APPENDIX D

MARKET CLEARING FOR INTERMEDIAT GOODS (SUPPLY SIDE)

At world market if assuming 2-economy, demand for intermediate goods is equal to supply of intermediate goods. That is,

Good 1: $Z_{1t}^{N} + Z_{1t}^{S} = X_{1t}^{N} + X_{1t}^{S}$

Good 2: $Z_{2t}^{N} + Z_{2t}^{S} = X_{2t}^{N} + X_{2t}^{S}$

We can solve relative price from only one of the above equation.

For good 2;

$$\left(\frac{\alpha_3}{1-\alpha_3}\frac{p_{2t}}{p_{1t}}\right)^{-\alpha_3} Y_t^N + \left(\frac{\alpha_3}{1-\alpha_3}\frac{p_{2t}}{p_{1t}}\right)^{-\alpha_3} Y_t^S = L_{2t}^N B^{\alpha_2}\left(\frac{p_{2t}}{p_{1t}}\right)^{\frac{\alpha_2}{\alpha_1-\alpha_2}} + L_{2t}^S B^{\alpha_2}\left(\frac{p_{2t}}{p_{1t}}\right)^{\frac{\alpha_2}{\alpha_1-\alpha_2}}$$

$$\left(\frac{\alpha_3}{1-\alpha_3}\right)^{-\alpha_3}\left(Y_t^N + Y_t^S\right)\left(\frac{p_{2t}}{p_{1t}}\right)^{-\alpha_3} = \left(L_{2t}^N + L_{2t}^S\right) B^{\alpha_2}\left(\frac{p_{2t}}{p_{1t}}\right)^{\frac{\alpha_2}{\alpha_1-\alpha_2}} \qquad \text{(D.1)}$$

Since $Y_t^W = Y_t^N + Y_t^S$, and $L_t^W = L_t^N + L_t^S$

Then, $$\left(\frac{\alpha_3}{1-\alpha_3}\right)^{-\alpha_3}\frac{Y_t^W}{L_t^W}\left(\frac{p_{2t}}{p_{1t}}\right)^{-\alpha_3} = \frac{L_{2t}^W}{L_t^W} B^{\alpha_2}\left(\frac{p_{2t}}{p_{1t}}\right)^{\frac{\alpha_2}{\alpha_1-\alpha_2}} \qquad \text{(D.2)}$$

Let $y_t^w = \frac{Y_t^W}{L_t^W}$ and $l_{2t}^w = \frac{L_{2t}^W}{L_t^W}$.

From factor market clearing condition,

$$k_t^w = \frac{K_t^W}{N_t^W} = \frac{K_{1t}^W}{L_{1t}^W}\frac{L_{1t}^W}{N_t^W} + \frac{K_{2t}^W}{L_{2t}^W}\frac{L_{2t}^W}{N_t^W} = \kappa_{1t} l_{1t}^w + \kappa_{2t} l_{2t}^w, \qquad \text{(D.3)}$$

and $l_t^w = l_{1t}^w + l_{2t}^w = 1$, where $\kappa_{it} = \frac{K_{it}^W}{L_{it}^W}$.

The price ratio as a function of the ratio of intermediate good at world production can be expressed as

$$\frac{p_{1t}}{p_{2t}} = \frac{\alpha_3}{1-\alpha_3}\frac{X_{2t}}{X_{1t}} \qquad . \qquad \text{(D.4)}$$

If $\frac{X_{2t}}{X_{1t}}$ can be expressed in terms of state variable k_t, then the equilibrium price can be stated as the function of k_t.

From equation (B.6), $l_{1t} = \frac{\alpha_3(1-\alpha_1)}{1-\gamma}$,

and from (B.7), $l_{2t} = 1 - l_{1t} = \frac{1-\alpha_1\alpha_3 - \alpha_2(1-\alpha_3) - \alpha_3 - \alpha_1\alpha_3}{1-\gamma} = \frac{(1-\alpha_2)(1-\alpha_3)}{1-\gamma}$.

That is, labor used in each sector can be expressed only in parameters.

Substituting $l_{2t}^w = \frac{(1-\alpha_2)(1-\alpha_3)}{1-\gamma}$ into $\left(\frac{\alpha_3}{1-\alpha_3}\right)^{-\alpha_3} y_t^w \left(\frac{p_{2t}}{p_{1t}}\right)^{-\alpha_3} = l_{2t}^w B^{\alpha_2}\left(\frac{p_{2t}}{p_{1t}}\right)^{\frac{\alpha_2}{\alpha_1-\alpha_2}}$,

then, $\left(\frac{p_{2t}}{p_{1t}}\right)^{\frac{\alpha_2+\alpha_1\alpha_3-\alpha_2\alpha_3}{\alpha_1-\alpha_2}} = \frac{(1-\gamma)y_t^w}{B^{\alpha_2}(1-\alpha_2)\alpha_3^{\alpha_3}(1-\alpha_3)^{(1-\alpha_3)}}$

$$\frac{p_{2t}}{p_{1t}} = \left[\frac{(1-\gamma)y_t^w}{B^{\alpha_2}(1-\alpha_2)\alpha_3^{\alpha_3}(1-\alpha_3)^{(1-\alpha_3)}}\right]^{\frac{\alpha_1-\alpha_2}{\gamma}} \qquad \text{(D.5)}$$

Since the integrated economy is considered as closed economy, then the production of final good can be solved as all intermediate goods produced in the integrated world are used in the production of world final good.

Substituting $y_t^w = f(k_t^w)$ from above result into price ratio, then

$$\frac{p_{2t}}{p_{1t}} = \left[\frac{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}{\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\left(\frac{1-\gamma}{\gamma}\right)^{\alpha_1-\alpha_2}\right]k_t^{\alpha_1-\alpha_2}, \qquad \text{(D.6)}$$

which is equal to equation (3.3.1).