/(xystep^2)) ...
            -eta(1,i,j)*v1(i-1,j)-eta(3,i,j)*v0(i+1,j) ...
            -theta(1,i,j)*v1(i,j-1)-theta(3,i,j)*v0(i,j+1)) ...
            /(eta(2,i,j)+theta(2,i,j));
            u1(i,j)=(v1(i,j) ...
            -(eta(4,i,j)*(u0(i-2,j)-2*u0(i-1,j)+u0(i,j))/(xystep^2) ...
            +eta(5,i,j)*(u0(i,j)-2*u0(i+1,j)+u0(i+2,j))/(xystep^2)) ...
            -(theta(4,i,j)*(u0(i,j-2)-2*u0(i,j-1)+u0(i,j))/(xystep^2) ...
            +theta(5,i,j)*(u0(i,j)-2*u0(i,j+1)+u0(i,j+2))/(xystep^2)) ...
            -eta(1,i,j)*u1(i-1,j)-eta(3,i,j)*u0(i+1,j) ...
            -theta(1,i,j)*u1(i,j-1)-theta(3,i,j)*u0(i,j+1)) ...
            /(eta(2,i,j)+theta(2,i,j));
        end;
    end;
    TOL=max(max(abs(u1-u0)));
    if TOL < epsilon
        break;
    end;
    v0=v1;
    u0=u1;
    disp([it TOL]);
end;
toc;
disp([it TOL]);
disp(max(max(abs(u1-uE))));
figure;
mesh(x,y,uE');
hold on;
surf(x,y,u1');
figure;
mesh(x,y,uE'-u1')
```

Source code of example 2 by using Finite Difference Method 13-point stencil

```
clear all;
format longG;
%% define parameter
a=0;
b=1;
c=0;
d=1;
epsilon=0.5*10^(-4);
xystep=0.05;
delt=0.001;
x=a:xystep:b;
y=c:xystep:d;
[tmp nx]=size(x);
[tmp ny]=size(y);
%% define exact solution
cof=1;
uuE=@(xx,yy) cof*((xx^2)*(xx-1)^2*(yy^2)*(yy-1)^2);
uu4xE=@(xx,yy) cof*(24*(yy^2)*(yy-1)^2);
uu4yE=@(xx,yy) cof*(24*(xx^2)*(xx-1)^2);
uuxxyyE=@(xx,yy)  cof*((2*(xx^2)+8*xx*(xx-1)+2*(xx-1)^2)*(2*(yy^2)…
            +8*yy*(yy-1)+2*(yy-1)^2));
ffE =@(xx,yy) uu4xE(xx,yy)+2*uuxxyyE(xx,yy)+uu4yE(xx,yy);uE=zeros(nx,ny);
f=zeros(nx,ny);
for i=1:nx
   for j=1:ny
      uE(i,j)=uuE(x(i),y(j));
      f(i,j)=ffE(x(i),y(j));
   end;
end;
%% compute IC
u0=zeros(nx,ny);
u0(1,:)=uE(1,:);
u0(2,:)=uE(2,:);
u0(nx,:)=uE(nx,:);
u0(nx-1,:)=uE(nx-1,:);
u0(:,1)=uE(:,1);
u0(:,2)=uE(:,2);
u0(:,ny)=uE(:,ny);
u0(:,ny-1)=uE(:,ny-1);
u1=u0;
it_max=10000000;
tic;
for it=1:it_max
   for i=3:nx-2;
      for j=3:ny-2
```

```
            u1(i,j)=u0(i,j)+delt*((xystep^4)*f(i,j) ...
            -(u0(i-2,j)+u0(i+2,j)+u0(i,j-2)+u0(i,j+2)) ...
            +8*(u0(i-1,j)+u0(i+1,j)+u0(i,j-1)+u0(i,j+1)) ...
            -2*(u0(i-1,j-1)+u0(i+1,j-1)+u0(i-1,j+1)+u0(i+1,j+1)) ...
            -20*u0(i,j));
        end;
    end;
    TOL=max(max(abs(u1-uE)));
    if TOL < epsilon
        break;
    end;
    u0=u1;
    disp([it TOL]);
end;
toc;
disp([it TOL]);
figure;
mesh(x,y,uE');
figure;
surf(x,y,u1');
```

Source code of example 2 by using Finite Difference Method 25-point stencil

```
clear all;
format longG;
%% define parameter
a=0;
b=1;
c=0;
d=1;
epsilon=0.5*10^(-4);
xystep=0.05;
delt=0.001;
x=a:xystep:b;
y=c:xystep:d;
[tmp nx]=size(x);
[tmp ny]=size(y);
%% define exact solution
cof=1;
uuE=@(xx,yy) cof*((xx^2)*(xx-1)^2*(yy^2)*(yy-1)^2);
uu4xE=@(xx,yy) cof*(24*(yy^2)*(yy-1)^2);
uu4yE=@(xx,yy) cof*(24*(xx^2)*(xx-1)^2);
uuxxyyE=@(xx,yy) cof*((2*(xx^2)+8*xx*(xx-1)+2*(xx-1)^2)*(2*(yy^2)…
          +8*yy*(yy-1)+2*(yy-1)^2));
ffE =@(xx,yy) uu4xE(xx,yy)+2*uuxxyyE(xx,yy)+uu4yE(xx,yy);
uE=zeros(nx,ny);
f=zeros(nx,ny);
for i=1:nx
    for j=1:ny
```

```
        uE(i,j)=uuE(x(i),y(j));
        f(i,j)=ffE(x(i),y(j));
    end;
end;
%% compute IC
u0=zeros(nx,ny);
u0(1,:)=uE(1,:);
u0(2,:)=uE(2,:);
u0(nx,:)=uE(nx,:);
u0(nx-1,:)=uE(nx-1,:);
u0(:,1)=uE(:,1);
u0(:,2)=uE(:,2);
u0(:,ny)=uE(:,ny);
u0(:,ny-1)=uE(:,ny-1);
u1=u0;
it_max=10000000;
tic;
for it=1:it_max
    for i=3:nx-2;
        for j=3:ny-2
            u1(i,j)=u0(i,j)+delt*((xystep^4)*(12*f(i,j)-f(i-2,j)-f(i+2,j)-f(i,j-2)
                -f(i,j+2)+16*(f(i-1,j)+f(i+1,j)+f(i,j-1)+f(i,j+1))) ...
                +5*(u0(i-2,j-2)+u0(i+2,j-2)+u0(i-2,j+2)+u0(i+2,j+2)) ...
                -44*(u0(i-1,j-2)+u0(i+1,j-2)+u0(i-1,j+2)+u0(i+1,j+2)+u0(i-2,j-1)
                +u0(i+2,j-1)+u0(i-2,j+1)+u0(i+2,j+1)) ...
                +6*(u0(i,j-2)+u0(i,j+2)+u0(i-2,j)+u0(i+2,j)) ...
                +128*(u0(i-1,j-1)+u0(i+1,j-1)+u0(i-1,j+1)+u0(i+1,j+1)) ...
                +120*(u0(i,j-1)+u0(i,j+1)+u0(i-1,j)+u0(i+1,j)) ...
                -684*u0(i,j));
        end;
    end;
    TOL=max(max(abs(u1-uE)));
    if TOL < epsilon
        break;
    end;
    u0=u1;
    disp([it TOL]);
end;
toc;
disp([it TOL]);
figure;
mesh(x,y,uE');
figure;
surf(x,y,u1');
```

Source code of example 2 by using Integrated Radial Basis Functions 5-point stencil

```
clear all;
format longG;
%% define parameter
G=@(xx,xxk,aak) sqrt((xx-xxk)^2+aak^2);
H0b=@(xx,xxk,aak) (1/4*(2*xx-2*xxk))*sqrt(xx^2-2*xx*xxk+xxk^2+aak^2)…
+(1/2)*aak^2*ln(xx-xxk+sqrt(xx^2-2*xx*xxk+xxk^2+aak^2));
H1b=@(xx,xxk,aak) (1/6)*(xx^2-2*xx*xxk+xxk^2+aak^2)^(3/2)…
+(1/2)*aak^2*log(xx-xxk+sqrt(xx^2-2*xx*xxk+xxk^2+aak^2))*xx…
-(1/2)*aak^2*sqrt(xx^2-2*xx*xxk+xxk^2+aak^2)…
-(1/2)*aak^2*xxk*log(xx-xxk+sqrt(xx^2-2*xx*xxk+xxk^2+aak^2));
% G=@(xx,xxk,aak) exp(-aak^2*(xx-xxk)^2);
% H0b=@(xx,xxk,aak) (1/2)*sqrt(pi)*erf(aak*xx-aak*xxk)/aak;
% H1b=@(xx,xxk,aak) (1/2)*sqrt(pi)*erf(aak*xx-aak*xxk)*xx/aak;
a=0;
b=1;
c=0;
d=1;
xystep=0.05;
x=a:xystep:b;
y=c:xystep:d;
[tmp nx]=size(x);
[tmp ny]=size(y);
ak=0.5;
epsilon=0.5*10^(-4);
eta=zeros(5,nx,ny);
theta=zeros(5,nx,ny);
for i=3:nx-2
   for j=3:ny-2
      x1=x(i-1);
      x3=x(i);
      x5=x(i+1);
      C=zeros(5,5);
      C(1,1)=H1b(x1,x1,ak);
      C(1,2)=H1b(x1,x3,ak);
      C(1,3)=H1b(x1,x5,ak);
      C(1,4)=x1;
      C(1,5)=1;
      C(2,1)=H1b(x3,x1,ak);
      C(2,2)=H1b(x3,x3,ak);
      C(2,3)=H1b(x3,x5,ak);
      C(2,4)=x3;
      C(2,5)=1;
      C(3,1)=H1b(x5,x1,ak);
      C(3,2)=H1b(x5,x3,ak);
      C(3,3)=H1b(x5,x5,ak);
      C(3,4)=x5;
      C(3,5)=1;
```

```
        C(4,1)=G(x1,x1,ak);
        C(4,2)=G(x1,x3,ak);
        C(4,3)=G(x1,x5,ak);
        C(4,4)=0;
        C(4,5)=0;
        C(5,1)=G(x5,x1,ak);
        C(5,2)=G(x5,x3,ak);
        C(5,3)=G(x5,x5,ak);
        C(5,4)=0;
        C(5,5)=0;
        iC=inv(C);
        eta(:,i,j)=[G(x3,x1,ak) G(x3,x3,ak) G(x3,x5,ak) 0 0]*iC;
        y1=y(i-1);
        y3=y(i);
        y5=y(i+1);
        C=zeros(5,5);
        C(1,1)=H1b(y1,y1,ak);
        C(1,2)=H1b(y1,y3,ak);
        C(1,3)=H1b(y1,y5,ak);
        C(1,4)=y1;
        C(1,5)=1;
        C(2,1)=H1b(y3,y1,ak);
        C(2,2)=H1b(y3,y3,ak);
        C(2,3)=H1b(y3,y5,ak);
        C(2,4)=y3;
        C(2,5)=1;
        C(3,1)=H1b(y5,y1,ak);
        C(3,2)=H1b(y5,y3,ak);
        C(3,3)=H1b(y5,y5,ak);
        C(3,4)=y5;
        C(3,5)=1;
        C(4,1)=G(y1,y1,ak);
        C(4,2)=G(y1,y3,ak);
        C(4,3)=G(y1,y5,ak);
        C(4,4)=0;
        C(4,5)=0;
        C(5,1)=G(y5,y1,ak);
        C(5,2)=G(y5,y3,ak);
        C(5,3)=G(y5,y5,ak);
        C(5,4)=0;
        C(5,5)=0;
        iC=inv(C);
        theta(:,i,j)=[G(y3,y1,ak) G(y3,y3,ak) G(y3,y5,ak) 0 0]*iC;
    end;
end;
%% define exact solution
cof=1;
uuE=@(xx,yy) cof*((xx^2)*(xx-1)^2*(yy^2)*(yy-1)^2);
uuxxE=@(xx,yy) cof*((2*(xx^2)+8*xx*(xx-1)+2*(xx-1)^2)*(yy^2)*(yy-1)^2);
uuyyE=@(xx,yy) cof*((2*(yy^2)+8*yy*(yy-1)+2*(yy-1)^2)*(xx^2)*(xx-1)^2);
```

```
uu4xE=@(xx,yy) cof*(24*(yy^2)*(yy-1)^2);
uu4yE=@(xx,yy) cof*(24*(xx^2)*(xx-1)^2);
uuxxyyE=@(xx,yy) cof*((2*(xx^2)+8*xx*(xx-1)+2*(xx-1)^2)*(2*(yy^2)…
        +8*yy*(yy-1)+2*(yy-1)^2));
vvE =@(xx,yy) uuxxE(xx,yy)+uuyyE(xx,yy);
vvxxE =@(xx,yy) uu4xE(xx,yy)+uuxxyyE(xx,yy);
vvyyE =@(xx,yy) uuxxyyE(xx,yy)+uu4yE(xx,yy);
ffE =@(xx,yy) uu4xE(xx,yy)+2*uuxxyyE(xx,yy)+uu4yE(xx,yy);
uE=zeros(nx,ny);
vE=zeros(nx,ny);
f=zeros(nx,ny);
for i=1:nx
   for j=1:ny
      uE(i,j)=uuE(x(i),y(j));
      vE(i,j)=vvE(x(i),y(j));
      f(i,j)=ffE(x(i),y(j));
   end;
end;
%% compute IC
v0=zeros(nx,ny);
v0(1,:)=vE(1,:);
v0(2,:)=vE(2,:);
v0(nx,:)=vE(nx,:);
v0(nx-1,:)=vE(nx-1,:);
v0(:,1)=vE(:,1);
v0(:,2)=vE(:,2);
v0(:,ny)=vE(:,ny);
v0(:,ny-1)=vE(:,ny-1);
u0=zeros(nx,ny);
u0(1,:)=uE(1,:);
u0(2,:)=uE(2,:);
u0(nx,:)=uE(nx,:);
u0(nx-1,:)=uE(nx-1,:);
u0(:,1)=uE(:,1);
u0(:,2)=uE(:,2);
u0(:,ny)=uE(:,ny);
u0(:,ny-1)=uE(:,ny-1);
v1=v0;
u1=u0;
it_max=100000;
tic;
for it=1:it_max
   for i=3:nx-2;
      for j=3:ny-2
         v1(i,j)=(f(i,j) ...
            -(eta(4,i,j)*(v0(i-2,j)-2*v0(i-1,j)+v0(i,j))/(xystep^2) ...
            +eta(5,i,j)*(v0(i,j)-2*v0(i+1,j)+v0(i+2,j))/(xystep^2)) ...
            -(theta(4,i,j)*(v0(i,j-2)-2*v0(i,j-1)+v0(i,j))/(xystep^2) ...
            +theta(5,i,j)*(v0(i,j)-2*v0(i,j+1)+v0(i,j+2))/(xystep^2)) ...
            -eta(1,i,j)*v1(i-1,j)-eta(3,i,j)*v0(i+1,j) ...
```

```
            -theta(1,i,j)*v1(i,j-1)-theta(3,i,j)*v0(i,j+1)) ...
            /(eta(2,i,j)+theta(2,i,j));
            u1(i,j)=(v1(i,j) -(eta(4,i,j)*(u0(i-2,j)-2*u0(i-1,j)+u0(i,j))/(xystep^2) ...
            +eta(5,i,j)*(u0(i,j)-2*u0(i+1,j)+u0(i+2,j))/(xystep^2)) ...
            -(theta(4,i,j)*(u0(i,j-2)-2*u0(i,j-1)+u0(i,j))/(xystep^2) ...
            +theta(5,i,j)*(u0(i,j)-2*u0(i,j+1)+u0(i,j+2))/(xystep^2)) ...
            -eta(1,i,j)*u1(i-1,j)-eta(3,i,j)*u0(i+1,j) ...
            -theta(1,i,j)*u1(i,j-1)-theta(3,i,j)*u0(i,j+1)) ...
            /(eta(2,i,j)+theta(2,i,j));
        end;
    end;
    TOL=max(max(abs(u1-u0)));
    if TOL < epsilon
        break;
    end;
    v0=v1;
    u0=u1;
    disp([it TOL]);
end;
toc;
disp([it TOL]);
disp(max(max(abs(u1-uE))));
figure;

mesh(x,y,uE');
hold on;
surf(x,y,u1');
figure;
mesh(x,y,uE'-u1')
```