APPENDIX C

WAGE AND RENTAL RATES
IN THE INTEGRATED ECONOMY

From equation (3.2.9),

$$w_t = \left(\frac{\alpha_2^{\alpha_1\alpha_2}(1-\alpha_2)^{\alpha_1(1-\alpha_2)}}{\alpha_1^{\alpha_1\alpha_2}(1-\alpha_1)^{(1-\alpha_1)\alpha_2}} \frac{p_{2t}^{\alpha_1}}{p_{1t}^{\alpha_2}} \right)^{\frac{1}{\alpha_1-\alpha_2}}$$

Substituting $p_{1t} = p(k_t)$ and $p_{2t} = p(k_t)$

$$w_t = \left(\frac{\alpha_2^{\alpha_1\alpha_2}(1-\alpha_2)^{\alpha_1(1-\alpha_2)}}{\alpha_1^{\alpha_1\alpha_2}(1-\alpha_1)^{(1-\alpha_1)\alpha_2}} \right)^{\frac{1}{\alpha_1-\alpha_2}} \cdot \left[(1-\alpha_3)^{\alpha_1} \left(\frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \frac{1-\alpha_3}{\alpha_3} \left(\frac{1-\gamma}{\gamma}\right)^{\alpha_2-\alpha_1} \right)^{-\alpha_1\alpha_3} k_t^{-(\alpha_2-\alpha_1)\alpha_1\alpha_3} \right]^{\frac{1}{\alpha_1-\alpha_2}} \cdot$$

$$\left[\alpha_3^{-\alpha_2} \left(\frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \frac{1-\alpha_3}{\alpha_3} \left(\frac{1-\gamma}{\gamma}\right)^{\alpha_2-\alpha_1} \right)^{-\alpha_2(1-\alpha_3)} k_t^{-(\alpha_2-\alpha_1)\alpha_2(1-\alpha_3)} \right]^{\frac{1}{\alpha_1-\alpha_2}}$$

$$= \alpha_3^{\alpha_3}(1-\alpha_3)^{1-\alpha_3} \left[\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1} \right]^{\frac{\alpha_1\alpha_3+\alpha_2(1-\alpha_3)}{\alpha_1-\alpha_2} - \frac{\alpha_2}{\alpha_1-\alpha_2}} \left[\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2} \right]^{\frac{\alpha_1}{\alpha_1-\alpha_2} - \frac{\alpha_1\alpha_3+\alpha_2(1-\alpha_3)}{\alpha_1-\alpha_2}} \cdot$$

$$\left(\frac{1-\gamma}{\gamma}\right)^{\alpha_1\alpha_3+\alpha_2(1-\alpha_3)} k_t^{\alpha_1\alpha_3+\alpha_2(1-\alpha_3)}$$

$$\therefore\ w_t = \left(\frac{1-\gamma}{\gamma}\right)^{\gamma} a k_t^{\gamma} \tag{C.1}$$

where $\gamma = \alpha_1\alpha_3 + \alpha_2(1-\alpha_3)$.

Similarly, from equation (3.2.10),

$$r_t = \left(\frac{\alpha_1^{\alpha_1(1-\alpha_2)}(1-\alpha_1)^{(1-\alpha_1)(1-\alpha_2)}}{\alpha_2^{(1-\alpha_1)\alpha_2}(1-\alpha_2)^{(1-\alpha_1)(1-\alpha_2)}} \frac{p_{2t}^{\alpha_1-1}}{p_{1t}^{\alpha_2-1}} \right)^{\frac{1}{\alpha_1-\alpha_2}}$$

Substituting $p_{1t} = p(k_t)$ and $p_{2t} = p(k_t)$

$$r_t = \frac{\left[\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}\right]^{\frac{\alpha_1-1}{\alpha_1-\alpha_2}}}{\left[\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\right]^{\frac{\alpha_2-1}{\alpha_1-\alpha_2}}} \cdot$$

$$\left[(1-\alpha_3)^{\alpha_1-1}\left(\frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}\frac{1-\alpha_3}{\alpha_3}\left(\frac{1-\gamma}{\gamma}\right)^{\alpha_2-\alpha_1}\right)^{(1-\alpha_1)\alpha_3} k_t^{-(\alpha_2-\alpha_1)(\alpha_1-1)\alpha_3}\right]^{\frac{1}{\alpha_1-\alpha_2}} \cdot$$

$$\left[\alpha_3^{1-\alpha_2}\left(\frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}\frac{1-\alpha_3}{\alpha_3}\left(\frac{1-\gamma}{\gamma}\right)^{\alpha_2-\alpha_1}\right)^{(1-\alpha_2)(1-\alpha_3)} k_t^{(\alpha_2-\alpha_1)(1-\alpha_2)(1-\alpha_3)}\right]^{\frac{1}{\alpha_1-\alpha_2}}$$

$$= \alpha_3^{\alpha_3}(1-\alpha_3)^{1-\alpha_3}\left[\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\right]^{\frac{(1-\alpha_2)-\alpha_3(1-\alpha_1)+(1-\alpha_2)(1-\alpha_3)}{\alpha_1-\alpha_2}}\left[\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}\right]^{\frac{(\alpha_1-1)+\alpha_3(1-\alpha_1)+(1-\alpha_2)(1-\alpha_3)}{\alpha_1-\alpha_2}} \cdot$$

$$\left(\frac{1-\gamma}{\gamma}\right)^{(1-\alpha_1)\alpha_3+(1-\alpha_2)(1-\alpha_3)} k_t^{\alpha_1\alpha_3+\alpha_2(1-\alpha_3)-1}$$

$$\therefore\ r_t = \left(\frac{1-\gamma}{\gamma}\right)^{\gamma-1} a k_t^{\gamma-1} \qquad \text{(C.2)}$$

Since the integrated economy is considered as closed economy, then the production of final good can be solved as all intermediate goods produced in the integrated world are used in the production of world final good.

$$y_t = x_{1t}^{\alpha_3}x_{2t}^{1-\alpha_3} = \left(k_{1t}^{\alpha_1}l_{1t}^{1-\alpha_1}\right)^{\alpha_3}\left(k_{2t}^{\alpha_2}l_{2t}^{1-\alpha_2}\right)^{1-\alpha_3}$$

$$= \left[\left(\frac{K_{1t}}{L_{1t}}\frac{L_{1t}}{N_t}\right)^{\alpha_1} l_{1t}^{1-\alpha_1}\right]^{\alpha_3}\left[\left(\frac{K_{2t}}{L_{2t}}\frac{L_{2t}}{N_t}\right)^{\alpha_2} l_{2t}^{1-\alpha_2}\right]^{1-\alpha_3}$$

$$= \left(\kappa_{1t}^{\alpha_1\alpha_3}l_{1t}^{\alpha_3}\right)\left(\kappa_{2t}^{\alpha_2(1-\alpha_3)}l_{2t}^{1-\alpha_3}\right)$$

$$= \left[\frac{\alpha_1(1-\gamma)}{\gamma(1-\alpha_1)}k_t\right]^{\alpha_1\alpha_3}\left[\frac{\alpha_3(1-\alpha_1)}{1-\gamma}\right]^{\alpha_3}\left[\frac{\alpha_2(1-\gamma)}{\gamma(1-\alpha_2)}k_t\right]^{\alpha_2(1-\alpha_3)}\left[\frac{(1-\alpha_3)(1-\alpha_2)}{1-\gamma}\right]^{1-\alpha_3}$$

$$= \alpha_3^{\alpha_3}(1-\alpha_3)^{1-\alpha_3}\left[\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\right]^{\alpha_3}\left[\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}\right]^{(1-\alpha_3)}\frac{(1-\gamma)^{\alpha_1\alpha_3-\alpha_3+\alpha_2(1-\alpha_3)-(1-\alpha_3)}}{\gamma^{\alpha_1\alpha_3+\alpha_2(1-\alpha_3)}}k_t^{\alpha_1\alpha_3+\alpha_2(1-\alpha_3)}$$

$$\therefore\ y_t = \frac{(1-\gamma)^{\gamma-1}}{\gamma^{\gamma}} a k_t^{\gamma} \tag{C.3}$$

Thus, $y_t = f(k_t)$

Since $r_t = f'(k_t)$,

$$\Rightarrow\ r_t = \gamma(1-\gamma)^{\gamma-1}\gamma^{-\gamma} a k_t^{\gamma-1} = \left(\frac{1-\gamma}{\gamma}\right)^{\gamma-1} a k_t^{\gamma-1}, \tag{C.4}$$

and $w_t = f(k_t) - k_t\, f'(k_t)$,

$$\begin{aligned} w_t &= (1-\gamma)^{\gamma-1}\gamma^{-\gamma} a k_t^{\gamma} - k_t \left(\frac{1-\gamma}{\gamma}\right)^{\gamma-1} a k_t^{\gamma-1} \\ \Rightarrow\quad &= (1-\gamma)^{\gamma-1} a k_t^{\gamma}\left[\gamma^{-\gamma} - \gamma^{1-\gamma}\right], \\ &= \left(\frac{1-\gamma}{\gamma}\right)^{\gamma} a k_t^{\gamma} \end{aligned} \tag{C.5}$$

These give the same results as solving prices endogenously in the integrated economy.